ETH zürich

Robot Dynamics

Fixed-wing UAVs: Dynamic Modeling and Control

151-0851-00 V

Marco Hutter, Roland Siegwart, and **Thomas Stastny**

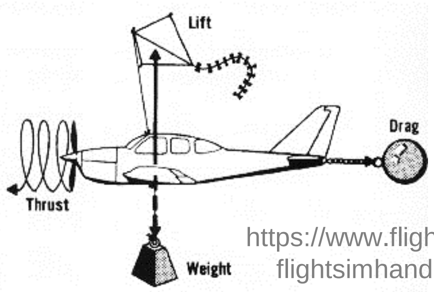
Contents | Fixed-wing UAVs

1. Introduction
2. Aerodynamic Basics
3. Aircraft Dynamic Modeling
4. Fixed-wing Control

Introduction

Introduction | Fixed-wing aircraft

- Definition: fixed-wing aircraft are capable of flight using wings that generate **lift** caused by the vehicle's **forward airspeed** and the **shape (geometry)** of the wings.



https://www.flightsimbooks.com/flightsimhandbook/43-1.jpg

"If the wings are traveling faster than the fuselage, it's probably a helicopter...and therefore, unsafe."

Advantages:
- Long range / endurance (cover larger areas, faster, and stay airborne longer – efficient!)
- Mechanically simple

Disadvantages:
- Typically requires some infrastructure (e.g. catapult for take-off, flat open landing area or net for landing)
- Unless hybrid – no hovering

Introduction | "Small" fixed-wing UAVs

Introduction | "Small" fixed-wing UAVs

- Wide variety of configurations for different purposes / applications
- No community standard – requires modeling and specific tuning per platform!

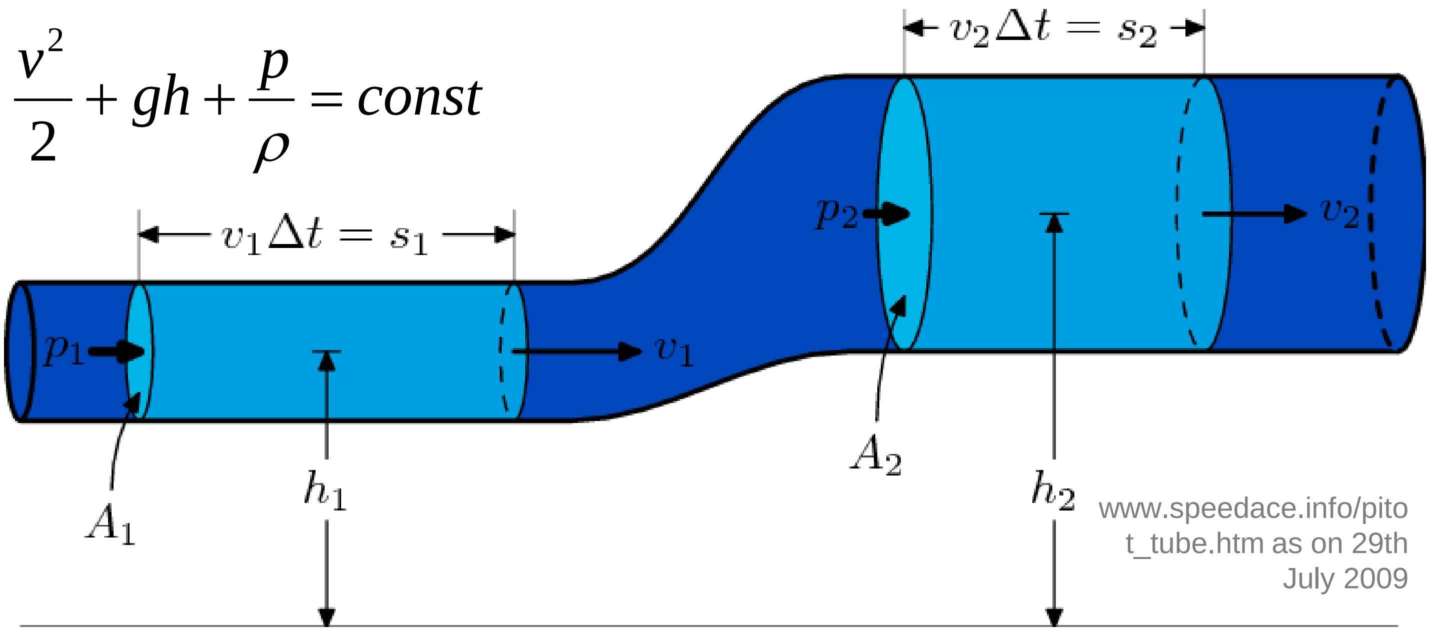
Aerodynamic Basics

Aerodynamic Basics | Basic Principles

Analysis on differential volumes:

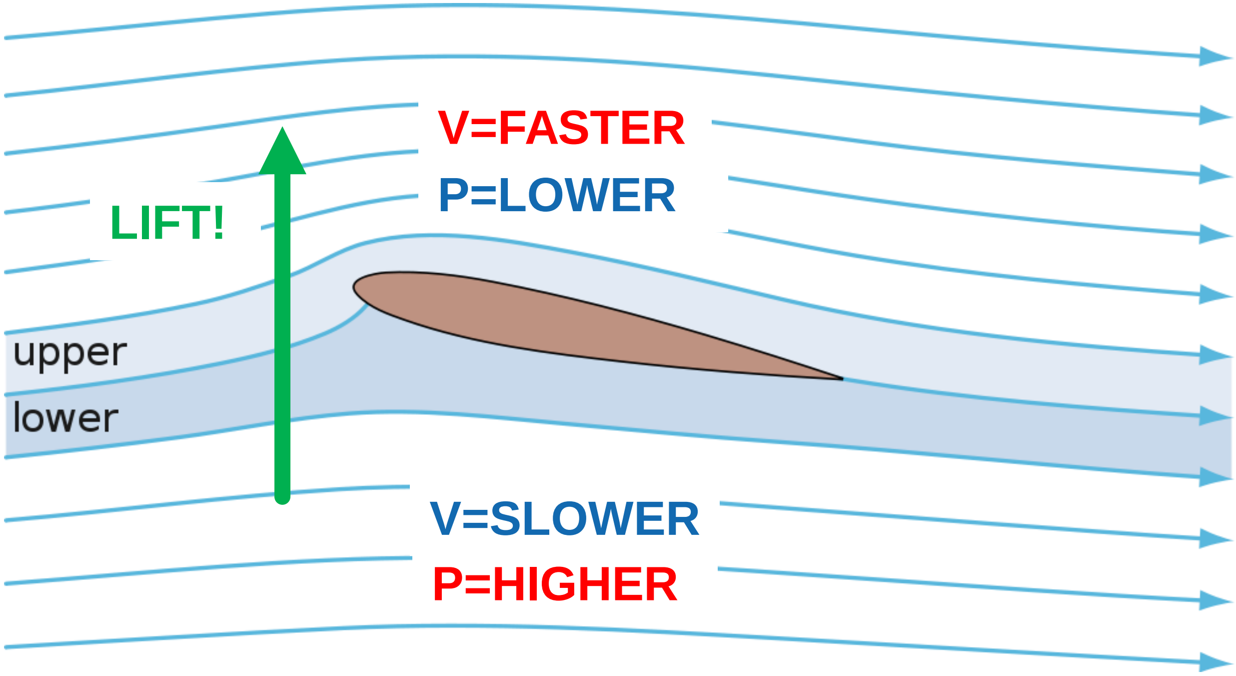
- With viscosity: Navier-Stokes Equation
- Without viscosity: Euler Equation
- Incompressible along streamline: **Bernoulli Equation**

$$\frac{v^2}{2} + gh + \frac{p}{\rho} = const$$



www.speedace.info/pitot_tube.htm as on 29th July 2009

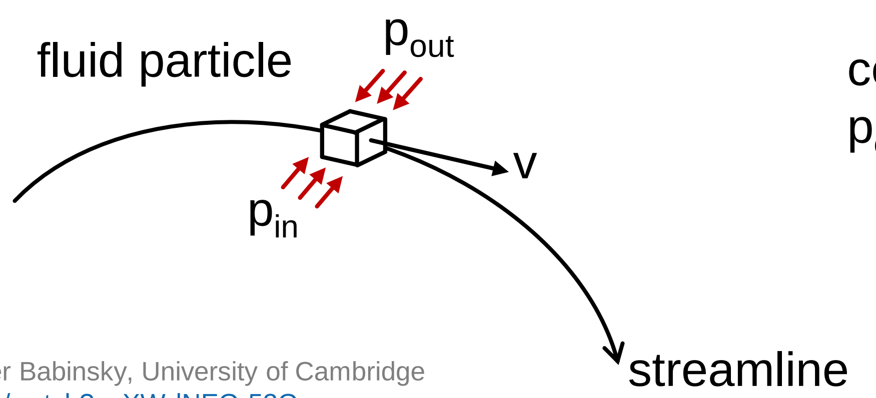
Aerodynamic Basics | Basic Principles



https://commons.wikimedia.org/wiki/File:Streamlines_around_a_NACA_0012.svg

Aerodynamic Basics | Basic Principles

- But…Bernoulli isn't the whole story!
- Watch out for common misconceptions of lift
  - E.g. the "distance traveled" argument for speed difference
- What is really going on? –streamline curvature induced pressure gradients.



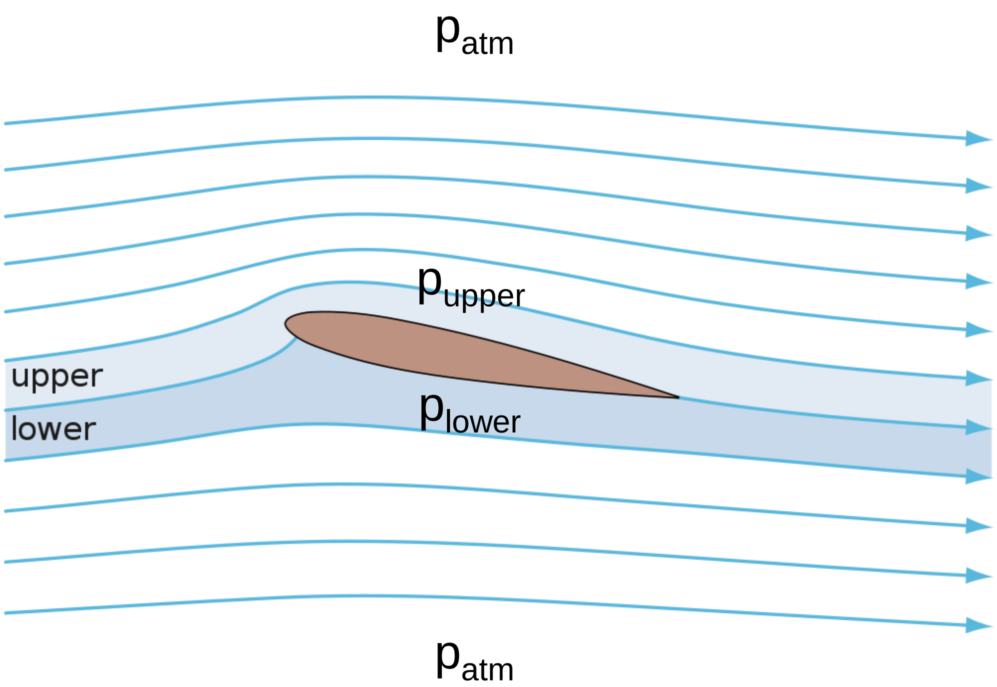
centripetal force:
$p_{out} > p_{in}$

Nice lecture by Dr. Holger Babinsky, University of Cambridge
https://www.youtube.com/watch?v=XWdNEGr53Gw

Aerodynamic Basics | Basic Principles

- $p_{upper} < p_{atm}$
- $p_{lower} > p_{atm}$
- Therefore: $p_{upper} < p_{lower}$

LIFT!

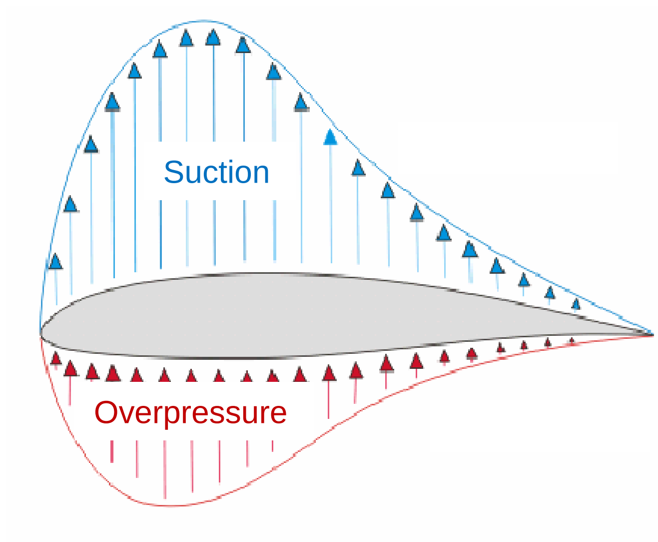


https://commons.wikimedia.org/wiki/File:Streamlines_around_a_NACA_0012.svg

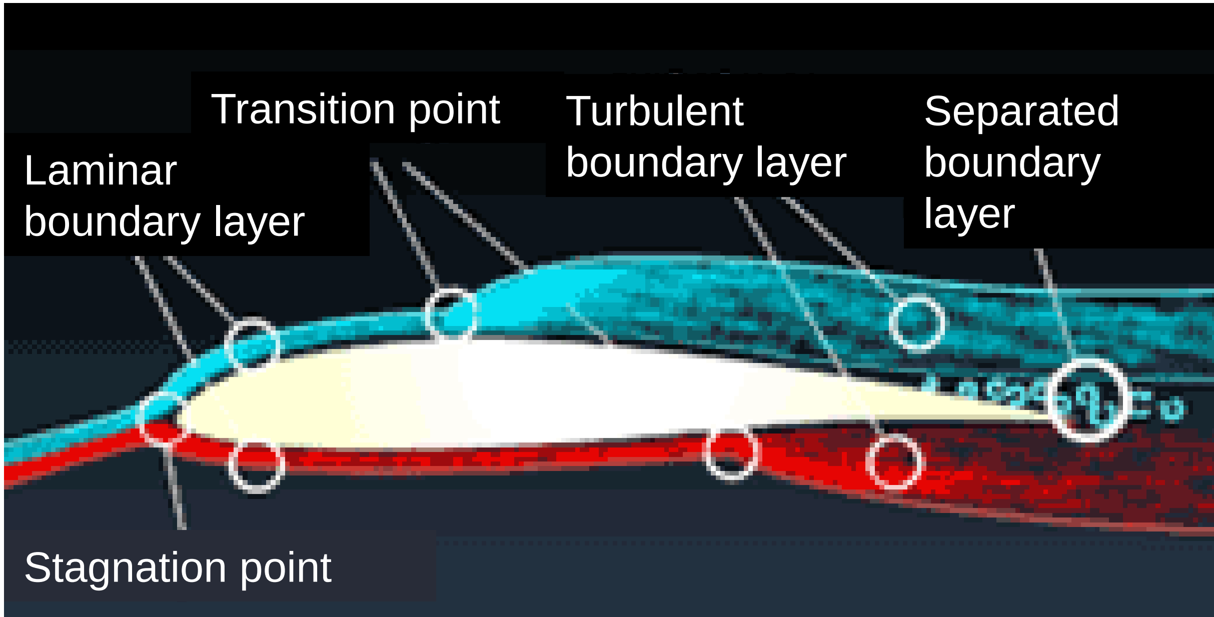
Aerodynamic Basics | Airfoils

2-Dimensional Flow Analysis

- Flow field (pressure distribution, laminar/turbulent) highly dependent on angle of attack, Reynolds number and Mach number

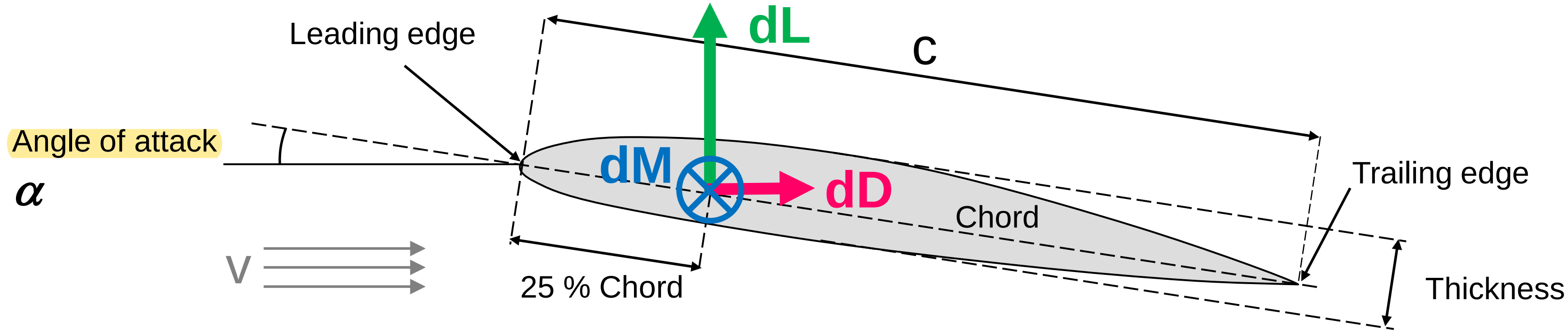


www.thuro.at/aerodynamik2.htm



http://www.thuro.at/anims/abloesung.gif

Aerodynamic Basics | Airfoils



Pressure distribution can be reduced to two forces and one moment ***per unit length***:

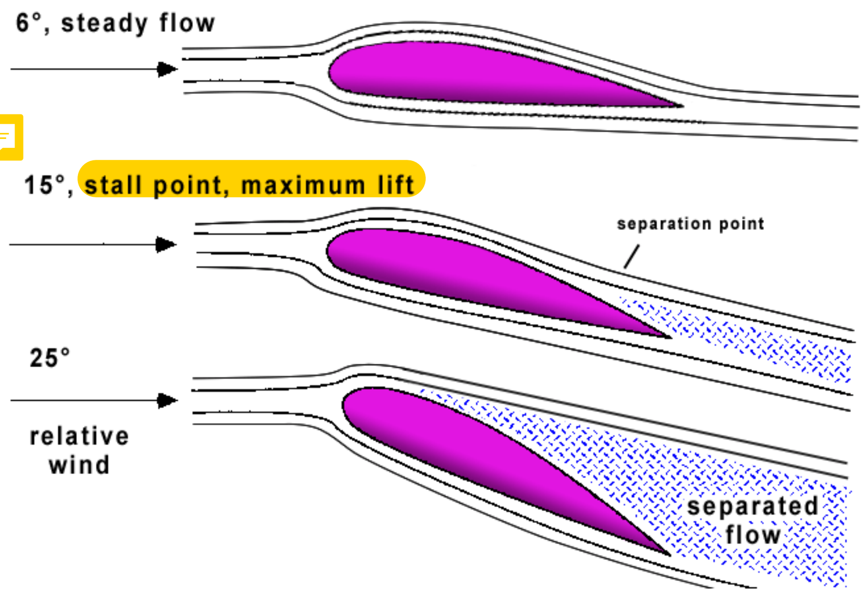
Lift force $$dL = C_l \frac{\rho}{2} c \cdot dy \cdot V^2$$

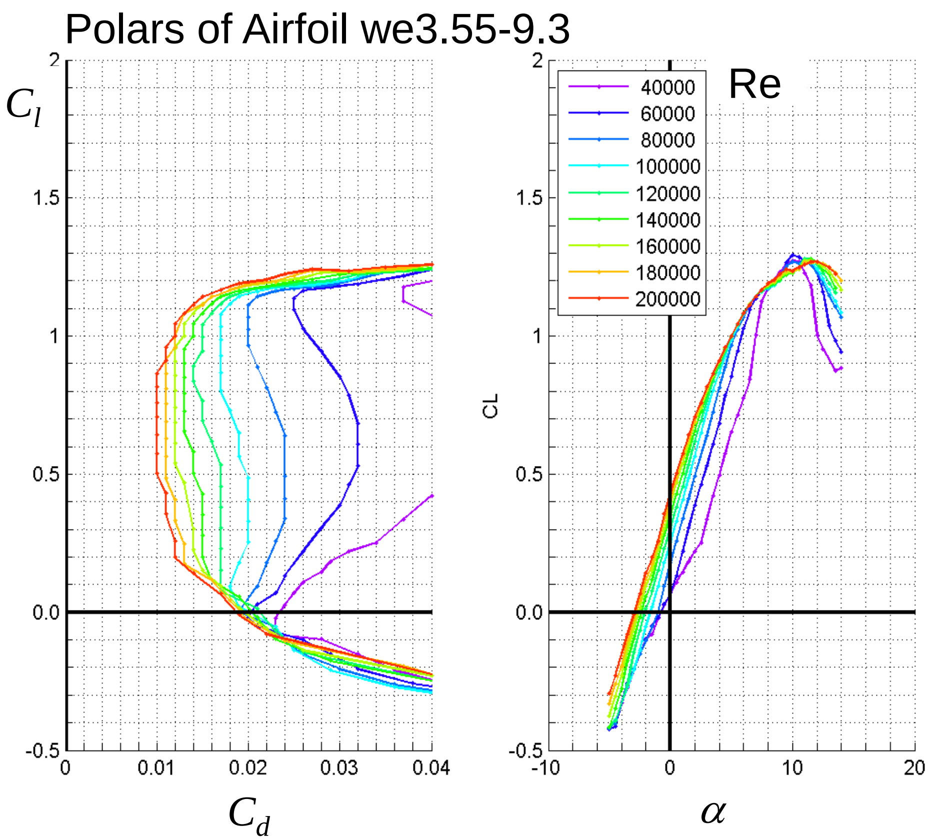
Drag force $$dD = C_d \frac{\rho}{2} c \cdot dy \cdot V^2$$

Moment $$dM = C_m \frac{\rho}{2} c^2 \cdot dy \cdot V^2$$

- ρ : Density of fluid (air) [kg/m³]
- c : Chord length [m]
- V : Flight speed (w.r.t. air) [m/s]
- C_l : Airfoil lift coefficient [-]
- C_d : Airfoil drag coefficient [-]
- C_m : Airfoil moment coefficient [-]

Aerodynamic Basics | Angle of attack / stall

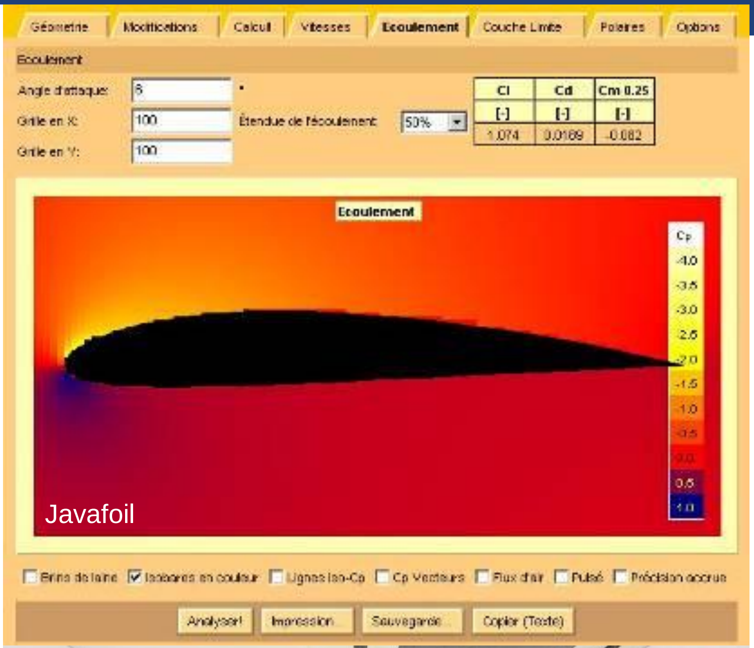


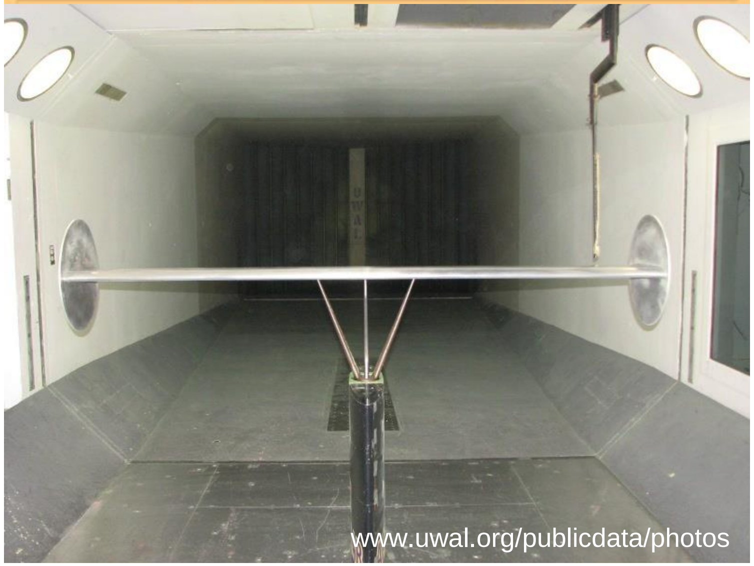


Airfoil Lift, Drag and Moment

Methods to determine airfoil lift, drag and moment coefficients:

- **Theoretically** using 2D-CFD software
  - Javafoil http://www.mh-aerotools.de/
  - Xfoil http://raphael.mit.edu/xfoil/
- **Experimentally** in a *wind tunnel*
  - Extruded airfoil mounted on a measurement system
  - Laminar flow produced by fans





T. Stastny, R. Siegwart. “Nonlinear Model Predictive Guidance for Fixed-wing UAVs Using Identified Control Augmented Dynamics”. International Conference on Unmanned Aerial Systems (ICUAS). 2018.

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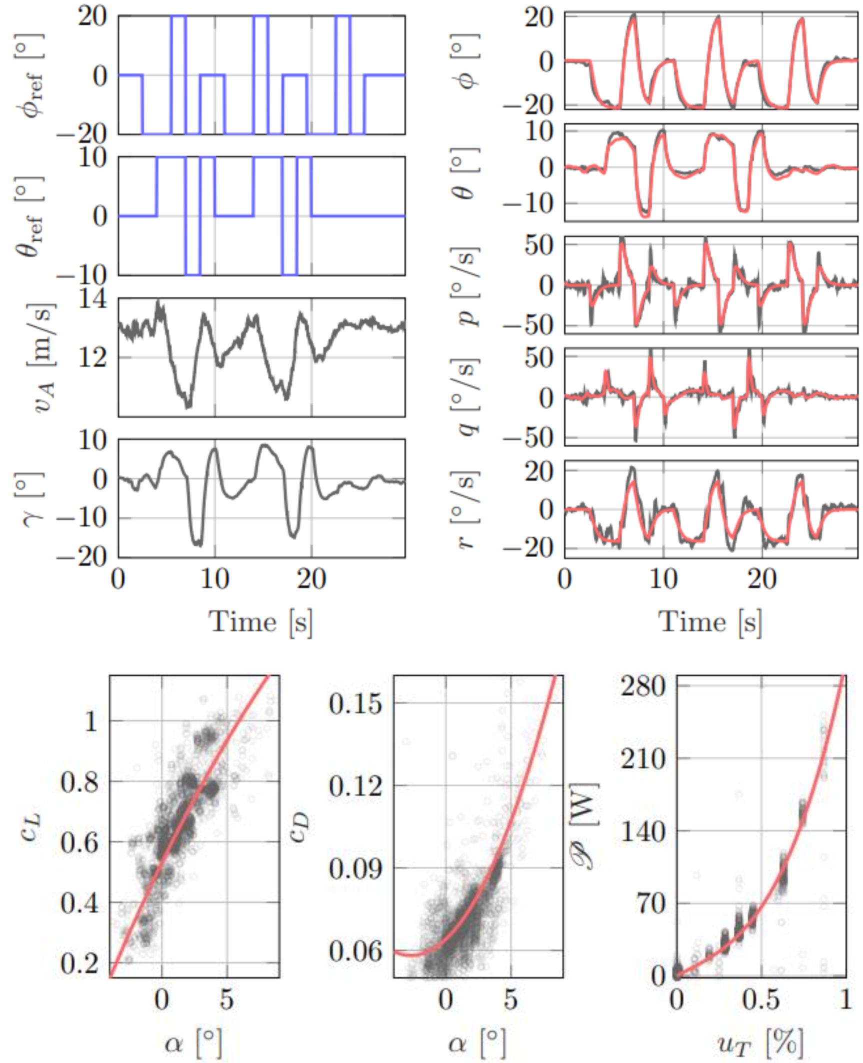
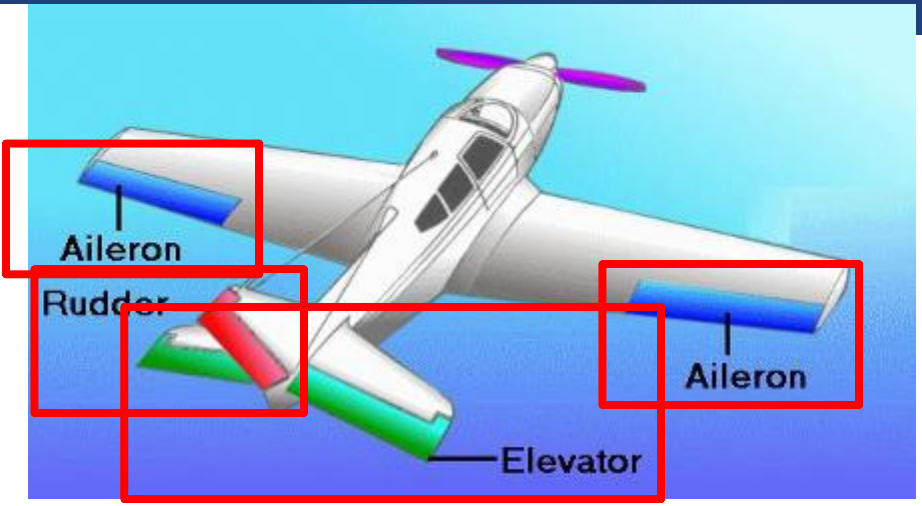
- **Experimentally** in a *wind tunnel*
  - Extruded airfoil mounted on a measurement system
  - Laminar flow produced by fans
- **Experimentally** from *flight data*
  - System identification on logged sensor measurements of static and dynamic maneuvers

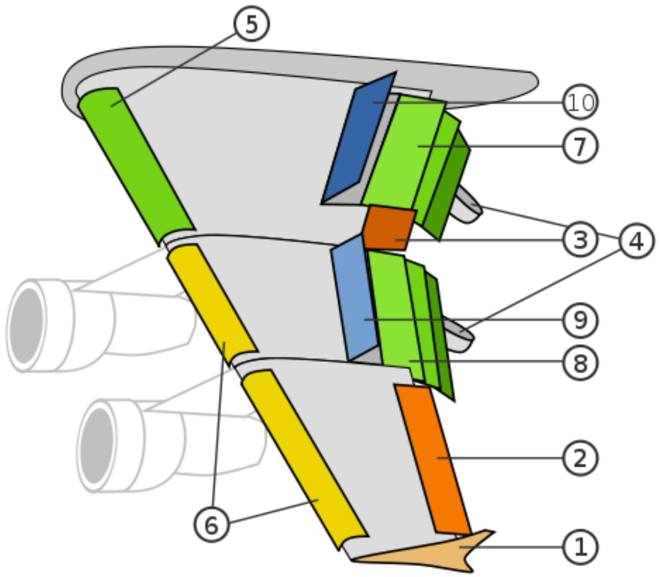


Control surfaces

- For **small** airplanes,
  the standard control surfaces are:
  - Ailerons (rolling)
  - Elevator (pitching)
  - Rudder (yawing)



- For larger airplanes, they can be more complex…

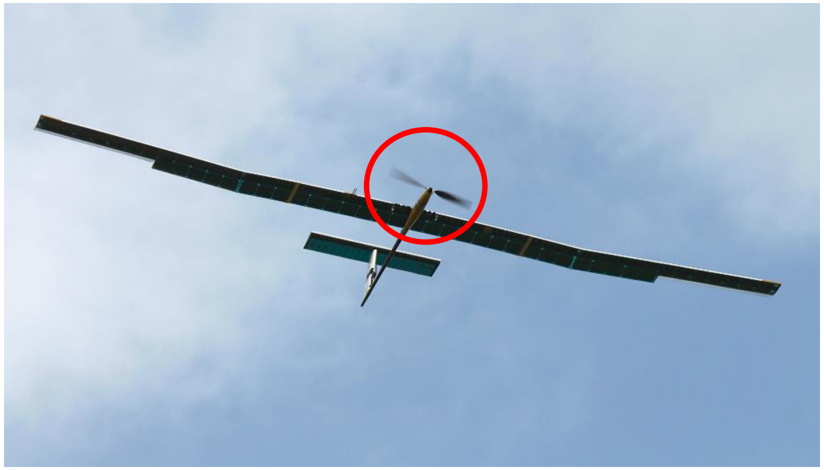


Ailerons:
2. Low-Speed Aileron
3. High-Speed Aileron

Lift increasing flaps and slats:
4. Flap track fairing
5. Krüger flaps
6. Slats
7. Three slotted inner flaps
8. Three slotted outer flaps

Spoilers:
9. Spoilers
10. Spoilers-Air brakes

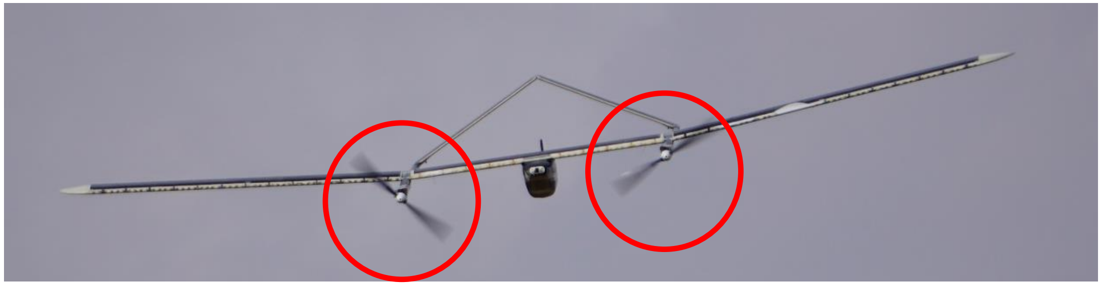
Propulsion Group | Placement



In the front…

In the back…

On the wings…



Aircraft Dynamic Modeling

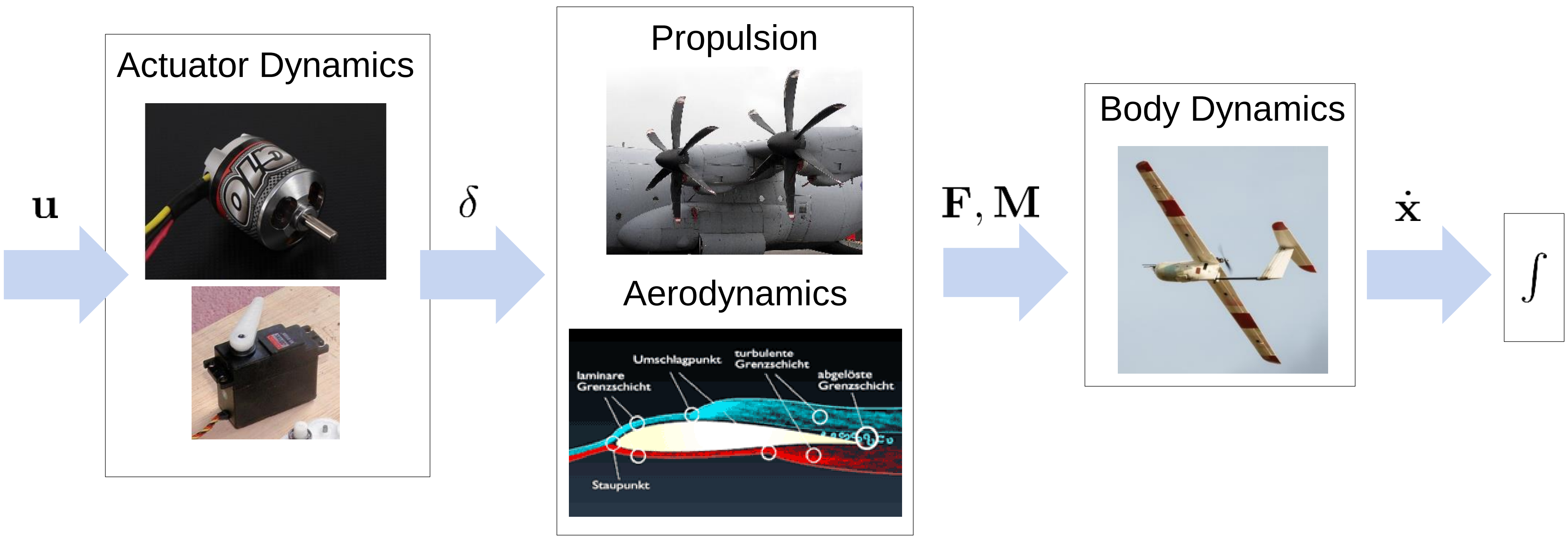
Why Model the Dynamics of an Airplane?

- **System analysis:** model allows evaluating flight characteristics
  - Stability
  - Controllability
  - Power required ➔ fuel needs
  - Controllability in the case of actuator failure
- **Autopilot design and simulation:** model allows comparing different control techniques and autopilot parameter tuning
  - Gain of time and money
  - Higher performance of the autopilot
  - No risk of damage compared to real tests
- **Pilot training (in Simulator)**
  - Allows simulating and training especially emergency situations

Aircraft Dynamic Modeling | Intro

- Dynamics of an airplane
  ... Are very different from an acrobatic aircraft to a jet-liner airplane
  ... but the principles remain the same for all
  - Wings, stabilizers
  - control surfaces (ailerons, rudder, elevator, flaps, spoilers, …)
  - propulsion group (motor-gearbox-propeller, turbine, rocket, …)
- In this lecture, we will model a typical fixed-wing UAV
  1. Assumptions/Simplifications
  2. Kinematics (frames of reference + state definitions)
  3. Dynamics (forces + moments)
  4. Equations of motion (Newton)

Aircraft Dynamic Modeling | Intro



Aircraft Dynamic Modeling | Assumptions and simplifications

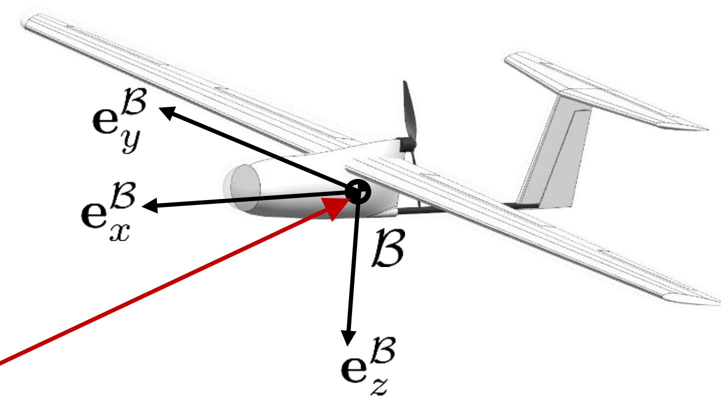
- **Definitions**
  - Origin of body-fixed coordinate frame set into center of gravity
- **Assumptions and simplifications**
  - **Rigid** and symmetric structure: constant, (almost) diagonal inertia matrix
  - Constant mass
  - Motor without dynamics and without gyroscopic effects (can be adapted)
  - Aerodynamics (list not complete):
    - We don’t enter stall (operation in the linear lift domain)
    - Neglect fuselage lift/sideslip force

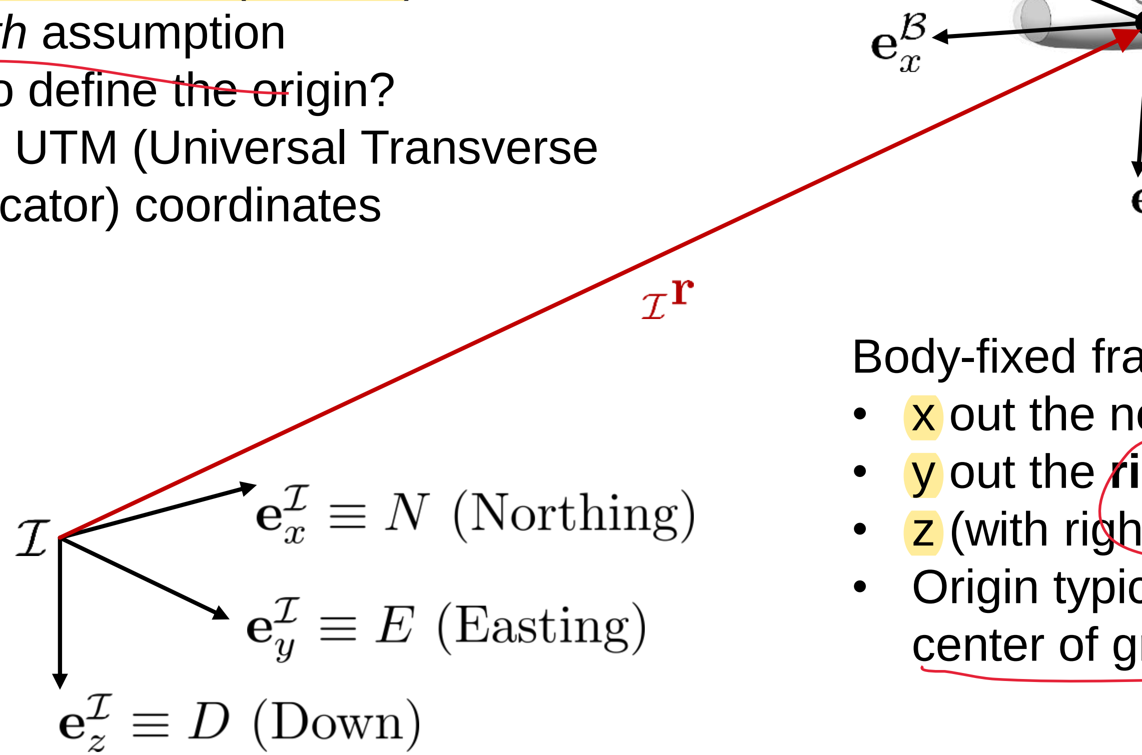
Aircraft Dynamic Modeling | On rigidity …

Aircraft Kinematics

Aircraft Kinematics | Reference axes

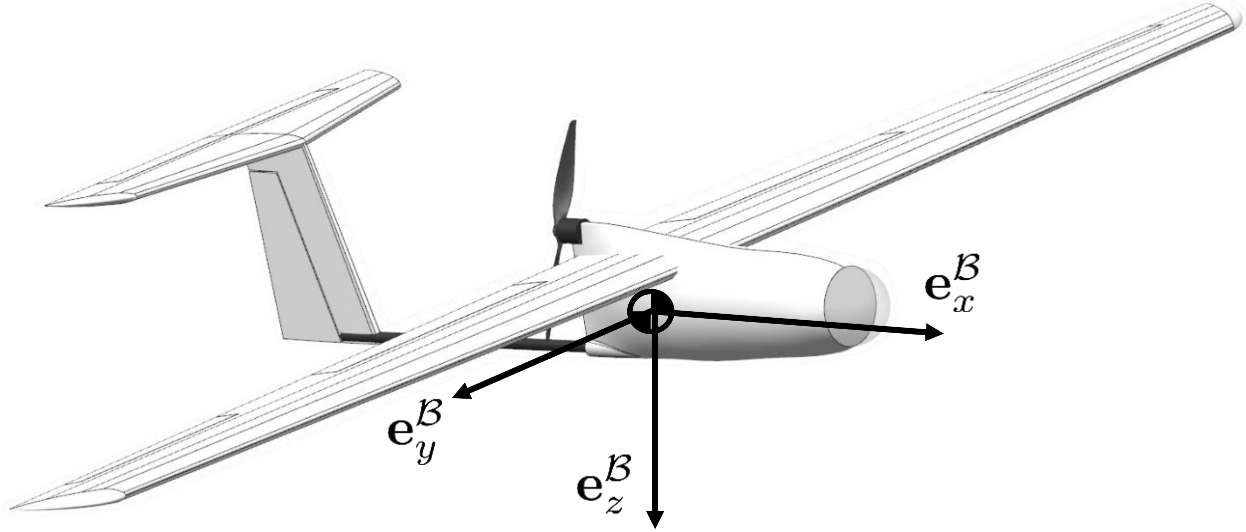
Inertial (local) reference frame
- North, East, **Down** (or NED)
- *Flat Earth* assumption
- Where to define the origin?
  - E.g. UTM (Universal Transverse Mercator) coordinates





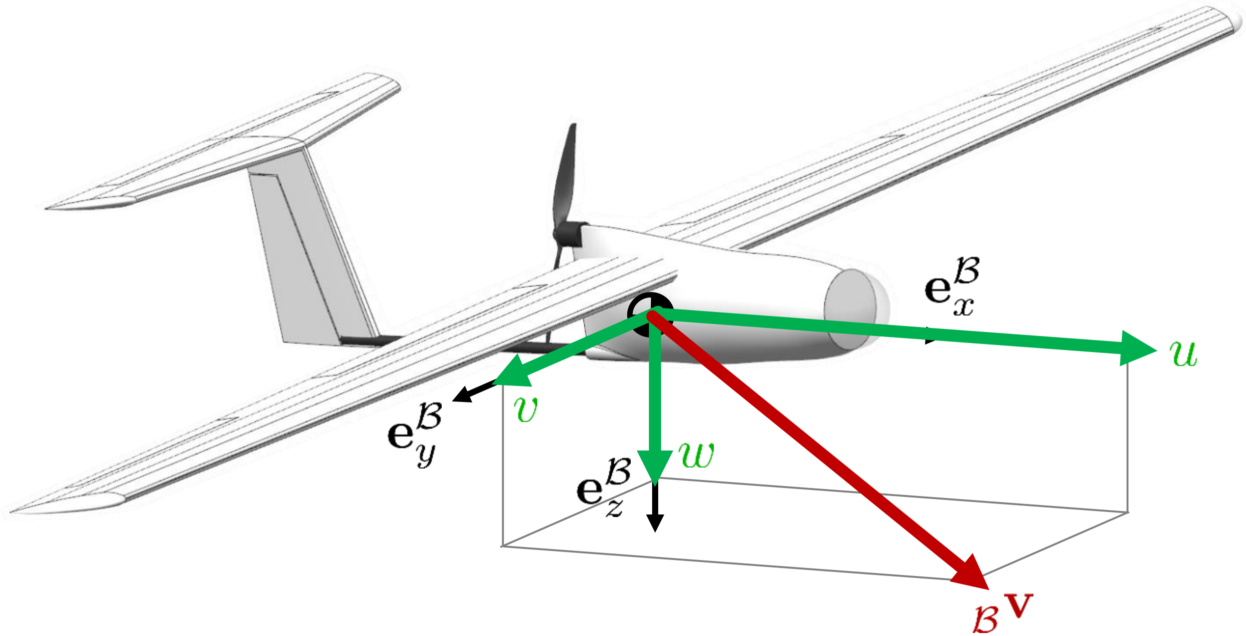
Body-fixed frame
- x out the nose
- y out the **right** wing
- z (with right hand rule) down
- Origin typically located at center of gravity

Aircraft Kinematics | Reference axes



Body-axis $\mathcal{B}$

Aircraft Kinematics | Reference axes



Body-axis $\mathcal{B}$

Body velocity:

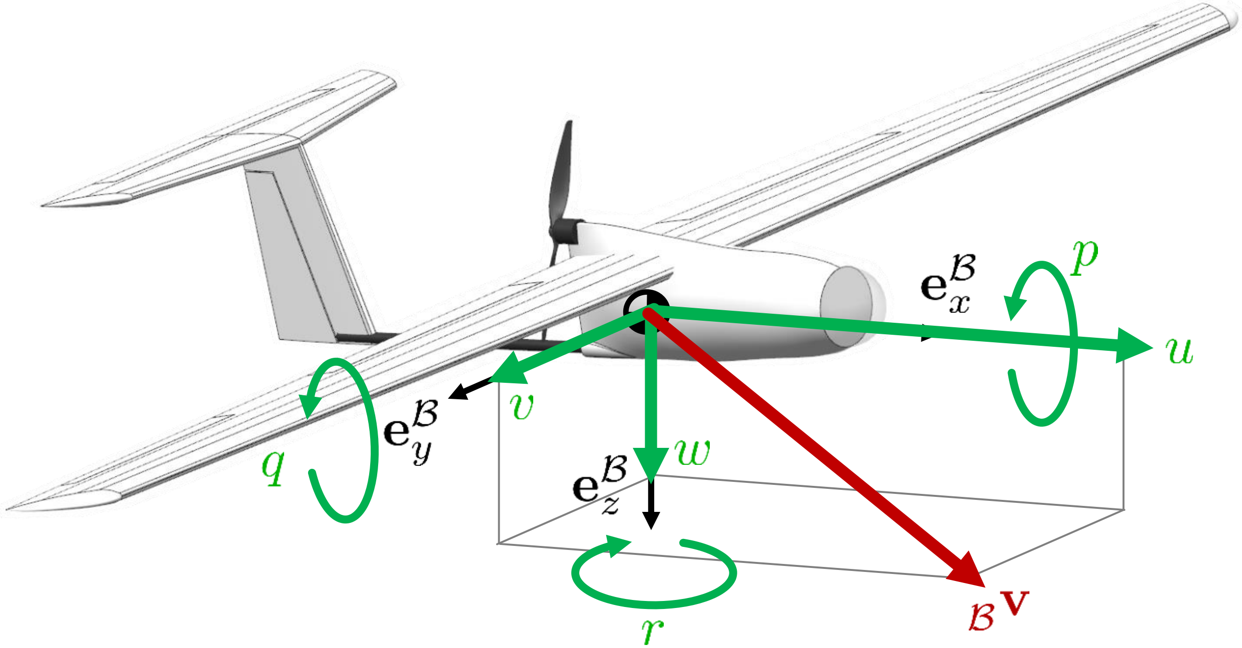
$$_{\mathcal{B}}\mathbf{v}_a = (u, v, w)^T$$

- Air-mass relative speed (airspeed):

$$V = \sqrt{u^2 + v^2 + w^2}$$

- V is always positive

Aircraft Kinematics | Reference axes



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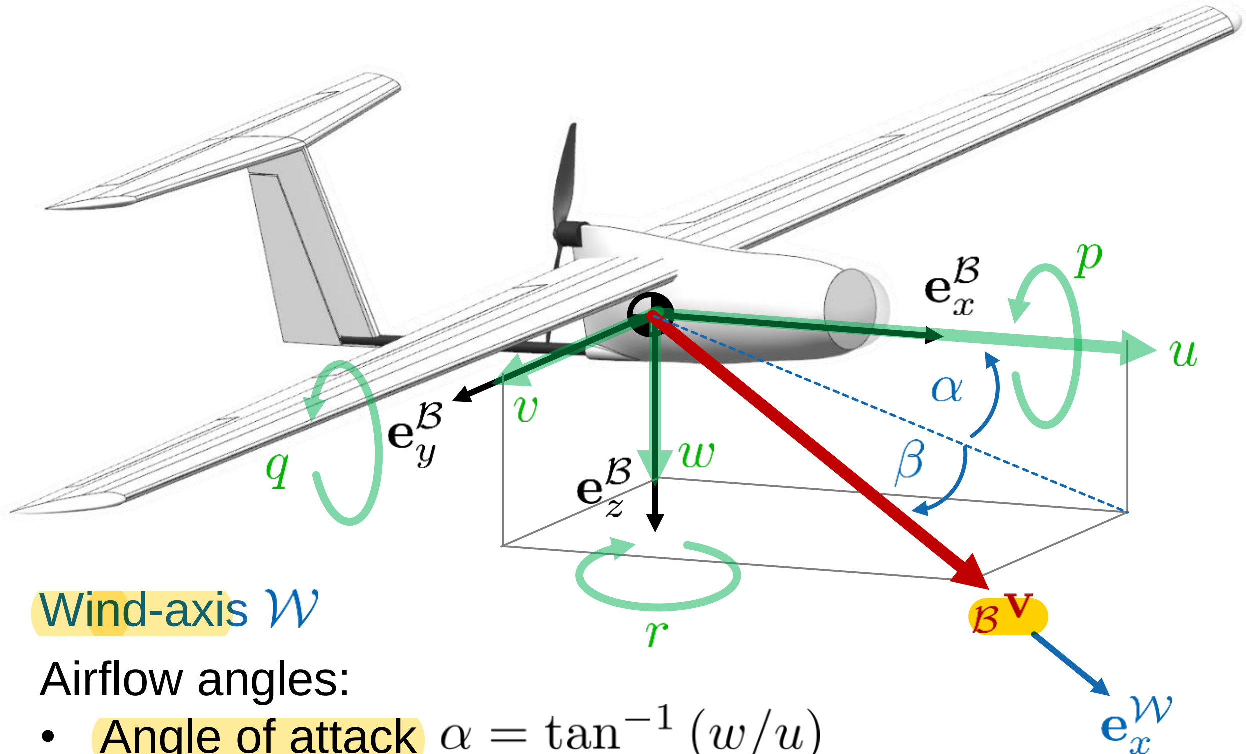
Body rates:

$$_{\mathcal{B}}\omega = (p, q, r)^T$$

Aircraft Kinematics | Reference axes



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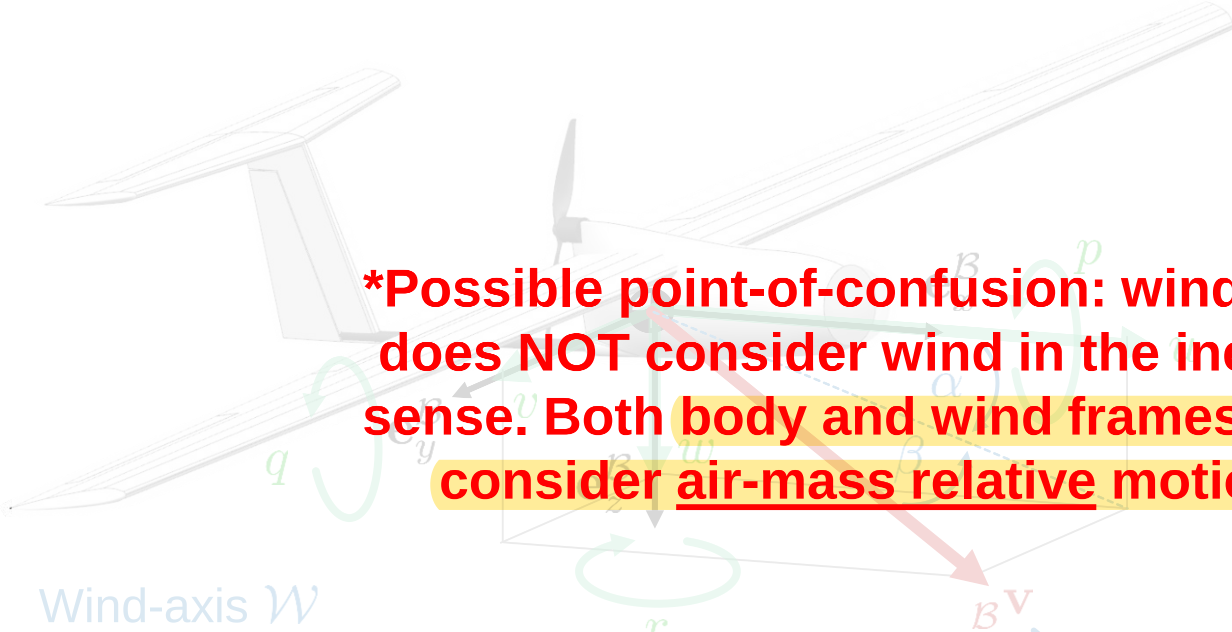
Wind-axis $\mathcal{W}$

Airflow angles:

- Angle of attack $\alpha = \tan^{-1}(w/u)$
- Sideslip angle $\beta = \sin^{-1}(v/V)$
- Wind frame is opposite to "free-stream" velocity, or "wind" (i.e. the air-mass)

Aircraft Kinematics | Reference axes



***Possible point-of-confusion: wind-axis does NOT consider wind in the inertial sense. Both body and wind frames only consider air-mass relative motion**

Body-axis $\mathcal{B}$

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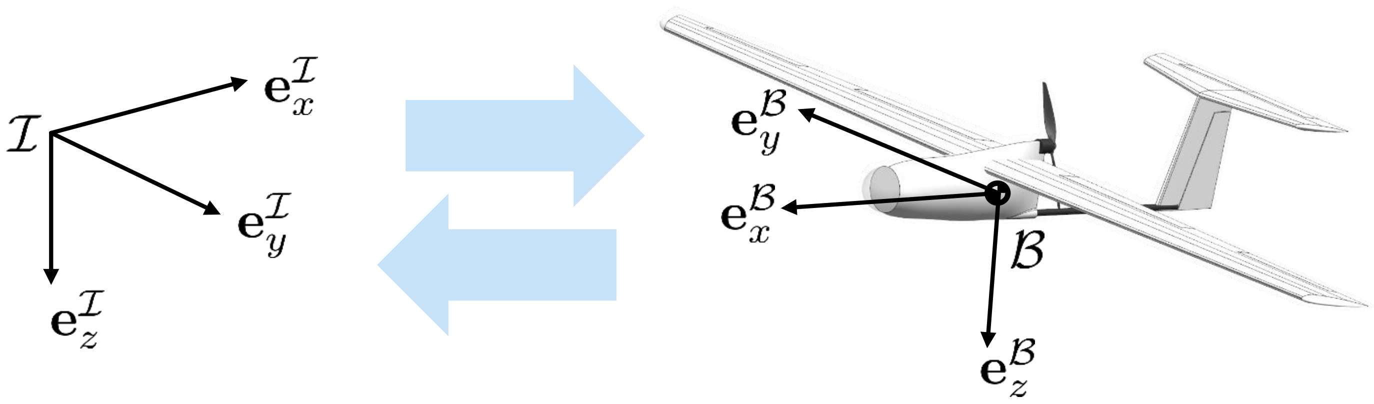
Aircraft Kinematics | Coordinate transformation

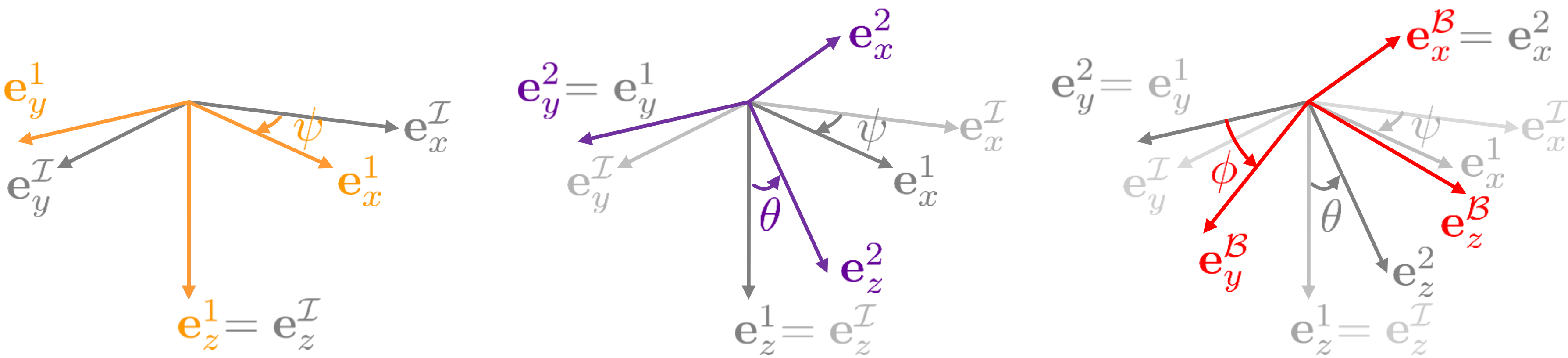
- Euler angles, roll, pitch, and yaw, are used to transform between inertial and body axes.

$$\boldsymbol{\Theta} = (\phi, \theta, \psi)^T$$



Aircraft Kinematics | Coordinate transformation

Rotation Matrix ($\mathcal{B}$ to $\mathcal{I}$) $\mathbf{C}_{\mathcal{I}\mathcal{B}}$ is parametrized with **3 successive rotations** using the **ZYX Tait-Brian Angles** (specific kind of Euler Angles):



1 **Yaw:**
around - $\mathbf{e}_z^{\mathcal{I}}$: $\mathbf{C}_{\mathcal{I}1}(\psi)$
$\Rightarrow$ Frame 1

2 **Pitch:**
around - $\mathbf{e}_y^1$: $\mathbf{C}_{12}(\theta)$
$\Rightarrow$ Frame 2

3 **Roll:**
around - $\mathbf{e}_x^2$: $\mathbf{C}_{2\mathcal{B}}(\phi)$
$\Rightarrow$ Frame B

$$\mathbf{C}_{\mathcal{I}\mathcal{B}} = \mathbf{C}_{\mathcal{I}1}(\psi)\,\mathbf{C}_{12}(\theta)\,\mathbf{C}_{2\mathcal{B}}(\phi)$$

Aircraft Kinematics | A note on angular rates

- **Angular Rates:**

Time variation of Tait-Bryan angles $\left(\dot{\phi}, \dot{\theta}, \dot{\psi}\right)$

$$\neq$$

Body angular rates $\left(p, q, r\right)$

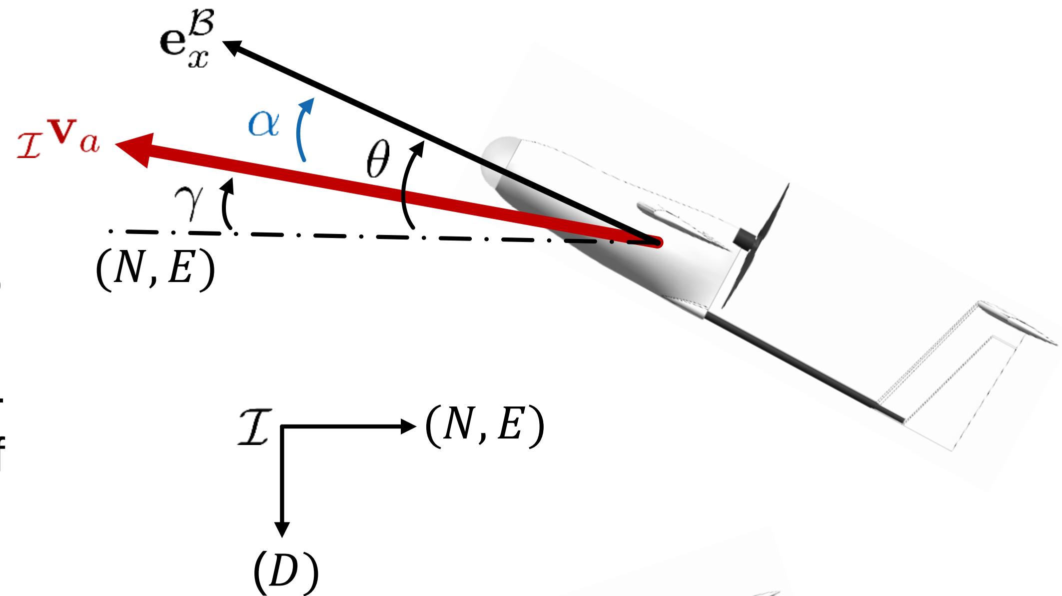
$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{J}_r \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad \mathbf{J}_r = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

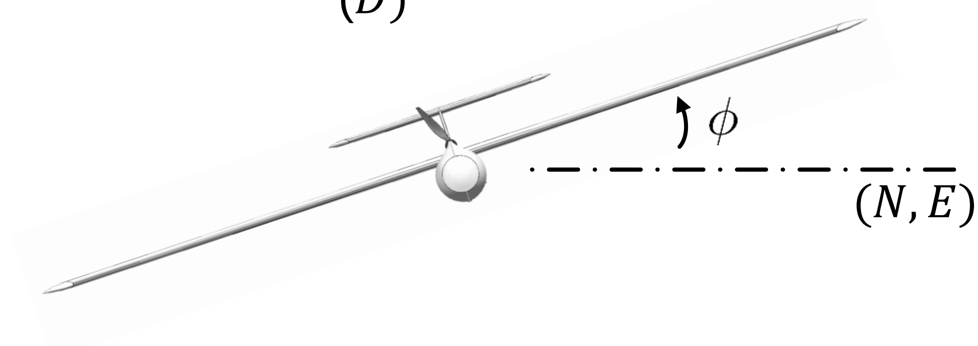
- **Singularity:** for $\theta = \pm\frac{\pi}{2}$ ($\mathbf{J}_r$ becomes singular)
  - « Gimbal Lock »

Aircraft Kinematics | Polar coordinates

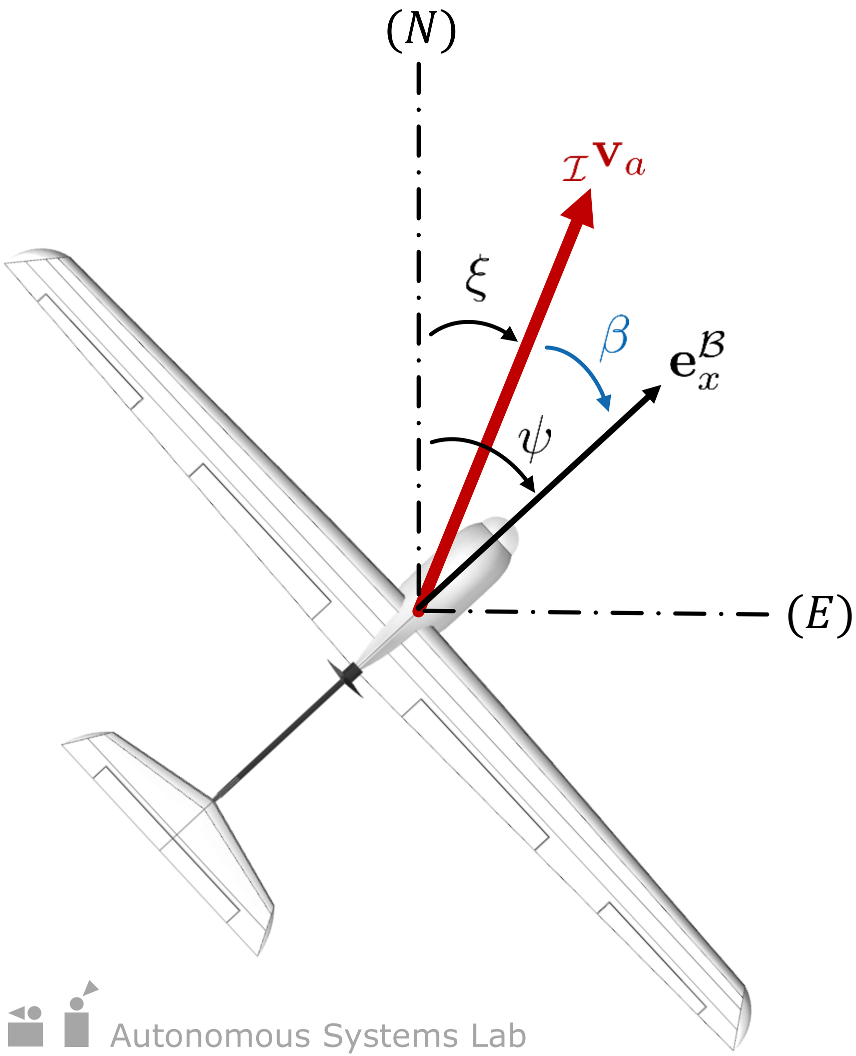
Longitudinal Inertial Frame (*side view)

- γ : Flight path angle, defined from horizon to ${}_{\mathcal{I}}\mathbf{v}_a$
- θ : Pitch angle, from horizon to x-body axis
- ϕ : Roll angle, rotation about x-body axis (note: pointing out of slide)
- ${}_{\mathcal{I}}\mathbf{v}_a = \mathbf{C}_{\mathcal{I}\mathcal{B}}\,{}_{\mathcal{B}}\mathbf{v}_a$





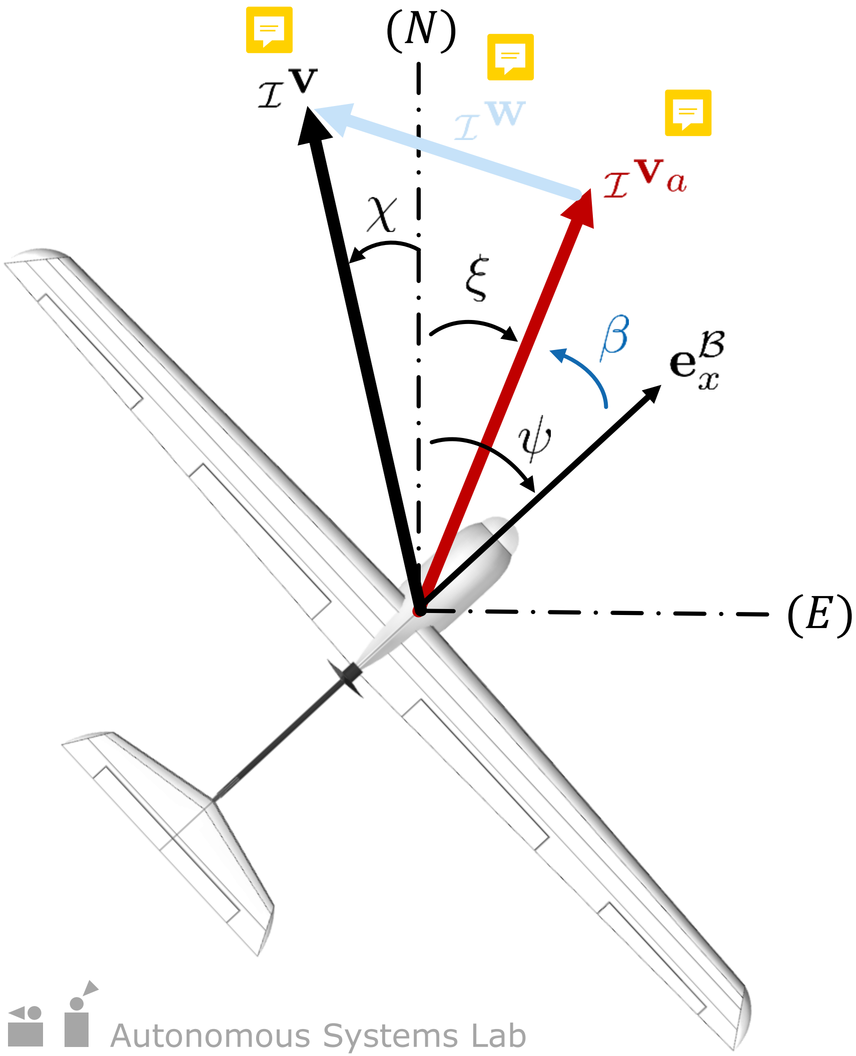
Aircraft Kinematics | Polar coordinates



Lateral-directional Inertial Frame (*top view)

- ξ : Heading angle, defined from North to ${}_{\mathcal{I}}\mathbf{v}_a$
- ψ: Yaw angle, from North to x-body axis
- Note: this STILL does not consider wind.

Aircraft Kinematics | Polar coordinates



Lateral-directional Inertial Frame (*top view)

- ξ : Heading angle, defined from North to ${}_{\mathcal{I}}\mathbf{v}_a$
- ψ: Yaw angle, from North to x-body axis

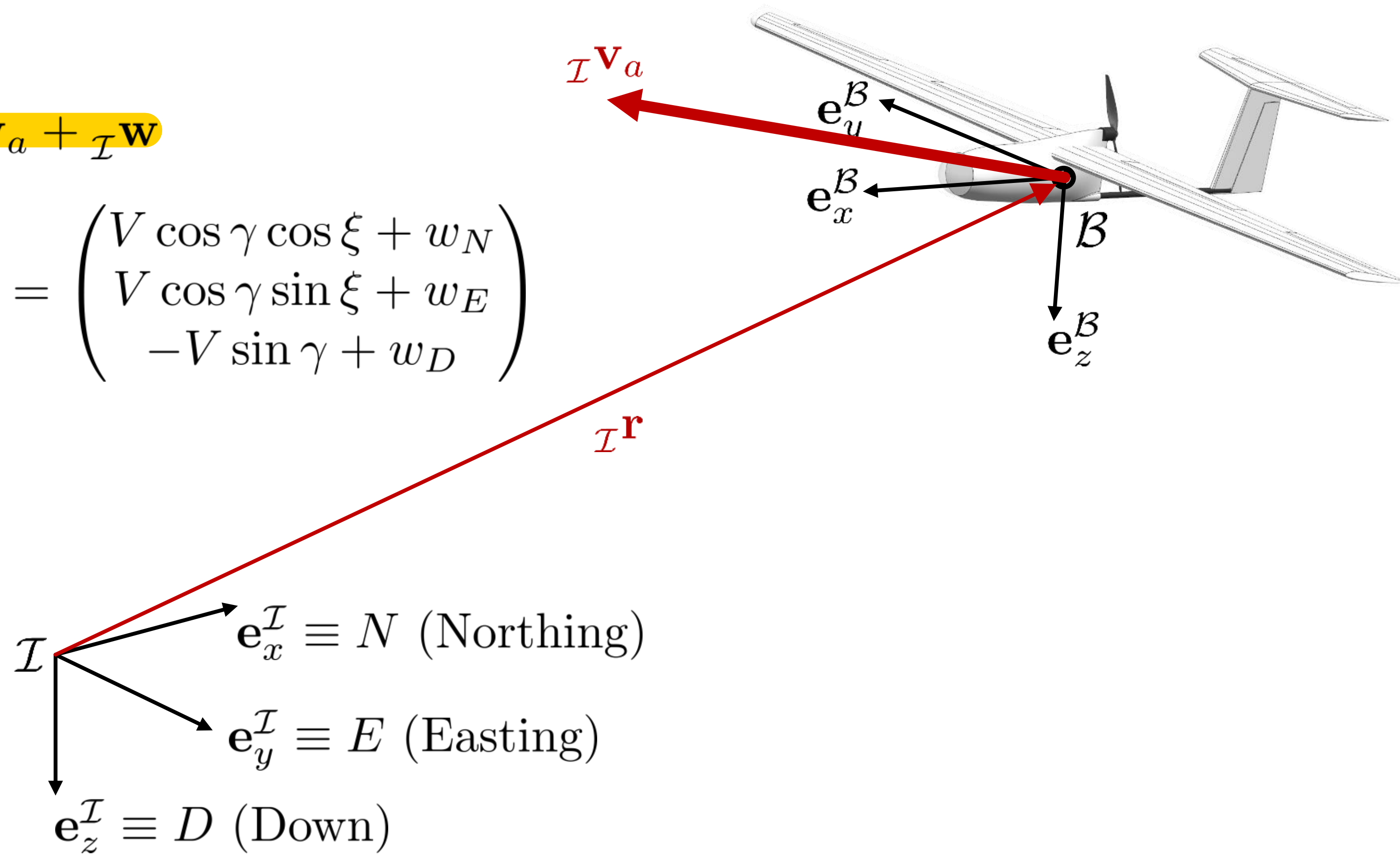
Adding a constant, horizontal wind:

- χ : Course angle, defined from North to ${}_{\mathcal{I}}\mathbf{v}$
- ${}_{\mathcal{I}}\mathbf{v}$: Ground-based inertial velocity (or "ground speed")
- ${}_{\mathcal{I}}\mathbf{w}$: Wind velocity
- Note: the constant wind assumption effects only position dynamics.

Aircraft Kinematics | Polar coordinates

$$
{}_{\mathcal{I}}\mathbf{v} = {}_{\mathcal{I}}\mathbf{v}_a + {}_{\mathcal{I}}\mathbf{w}
$$

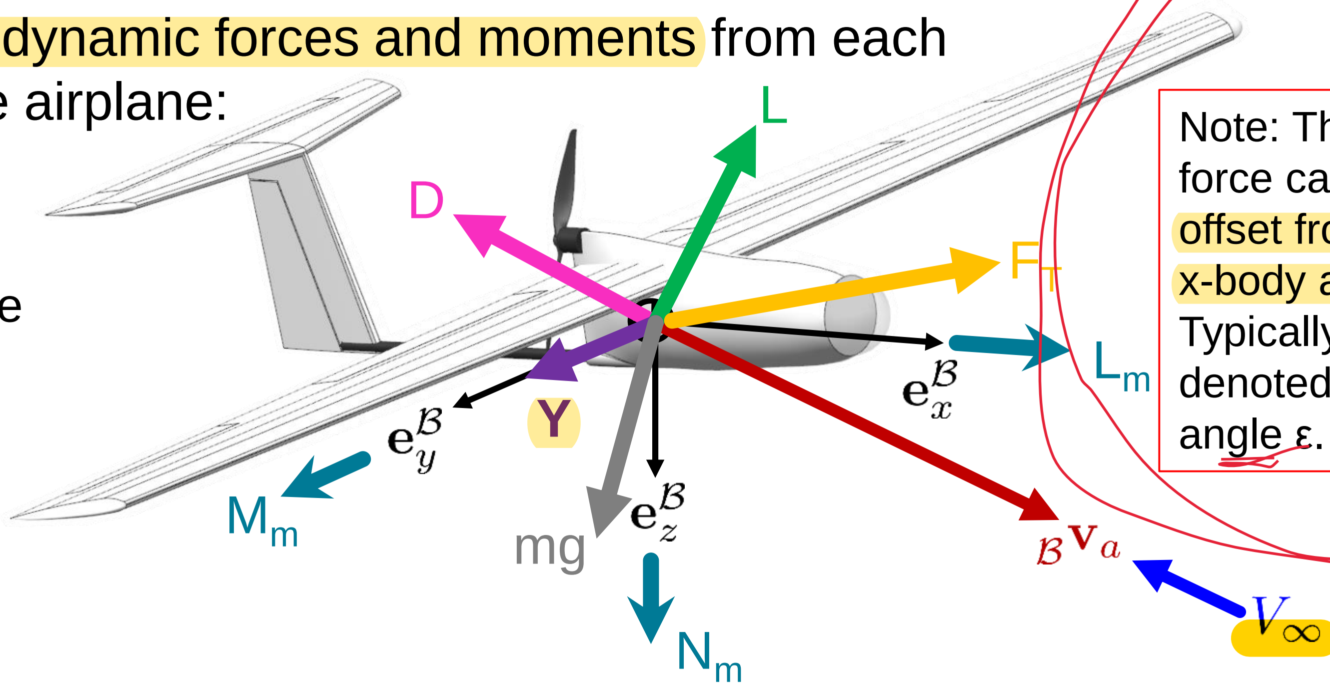
$$
= {}_{\mathcal{I}}\dot{\mathbf{r}} = \begin{pmatrix} V\cos\gamma\cos\xi + w_N \\ V\cos\gamma\sin\xi + w_E \\ -V\sin\gamma + w_D \end{pmatrix}
$$



Aircraft Dynamics

Aircraft Dynamics | Forces & Moments

- **Forces and moments acting on the airplane**
  - Weight at the center of gravity
  - Thrust of propeller: complex task → will not be presented here
  - Sum aerodynamic forces and moments from each part of the airplane:
    - Wing
    - Tail
    - Fuselage



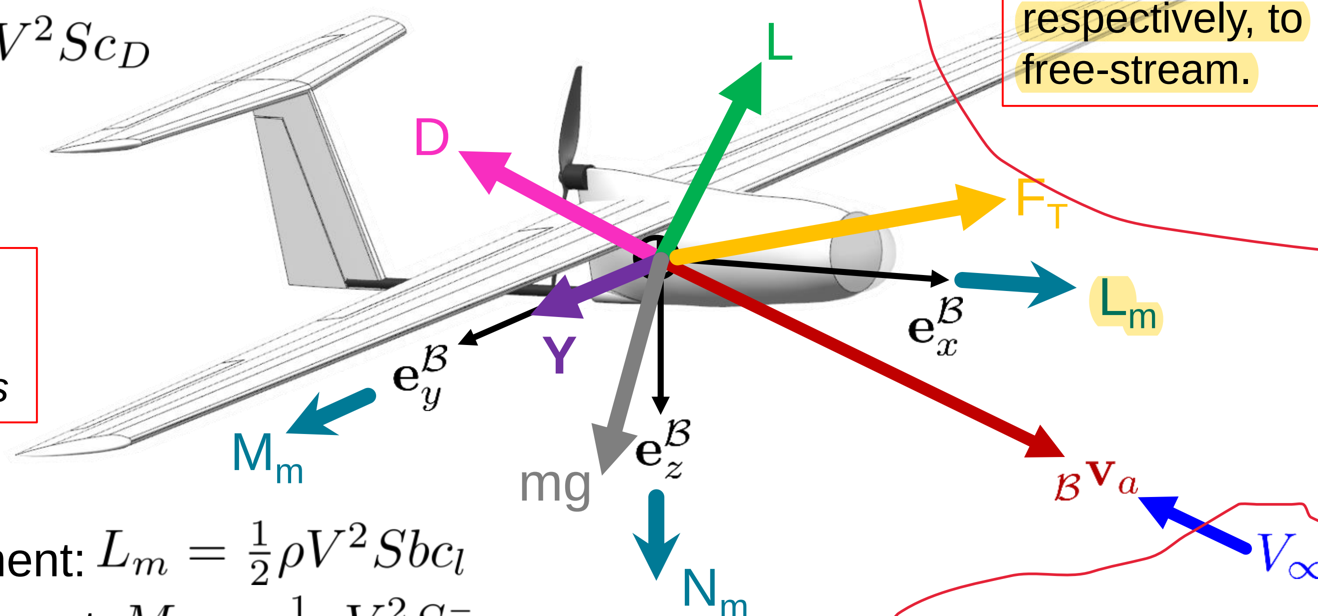
Note: Thrust force can be offset from x-body axis. Typically denoted by angle ε.

Aircraft Dynamics | Forces & Moments

- **Forces and moments acting on the airplane**
  - Lift $L = \frac{1}{2}\rho V^2 S c_L$
  - Drag $D = \frac{1}{2}\rho V^2 S c_D$
  - Thrust
  - Gravity

For geometry definitions (i.e. S, b, $\bar{c}$) see "Wing Geometry" in *backup slides*

Lift and Drag **always** perpendicular and parallel, respectively, to free-stream.



Free-stream velocity

  - Moments
    - Rolling moment: $L_m = \frac{1}{2}\rho V^2 S b c_l$
    - Pitching moment: $M_m = \frac{1}{2}\rho V^2 S \bar{c} c_m$
    - Yawing moment: $N_m = \frac{1}{2}\rho V^2 S b c_n$

Aircraft Dynamics | Component build-up

- The aerodynamic forces and moments are built up from both static and dynamic components, summed from each part of the aircraft.
  - E.g.

$$
\begin{aligned}
c_L &= f\left(\alpha, q, \delta_e\right) \\
&= c_{L_0} + c_{L_\alpha}\alpha + c_{L_q}\hat{q} + c_{L_{\delta_e}}\delta_e \\
&= c_{L_0} + c_{L_\alpha}\alpha + c_{L_\alpha^2}\alpha^2 + c_{L_q}\hat{q} + c_{L_{\delta_e}}\delta_e + c_{L_{\alpha\delta_e}}\alpha\delta_e
\end{aligned}
$$

1st order Taylor Expansion (only linear terms)

$$c_{L_\alpha} = \frac{\partial c_L}{\partial \alpha}$$

2nd order Taylor Expansion (some coupled terms)

  - Other model structures could be used, but the build-up approach **generalizes well in practice**, when in nominal flight regimes

Aircraft Dynamics | Forces and moments

- Represented in body frame, at the CoG:

$$\mathbf{F}_{tot} = \begin{pmatrix} -D\cos\alpha + L\sin\alpha \\ Y \\ -D\sin\alpha - L\cos\alpha \end{pmatrix} + \begin{pmatrix} F_T\cos\varepsilon \\ 0 \\ F_T\sin\varepsilon \end{pmatrix} + m\mathbf{C}_{BI}\begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$$

$$= \begin{pmatrix} F_T\cos\varepsilon - D\cos\alpha + L\sin\alpha - mg\sin\theta \\ Y + mg\sin\phi\cos\theta \\ F_T\sin\varepsilon - D\sin\alpha - L\cos\alpha + mg\cos\phi\cos\theta \end{pmatrix}$$

$$\mathbf{M}_{tot} = \begin{pmatrix} L_m \\ M_m \\ N_m \end{pmatrix} + \begin{pmatrix} L_{m_T} \\ M_{m_T} \\ N_{m_T} \end{pmatrix}$$

Moments due to propulsion

Aircraft Dynamic Modeling | Equations of motion

- Application of Newton's Second Law

$$\mathbf{F}_{tot} = \frac{d}{dt}\left(m \cdot {}_B\mathbf{v}\right) = \quad m\left[\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} + \begin{pmatrix}p\\ q\\ r\end{pmatrix} \times \begin{pmatrix}u\\ v\\ w\end{pmatrix}\right] = m\begin{bmatrix}\dot{u} + qw - rv\\ \dot{v} + ru - pw\\ \dot{w} + pv - qu\end{bmatrix}$$

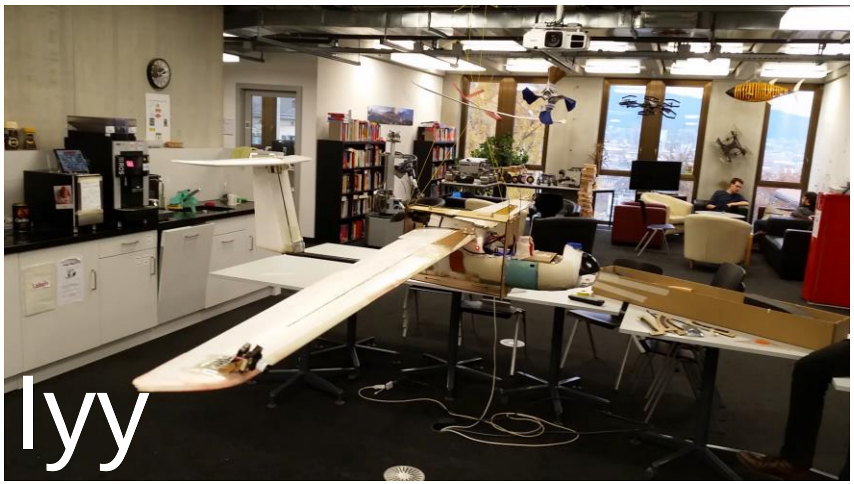
$$\mathbf{M}_{tot} = \frac{d}{dt}\left(\mathbf{I} \cdot {}_B\boldsymbol{\omega}\right) = \frac{d}{dt}\left(\begin{bmatrix}I_{xx} & 0 & I_{xz}\\ 0 & I_{yy} & 0\\ I_{xz} & 0 & I_{zz}\end{bmatrix} \cdot \begin{pmatrix}p\\ q\\ r\end{pmatrix}\right) =$$

Typically small

$$\begin{bmatrix}I_{xx} & 0 & I_{xz}\\ 0 & I_{yy} & 0\\ I_{xz} & 0 & I_{zz}\end{bmatrix} \cdot \begin{pmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{pmatrix} + \begin{pmatrix}p\\ q\\ r\end{pmatrix} \times \left(\begin{bmatrix}I_{xx} & 0 & I_{xz}\\ 0 & I_{yy} & 0\\ I_{xz} & 0 & I_{zz}\end{bmatrix} \cdot \begin{pmatrix}p\\ q\\ r\end{pmatrix}\right) = \begin{pmatrix}I_{xx}\dot{p} + I_{xz}\dot{r} + qr\left(I_{zz} - I_{yy}\right) + qpI_{xz}\\ I_{yy}\dot{q} + pr\left(I_{xx} - I_{zz}\right) + \left(r^2 - p^2\right)I_{xz}\\ I_{xz}\dot{p} + I_{zz}\dot{r} + pq\left(I_{yy} - I_{xx}\right) - qrI_{xz}\end{pmatrix}$$

Aircraft Dynamic Modeling | Equations of motion

- Side-note on *identification* of mass moments of inertia:



SWING TEST

Aircraft Dynamic Modeling | Equations of motion

- Side-note on *identification* of mass moments of inertia:

SWING TEST

Aircraft Dynamic Modeling | Equations of motion

- Translation

$$\dot{u} = rv - qw + \frac{1}{m}\left(F_T \cos\varepsilon - D\cos\alpha + L\sin\alpha\right) - g\sin\theta$$

$$\dot{v} = pw - ru + \frac{1}{m}Y + g\sin\phi\cos\theta$$

$$\dot{w} = qu - pv + \frac{1}{m}\left(F_T \sin\varepsilon - D\sin\alpha - L\cos\alpha\right) + g\cos\phi\cos\theta$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \mathbf{C}_{IB}\begin{bmatrix} u \\ v \\ w \end{bmatrix} + {}_I\mathbf{w}$$

Aircraft Dynamic Modeling | Equations of motion

- Rotation (simplified with $I_{xz} \approx 0$):

$$\dot{p} = \frac{1}{I_{xx}} \left[L_m + L_{m_T} - qr\left(I_{zz} - I_{yy}\right) \right]$$

$$\dot{q} = \frac{1}{I_{yy}} \left[M_m + M_{m_T} - pr\left(I_{xx} - I_{zz}\right) \right]$$

$$\dot{r} = \frac{1}{I_{zz}} \left[N_m + N_{m_T} - pq\left(I_{yy} - I_{xx}\right) \right]$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{J}_r^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p + q \tan\theta \sin\phi + r \tan\theta \cos\phi \\ q \cos\phi - r \sin\phi \\ q \dfrac{\sin\phi}{\cos\theta} + r \dfrac{\cos\phi}{\cos\theta} \end{bmatrix}$$

Fixed-wing Control

Fixed-wing Control | Introduction

- Control of airplanes is not so easy!:
  - Inherently non-linear (especially in longitudinal axis)
  - Low control authority
  - Actuator saturation
  - Double integrator characteristics
  - MIMO: 4 inputs, 6 DoF, thus underactuated

Fixed-wing Control | Control Concepts

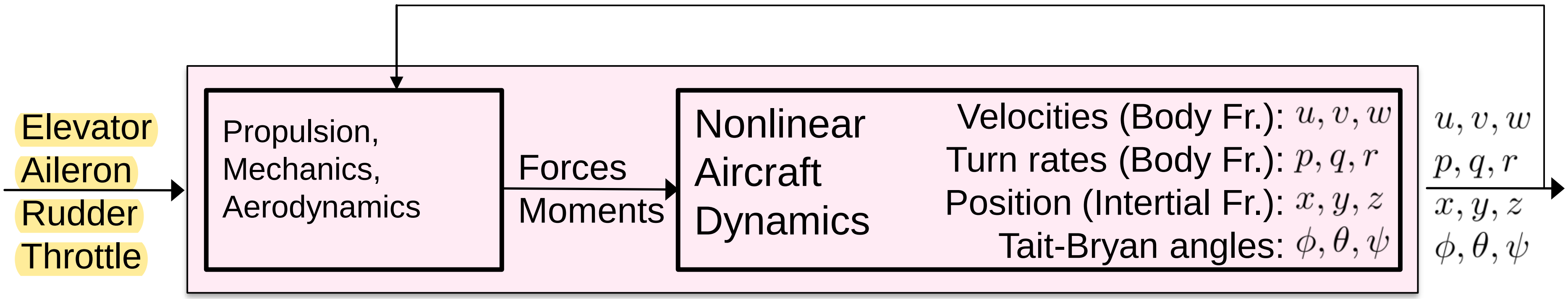
Many control techniques:

- Cascaded PID loops
- Optimal control
  - LQR
- Robust control
  - H-infinity
  - H-2 loop-shaping
- Adaptive control
- Model Predictive Control
  - Linear/Nonlinear
- (Nonlinear) Dynamic Inversion

Chose according to:

- Computational Power
- Type of flight (e.g. aerobatics vs. level flight)
- Availability / fidelity of model

Fixed-wing Control | The plant



Input vector:

$$\mathbf{u} = \begin{pmatrix} \delta_T \\ \delta_e \\ \delta_a \\ \delta_r \end{pmatrix}$$

State vector:

$$\mathbf{x} = [u, v, w, p, q, r, x, y, z, \phi, \theta, \psi]^T$$

Output: e.g.

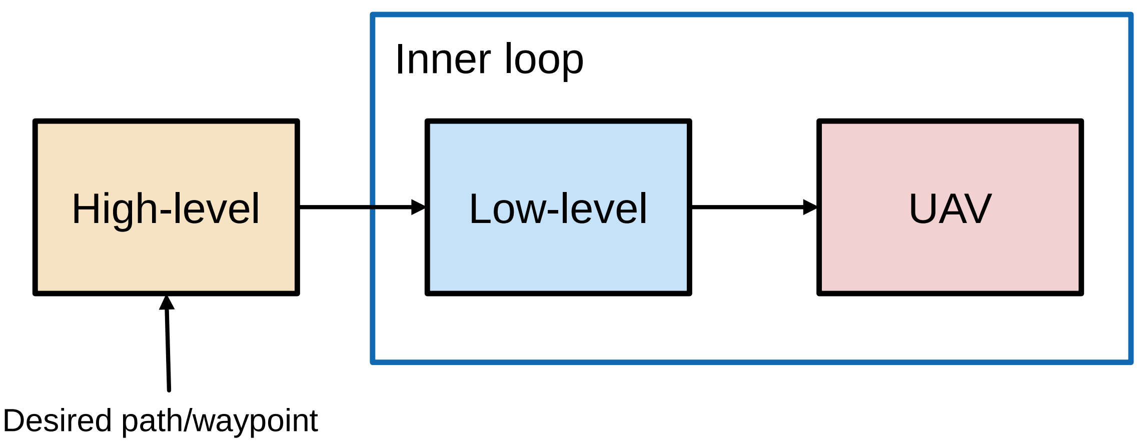
$$\mathbf{y} = \begin{pmatrix} V \\ v \\ \phi \\ \theta \end{pmatrix} \text{ or } \beta$$

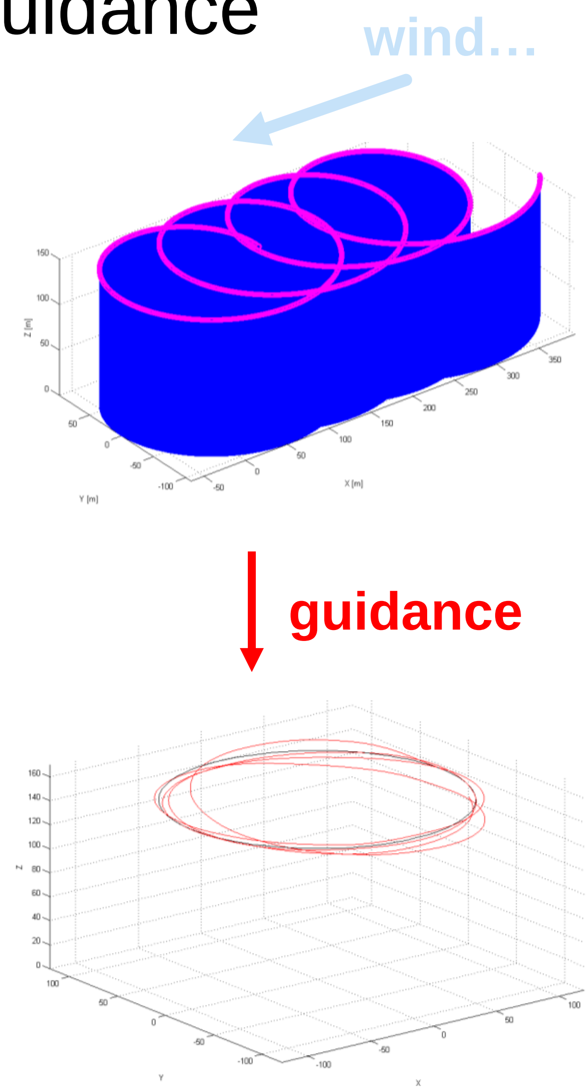
- Some remarks about the conventions used in this lecture:
  - Input limits: $\delta_e \in [-1,1], \delta_a \in [-1,1], \delta_r \in [-1,1], \delta_T \in [0,1]$
  - Ailerons: $\delta_{a,\mathrm{right}} = -\delta_{a,\mathrm{left}} = \delta_a$
    - may be mixed to symmetric or differential deflections
  - We define positive actuator inputs as those that produce positive moments

Fixed-wing Control | Control & Guidance

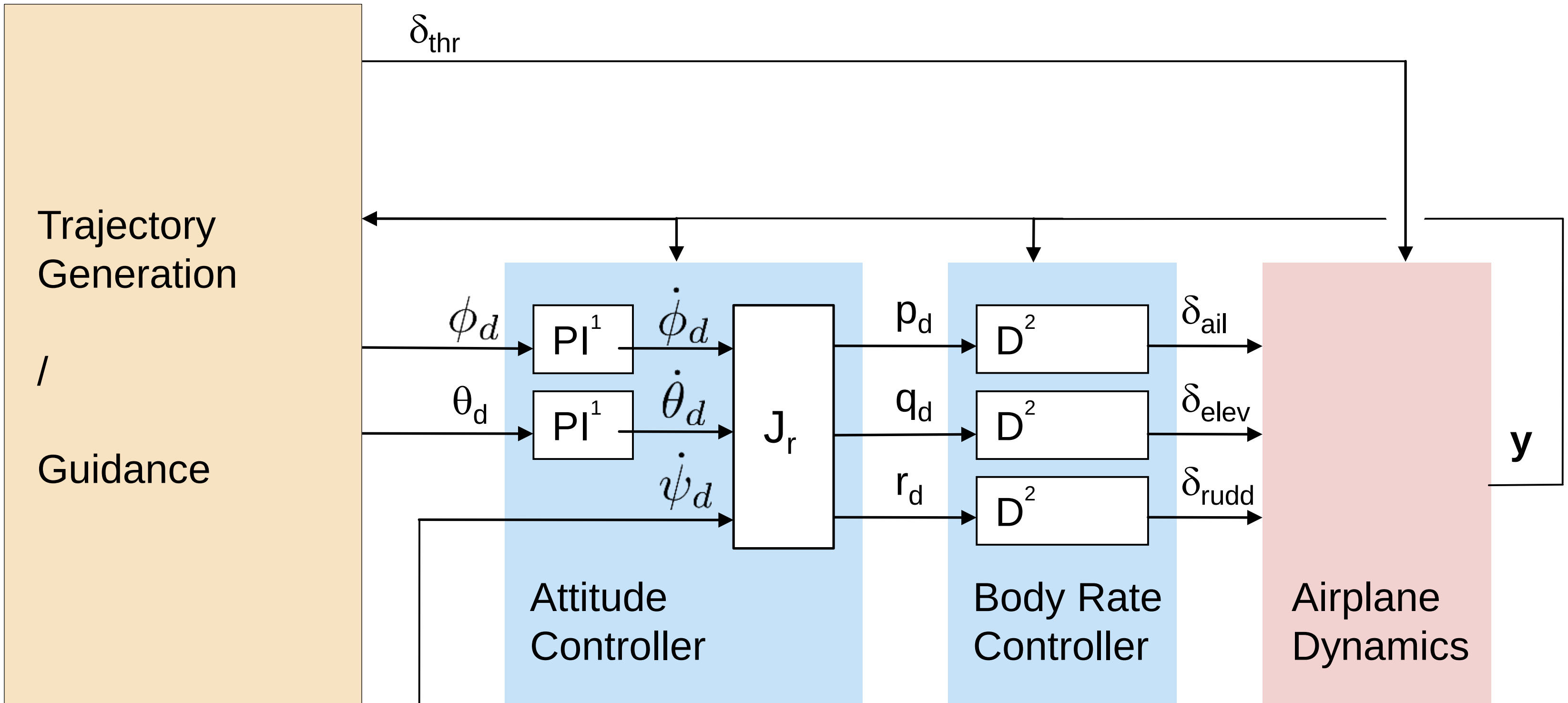
A popular concept: cascaded control loops

- **Control** = low level part
  - Stabilize attitude (and sometimes airspeed)
- **Guidance** = high level part
  - Follow paths or trajectory (position control)
    Effect: Reject constant low frequency perturbation (constant wind)





Fixed-wing Control | Simple cascaded control



Constrain to coordinated turn:

$$\Rightarrow \quad \dot{\psi}_d = \frac{g \tan \phi_d}{V}$$

[1] PI with anti-reset wind-up
[2] Gain scaled with $1/V_T^2$

- Bandwidths of inner Loops must be sufficiently larger!

Fixed-wing Control | Steady level turning flight

Assuming NO sideslip, i.e. $\xi = \psi$

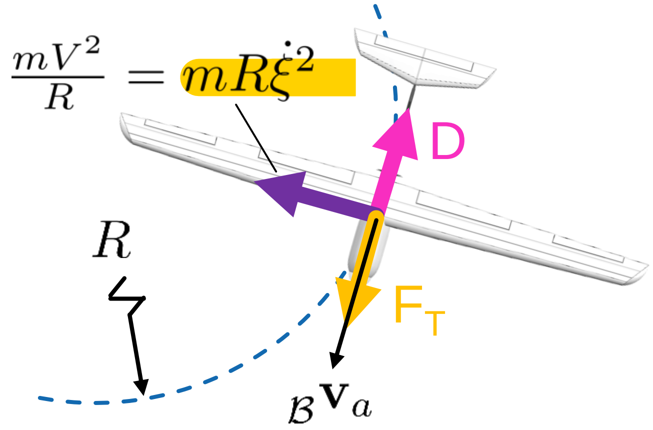
- Turning (not straight)

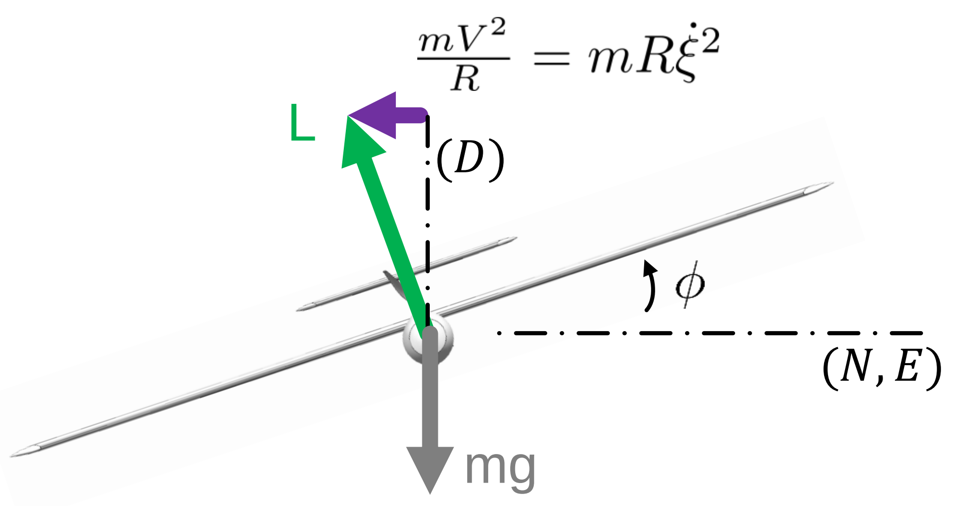
$${}_{\mathcal{B}}\dot{\mathbf{v}}_a = \mathbf{0},\ {}_{\mathcal{B}}\dot{\omega} = \mathbf{0}$$ <- steady (unaccelerated)

$$\theta = \alpha \rightarrow \gamma = 0$$ <- level

$$\phi = \text{const.} \neq 0$$ <- turning

  - Demand for **coordinated** turn: Y=0
  - L increases with $\frac{1}{\cos\phi}$
  - V_{min} increases with $\sqrt{\frac{1}{\cos\phi}}$





Recall Y is composed of only aerodynamic forces, which must be zero, thus the lateral force here only comes from centripetal acceleration.

Fixed-wing Control | Steady level turning flight

Assuming NO sideslip, ...

- Turning (not straight)

$_\mathcal{B}\dot{\mathbf{v}}_a = \mathbf{0},\ _\mathcal{B}\dot{\omega} = \mathbf{0}$ <- steady (unacceler...

$\theta = \alpha \rightarrow \gamma = 0$ <- level

$\phi = \text{const.} \neq 0$ <- turning

  - Demand for **coordinated** turn: Y ...

$$L\cos\phi = mg$$

$$L = \frac{mg}{\cos\phi} \sim \frac{1}{\cos\phi}$$

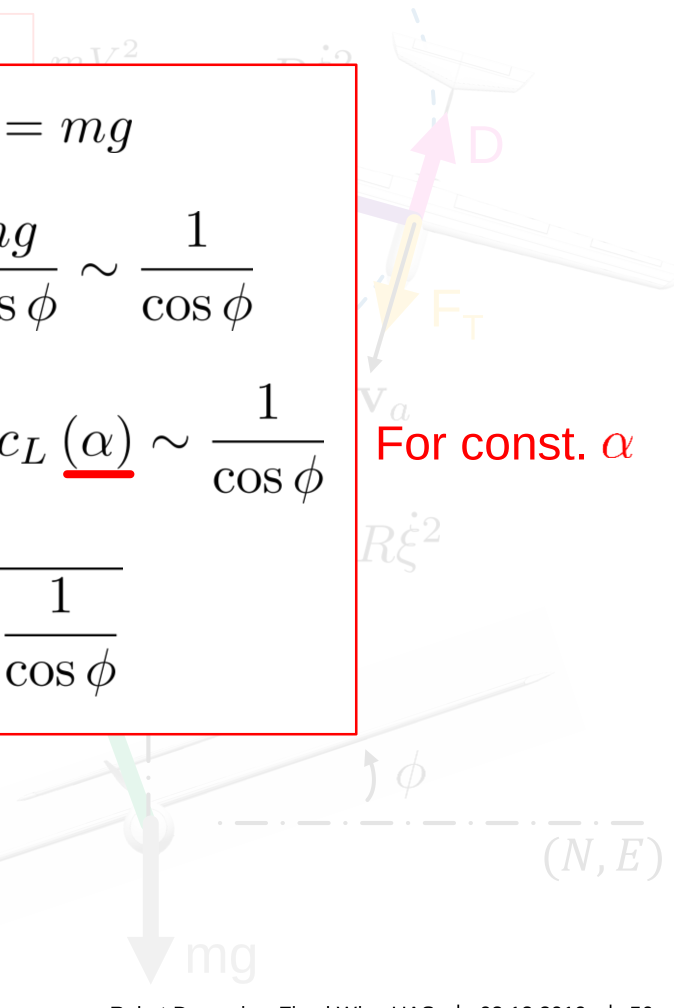
$$\frac{1}{2}\rho V^2 S c_L\left(\underline{\alpha}\right) \sim \frac{1}{\cos\phi}$$

For const. α

$$V \sim \sqrt{\frac{1}{\cos\phi}}$$

- L increases with $\frac{1}{\cos\phi}$
- V_{min} increases with $\sqrt{\frac{1}{\cos\phi}}$

Recall Y is composed of only aerodynamic forces, which must be zero, thus the lateral force here only comes from centripetal acceleration.



Fixed-wing Control | Steady level turning flight

- Heading-rate can be found for a given roll angle with a force balance (assuming $\dot{\psi} \approx \dot{\xi}$)

Note this assumes we have thrust force only acting in the same axis as drag. In reality, thrust force likely will add a small vertical component to the lift (e.g. if we fly at any pitch/angle of attack).

Force balance:

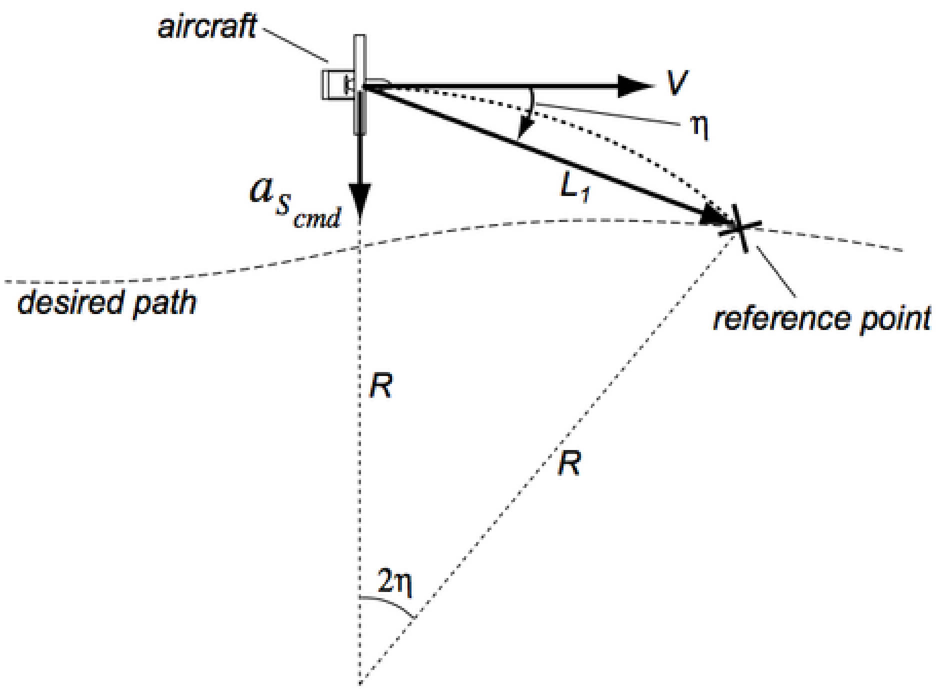
$$L\cos\phi = mg$$
$$D = F_T$$
$$m\frac{V^2}{R} = L\sin\phi$$

$$\Rightarrow$$

$$\boxed{\begin{aligned} \frac{L\sin\phi}{L\cos\phi} &= \frac{m\frac{V^2}{R}}{mg} \\ \tan\phi &= \frac{V\dot{\xi}}{g} \\ \rightarrow \dot{\xi} &= g\tan\phi/V \\ \rightarrow \dot{\psi} &= g\tan\phi/V \end{aligned}}$$

Fixed-wing Control | $\mathcal{L}_1$ Guidance

Following a Trajectory on the Horizontal Plane



$$\sin\eta = \frac{L_1}{2R} \Rightarrow R = \frac{L_1}{2\sin\eta}$$

$$a_{s_{cmd}} = \frac{V^2}{R} = 2\frac{V^2\sin\eta}{L_1} = \dot{\xi}_d V$$

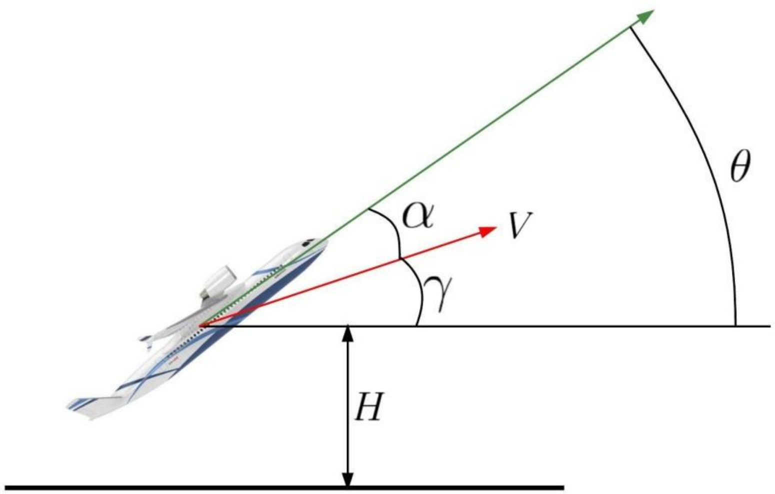
$$\Rightarrow \dot{\xi}_d = \frac{a_{s_{cmd}}}{V}$$

$$\dot{\xi}_d = \frac{g\tan\phi_d}{V} \Rightarrow \phi_d = \tan^{-1}\left(\frac{a_{s_{cmd}}}{g}\right)$$

Theroy and Graphics from:
S. Park, J. Deyst, and J. P. How, “A New Nonlinear Guidance Logic for Trajectory Tracking”, Proceedings of the AIAA Guidance, Navigation and Control Conference, Aug 2004. AIAA-2004-4900

Fixed-wing Control | TECS (Total Energy Control System)

Control Altitude and Airspeed



$$E_{tot} = E_{kin} + E_{pot} = \frac{1}{2}mV^2 + mgH$$

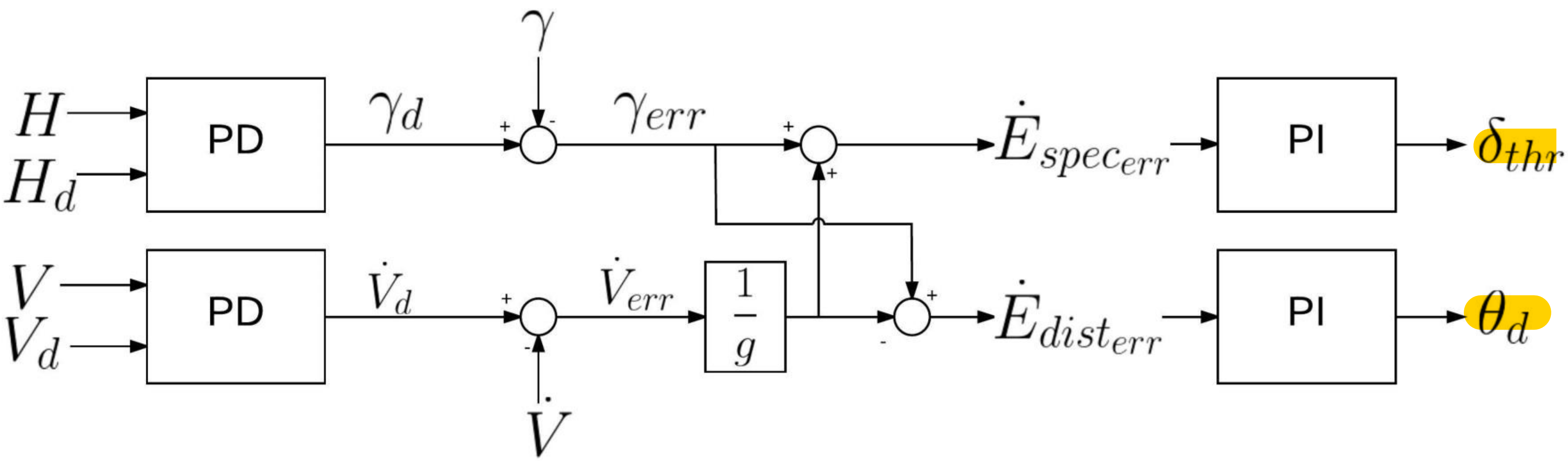
$$\frac{\dot{E}_{tot}}{mg} = \frac{mV\dot{V}}{mg} + \frac{\dot{H}mg}{mg} = \frac{V\dot{V}}{g} + \dot{H}$$

$$\dot{E}_{spec} = \frac{\dot{E}_{tot}}{mgV} = \frac{\dot{V}}{g} + \frac{\dot{H}}{V} = \frac{\dot{V}}{g} + \sin\gamma$$

$$\dot{E}_{spec} = \frac{\dot{V}}{g} + sin\gamma \approx \frac{\dot{V}}{g} + \gamma$$

$$\dot{E}_{dist} = \gamma - \frac{\dot{V}}{g}$$

Fixed-wing Control | TECS (Total Energy Control System)



Fixed-wing Control | Note the model abstraction

- In lower-level loops, dynamics are modeled from actuators→attitude/airspeed
  - Note that aside from computaitonal costs, these dynamics are challenging to globally identify in a nonlinear, high fidelity form. Thus **linearizations** are often made.
- Higher-level loops often model the aircraft in a three-degrees-of-freedom (3DoF) sense, mapping attitude/airspeed→position
  - Modeling of high-level dynamics does not require identification, as typically only kinematics are used.

Next week

- 10.12: Fixed-wing case studies!
  - **Design, modeling, and control of hybrid (VTOL) fixed-wing platforms**

    Sebastian Verling, Wingtra
  - **Autonomy for solar-powered UAVs beyond the horizon**

    Thomas Stastny, ASL

- 11.12: Fixed-wing exercise
  - Control/simulation

Backup Slides

References

- B.W. McCormick. Aerodynamics, Aeronautics, and Flight Mechanics. Wiley, 1979. ISBN: 9780471030324.
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- G.J.J. Ducard. Fault-Tolerant Flight Control and Guidance Systems: Practical Methods for Small Unmanned Aerial Vehicles . Advances in Industrial Control. Springer, 2009. ISBN: 9781848825611.
- R.W. Beard and T.W. McLain. Small Unmanned Aircraft: Theory and Practice. Princeton University Press, 2012. ISBN: 9780691149219.
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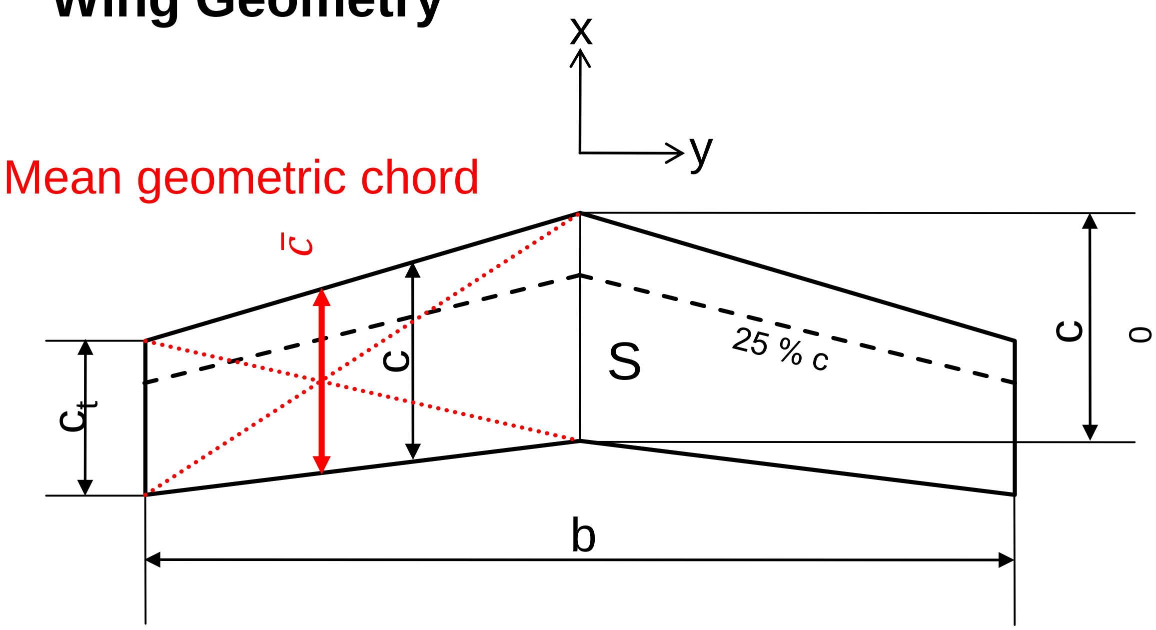
Basics of Aerodynamics | Wing Geometry

Wing Geometry



b:	Wingspan
c:	Chord
c_0:	Root Chord
c_t:	Tip Chord
S:	Reference Area
AR:	Aspect Ratio

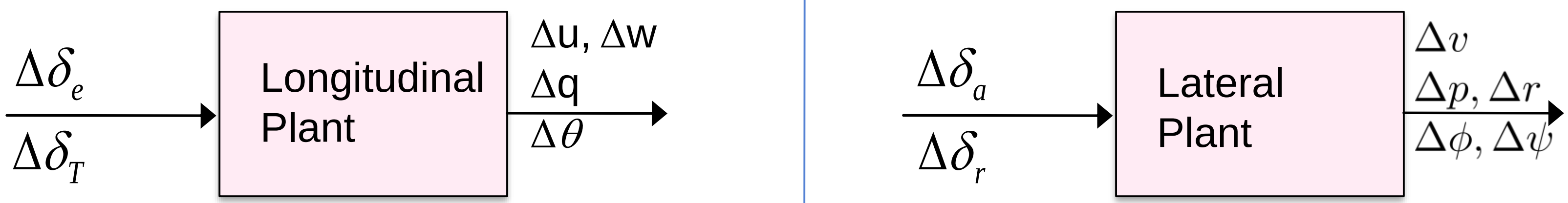
$$AR = \frac{b^2}{S}$$

Modeling for Control | Linearization

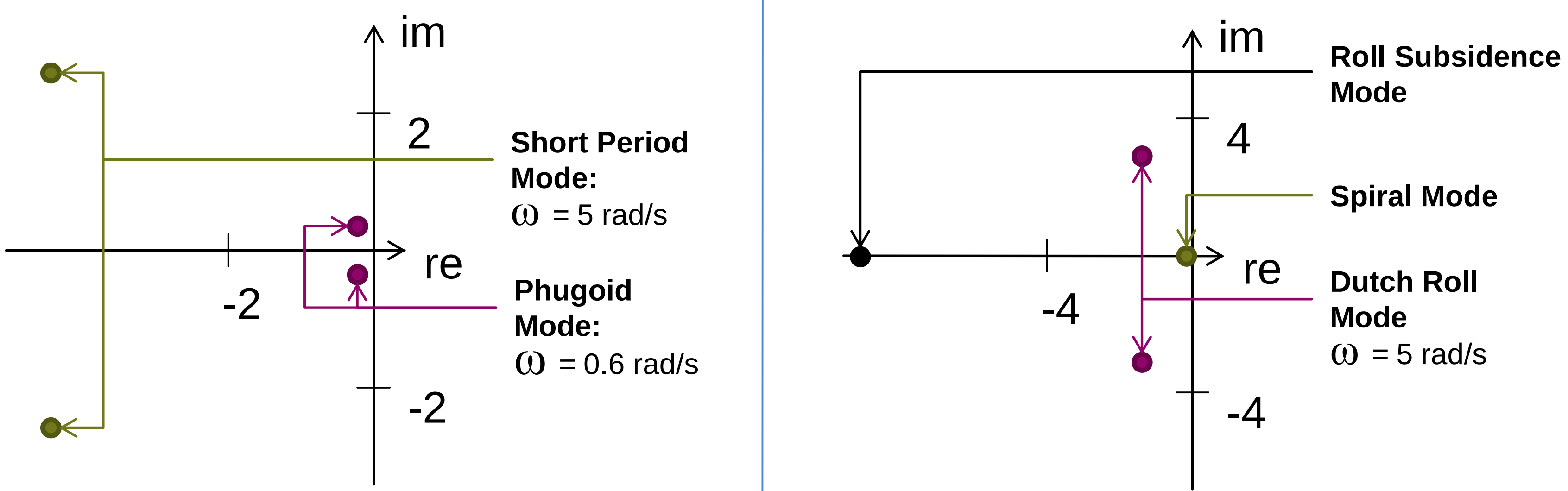
- Decouple plant into lateral-directional and longitudinal states
- Linearize about trim airspeed
  - Typically straight and level steady flight

Modeling for Control | The (linearized) plant

Subsystem



Corresponding Poles (Aerobatic Model Airplane)

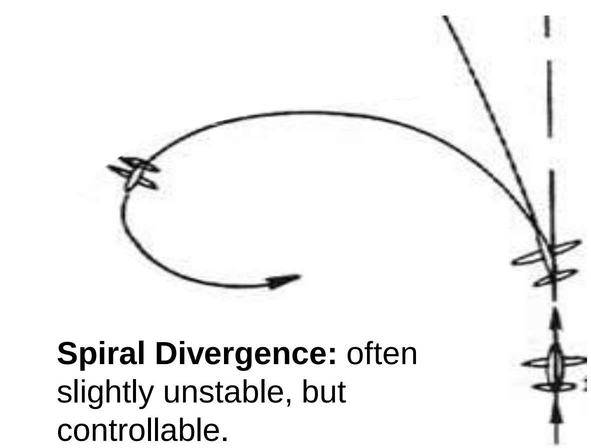


Modeling for Control | The (linearized) plant

Phugoid mode: exchange between kinetic and potential energy. **SLOW**

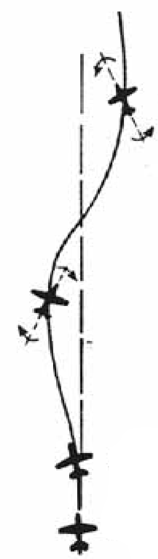
Longitudinal Modes

Short Period Mode: oscillation of angle of attack. **FAST**

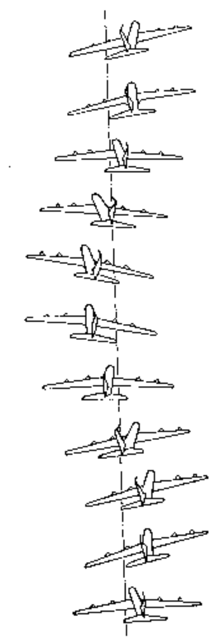


Spiral Divergence: often slightly unstable, but controllable.

Lateral-directional Modes



Dutch Roll Mode: combined yaw-roll oscillation



Grafics adapted from:
http://history.nasa.gov/SP-367/chapt9.htm and
http://www.fzt.haw-hamburg.de/pers/Scholz/Flugerprobung.html