A GRADUATE COURSE

IN

OPTIMIZATION

A GRADUATE COURSE

IN

OPTIMIZATION

CIE6010 Notebook

Prof. Yin Zhang

The Chinese University of Hongkong, Shenzhen

香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

Contents

Acknowledgments

This book is from the CIE6010 in fall semester, 2018.

CUHK(SZ)

Notations and Conventions

X	Set
$\inf X \subseteq \mathbb{R}$	Infimum over the set X
$\mathbb{R}^{m\times n}$	set of all $m \times n$ real-valued matrices
$\mathbb{C}^{m\times n}$	set of all $m \times n$ complex-valued matrices
x_i	ith entry of column vector $\boldsymbol{x}$
a_{ij}	(i,j)th entry of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i$	ith column of matrix $\boldsymbol{A}$
$\boldsymbol{a}_i^{\mathrm{T}}$	ith row of matrix $\boldsymbol{A}$
$\mathbb{S}^n$	set of all $n \times n$ real symmetric matrices, i.e., $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $a_{ij} = a_{ji}$ for all i,j
$\mathbb{H}^n$	set of all $n \times n$ complex Hermitian matrices, i.e., $\boldsymbol{A} \in \mathbb{C}^{n\times n}$ and $\bar{a}_{ij} = a_{ji}$ for all i,j
$\boldsymbol{A}^{\mathrm{T}}$	transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{T}}$ means $b_{ji} = a_{ij}$ for all i,j
$\boldsymbol{A}^{\mathrm{H}}$	Hermitian transpose of $\boldsymbol{A}$, i.e, $\boldsymbol{B} = \boldsymbol{A}^{\mathrm{H}}$ means $b_{ji} = \bar{a}_{ij}$ for all i,j
$\mathrm{trace}(\boldsymbol{A})$	sum of diagonal entries of square matrix $\boldsymbol{A}$
$\mathbf{1}$	A vector with all 1 entries
$\mathbf{0}$	either a vector of all zeros, or a matrix of all zeros
$\boldsymbol{e}_i$	a unit vector with the nonzero element at the ith entry
$\mathcal{C}(\boldsymbol{A})$	the column space of $\boldsymbol{A}$
$\mathcal{R}(\boldsymbol{A})$	the row space of $\boldsymbol{A}$
$\mathcal{N}(\boldsymbol{A})$	the null space of $\boldsymbol{A}$
$\mathrm{Proj}_{\mathcal{M}}(\boldsymbol{A})$	the projection of $\boldsymbol{A}$ onto the set $\mathcal{M}$

Chapter 1

Week1

1.1. Monday

1.1.1. Introduction to Optimizaiton

The usual optimization formulation is given by:

$$\begin{aligned} &\min f(\boldsymbol{x}), && \text{where } f : \mathbb{R}^n \mapsto \mathbb{R} \\ &\text{such that} && \boldsymbol{x} \in X \subseteq \mathbb{R}^n \end{aligned}$$

One example of the set X is given by:

$$X = \left\{ \boldsymbol{x} \in \mathbb{R}^n \,\middle|\, \begin{array}{l} C_i(\boldsymbol{x}) = \boldsymbol{0}, i = 1, 2, \dots, m \leq n \\ h_i(\boldsymbol{x}) \geq \boldsymbol{0}, i = 1, 2, \dots, p \end{array} \right\}$$

Linear programming can be easily solved, but Integer linear programming is much harder. The equivalent LP formulation is given by:

$$\begin{aligned} \min \quad & \boldsymbol{c}^{\mathrm{T}} \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{c} \leq \boldsymbol{B}\boldsymbol{x} \leq \boldsymbol{c}' \end{aligned}$$