Sparse Cholesky Factorization

Cholesky decomposition is the most efficient way to solve linear systems with symmetric, positive definite coefficient matrices. If A is symmetric, positive definite, then it can be factorized into $A = R^T R$ where R is upper triangular. Therefore, the solution to the linear system $Ax = b$ can be expressed, in Matlab notation, as

```
R = chol(A);  x = R\(R'\b).
```

Matlab is smart enough to solve the triangular systems efficiently. If A is sparse, then rows and columns of A usually need to be reordered to make the factor R sparse (in solving $Ax = b$, x and b need to be reordered accordingly). One of the ordering functions in Matlab is `symamd`. The following script generates pictures showing how ordering affects sparsity.

```
% minimum degree ordering and sparse cholesky
load ship04s;               % load a sparse matrix A
B = A*A' + speye(402);      % Guarantee positive definiteness
p = symamd(B);   % SYMmetric Approximate Minimum Degree ordering

% spy, spy, spy and spy
subplot(221); spy(B);            title('B');
subplot(222); spy(B(p,p));       title('B(p,p)');
subplot(223); spy(chol(B));      title('chol(B)');
subplot(224); spy(chol(B(p,p))); title('chol(B(p,p))');
```

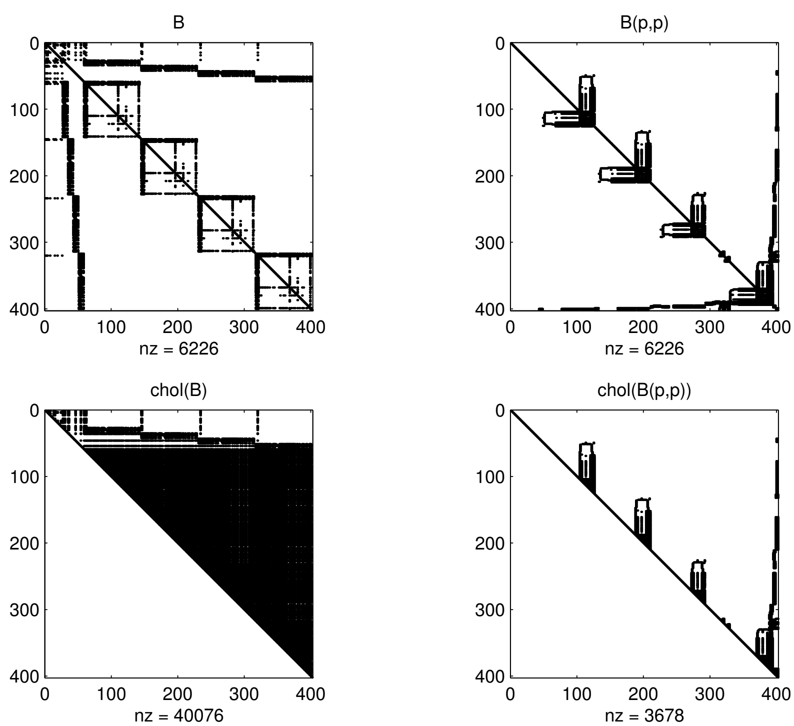

Figure 1: A Comparison of Sparsity (nz = number of nonzeros)