Robot Dynamics Quiz 1

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October 16, 2019

Duration: 1h 15min
Permitted Aids: The exam is open book, which means you can use the script, slides, exercises, etc; The use of internet (beside for licenses) is forbidden; no communication among students during the test.

1 Instructions

1. Download the ZIP file for quiz 1 from Piazza. Extract all contents of this file into a new folder and set MATLAB's[1] current path to this folder.
2. Run `init_workspace` in the Matlab command line
3. All problem files that you need to complete are located in the `problems` folder
4. Run `evaluate_problems` to check if your functions run. This script does not test for correctness. You will get 0 points if a function does not run (e.g., for syntax errors).
5. When the time is up, zip the entire folder and name it
   `ETHStudentID_StudentName.zip`
   Upload this zip-file through the following link
   `https://www.dropbox.com/request/CkoAClzaACSfpz0R6MzV`.
   You will receive a confirmation of receipt.
6. If the previous step did not succeed, you can email your file to
   `robotdynamics@leggedrobotics.com`
   from your ETH email address with the subject line
   `[RobotDynamics] ETHStudentID - StudentName`

[1] Online version of MATLAB at `https://matlab.mathworks.com/`

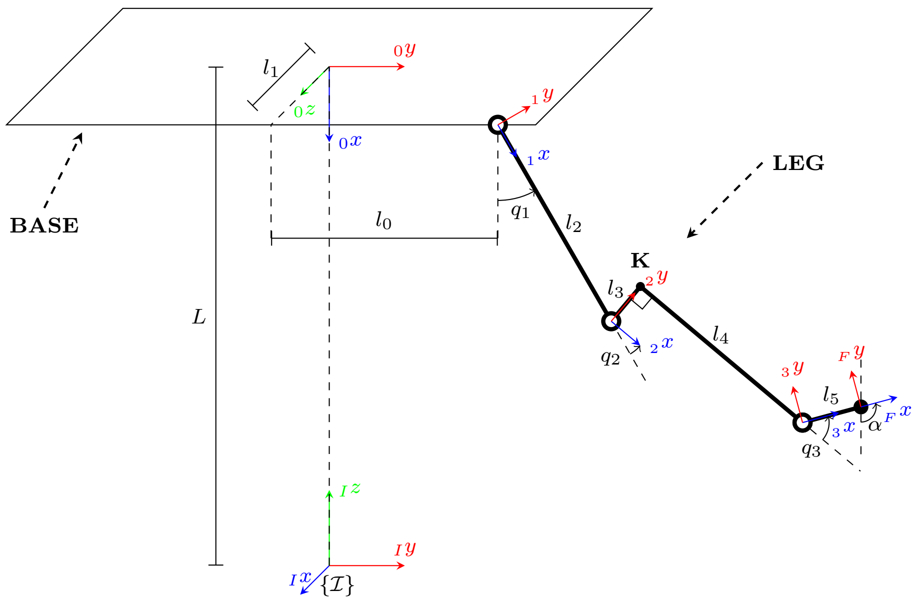

Figure 1: Three degree of freedom robotic leg attached to a fixed base. All joints rotate around their local positive z axis. The z axis of the frames $\{1\}$, $\{2\}$, $\{3\}$ is parallel to the ${}_0z$ axis.

2 Questions

In this quiz, you will model the forward, differential, and inverse kinematics of the robotic leg shown in Fig. 1. It is a 3 degrees of freedom leg connected to a **fixed** base.

Let $\{0\}$ be the base frame, which is displaced by L from the inertial frame $\{I\}$ along the ${}_Iz$ axis. The leg is composed of three links and is displaced by l_0 and l_1 from the base frame $\{0\}$ along the ${}_0y$, and ${}_0z$ axis, respectively. The links' segments have lengths l_2, l_3, l_4, l_5.

The generalized coordinates are defined as

$$\boldsymbol{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top . \tag{1}$$

In the following questions, all required parameters are passed to your functions in a structure called `params`. You can access it as follows:

```
l0 = params.l0;
l1 = params.l1;
l2 = params.l2;
l3 = params.l3;
l4 = params.l4;
l5 = params.l5;
L  = params.L;
```

Question 1. 6 P.

Let $\{F\}$ be the end-effector (foot) frame. Find the homogeneous transform between the inertial frame $\{I\}$ and the foot frame $\{F\}$, i.e., the matrix $\mathbf{T}_{IF}$ as a function of the generalized coordinates $\boldsymbol{q}$.
Hint: Try to find the transforms of subsequent frames first.

You should implement your solution in the function `jointToEndeffectorPose.m`

NOTE: Do NOT solve Question 1 using the transforms provided in the solutions folder. The provided solutions correspond to a choice of coordinate frames with a (secret) rotational offset from those shown in Fig. 1. Questions 2 to 4 MUST be solved using the homogeneous transforms provided in the solutions folder (`T_IO_solution`, `T_01_solution`, etc.), do not use your own implementation from Question 1.

Question 2. 6 P.

Derive the geometric Jacobian ${}_I\mathbf{J}_0 \in \mathbb{R}^{6\times3}$ of the **knee**, at point **K**. This Jacobian should map the generalized velocities $\dot{\boldsymbol{q}}$ to linear and angular velocities of the knee in $\{I\}$ frame, i.e.,

$$\begin{pmatrix} {}_I\boldsymbol{v}_K \\ {}_I\boldsymbol{\omega}_K \end{pmatrix} = {}_I\mathbf{J}_0 \, \dot{\boldsymbol{q}} \; . \tag{2}$$

Hints:

1. Use our provided functions for the transforms
   (Note: Although these transforms relate to a different choice of coordinate frames with respect to Fig. 1, the inertial frame and thus the rotation axis is the same).

2. The MATLAB function for cross product $\boldsymbol{a} \times \boldsymbol{b}$ is `cross(a,b)`

You should implement your solution in the function `jointToKneeGeometricJacobian.m`

Question 3. 3 P.

Implement an inverse-kinematic motion control algorithm for the robot leg. Your implementation should return the joint angles $\boldsymbol{q}_{\text{next}}$, which are the joint angles after one sampling time, t_s, i.e. $\boldsymbol{q}_{\text{next}} = \boldsymbol{q}(t + t_s)$. The returned angles should make the **foot** track a pre-defined reference trajectory.

We indicate with ${}_I\boldsymbol{p} \in \mathbb{R}^3$ and ${}_I\boldsymbol{w} \in \mathbb{R}^3$ the following vectors:

$${}_I\boldsymbol{p} = \begin{bmatrix} {}_Iy_F \\ {}_Iz_F \\ \alpha \end{bmatrix} , \quad {}_I\boldsymbol{w} = \begin{bmatrix} {}_I\dot{y}_F \\ {}_I\dot{z}_F \\ \dot{\alpha} \end{bmatrix} , \tag{3}$$

where the angle α is indicated in Fig. 1. It is the rotation angle around ${}_Ix$ between the x-axis of the foot frame, ${}_Fx$, and the drawn vertical axis (aligned with ${}_Iz$). The desired vectors ${}_I\boldsymbol{p}_{\text{des}}$, ${}_I\boldsymbol{w}_{\text{des}}$ are passed as inputs to the matlab function.

For this question, we provide

- the current vector ${}_I\boldsymbol{p}$, derived from the provided generalized coordinates.
- the analytical Jacobian $\mathbf{J}_A \in \mathbb{R}^{3\times3}$ that fulfills

$$\begin{pmatrix} {}_I\dot{y}_F \\ {}_I\dot{z}_F \\ \dot{\alpha} \end{pmatrix} = \mathbf{J}_A \, \dot{\boldsymbol{q}} \; . \tag{4}$$

You can call it with `jointToEndeffectorAnalyticJacobian_solution(q, params)`

- a function for calculating damped pseudo-inverses, as you have seen in the exercise: `pseudoInverseMat_solution(J, lambda)`

You should implement your solution in the function `inverseKinematicControl.m`

Your implementation is judged based on how well it tracks a predefined trajectory. Execute `main_control_loop.m` to see what the solution should look like. To visualize your implementation, set the variable `use_solution` to 0.

Question 4. 3 P.

Assume now that the base has an additional rotation described by the XYZ Euler parametrization. The rotation is expressed as three consecutive rotations; first around the x-axis of frame $\{0\}$ resulting in the frame $\{0'\}$, afterwards a rotation around the y-axis of frame $\{0'\}$, resulting in frame $\{0''\}$, and finally a rotation around the z-axis of frame $\{0''\}$ resulting in the frame $\{B\}$.

Find the orientation of the foot frame with respect to the inertial frame C_{IF}.

The transform from frame $\{B\}$ to frame $\{1\}$ is provided by `T_B1_solution(q, params)`.

You should implement your solution in the function `footFrameOrientationWithBaseRotation.m`