Optimization Theory and Algorithms

Incremental Gradient Method

Problem

For given data (A, b), where $A \in \mathbb{R}^{m\times n}$ $(m > n)$ and $b \in \mathbb{R}^m$, let a_i^T be the i-th row of A and b_i the i-th element of b. Consider solving the linear least squares problem

$$\min_{x\in\mathbb{R}^n} \sum_{i=1}^{m} \frac{\left(a_i^T x - b_i\right)^2}{2} \equiv \sum_{i=1}^{m} f_i(x) \tag{1}$$

by the incremental gradient method. The algorithm generates an outer iteration sequence $\{x^k\}$ where x^{k+1} is obtained from x^k by taking a step in the negative gradient direction of each component function, one after another and always evaluated at the latest point available, while the step size $\alpha_k = \theta/k$ goes to 0 as the outer iteration number goes to infinity. Here $\theta > 0$ is a fixed constant that you can choose. Mathematically, we may write with a slight abuse of notation:

$$x^{k+1} = x^k - \alpha_k g_1^k - \alpha_k g_2^k \cdots - \alpha_k g_i^k \cdots - \alpha_k g_m^k \tag{2}$$

where, with the convention $g_0^k = 0$,

$$g_i^k = \nabla f_i(x^k - \alpha_k g_1^k \cdots - \alpha_k g_{i-1}^k),\ i = 1, 2, \cdots, m. \tag{3}$$

Matlab

- Implement the incremental gradient method described in the lecture notes by writing a Matlab function

```
x = myIncremental(A,b,x0,tol,maxit)
```

  where (A, b) is the given dataset, `x0` is an initial guess, `tol` is an tolerance value for termination, and `maxit` is the maximum number of iterations allowed. The termination criterion is

$$\Delta = \frac{\|x^k - x^{k-1}\|_2}{\|x^{k-1}\|_2} \le \texttt{tol}. \tag{4}$$

- Download the file `handout_incremental.zip` and run `test_incremental.m` (with or without your code).

- Your code should have the same output format as the instructor's code. Submit your code and the print-out/outputs from the test run.

- Submit a half page report on this part of the assignment to summarize a couple of points that you consider to be the most important.