Q9 Discussion:

As we can conclude from the plots, with , the resulted closed-loop systems of LQR and MPC controller are very similar. Both satisfied input and state constraints, and approached the reference reasonably fast. However, with , the MPC controller could still do a good job, with all constraints satisfied and the stable reference state achieved, but the LQR controller failed to satisfy the state constraints. The LQR controller underwent weird changes on state and input, the constraints for  were violated, and the final state was not as stable or satisfying as the MPC controller did. This is probably because that the deviation from the steady-state  is a little larger, and the LQR controller cannot explicitly satisfy state or input constraints, thus constraints were violated and the performance was bad.

Q11

The proof is very similar to what we have done in the lecture. Firstly, it is obvious the origin is an equilibrium point since it satisfies the dynamics  setting inputs to 0. Given that (4) is feasible for x(0), and (4) guarantees that , i.e., terminal constraint at the origin, it corresponds to the case where , and from the proof in the lecture notes 06 P32-35, the system is recursively feasible and Lyapunov stable, thus asymptotically stable.

Q13

Difference:  is less than . This is probably because forcing the terminal state to origin is a stricter requirement to satisfy and results in a trajectory with more stage cost and thus the overall cost in this case is more than that without terminal state being the origin but with an extra terminal cost.

For initial condition, the MPC problem is infeasible and state constraints are violated.

Q14

Choose  where  is the solution to the discrete-time algebraic Riccati equation: 

The proof is the same as in the lecture notes.

Q15

The closed-loop trajectories are very similar to the results from Task 9 despite some tiny differences. Both are feasible, stable, and converge to the reference similarly fast.

Q16

The region of attraction of the MPC controller (5) is much larger than that of (4), since (4) requires the terminal state to be exactly the origin but (5) simply requires it to be in the set . This is why  is infeasible for (4) but could be contained in the region of attraction of (5).