Robot Dynamics Quiz 2

Prof. Marco Hutter
Teaching Assistants: Jan Carius, Dario Bellicoso

November 7, 2018

First Name(s): ____________________

Surname: ____________________

Student ID: ____________________

ETHZ-Email: ____________________`@student.ethz.ch`

Duration: 1h 15min
Permitted Aids: Everything; no communication among students during the test

1 Instructions

1. Download the ZIP file for quiz 2 from Piazza. Extract all contents of this file into a new folder and set MATLAB's[1] current path to this folder.
2. Run `init_workspace` in the MATLAB command line
3. **All answers must be written on this document. Please write numerical values with four decimal digits.** You should use the provided `student_script.m` to implement your calculations. You may create additional functions if needed.
4. When the time is up, email your solution file(s) to
   `robotdynamics@leggedrobotics.com`
   from your ETH email address with the subject line
   `[RobotDynamics] ETHStudentID - StudentName`
5. Hand in this sheet with your solutions at the end.

[1] Online version of MATLAB at `https://matlab.mathworks.com/`

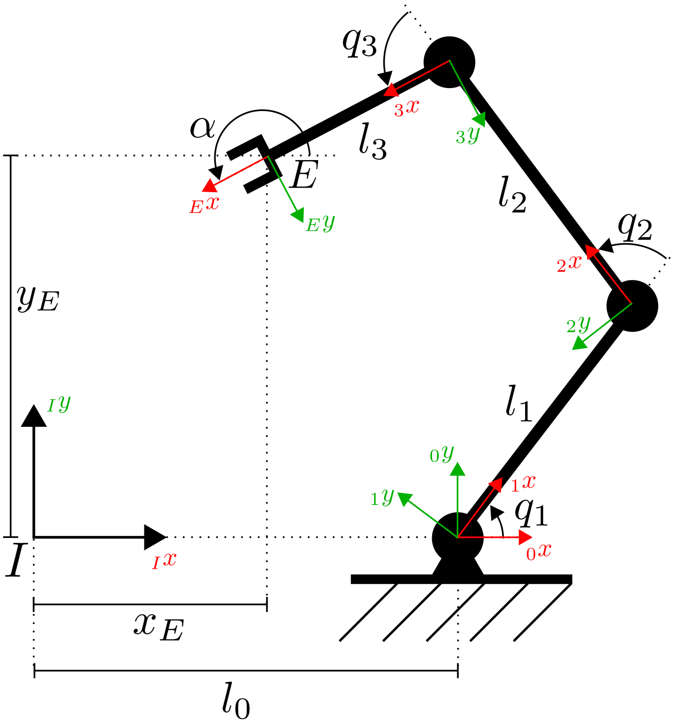

Figure 1: The three degree of freedom planar robotic arm considered in this document.

2 Questions

In this quiz, you will model the dynamics of the robot arm shown in Fig. 1 and use it for control. It is a fixed base manipulator with 3 degrees of freedom. All joints rotate around the positive z axis. The generalized coordinates are defined as

$$\boldsymbol{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top .$$

The arm is composed by three links with lengths l_1, l_2, and l_3 respectively and is displaced by l_0 from the inertial frame I along the ${}_I x$ axis.
In the following questions, we have already provided the kinematics (transforms, Jacobians, Jacobian time derivatives) for you. You may access them as follows:

```
params.l1; % length of link 1
params.m{1}; % mass of link 1
params.k_r_ks{1}; % position of center of mass of link 1 in link 1 ...
    frame
params.k_I_s{1}; % intertia tensor of link 1 in link 1 frame
params.k_n_k{1}; % rotation axis of joint 1 in link 1 frame
I_Jr{1}; % rotational Jacobian of link 1 in I frame
I_dJr{1}; % time derivative of I_Jr{1}
I_Jp_s{1}(q); % positional Jacobian of center of mass of link 1 in ...
    I frame
I_dJp_s{1}(q,dq); % time derivative of I_Jp_s{1}
I_Jp_E(q); % positional Jacobian of the end-effector in I frame
I_dJp_E(q,dq); % time derivative of I_Jp_E
```

Question 1. 7 P.

Calculate the mass matrix, nonlinear terms (Coriolis and centrifugal) and gravity terms for joint configuration and speed

$$\boldsymbol{q} = \begin{bmatrix}\pi/4 & \pi/8 & \pi/2\end{bmatrix}^\top$$
$$\dot{\boldsymbol{q}} = \begin{bmatrix}0.1 & 0.2 & -0.2\end{bmatrix}^\top$$

Assume the gravitational constant is $g = 9.81\,\mathrm{m/s^2}$.
Hint: If MATLAB prints out fractions instead of decimals, use the `double` function.

$$\boldsymbol{M} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

$$\boldsymbol{b} = [\qquad\qquad\qquad\qquad]^\top$$

$$\boldsymbol{g} = [\qquad\qquad\qquad\qquad]^\top$$

Question 2. 6 P.

We now take a different joint configuration and velocity

$$\boldsymbol{q} = \begin{bmatrix}\pi/4 & -\pi/4 & \pi/4\end{bmatrix}^\top ,$$
$$\dot{\boldsymbol{q}} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^\top .$$

We have already computed the following values using this state:

$$\boldsymbol{M} = \begin{bmatrix} 1.6369 & 0.6641 & 0.1182 \\ 0.6641 & 0.5013 & 0.1057 \\ 0.1182 & 0.1057 & 0.1013 \end{bmatrix}$$
$$\boldsymbol{b} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^\top$$
$$\boldsymbol{g} = \begin{bmatrix}11.9731 & 4.3427 & 0.1734\end{bmatrix}^\top$$
$${}_I\mathbf{J}_E = \begin{bmatrix} -0.5303 & -0.1768 & -0.1768 \\ 0.7803 & 0.4268 & 0.1768 \\ 0 & 0 & 0 \end{bmatrix}$$
$${}_I\dot{\mathbf{J}}_E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where ${}_I\mathbf{J}_E$ is the positional geometric Jacobian of the end-effector. Those values are already provided in your solution script. Use these given quantities to answer the following independent questions:

1. What is the quasi-static torque required for gravity compensation and lifting 5 kg with the end-effector?

$$\boldsymbol{\tau} = [\qquad\qquad\qquad\qquad]^\top$$

2. What is the torque required to accelerate the end-effector with $1\,\mathrm{m/s^2}$ in vertical direction upwards if we minimize $||\ddot{\boldsymbol{q}}||^2$ and do not care about horizontal and rotational acceleration?

$$\boldsymbol{\tau} = [\qquad\qquad\qquad\qquad]^\top$$

3. What is the torque required to accelerate the end-effector with 1 $^{m}/_{s^2}$ in vertical direction upwards if we minimize $||\boldsymbol{\tau}||^2$ and do not care about horizontal and rotational acceleration?

$$\boldsymbol{\tau} = [\qquad\qquad\qquad\qquad\qquad\qquad]^\top$$

Question 3. 4 P.

Each multiple choice question gives +1 point if correct and -1 point if wrong. The minimum for all multiple choice questions is zero. Please circle your answer.

1. In contrast to gravity compensation, controllers that use inverse dynamics achieve always zero tracking error. TRUE / FALSE

2. When a robotic arm is in stationary equilibrium (zero velocity, zero acceleration), then the joint torques are proportional to the gravitational acceleration. TRUE / FALSE

3. In zero gravity environment, the system's Hamiltonian changes when actuation torques are applied. TRUE / FALSE

4. Assume the robot arm in Fig. 1 is initially stationary, i.e., no movement, no acceleration, and the joint torques perfectly compensate for gravity. If you now apply an external force at the end-effector, the resulting acceleration of the end-effector will be instantaneously parallel to the force. TRUE / FALSE