A FIRST COURSE

IN

NUMERICAL ANALYSIS

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MAT4001 Notebook

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Contents

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CUHK(SZ)

Notations and Conventions

$\mathbb{R}^n$	n-dimensional real space
$\mathbb{C}^n$	n-dimensional complex space
$\mathbb{R}^{m\times n}$	set of all $m \times n$ real-valued matrices
$\mathbb{C}^{m\times n}$	set of all $m \times n$ complex-valued matrices
x_i	ith entry of column vector $\boldsymbol{x}$
a_{ij}	(i,j)th entry of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i$	ith column of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i^{\mathrm{T}}$	ith row of matrix $\boldsymbol{A}$
$\mathbb{S}^n$	set of all $n \times n$ real symmetric matrices, i.e., $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $a_{ij} = a_{ji}$ for all i,j
$\mathbb{H}^n$	set of all $n \times n$ complex Hermitian matrices, i.e., $\boldsymbol{A} \in \mathbb{C}^{n\times n}$ and $\bar{a}_{ij} = a_{ji}$ for all i,j
$\boldsymbol{A}^{\mathrm{T}}$	transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{T}}$ means $b_{ji} = a_{ij}$ for all i,j
$\boldsymbol{A}^{\mathrm{H}}$	Hermitian transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{H}}$ means $b_{ji} = \bar{a}_{ij}$ for all i,j
$\mathrm{trace}(\boldsymbol{A})$	sum of diagonal entries of square matrix $\boldsymbol{A}$
$\mathbf{1}$	A vector with all 1 entries
$\mathbf{0}$	either a vector of all zeros, or a matrix of all zeros
$\boldsymbol{e}_i$	a unit vector with the nonzero element at the ith entry
$\mathcal{C}(\boldsymbol{A})$	the column space of $\boldsymbol{A}$
$\mathcal{R}(\boldsymbol{A})$	the row space of $\boldsymbol{A}$
$\mathcal{N}(\boldsymbol{A})$	the null space of $\boldsymbol{A}$
$\mathrm{Proj}_{\mathcal{M}}(\boldsymbol{A})$	the projection of $\boldsymbol{A}$ onto the set $\mathcal{M}$

Chapter 1

Week1

1.1. Tuesday

The Markov Chains course mainly focus on the performance analysis for which the Markov decision is made.

1.1.1. News Vendor Problems

A store sells perishable iterms (newpapers) with:

- Selling price $c_p = 1$
- Variable cost $c_v = 0.25$
- Salvage value $c_s = 0$

The aim is to decide how many copies should be ordered. Before making the decision, we need to estimate the demand.

Suppose the demand D has the following distribution:

d	10	15	20	25	30
$\mathbb{P}$	$1/4$	$1/8$	$1/8$	$1/4$	$1/4$

Hence, the profit for the day i is given by:

$$\text{profit per day} = \min(q, D_i)c_p - qc_v,$$

with q being the number of copies ordered.

More generally, for $c_s \neq 0$, the profit for the day i is

$$\text{profit per day} = \min(q, D_i)c_p - qc_v + \max(q - D_i, 0)c_s,$$

Hence, our objeective is to maximize the expected profit for a day

$$\begin{aligned} h(q) &= \mathbb{E}_D \text{Profit}(q, D) \\ &= c_p \mathbb{E}_D \min(q, D) - c_v q + c_s \mathbb{E}_D \max(q - D, 0) \end{aligned} \tag{1.1}$$