ETH zürich

Exercise 4 – Modeling and Control of Micro Aerial Vehicles (MAVs)

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Robot Dynamics

Overview

- Dynamic Model
  - MAV dynamic model (review)
  - Allocation matrix

- Exercise Overview

Dynamic Model | MAV Dynamic Model

$$\mathbf{M}(\varphi)\ddot{\varphi} + b(\varphi,\dot{\varphi}) + g(\varphi) + \mathbf{J}_{ex}^T F_{ex} = \mathbf{S}^T \tau_{act}$$

- Change of momentum and spin in the body frame

$$\begin{bmatrix} mE_{3x3} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} {}_B\dot{\boldsymbol{v}} \\ {}_B\dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} {}_B\boldsymbol{\omega} \times m \; {}_B\boldsymbol{v} \\ {}_B\boldsymbol{\omega} \times \boldsymbol{I} \, {}_B\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} {}_B\boldsymbol{F} \\ {}_B\boldsymbol{M} \end{bmatrix}$$

E_{3x3}: Identity matrix

Torque!!

- Position in the inertial frame and the attitude

$${}_E\dot{\boldsymbol{x}} = \boldsymbol{C}_{EB} \, {}_B\boldsymbol{v} \qquad E_r \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = {}_B\boldsymbol{\omega}$$

- Forces and moments
  - Aerodynamics and gravity

$${}_B\boldsymbol{F} = {}_B\boldsymbol{F}_G + {}_B\boldsymbol{F}_{Aero}$$

$${}_B\boldsymbol{M} = {}_B\boldsymbol{M}_{Aero}$$

$${}_B\boldsymbol{F}_G = \boldsymbol{C}_{EB}^T \; {}_E\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

Dynamic Model | MAV Dynamic Model

- $F_i = b\omega_{p,i}^2$ $\tau_i = \pm d\omega_{p,i}^2$
- Virtual control inputs:
  - Total thrust U_1

$$U_1 = \sum_{i=1}^{4} F_i \Longrightarrow {}_B\boldsymbol{F}_{Aero} = -\begin{pmatrix} 0 \\ 0 \\ U_1 \end{pmatrix}$$

  - Moments around body axes U_2, U_3, U_4

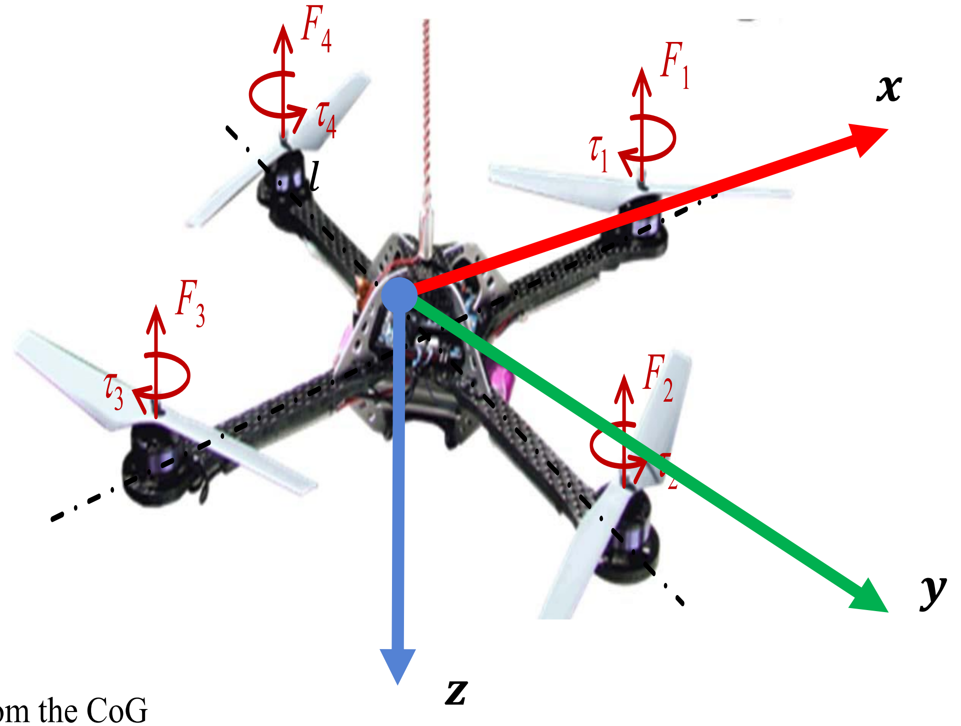
$$\begin{cases} U_2 = l(F_4 - F_2) \\ U_3 = l(F_1 - F_3) \\ U_4 = -\tau_1 + \tau_2 - \tau_3 + \tau_4 \end{cases} \Rightarrow {}_B\boldsymbol{M}_{Aero} = \begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix}$$

b: thrust constant

d: drag constant

l: distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i



Dynamic Model | MAV Dynamic Model

- $F_i = b\omega_{p,i}^2$ $\qquad\qquad$ $\tau_i = \pm d\omega_{p,i}^2$
- Virtual control inputs:
  - Arrange the previous equations

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \boldsymbol{A}_{quad} \begin{pmatrix} \omega_{p,1}^2 \\ \omega_{p,2}^2 \\ \omega_{p,3}^2 \\ \omega_{p,4}^2 \end{pmatrix}$$

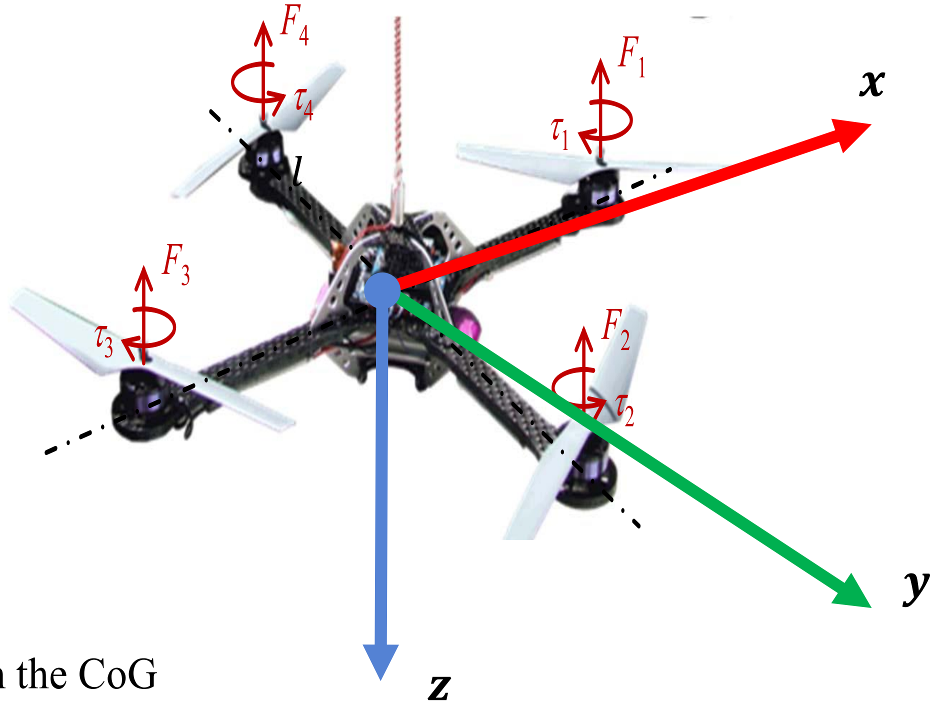
Allocation Matrix

b : thrust constant

d : drag constant

l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i



Dynamic Model | MAV Dynamic Model

- $F_i = b\omega_{p,i}^2 \qquad\qquad \tau_i = \pm d\omega_{p,i}^2$
- Virtual control inputs:
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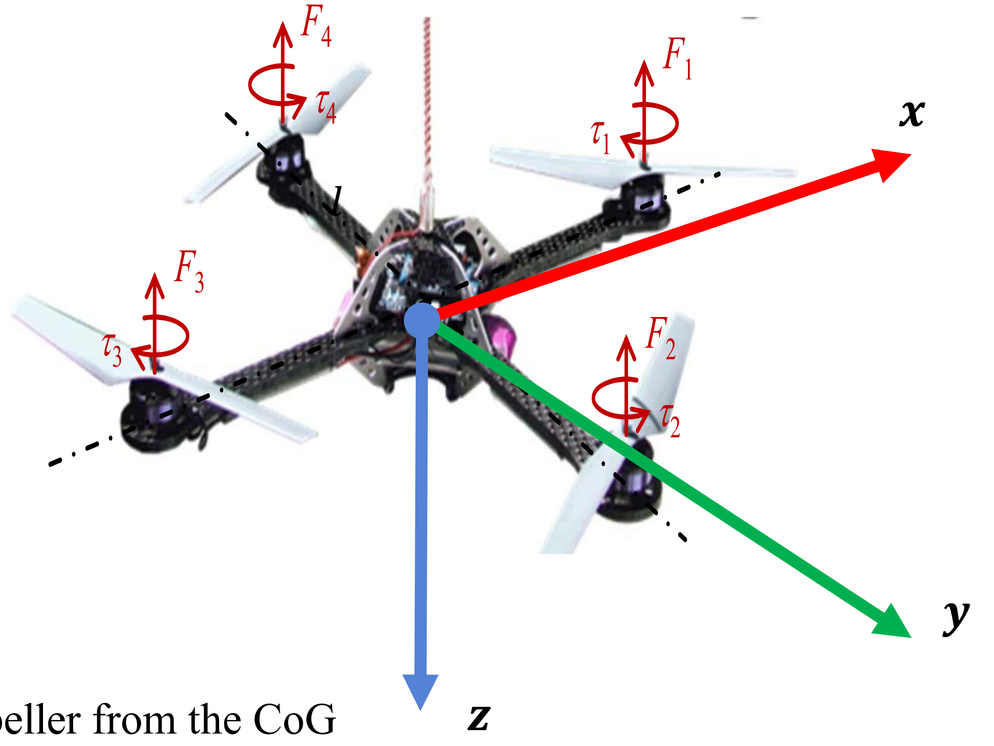
$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \boldsymbol{A}_{quad} \begin{pmatrix} \omega_{p,1}^2 \\ \omega_{p,2}^2 \\ \omega_{p,3}^2 \\ \omega_{p,4}^2 \end{pmatrix}$$

$$\boldsymbol{A}_{quad} = \begin{pmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ lb & 0 & -lb & 0 \\ -d & d & -d & d \end{pmatrix}$$



b : thrust constant

d : drag constant

l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i

Dynamic Model | MAV Dynamic Model

- $F_i = b\omega_{p,i}^2 \qquad \tau_i = \pm d\omega_{p,i}^2$
- Virtual control inputs:
  - Can generalize to any multirotor system

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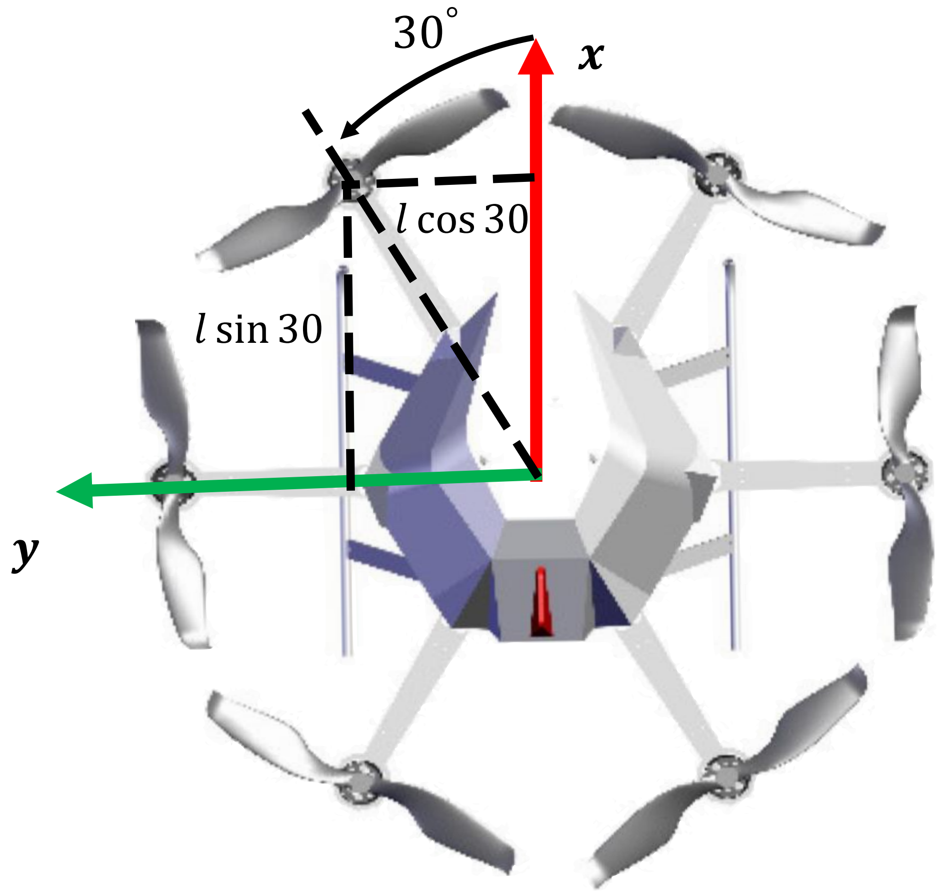
$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \omega_{p,1}^2 \\ \vdots \\ \omega_{p,n}^2 \end{pmatrix}$$



b : thrust constant

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l : distance of propeller from the CoG

$\omega_{p,i}$: rotational speed of propellor i