ETH zürich

Lecture «Robot Dynamics»: Dynamics and Control

151-0851-00 V

lecture:	HG F3	Tuesday 10:15 – 12:00, every week
exercise:	HG D7.1	Wednesday 8:15 – 10:00, according to schedule

Marco Hutter, Roland Siegwart, and Thomas Stastny

17.09.2019	Intro and Outline	Course Introduction; Recapitulation Position, Linear Velocity			
24.09.2019	Kinematics 1	Rotation and Angular Velocity; Rigid Body Formulation, Transformation	25.09.2019	Exercise 1a	Kinematics Modeling the ABB arm
01.10.2019	Kinematics 2	Kinematics of Systems of Bodies; Jacobians	02.10.2019	Exercise 1a	Differential Kinematics of the ABB arm
08.10.2019	Kinematics 3	Kinematic Control Methods: Inverse Differential Kinematics, Inverse Kinematics; Rotation Error; Multi-task Control	09.10.2019	Exercise 1b	Kinematic Control of the ABB Arm
15.10.2019	Dynamics L1	Multi-body Dynamics	16.10.2019	Midterm 1	Programming kinematics with matlab
22.10.2019	Dynamics L2	Floating Base Dynamics	23.10.2019	Exercise 2a	Dynamic Modeling of the ABB Arm
29.10.2019	Dynamics L3	Dynamic Model Based Control Methods	30.10.2019	Exercise 2b	Dynamic Control Methods Applied to the ABB arm
05.11.2019	Legged Robot	Dynamic Modeling of Legged Robots & Control	06.11.2019	Midterm 2	Programming dynamics with matlab
12.11.2019	Case Studies 1	Legged Robotics Case Study	13.11.2019	Exercise 3	Legged robot
19.11.2019	Rotorcraft	Dynamic Modeling of Rotorcraft & Control	20.11.2019		
26.11.2019	Case Studies 2	Rotor Craft Case Study	27.11.2019	Exercise 4	Modeling and Control of Multicopter
03.12.2019	Fixed-wing	Dynamic Modeling of Fixed-wing & Control	04.12.2019		
10.12.2019	Case Studies 3	Fixed-wing Case Study (Solar-powered UAVs - AtlantikSolar, Vertical Take-off and Landing UAVs – Wingtra)	11.12.2019	Exercise 5	Fixed-wing Control and Simulation
17.12.2019	Summery and Outlook	Summery; Wrap-up; Exam			

Recapitulation

- We learned how to get the equation of motion in joint space
  - Newton-Euler
  - Projected Newton-Euler
  - Lagrange II

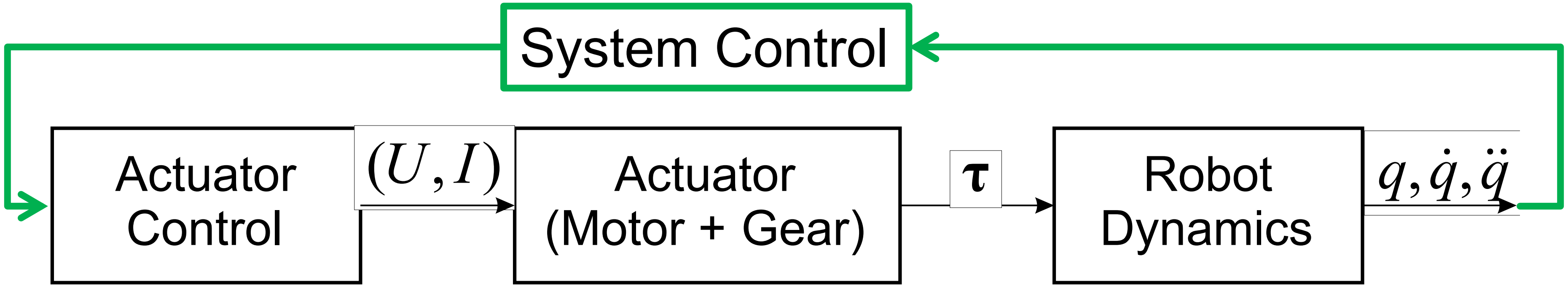
- Today:
  - How can we use this information in order to control the robot

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}+\mathbf{J}_c^T\mathbf{F}_c$$

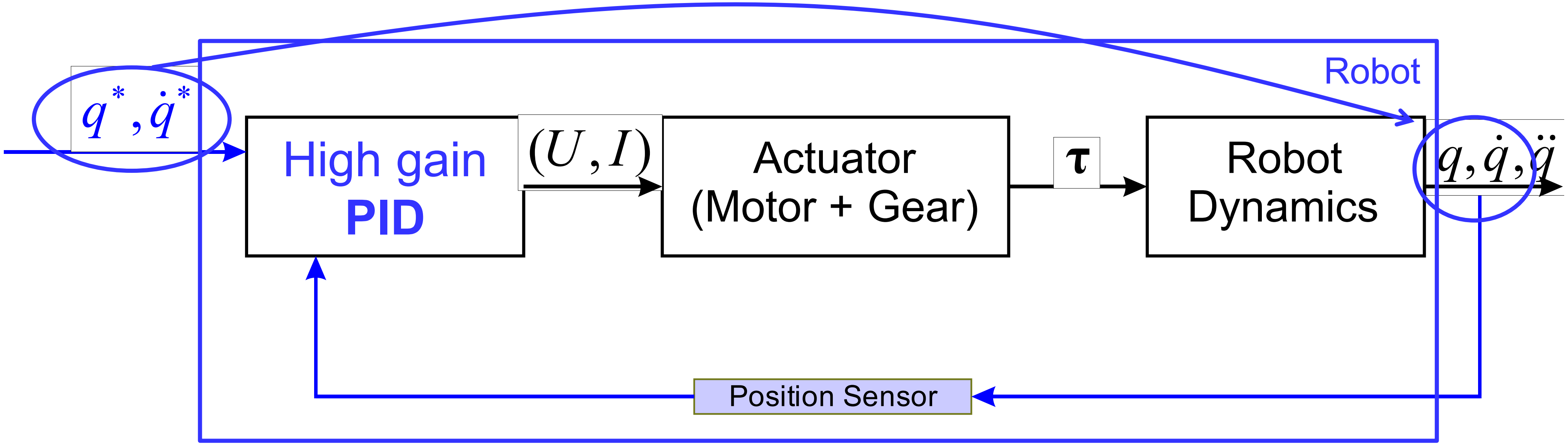
$\ddot{\mathbf{q}}$	Generalized coordinates
$\mathbf{M}(\mathbf{q})$	Mass matrix
$\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})$	Centrifugal and Coriolis forces
$\mathbf{g}(\mathbf{q})$	Gravity forces
$\boldsymbol{\tau}$	Generalized forces
$\mathbf{F}_c$	External forces
$\mathbf{J}_c$	Contact Jacobian

Position vs. Torque Controlled Robot Arms

Setup of a Robot Arm

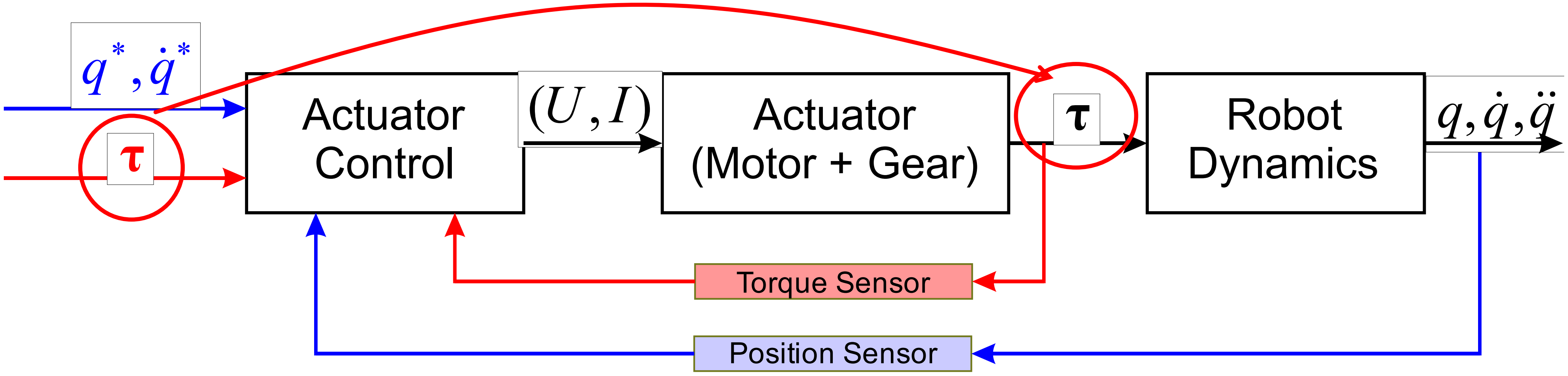


Classical Position Control of a Robot Arm



- Position feedback loop on joint level
  - Classical, position controlled robots don’t care about dynamics
  - High-gain PID guarantees good joint level tracking
  - Disturbances (load, etc) are compensated by PID
  - => interaction force can only be controlled with compliant surface

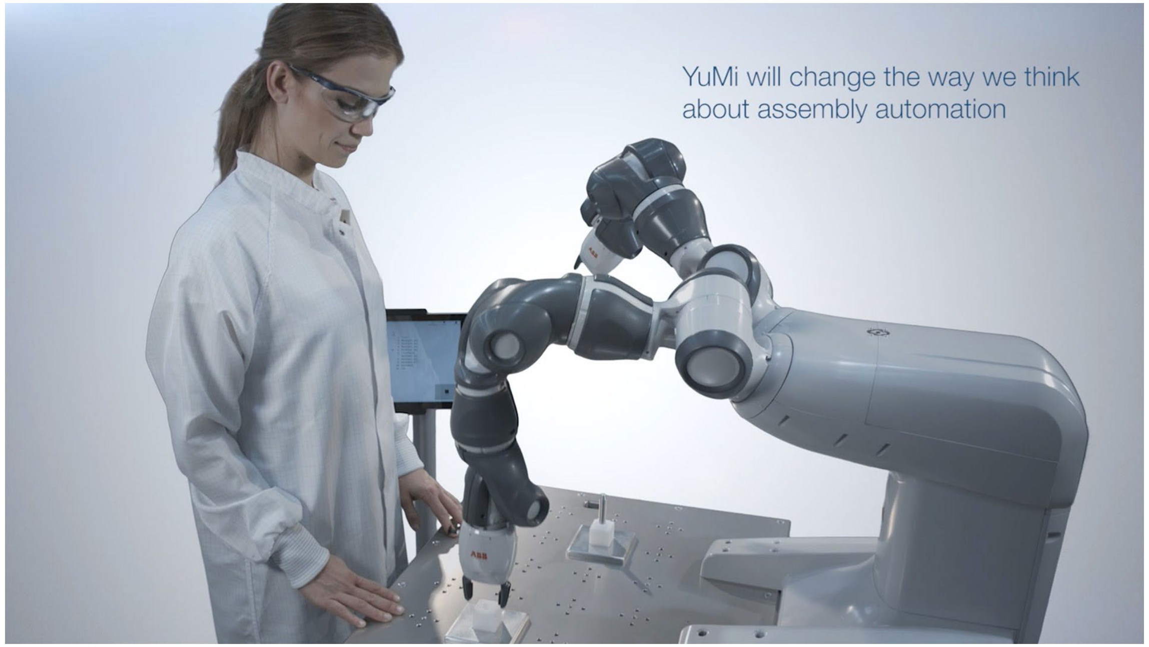
Joint Torque Control of a Robot Arm



- Integrate force-feedback
  - Active regulation of system dynamics
  - Model-based load compensation
  - Interaction force control

Setup of Modern Robot Arms

- Modern robots have force sensors
  - Dynamic control
  - Interaction control
  - Safety for collaboration



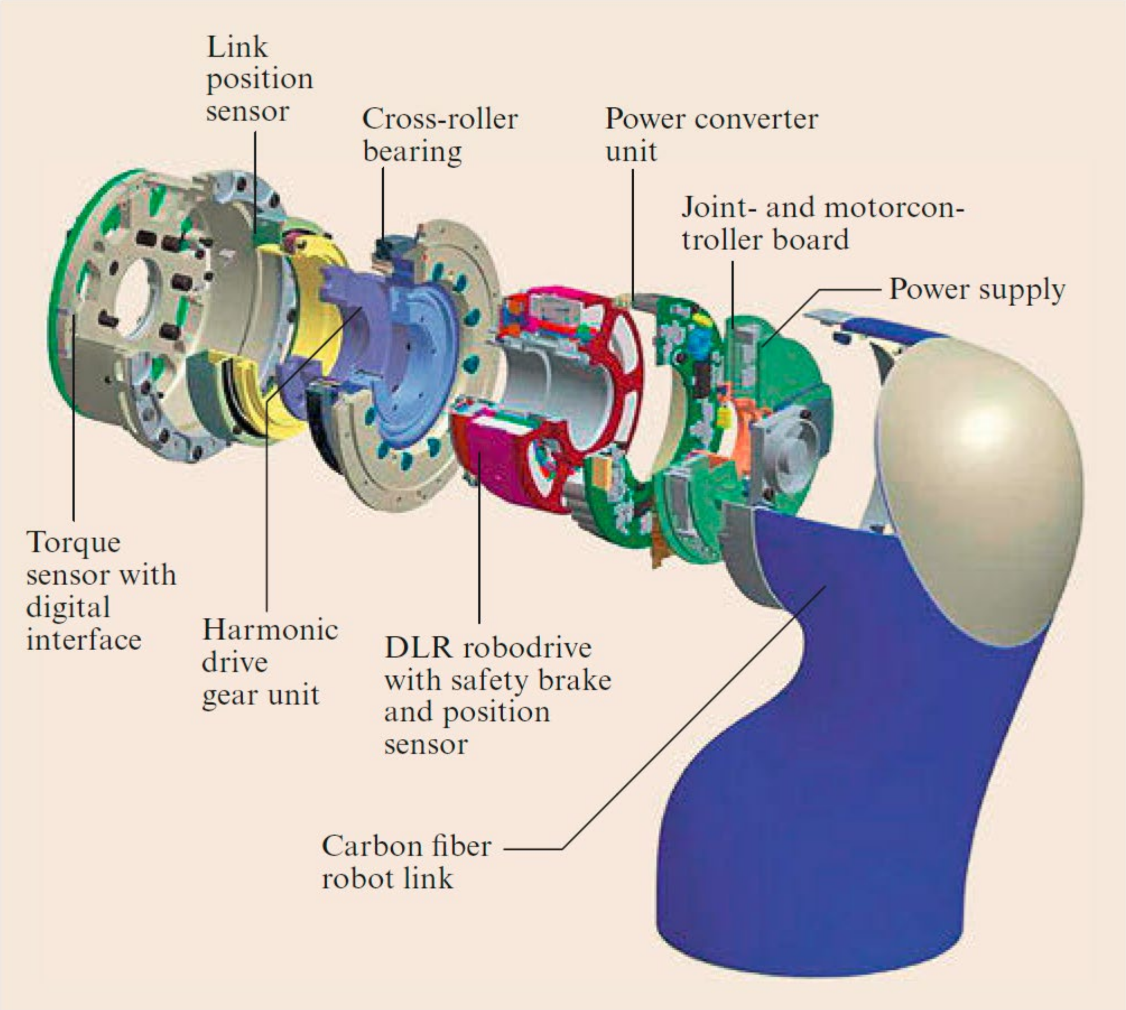

Link position sensor
Cross-roller bearing
Power converter unit
Joint- and motorcon- troller board
Power supply
Torque sensor with digital interface
Harmonic drive gear unit
DLR robodrive with safety brake and position sensor
Carbon fiber robot link

Fig. 11.8 Exploded view of a joint of the *DLR LWR-III* lightweight manipulator and its sensor suite

FRANKA – an example of a force controllable robot arm

CHAPTER I

—

THIS IS FRANKA

Joint Impedance Control

$$\boxed{\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}}$$

- Torque as function of position and velocity error $\boldsymbol{\tau}^* = \mathbf{k}_p(\mathbf{q}^* - \mathbf{q}) + \mathbf{k}_d(\dot{\mathbf{q}}^* - \dot{\mathbf{q}})$

- Closed loop behavior

$$\cancel{\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}} + \cancel{\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} = \mathbf{k}_p(\mathbf{q}^* - \mathbf{q}) + \mathbf{k}_d(\dot{\mathbf{q}}^* - \dot{\mathbf{q}})$$

  ➢ Static offset due to gravity

- Impedance control and gravity compensation

$$\boldsymbol{\tau}^* = \mathbf{k}_p(\mathbf{q}^* - \mathbf{q}) + \mathbf{k}_d(\dot{\mathbf{q}}^* - \dot{\mathbf{q}}) + \hat{\mathbf{g}}(\mathbf{q})$$

Estimated gravity term

Simple setup…
but configuration dependent load

Inverse Dynamics Control

$$\boxed{\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}}$$

- Compensate for system dynamics

$$\boldsymbol{\tau} = \hat{\mathbf{M}}(\mathbf{q})\,\ddot{\mathbf{q}}^* + \hat{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) + \hat{\mathbf{g}}(\mathbf{q})$$

- In case of no modeling errors,
  - the desired dynamics can be perfectly prescribed

$$\mathbb{I}\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^*$$

- PD-control law

$$\mathbb{I}\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^* = \mathbf{k}_p\left(\mathbf{q}^* - \mathbf{q}\right) + \mathbf{k}_d\left(\dot{\mathbf{q}}^* - \dot{\mathbf{q}}\right)$$

  - Every joint behaves like a decoupled mass-spring-damper with unitary mass

$$\omega = \sqrt{k_p} \qquad D = \frac{k_d}{2\sqrt{k_p}}$$

Can achieve great performance…
but requires accurate modeling

Inverse Dynamics Control with Multiple Tasks

$$\boxed{\boldsymbol{\tau} = \hat{\mathbf{M}}(\mathbf{q})\,\ddot{\mathbf{q}}^* + \hat{\mathbf{b}}(\mathbf{q},\dot{\mathbf{q}}) + \hat{\mathbf{g}}(\mathbf{q})}$$

Motion in joint space is often hard to describe => use task space

- A single task can be written as $\dot{\mathbf{w}}_e = \begin{pmatrix}\ddot{\mathbf{r}}\\ \dot{\boldsymbol{\omega}}\end{pmatrix}_e = \mathbf{J}_e\ddot{\mathbf{q}} + \dot{\mathbf{J}}_e\dot{\mathbf{q}}$
- In complex machines, we want to fulfill multiple tasks
- (As introduced already for velocity control)
  - Same priority, multi-task inversion $$\ddot{\mathbf{q}} = \begin{bmatrix}\mathbf{J}_1\\ \vdots\\ \mathbf{J}_{n_t}\end{bmatrix}^+ \left(\begin{pmatrix}\dot{\mathbf{w}}_1\\ \vdots\\ \dot{\mathbf{w}}_{n_t}\end{pmatrix} - \begin{bmatrix}\dot{\mathbf{J}}_1\\ \vdots\\ \dot{\mathbf{J}}_{n_t}\end{bmatrix}\dot{\mathbf{q}}\right)$$
  - Hierarchical $$\ddot{\mathbf{q}} = \sum_{i=1}^{n_T}\mathbf{N}_i\ddot{\mathbf{q}}_i, \quad \text{with} \quad \ddot{\mathbf{q}}_i = (\mathbf{J}_i\mathbf{N}_i)^+\left(\mathbf{w}_i^* - \dot{\mathbf{J}}_i\dot{\mathbf{q}} - \mathbf{J}\sum_{k=1}^{i-1}\mathbf{N}_k\dot{\mathbf{q}}_k\right)$$

Quiz 1

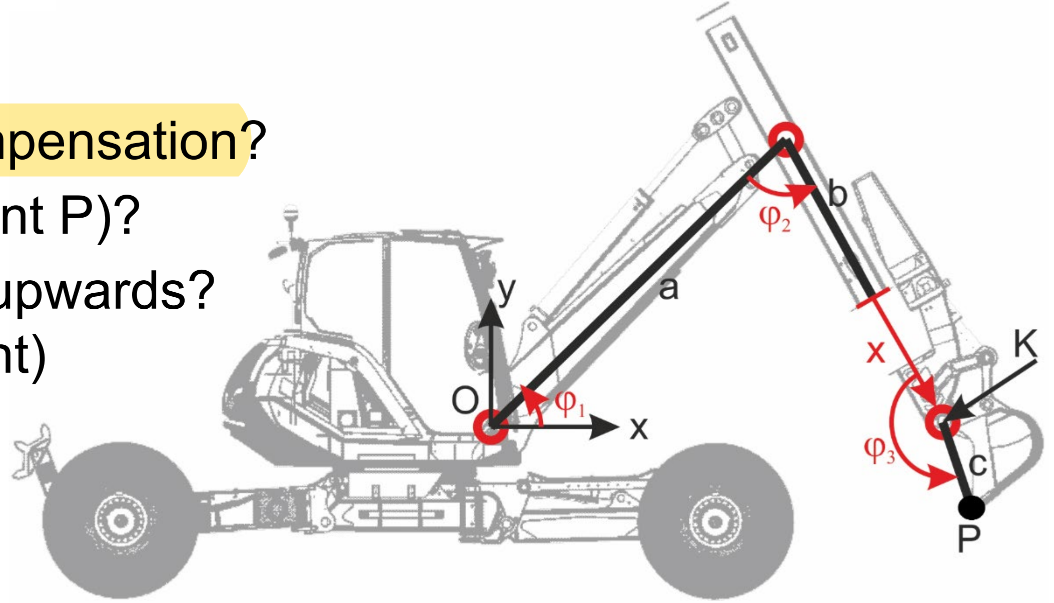
5

5

Quiz
👤 3min
👥 1min

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions
  a. What joint torque is required for static gravity compensation?
  b. + if you additionally lift 100kg in the bucket (at point P)?
  c. What torque is required to accelerate with 1m/s^2 upwards? (orientation and x-position should be kept constant)
  d. Is this solution unique?

What is the dimensionality?



Solution 1

- a) Static gravity compensation $\tau = \mathbf{M}(\cancel{\mathbf{q}})\ddot{\mathbf{q}}^* + \mathbf{b}(\cancel{\mathbf{q},\dot{\mathbf{q}}}) + \mathbf{g}$

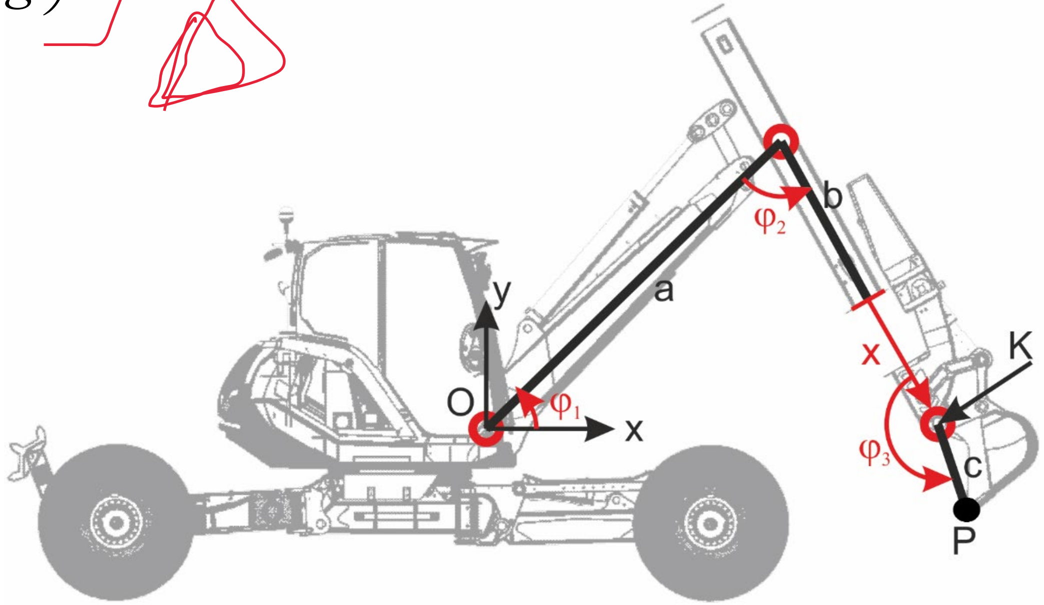
- b) lifting 100kg: $\tau = \mathbf{g} + \mathbf{J}_p^T \begin{pmatrix} 0 \\ 100g \end{pmatrix}$ $\Leftarrow$ $-\tau + \begin{pmatrix} 0 \\ -100g \end{pmatrix} = g$

- c) $\mathbf{J}_t = \begin{bmatrix} \mathbf{J}_P \\ \mathbf{J}_R \end{bmatrix}, \dot{\mathbf{w}}_t^* = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\ddot{\mathbf{q}}^* = \mathbf{J}_t^+ \left(\dot{\mathbf{w}}_t^* - \dot{\mathbf{J}}_t \dot{\mathbf{q}} \right)$$

$$\tau = \mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{b} + \mathbf{g}$$

- There exist multiple solutions for $\ddot{\mathbf{q}}^*$
- Given $\ddot{\mathbf{q}}^*$, the torque is unique



Task Space Dynamics

- Joint-space dynamics

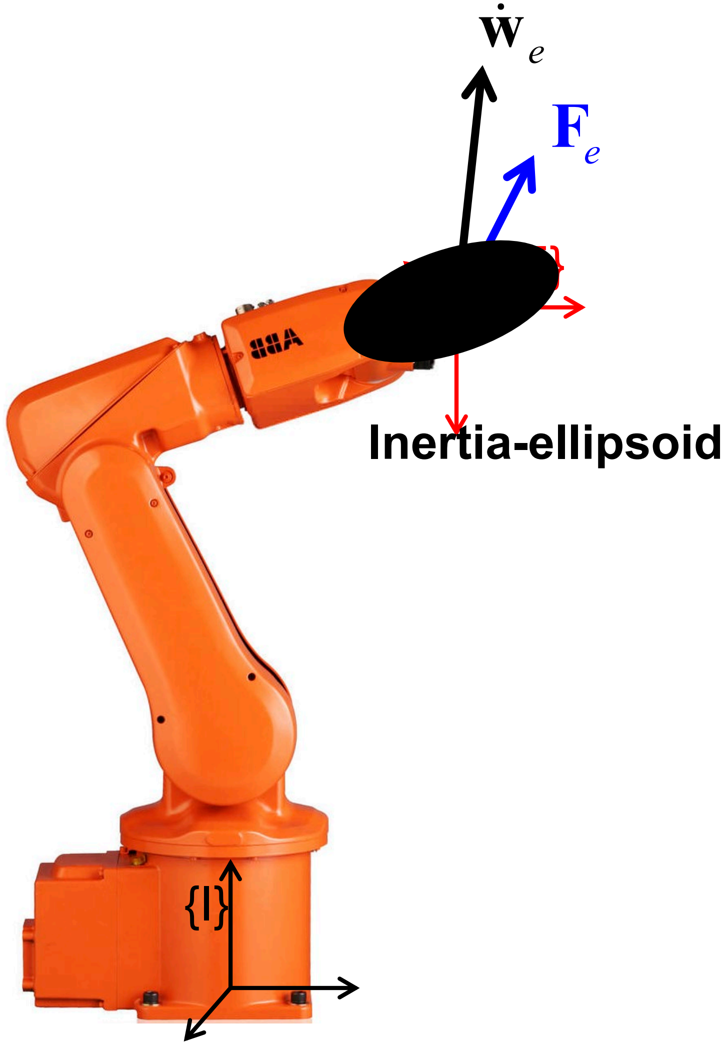
$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

End-effector dynamics

$$\boldsymbol{\Lambda}\dot{\mathbf{w}}_e + \boldsymbol{\mu} + \mathbf{p} = \mathbf{F}_e$$

  - Torque to force mapping $\boldsymbol{\tau} = \mathbf{J}_e^T \mathbf{F}_e$
  - Kinematic relation $\dot{\mathbf{w}}_e = \begin{pmatrix} \ddot{\mathbf{r}} \\ \dot{\boldsymbol{\omega}} \end{pmatrix}_e = \mathbf{J}_e\ddot{\mathbf{q}} + \dot{\mathbf{J}}_e\dot{\mathbf{q}}$
  - Substitute acceleration $\dot{\mathbf{w}}_e = \mathbf{J}_e\mathbf{M}^{-1}(\boldsymbol{\tau} - \mathbf{b} - \mathbf{g}) + \dot{\mathbf{J}}_e\dot{\mathbf{q}}$

$$\Rightarrow \boxed{\begin{aligned} \boldsymbol{\Lambda} &= \left(\mathbf{J}_e\mathbf{M}^{-1}\mathbf{J}_e^T\right)^{-1} \\ \boldsymbol{\mu} &= \boldsymbol{\Lambda}\mathbf{J}_e\mathbf{M}^{-1}\mathbf{b} - \boldsymbol{\Lambda}\dot{\mathbf{J}}_e\dot{\mathbf{q}} \\ \mathbf{p} &= \boldsymbol{\Lambda}\mathbf{J}_e\mathbf{M}^{-1}\mathbf{g} \end{aligned}}$$



End-effector Motion Control

- Determine a desired end-effector acceleration

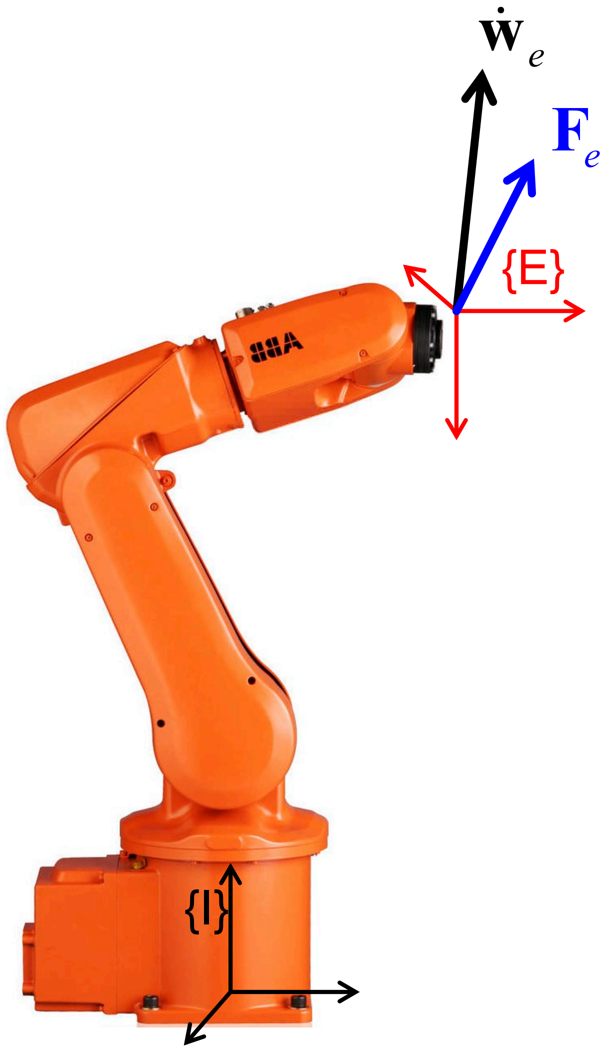
$$\dot{\mathbf{w}}_e^* = \mathbf{k}_p \mathbf{E}\left(\chi_e^* - \chi_e\right) + \mathbf{k}_d\left(\mathbf{w}_e^* - \mathbf{w}_e\right) + \dot{\mathbf{w}}_e(t)$$

Trajectory control

Note: a rotational error can be related to differenced in representation by $\Delta\phi = E_R(\chi_R)\Delta\chi_R$

- Determine the corresponding joint torque

$$\boldsymbol{\tau}^* = \hat{\mathbf{J}}^T\left(\hat{\boldsymbol{\Lambda}}_e \dot{\mathbf{w}}_e^* + \hat{\boldsymbol{\mu}} + \hat{\mathbf{p}}\right)$$



Quiz 2

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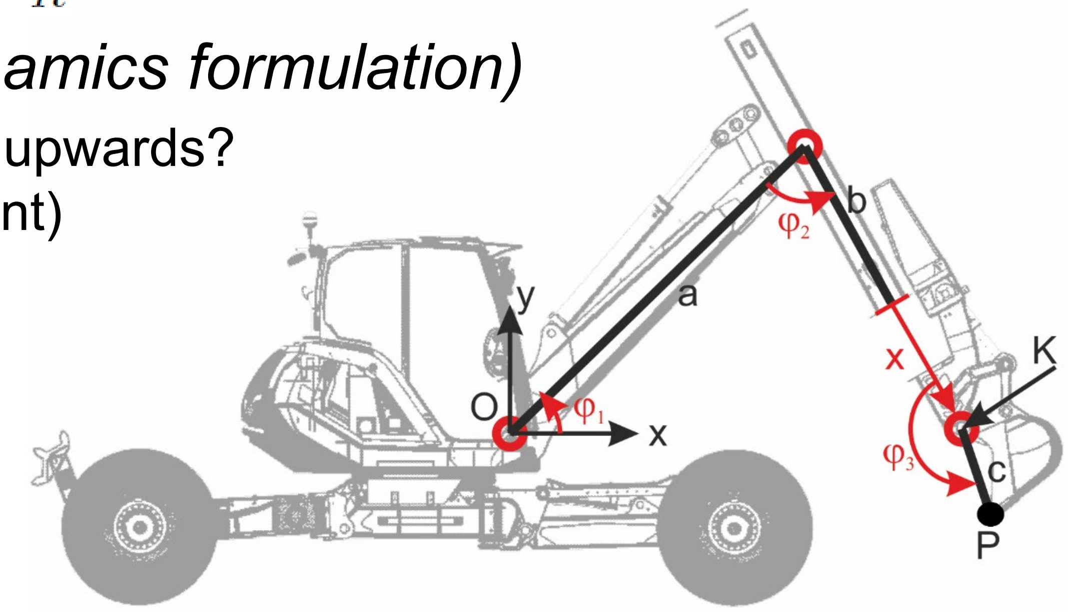
Quiz
2min
1min

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions *(solve this using tasks space dynamics formulation)*
  a. What torque is required to accelerate with 1m/s^2 upwards? (orientation and x position should be kept constant)
  b. Is this solution the same as the last one?



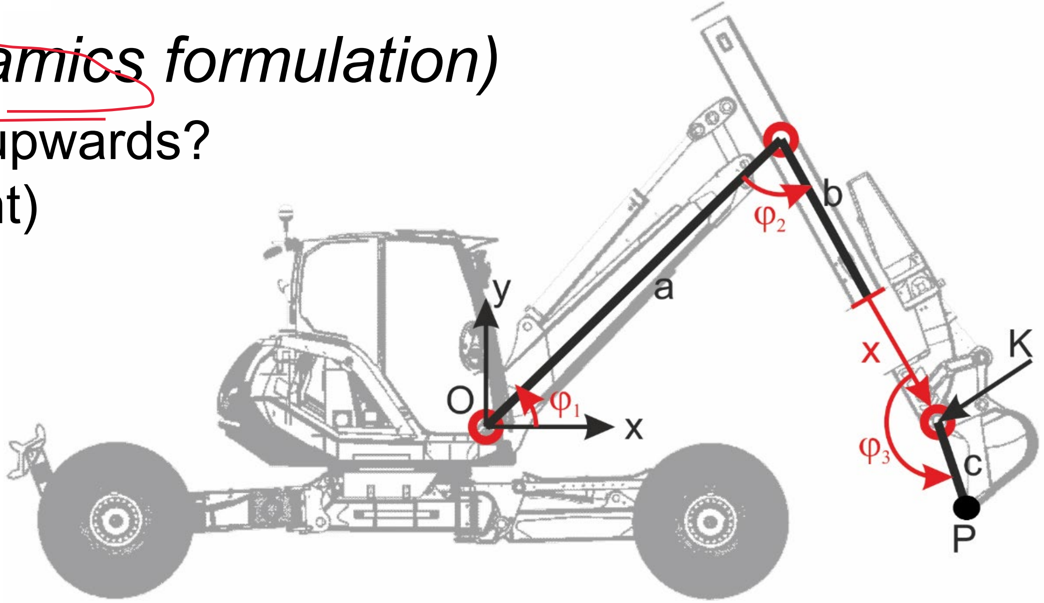
Solution 2

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions *(solve this using tasks space dynamics formulation)*
  a. What torque is required to accelerate with 1m/s^2 upwards? (orientation and x position should be kept constant)
  b. Is this solution the same as the last one?

$$\dot{\mathbf{w}}_t^* = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{J}_e = \mathbf{J}_t = \begin{bmatrix} \mathbf{J}_P \\ \mathbf{J}_R \end{bmatrix}$$

$$\boldsymbol{\tau}^* = \mathbf{J}_t^T \left(\boldsymbol{\Lambda} \dot{\mathbf{w}}_t^* + \boldsymbol{\mu} + \mathbf{p} \right)$$

$$\boxed{\begin{aligned} \boldsymbol{\Lambda} &= \left(\mathbf{J}_e \mathbf{M}^{-1} \mathbf{J}_e^T\right)^{-1} \\ \boldsymbol{\mu} &= \boldsymbol{\Lambda} \mathbf{J}_e \mathbf{M}^{-1} \mathbf{b} - \boldsymbol{\Lambda} \dot{\mathbf{J}}_e \dot{\mathbf{q}} \\ \mathbf{p} &= \boldsymbol{\Lambda} \mathbf{J}_e \mathbf{M}^{-1} \mathbf{g} \end{aligned}}$$



Robots in Interaction

There is a long history in robots controlling motion and interaction

Operational Space Control

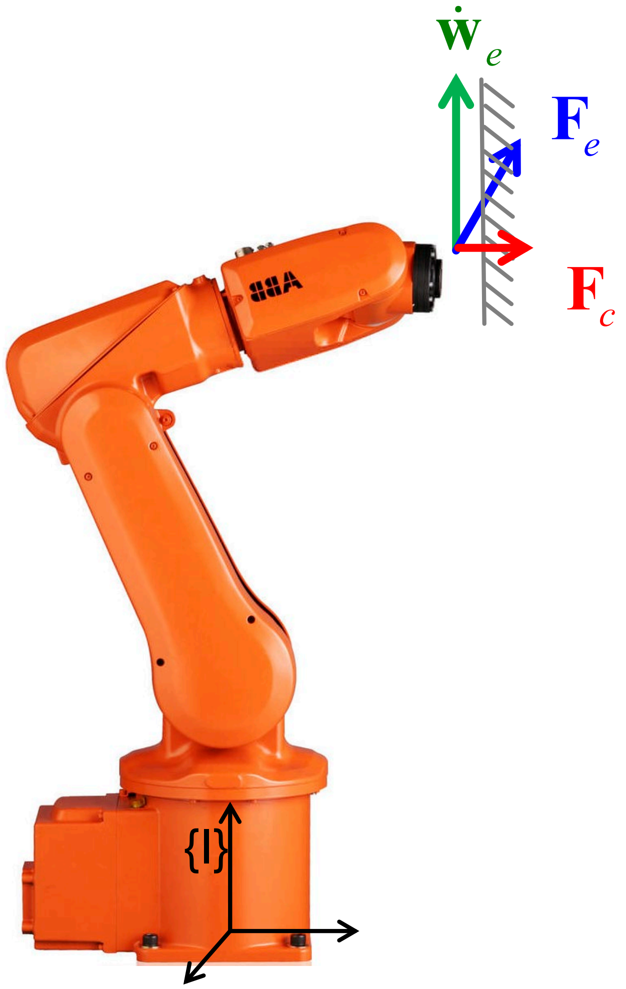
Generalized framework to control motion and force

- Extend end-effector dynamics in contact with contact force

$$\underline{\mathbf{F}_c} + \boldsymbol{\Lambda}\boxed{\dot{\mathbf{w}}_e} + \boldsymbol{\mu} + \mathbf{p} = \boxed{\mathbf{F}_e}$$

- Introduce selection matrices to separate motion force directions

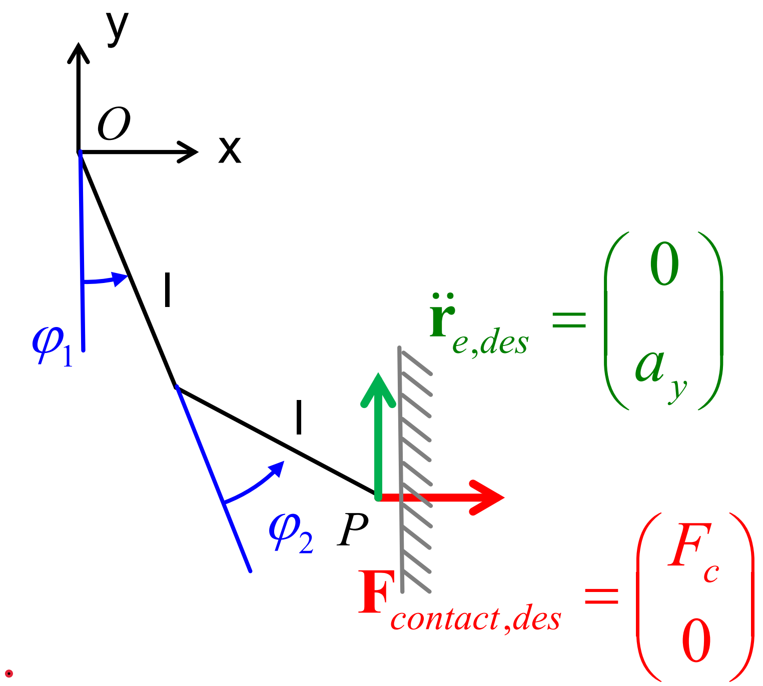
$$\boldsymbol{\tau}^* = \hat{\mathbf{J}}^T \left(\hat{\boldsymbol{\Lambda}}\underline{\mathbf{S}_M}\dot{\mathbf{w}}_e + \underline{\mathbf{S}_F}\mathbf{F}_c + \hat{\tilde{\boldsymbol{\mu}}} + \hat{\mathbf{p}}\right)$$



Operational Space Control

2-link example

- Given: $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$
- Find $\boldsymbol{\tau}$, s.t. the end-effector
  - accelerates with $\ddot{\mathbf{r}}_{e,des} = \begin{pmatrix} 0 & a_y \end{pmatrix}^T$
  - exerts the contact force $\mathbf{F}_{contact,des} = \begin{pmatrix} F_c & 0 \end{pmatrix}^T$



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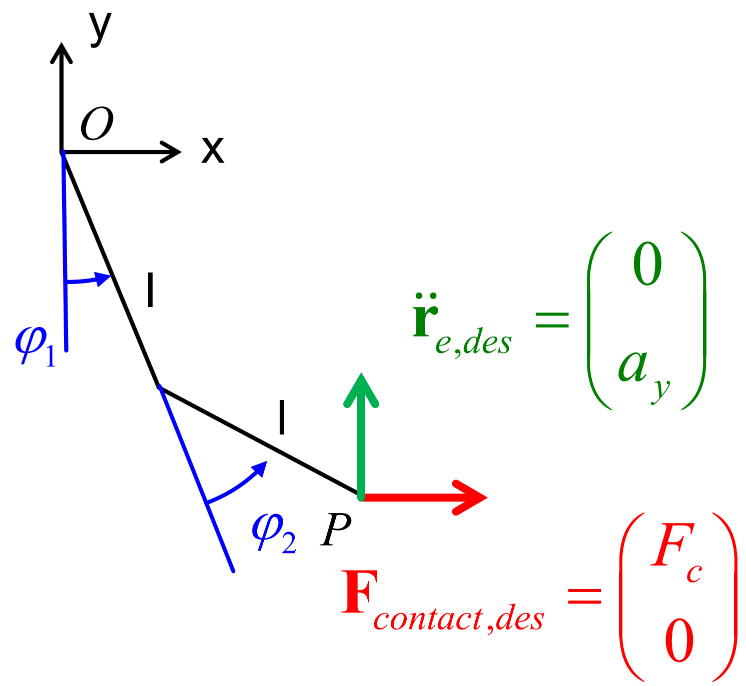
P
$\ddot{\mathbf{r}}_{e,des} = \begin{pmatrix} 0 \\ a_y \end{pmatrix}$
$\mathbf{F}_{contact,des} = \begin{pmatrix} F_c \\ 0 \end{pmatrix}$

Operational Space Control

2-link example

- Given: $\mathrm{M}(\mathrm{q})\,\ddot{\mathrm{q}} + \mathrm{b}(\mathrm{q},\dot{\mathrm{q}}) + \mathrm{g}(\mathrm{q}) = \boldsymbol{\tau}$
- Find $\boldsymbol{\tau}$, s.t. the end-effector
  - accelerates with $\ddot{\mathbf{r}}_{e,des} = \begin{pmatrix} 0 & a_y \end{pmatrix}^T$
  - exerts the contact force $\mathbf{F}_{contact,des} = \begin{pmatrix} F_c & 0 \end{pmatrix}^T$



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P
$\ddot{\mathbf{r}}_{e,des} = \begin{pmatrix} 0 \\ a_y \end{pmatrix}$
$\mathbf{F}_{contact,des} = \begin{pmatrix} F_c \\ 0 \end{pmatrix}$

- End-effector position and Jacobian

$$\mathbf{r}_E = \begin{pmatrix} ls_1 + ls_{12} \\ -lc_1 - lc_{12} \end{pmatrix} \qquad \mathbf{J}_e = \begin{bmatrix} lc_1 + lc_{12} & lc_{12} \\ ls_1 + ls_{12} & ls_{12} \end{bmatrix}$$

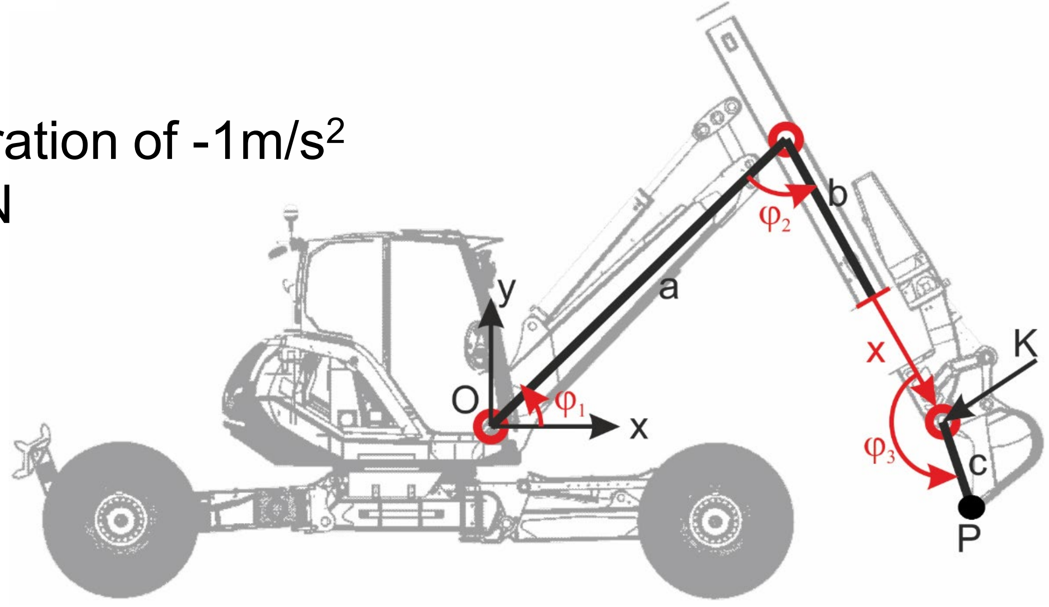
$$\boxed{\begin{aligned} \boldsymbol{\Lambda} &= \left(\mathbf{J}_e \mathbf{M}^{-1} \mathbf{J}_e^T\right)^{-1} \\ \boldsymbol{\mu} &= \boldsymbol{\Lambda} \mathbf{J}_e \mathbf{M}^{-1} \mathbf{b} - \boldsymbol{\Lambda} \dot{\mathbf{J}} \dot{\mathbf{q}} \\ \mathbf{p} &= \boldsymbol{\Lambda} \mathbf{J}_e \mathbf{M}^{-1} \mathbf{g} \end{aligned}}$$

  - Desired end-effector dynamics

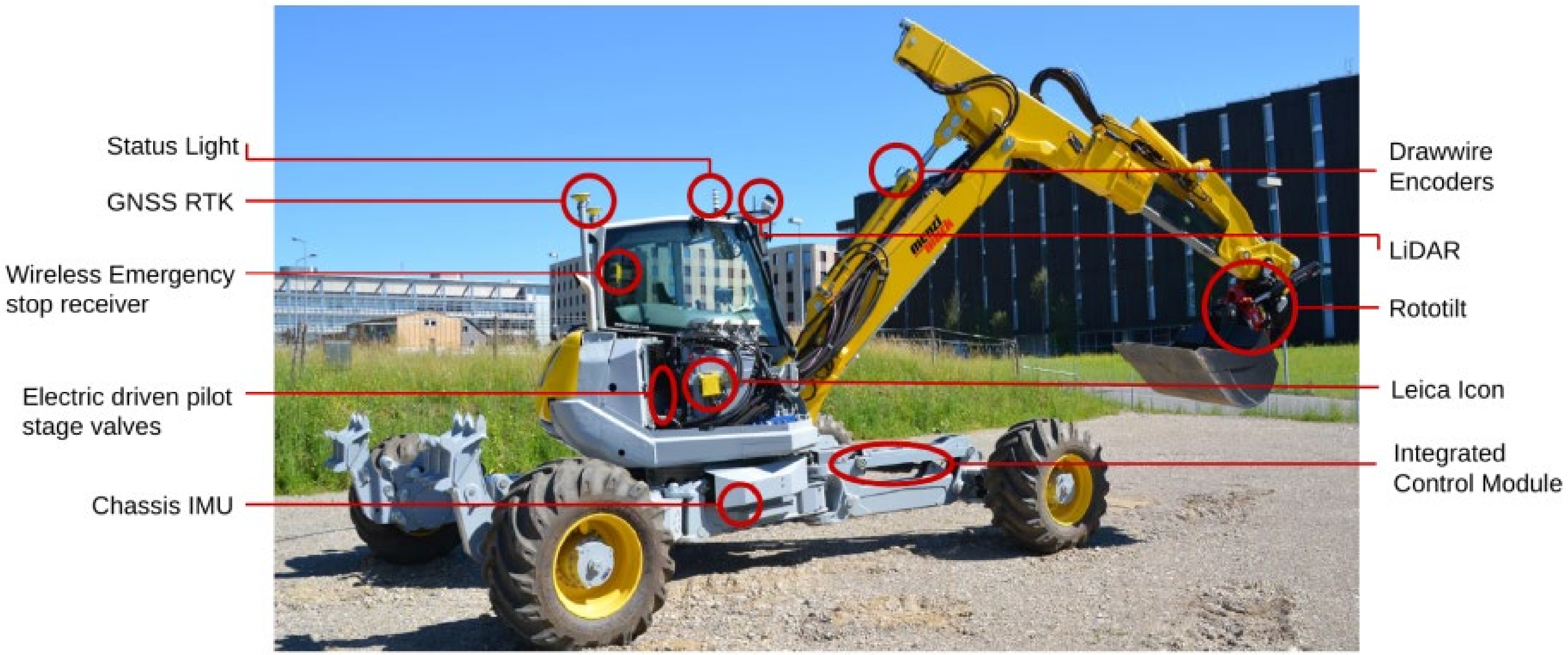
$$\boldsymbol{\tau}^* = \hat{\mathbf{J}}_e^T \left(\hat{\boldsymbol{\Lambda}} \ddot{\mathbf{r}}_{e,des} + \hat{\boldsymbol{\mu}} + \hat{\mathbf{p}} + \mathbf{F}_{contace,des} \right)$$

Quiz 3

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions
  a. You are filling the bucket with a horizontal acceleration of -1m/s^2 and a drag force of 1000N while producing 1000N normal contact force. What is the joint torque?



Quiz 3 (background)



Status Light

GNSS RTK

Wireless Emergency stop receiver

Electric driven pilot stage valves

Chassis IMU

Drawwire Encoders

LiDAR

Rototilt

Leica Icon

Integrated Control Module

Quiz 3 (background)

Quiz 3 (background)

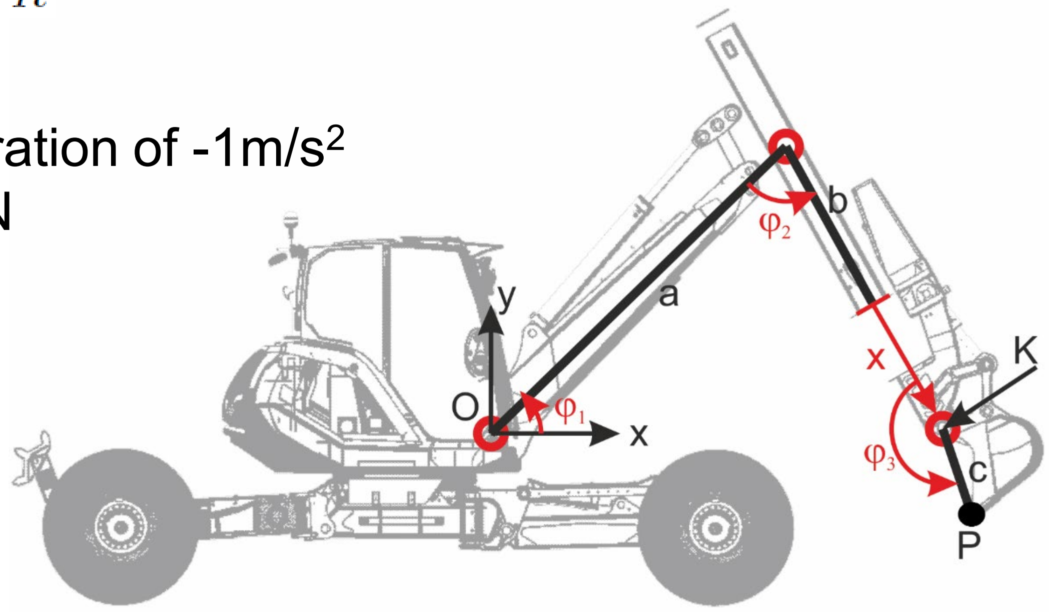
5

5

Quiz
3min
1min

Quiz 3

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions
  1. You are filling the bucket with a horizontal acceleration of -1m/s^2 and a drag force of 1000N while producing 1000N normal contact force. What is the joint torque?

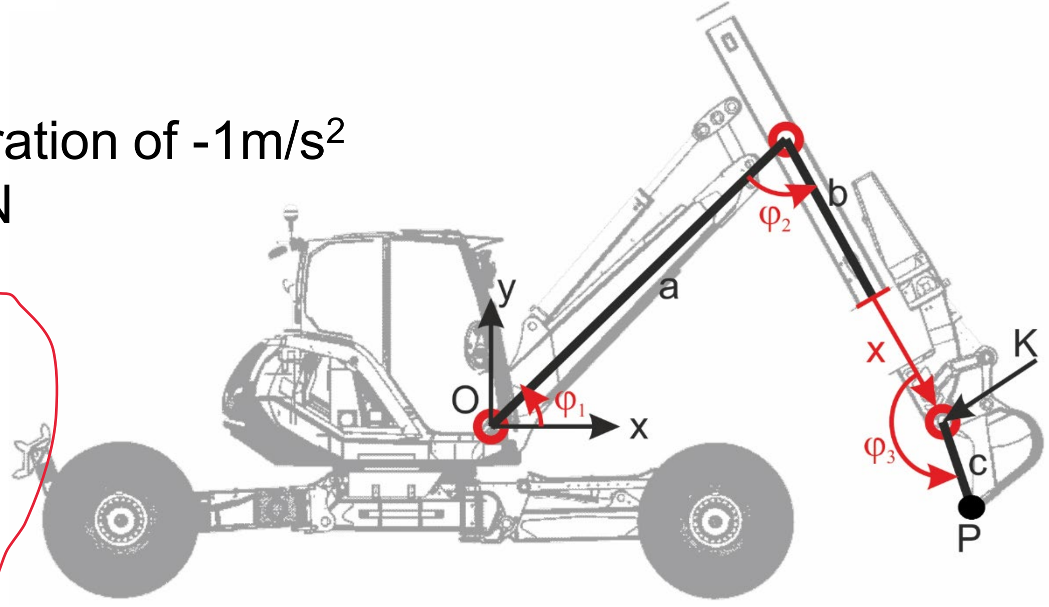


Solution 3

- Given:
  - Generalized coordinates $\mathbf{q} = (\varphi_1 \quad \varphi_2 \quad x \quad \varphi_3)^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions

a. You are filling the bucket with a horizontal acceleration of -1m/s^2 and a drag force of 1000N while producing 1000N normal contact force. What is the joint torque?

$$\dot{\mathbf{w}}_t^* = \begin{pmatrix}-1\\0\\0\end{pmatrix} \qquad \mathbf{J}_t = \begin{bmatrix}\mathbf{J}_P\\ \mathbf{J}_R\end{bmatrix} \qquad \boldsymbol{\tau}^* = \mathbf{J}_t^T\left(\boldsymbol{\Lambda}\dot{\mathbf{w}}_t^* + \boldsymbol{\mu} + \mathbf{p} + \begin{pmatrix}-1000\\-1000\\0\end{pmatrix}\right)$$



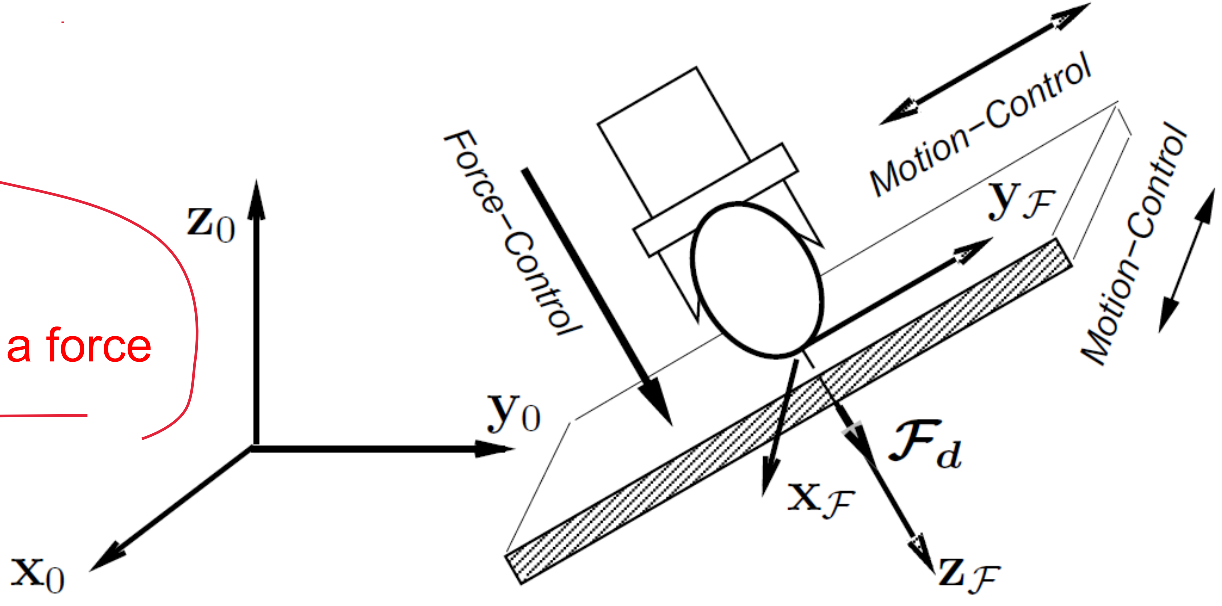
内发力。无论是joint space 还是 task space， 只要contact force在等式左边，就是机器的自发力，即外界对物体作用力的反向力。

How to Find a Selection Matrix

- Selection matrix in local frame

1: it can move
0: it can apply a force

$$\boldsymbol{\Sigma}_p = \begin{bmatrix} \sigma_{px} & 0 & 0 \\ 0 & \sigma_{py} & 0 \\ 0 & 0 & \boxed{\sigma_{pz}} \end{bmatrix}$$



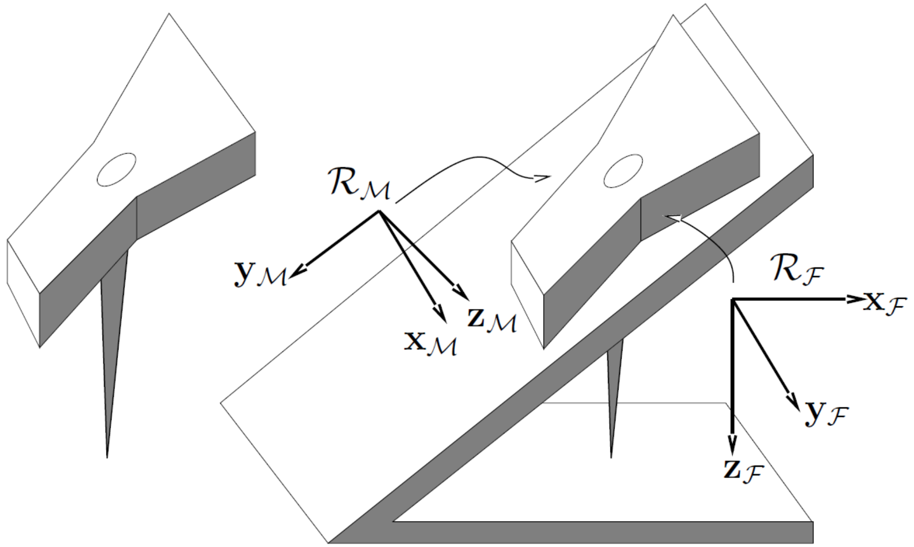
- Rotation between contact force and world frame

$$\mathbf{S}_M = \mathbf{C}^T \boldsymbol{\Sigma}_p \mathbf{C} \qquad \mathbf{S}_F = \mathbf{C}^T \left(\mathbb{I}_3 - \boldsymbol{\Sigma}_p \right) \mathbf{C}$$

How to Find a Selection Matrix

- Selection matrix in local frame

$$\mathbf{\Sigma}_p = \begin{bmatrix} \sigma_{px} & 0 & 0 \\ 0 & \sigma_{py} & 0 \\ 0 & 0 & \sigma_{pz} \end{bmatrix} \quad \mathbf{\Sigma}_r = \begin{bmatrix} \sigma_{rx} & 0 & 0 \\ 0 & \sigma_{ry} & 0 \\ 0 & 0 & \sigma_{rz} \end{bmatrix}$$



- Rotation between contact force and world frame

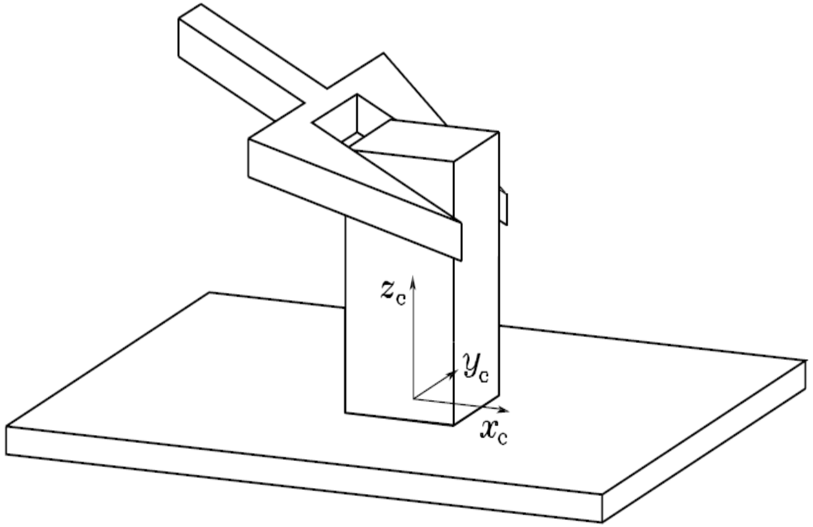
$$\mathbf{S}_M = \begin{bmatrix} \mathbf{C}^T \mathbf{\Sigma}_p \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^T \mathbf{\Sigma}_r \mathbf{C} \end{bmatrix} \qquad \mathbf{S}_F = \begin{bmatrix} \mathbf{C}^T \left(\mathbb{I}_3 - \mathbf{\Sigma}_p\right) \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^T \left(\mathbb{I}_3 - \mathbf{\Sigma}_r\right) \mathbf{C} \end{bmatrix}$$

Sliding a Prismatic Object Along a Surface

- Assume friction less contact surface

$$\boldsymbol{\Sigma}_{Mp} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{\Sigma}_{Mr} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{Fp} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{\Sigma}_{Fr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

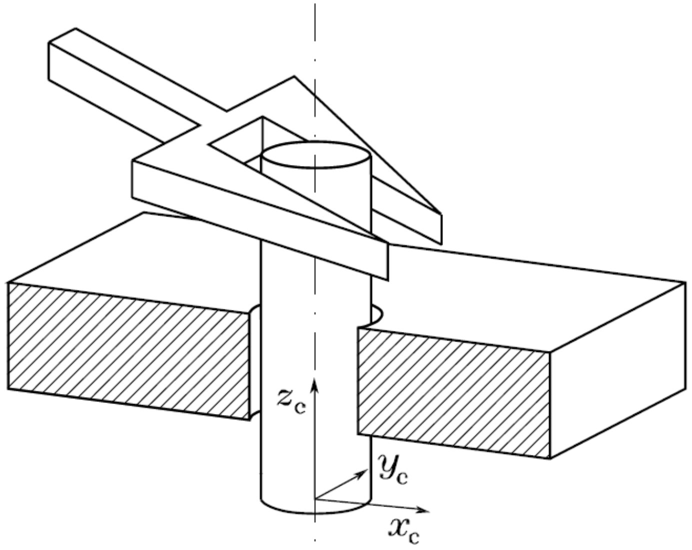


Inserting a Cylindrical Peg in a Hole

- Find the selection matrix (in local frame)

$$\mathbf{\Sigma}_{Mp} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{\Sigma}_{Mr} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{\Sigma}_{Fp} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{\Sigma}_{Fr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Inverse Dynamics, OSC, QP optimization
3 variants

- Different «inverse dynamics»-based methods

1. Classic ID: $\boldsymbol{\tau}^* = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}^* + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}$ with $\ddot{\mathbf{q}}^* = \mathbf{J}^+(\dot{\mathbf{w}} - \dot{\mathbf{J}}\dot{\mathbf{q}})$ => $\min\|\ddot{\mathbf{q}}\|$

2. OSC $\boldsymbol{\tau}^* = \mathbf{J}_t^T(\boldsymbol{\Lambda}\dot{\mathbf{w}}_t^* + \boldsymbol{\mu} + \mathbf{p}) + N(\mathbf{J}_t^T)\boldsymbol{\tau}_0$

$$N(\mathbf{J}_t^T) = (\mathbf{I} - \mathbf{J}^T\mathbf{J}^{T\#}) = (\mathbf{I} - \mathbf{J}^T(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1}\mathbf{J}\mathbf{M}^{-1})$$

If the torque is applied in this null-space, there is no acceleration at the end-effector

$$\boldsymbol{\tau}^* = \mathbf{J}_t^T\left(\left(\mathbf{J}_t\mathbf{M}^{-1}\mathbf{J}_t^T\right)^{-1}\dot{\mathbf{w}} + \boldsymbol{\mu} + \mathbf{p}\right)$$

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\boldsymbol{\tau}^* - \mathbf{b} - \mathbf{g}) = \boxed{\mathbf{M}^{-1}(\mathbf{J}_t^T((\mathbf{J}_t\mathbf{M}^{-1}\mathbf{J}_t^T)^{-1}}\dot{\mathbf{w}} + \boldsymbol{\mu} + \mathbf{p}) - \mathbf{b} - \mathbf{g})$$

“some sort of mass-matrix weighted pseudo-inverse”

3. Quadratic optimization

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g}$$

$$\mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} = \dot{\mathbf{w}}$$

$\min\|\ddot{\mathbf{q}}\|$ or $\min\|\boldsymbol{\tau}\|$

$$\left.\begin{array}{l} \begin{bmatrix}\mathbf{M} & -\mathbf{I}\end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}} \\ \boldsymbol{\tau}\end{pmatrix} + \mathbf{b} + \mathbf{g} = \mathbf{0} \\ \begin{bmatrix}\mathbf{J}_e & \mathbf{0}\end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}} \\ \boldsymbol{\tau}\end{pmatrix} + \dot{\mathbf{J}}\dot{\mathbf{q}} = \dot{\mathbf{w}}_e^* \end{array}\right\}$$

Single task $\min\limits_{\ddot{\mathbf{q}},\boldsymbol{\tau}}\left\|\begin{bmatrix}\mathbf{M} & -\mathbf{I} \\ \mathbf{J}_e & \mathbf{0}\end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}} \\ \boldsymbol{\tau}\end{pmatrix} - \begin{pmatrix}-\mathbf{b}-\mathbf{g} \\ \dot{\mathbf{w}}_e^* - \dot{\mathbf{J}}\dot{\mathbf{q}}\end{pmatrix}\right\|_2$

Priority $\left\{\begin{array}{l} \min\limits_{\ddot{\mathbf{q}},\boldsymbol{\tau}}\left\|\begin{bmatrix}\mathbf{J}_e & \mathbf{0}\end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}} \\ \boldsymbol{\tau}\end{pmatrix} - (\dot{\mathbf{w}}_e^* - \dot{\mathbf{J}}\dot{\mathbf{q}})\right\|_2 \\ s.t.\begin{bmatrix}\mathbf{M} & -\mathbf{I}\end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}} \\ \boldsymbol{\tau}\end{pmatrix} - (-\mathbf{b} - \mathbf{g}) = \mathbf{0} \end{array}\right.$

Least Square Optimization

some notes on quadratic optimization

$$\mathbf{A}\mathbf{x}-\mathbf{b}=\mathbf{0} \longrightarrow \mathbf{x}=\mathbf{A}^{+}\mathbf{b} \qquad \min_{\mathbf{x}}\left\|\mathbf{A}\mathbf{x}-\mathbf{b}\right\|_2 \qquad \min\left\|\mathbf{x}\right\|_2$$

$$\mathbf{A}_1\mathbf{x}_1-\mathbf{b}=\mathbf{A}_2\mathbf{x}_2 \longrightarrow \begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix}=\begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\end{bmatrix}^{+}\mathbf{b} \qquad \min_{\mathbf{x}_1,\mathbf{x}_2}\left\|\begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\end{bmatrix}\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix}-\mathbf{b}\right\|_2 \qquad \min\left\|\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix}\right\|_2$$

$$\begin{aligned}\mathbf{A}_1\mathbf{x}-\mathbf{b}_1&=\mathbf{0}\\ \mathbf{A}_2\mathbf{x}-\mathbf{b}_2&=\mathbf{0}\end{aligned} \xrightarrow{\text{Equal priority}} \mathbf{x}=\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix}^{+}\begin{pmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{pmatrix} \qquad \min_{\mathbf{x}}\left\|\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix}\mathbf{x}-\begin{pmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{pmatrix}\right\|_2 \qquad \min\left\|\mathbf{x}\right\|_2$$

$$\xrightarrow{\text{Hierarchy}}$$

$$\mathbf{x}=\mathbf{A}_1^{+}\mathbf{b}_1+\mathcal{N}\left(\mathbf{A}_1\right)\mathbf{x}_0 \qquad \min_{\mathbf{x}}\left\|\mathbf{A}_1\mathbf{x}-\mathbf{b}_1\right\|_2$$

$$\left.\begin{aligned}\mathbf{A}_2\mathbf{x}-\mathbf{b}_2=\mathbf{A}_2\left(\mathbf{A}_1^{+}\mathbf{b}_1+\mathcal{N}\left(\mathbf{A}_1\right)\mathbf{x}_0\right)-\mathbf{b}_2=\mathbf{0}\\ \mathbf{x}_0=\left(\mathbf{A}_2\mathcal{N}\left(\mathbf{A}_1\right)\right)^{+}\left(\mathbf{b}_2-\mathbf{A}_2\mathbf{A}_1^{+}\mathbf{b}_1\right)\end{aligned}\right\} \qquad \left\{\begin{aligned}&\min_{\mathbf{x}}\left\|\mathbf{A}_2\mathbf{x}-\mathbf{b}_2\right\|_2\\ &s.t.\left\|\mathbf{A}_1\mathbf{x}-\mathbf{b}_1\right\|=c_1\end{aligned}\right.$$

Solving a Set of QPs

$$\min_{\mathbf{x}} \quad \|\mathbf{A}_i\mathbf{x}-\mathbf{b}_i\|_2$$

$$s.t. \quad \underbrace{\begin{bmatrix}\mathbf{A}_1\\ \vdots \\ \mathbf{A}_{i-1}\end{bmatrix}}_{\hat{\mathbf{A}}_{i-1}} \mathbf{x} - \underbrace{\begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_{i-1}\end{pmatrix}}_{\hat{\mathbf{b}}_{i-1}} = \mathbf{c}$$

- QPs need different priority!!
- Exploit Null-space of tasks with higher priority
- Every step = quadratic problem with constraints
- Use iterative null-space projection *(formula in script)*

```
$n_T$ = Number of Tas
$\mathbf{x} = \mathbf{0}$                                   l optimal solution
$\mathbf{N}_1 = \mathbb{I}$                                  ll-space projector
for $i = 1 \rightarrow n_T$ do
    $\mathbf{x}_i = (\mathbf{A}_i\mathbf{N}_i)^+ (\mathbf{b}_i$
    $\mathbf{x} = \mathbf{x} + \mathbf{N}_i\mathbf{x}_i$
    $\mathbf{N}_{i+1} = \mathcal{N}\left([\mathbf{A}_1^T$          $_i]\right)$
end for
```

Use a **numeric solver**

- e.g. quadprog, OOCP, …
- quadratic optimization
  - equality constraints
  - inequality constraints

5

5

Quiz
👤 3min
👥 1min

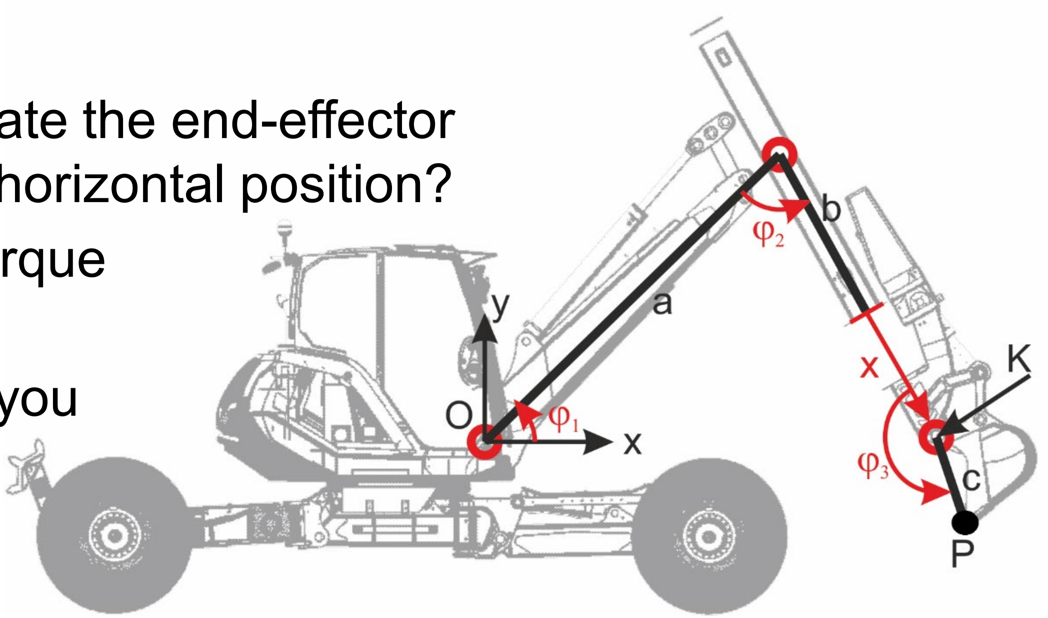
Quiz 4

- Given:
  - Generalized coordinates $\mathbf{q} = \begin{pmatrix} \varphi_1 & \varphi_2 & x & \varphi_3 \end{pmatrix}^T$
  - Equations of motion $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau}$
  - End-effector position and rotation Jacobian $\mathbf{J}_P \quad \mathbf{J}_R$
- Questions *(write down the tasks)*

a. What is the least square minimal torque to accelerate the end-effector with 1m/s^2 upwards while keeping orientation and horizontal position?

b. Assume the first cylinder is blocked. What is the torque for the other joints to achieve the same motion?

c. Assume the first cylinder has a hose rupture. Can you achieve the vertical motion? How/why not?



Quiz 4

a. What is the least square minimal torque to accelerate the end-effector with 1m/s^2 upwards while keeping orientation and horizontal position?

$$[\mathbf{M}\quad -\mathbf{I}]\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(-\mathbf{b}-\mathbf{g}\right)=\mathbf{0}$$

$$\begin{bmatrix}\mathbf{J}_P & \mathbf{0}\\ \mathbf{J}_R & \end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{bmatrix}\dot{\mathbf{J}}_P\\ \dot{\mathbf{J}}_R\end{bmatrix}\dot{\mathbf{q}}\right)=\mathbf{0}$$

$[\mathbf{0}\quad \mathbf{I}]\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}=\mathbf{0}$ as the last task => $\min\|\boldsymbol{\tau}\|$

b. Assume the first cylinder is blocked. What is the torque for the other joints to achieve the same motion?

$\ddot{\varrho}_1 = 0$

$$[\mathbf{M}\quad -\mathbf{I}]\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(-\mathbf{b}-\mathbf{g}\right)=\mathbf{0}$$

$$\begin{bmatrix}\mathbf{J}_P & \\ \mathbf{J}_R & \mathbf{0}_{4x4}\\ [1\quad 0\quad 0\quad 0] & \end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(\begin{pmatrix}0\\1\\0\\0\end{pmatrix}-\begin{bmatrix}\dot{\mathbf{J}}_P\\ \dot{\mathbf{J}}_R\\ [0\quad 0\quad 0\quad 0]\end{bmatrix}\dot{\mathbf{q}}\right)=\mathbf{0}$$

c. Assume the first cylinder has a hose rupture. Can you achieve the vertical motion? How/why not?

$\tau_1 = 0$

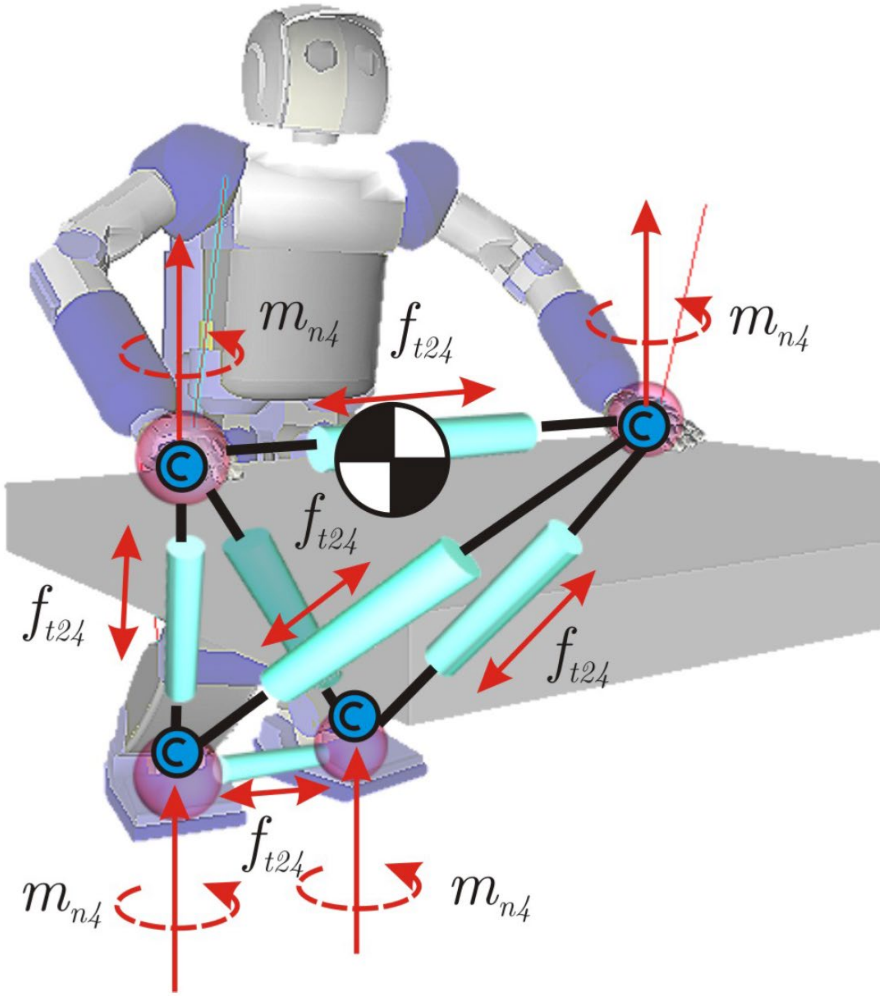
$$[\mathbf{M}\quad -\mathbf{I}]\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(-\mathbf{b}-\mathbf{g}\right)=\mathbf{0}$$

$$\left[\mathbf{0}\quad [1\quad 0\quad 0\quad 0]\right]\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}=\mathbf{0}$$

$$\begin{bmatrix}\mathbf{J}_P & \mathbf{0}\\ \mathbf{J}_R & \end{bmatrix}\begin{pmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\tau}\end{pmatrix}-\left(\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{bmatrix}\dot{\mathbf{J}}_P\\ \dot{\mathbf{J}}_R\end{bmatrix}\dot{\mathbf{q}}\right)=\mathbf{0}$$

Next Week: Application to Floating base Robots

Some Examples of Using Internal Forces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