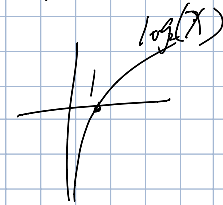
Problem Set #1 5 Questions

2) if given $f(n) = O(g(n)) \Rightarrow f(n) \le C\cdot g(n) \Rightarrow \log(f(n)) = O(\log(g(n)))$

$\Rightarrow \log f \le \log g + C$

is $f(n) * \log_2(f(n)^c) = O(g(n) * \log_2(g(n)))$? ($c > 0$, $f, g > 1$.)



Assume $f(n) * \log_2(f(n)^c) = O(g(n) * \log_2(g(n)))$

$\Rightarrow f(n) * \log_2(f(n)^c) \le C\cdot g(n) * \log_2(g(n))$

Note $= c\cdot f(n) * (\log_2 f(n)) = O(g(n)\cdot \log_2(g(n)))$

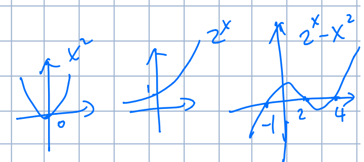
3). $f(n) = O(g(n))$ is $2^{f(n)} = O(2^{g(n)})$?

$f(n) \le C\cdot g(n)$ $\quad 2^{f(n)} \le C\cdot 2^{g(n)}$

$g(n) \ge \frac{f(n)}{C}$

$C\cdot 2^{g(n)} \ge 2^{g(n)} \ge 2^{\frac{f(n)}{c}}$

4). k-way merge (multiway merges)



5). increasing order of growth rate for

a) 2^{2^n} b) 2^{n^2} c). $n^2\log(n)$ d). n e). n^{2^n}

a $(2^{2^n})' = 2^{n+2^n}\log^2(2)$; $(2^{n^2})' = 2^{n^2+1}\cdot n\log 2$; $(n^{2^n})' = 2^n\cdot n^{2^n-1}(n\cdot\log 2\cdot \log n + 1)$

c $(n^2\log(n))' = 0$; $(n)' = 1$

dcbae

$2^{n+2^n} = 2^{2^n}\cdot 2^n$; $2^{n^2+1} = 2\cdot 2^{n^2}$; $2^n\cdot 2^{2^n-1} = 2^{n-1}\cdot 2^{2^n}$

Programming Assignment #1

Karatsuba multiplication

x (n-digit integer)

$x = a \times 10^{\frac{n}{2}} + b$

$2925 = 29 \times 10^{2} + 25$

$y = c \times 10^{\frac{n}{2}} + d$

$xy = (a \times 10^{\frac{n}{2}} + b)(c \times 10^{\frac{n}{2}} + d)$

$= ac \times 10^{n} + \underbrace{(ad + bc)} \times 10^{\frac{n}{2}} + bd$

$ad + bc = (a+b)(c+d) - ac - bd$

Karatsuba running time:

if $T(n) = aT(\frac{n}{b}) + O(n^{d})$

Then

$$T(n) = \begin{cases} O(n^{d} \log n) & \text{if } a = b^{d} \\ O(n^{a}) & \text{if } a < b^{d} \\ O(n^{\log_b a}) & \text{if } a > b^{d} \end{cases}$$