A FIRST COURSE

IN

NUMERICAL ANALYSIS

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MAT4001 Notebook

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Contents

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CUHK(SZ)

Notations and Conventions

$\mathbb{R}^n$	n-dimensional real space
$\mathbb{C}^n$	n-dimensional complex space
$\mathbb{R}^{m\times n}$	set of all $m \times n$ real-valued matrices
$\mathbb{C}^{m\times n}$	set of all $m \times n$ complex-valued matrices
x_i	ith entry of column vector $\boldsymbol{x}$
a_{ij}	(i,j)th entry of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i$	ith column of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i^{\mathrm{T}}$	ith row of matrix $\boldsymbol{A}$
$\mathbb{S}^n$	set of all $n \times n$ real symmetric matrices, i.e., $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $a_{ij} = a_{ji}$ for all i,j
$\mathbb{H}^n$	set of all $n \times n$ complex Hermitian matrices, i.e., $\boldsymbol{A} \in \mathbb{C}^{n\times n}$ and $\bar{a}_{ij} = a_{ji}$ for all i,j
$\boldsymbol{A}^{\mathrm{T}}$	transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{T}}$ means $b_{ji} = a_{ij}$ for all i,j
$\boldsymbol{A}^{\mathrm{H}}$	Hermitian transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{H}}$ means $b_{ji} = \bar{a}_{ij}$ for all i,j
$\mathrm{trace}(\boldsymbol{A})$	sum of diagonal entries of square matrix $\boldsymbol{A}$
$\mathbf{1}$	A vector with all 1 entries
$\mathbf{0}$	either a vector of all zeros, or a matrix of all zeros
$\boldsymbol{e}_i$	a unit vector with the nonzero element at the ith entry
$\mathcal{C}(\boldsymbol{A})$	the column space of $\boldsymbol{A}$
$\mathcal{R}(\boldsymbol{A})$	the row space of $\boldsymbol{A}$
$\mathcal{N}(\boldsymbol{A})$	the null space of $\boldsymbol{A}$
$\mathrm{Proj}_{\mathcal{M}}(\boldsymbol{A})$	the projection of $\boldsymbol{A}$ onto the set $\mathcal{M}$

Chapter 1

Week1

1.1. Wednesday

1.1.1. Introduction to Imaginary System

Actually, $(\mathbb{C}, +)$ forms a group:

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$
$$z_1 + z_2 = z_2 + z_1$$
$$z + 0 = 0 + z = z$$
$$z + (-z) = (-z) + z = 0$$

Also, $(\mathbb{C} \setminus \{0\}, \cdot)$ forms a group.

The product for imaginary numbers is different from vector product:

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2$$

Also, we can define the crossover product $\vec{v} \times \vec{w}$.

modulus:

$$|z| = \sqrt{x^2 + y^2}$$

direction angle (Argument):

$$\tan \theta = \frac{y}{x}$$

Using the polar coordination, we find $z = x + iy$ can be transformed into

$$\begin{aligned} z &= r\cos\theta + ir\sin\theta \\ &= r(\cos\theta + i\sin\theta) \\ &= r\left[\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)\right] \end{aligned}$$

Principal argument:

$$-\pi < \mathrm{Arg} z \leq \pi$$

Conjugate form of imaginary number:

$$\bar{z} = x - iy$$

$$\overline{z + w} = \bar{z} + \bar{w}$$

$$z \cdot \bar{z} = |z|^2$$

$$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

Proposition 1.1 $|z + w| \leq |z| + |w|$

Proposition 1.2 $|z + w|^2 + |z - w|^2 = 2|z|^2 + 2|w|^2$.

Proposition 1.3

$$\Re z = \frac{z + \bar{z}}{2}, \ \Im z = \tfrac{z - \bar{z}}{2i}$$