

# Exercise 1b: Differential Kinematics of the ABB IRB 120

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## Abstract

The aim of this exercise is to calculate the differential kinematics of an ABB robot arm. You will practice on the derivation of velocities for a multi-body system, as well as derive the mapping of between generalized velocities and end-effector velocities. A separate MATLAB script will be provided for the 3D visualization of the robot arm.



Figure 1: The ABB IRW 120 robot arm.

## 1 Introduction

The following exercise is based on an ABB IRB 120 depicted in figure 2. It is a 6-link robotic manipulator with a fixed base. During the exercise you will implement several different MATLAB functions, which you should test carefully since the

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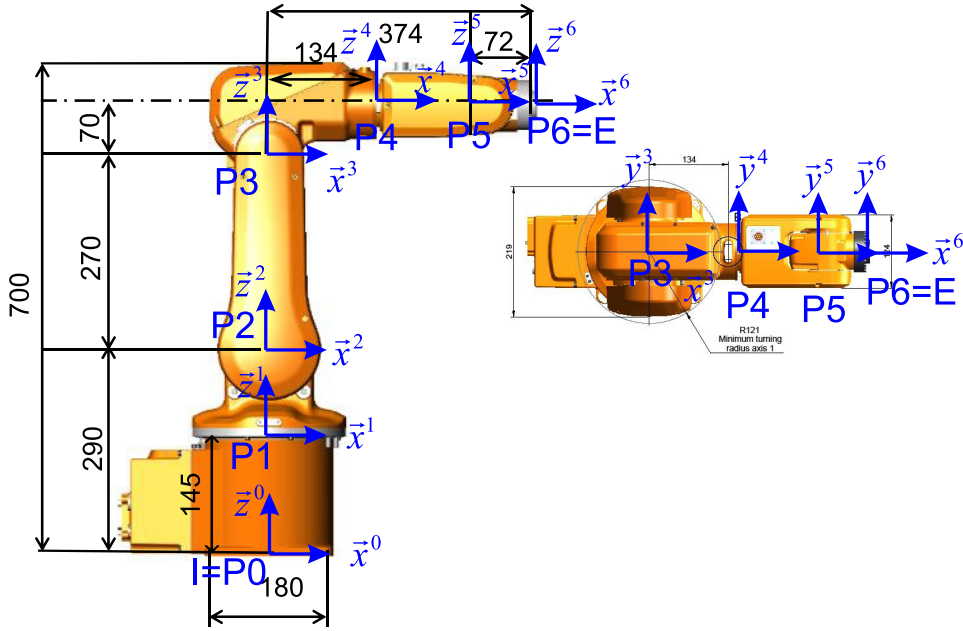


Figure 2: ABB IRB 120 with coordinate systems and joints.

next exercises will depend on them. To help you with this, we have provided the script prototypes at <http://www.rsl.ethz.ch/education-students/lectures/robotdynamics.html> together with a visualizer of the manipulator.

Throughout this document, we will employ  $I$  for denoting the inertial world coordinate system (which has the same pose as the coordinate system P0 in figure 2) and  $E$  for the coordinate system attached to the end-effector (which has the same pose as the coordinate system P6 in figure 2).

## 2 Differential Kinematics

### Exercise 2.1

In this exercise, we seek to compute an analytical expression for the twist  ${}^I\mathbf{w}_E = [{}^I\mathbf{v}_E^T \ {}^I\boldsymbol{\omega}_E^T]^T$  of the end-effector. To this end, find the analytical expression of the end-effector linear velocity vector  ${}^I\mathbf{v}_E$  and angular velocity vector  ${}^I\boldsymbol{\omega}_E$  as a function of the linear and angular velocities of the coordinate frames attached to each link.

*Hint: start by writing the **rigid body motion theorem** and extend it to the case of a 6DoF arm.*

### Exercise 2.2

This exercise focuses on deriving the mapping between the generalized velocities  $\dot{\mathbf{q}}$  and the end-effector twist  ${}^I\mathbf{w}_E$ , namely the *basic* or *geometric* Jacobian  ${}^I\mathbf{J}_{e0} = [{}^I\mathbf{J}_P^T \ {}^I\mathbf{J}_R^T]^T$ . To this end, you should derive the translational and rotational Jacobians of the end-effector, respectively  ${}^I\mathbf{J}_P$  and  ${}^I\mathbf{J}_R$ . To do this, you can start from the derivation you found in exercise 1. The Jacobians should depend on the minimal coordinates  $\mathbf{q}$  only. Remember that Jacobians map joint space generalized velocities to operational space generalized velocities:

$${}^I\mathbf{v}_{IE} = {}^I\mathbf{J}_P(\mathbf{q})\dot{\mathbf{q}} \quad (1)$$

$${}^I\boldsymbol{\omega}_{IE} = {}^I\mathbf{J}_R(\mathbf{q})\dot{\mathbf{q}} \quad (2)$$

Please implement the following two functions:

```
1 function J_P = jointToPosJac(q)
2     % Input: vector of generalized coordinates (joint angles)
3     % Output: Jacobian of the end-effector translation which maps joint
4     % velocities to end-effector linear velocities in I frame.
5
6     % Compute the translational jacobian.
7     J_P = zeros(3, 6);
8 end
9
10 function J_R = jointToRotJac(q)
11     % Input: vector of generalized coordinates (joint angles)
12     % Output: Jacobian of the end-effector orientation which maps joint
13     % velocities to end-effector angular velocities in I frame.
14
15     % Compute the rotational jacobian.
16     J_R = zeros(3, 6);
17 end
```