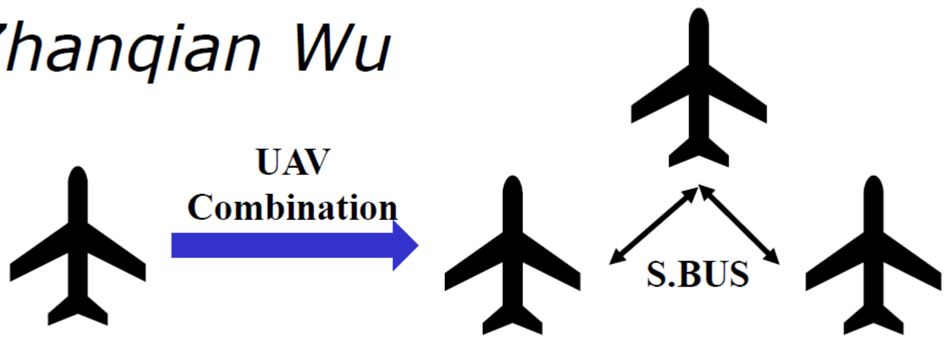
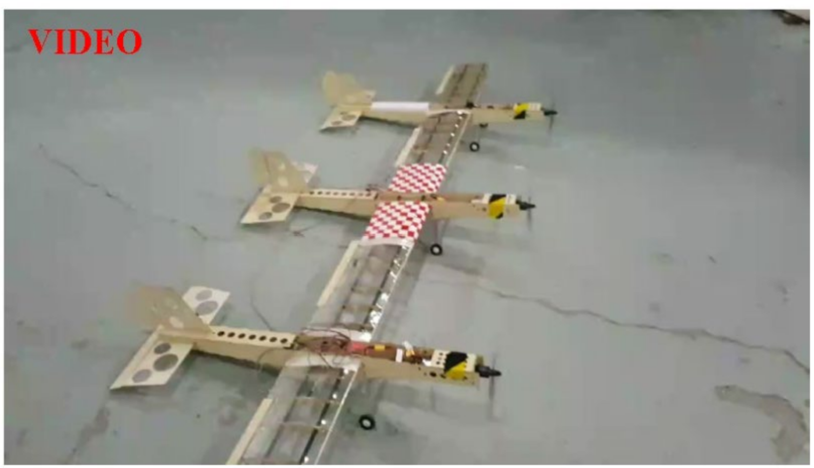
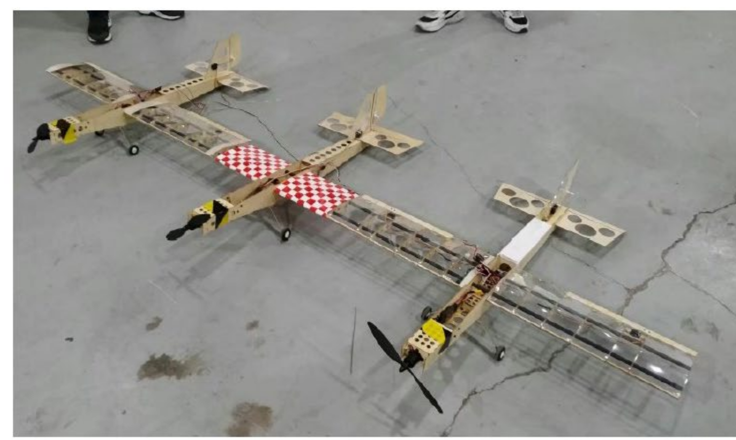
Final Portfolio

MEAM 543
2024A

Zhanqian Wu







Penn Engineering

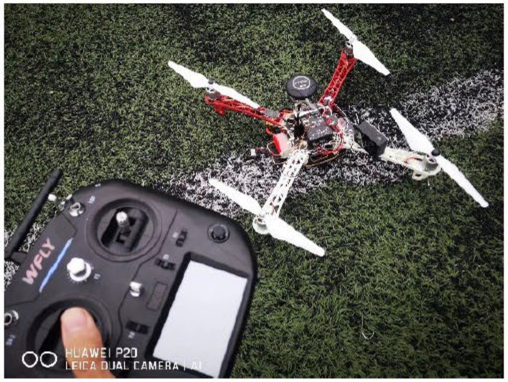
Introduce Yourself (Optional)

- China
- MEAM MS, Spring 2025
- My Story

2010
My first RC plane.
Crash 2s after take off

2015
Flight Simulator X.
With my old computer

2018-2019
Undergraduate Programs
Designed a RC plane

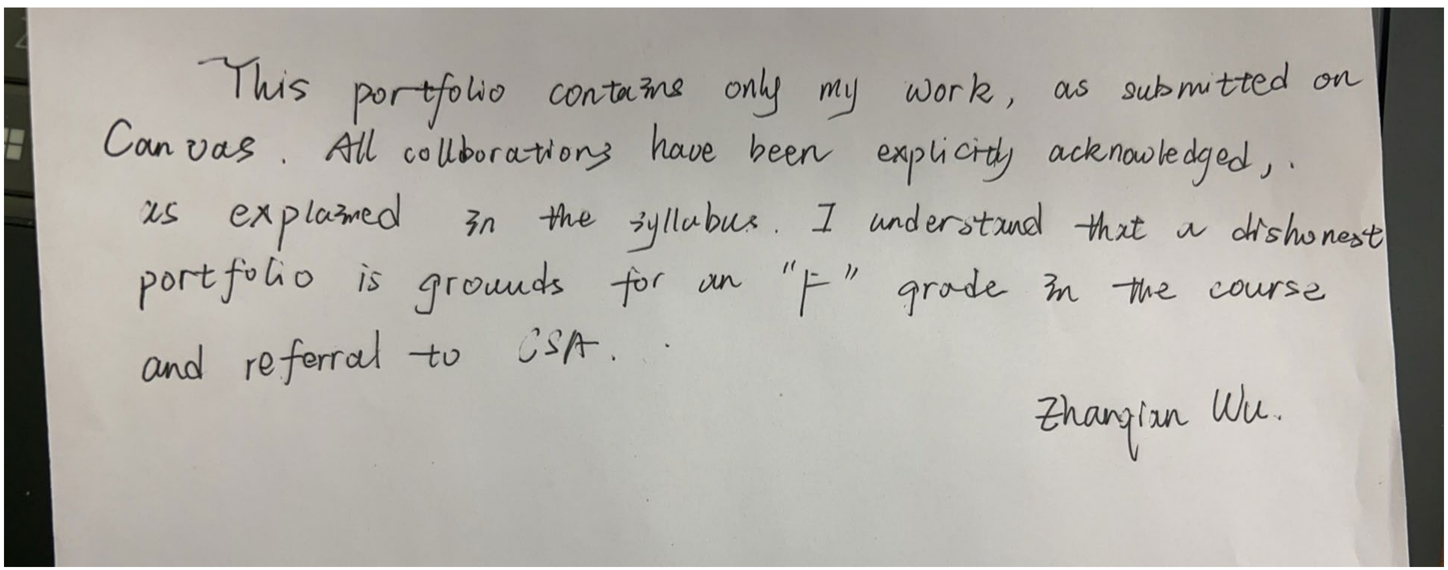


2020
My DIY quadcopter drone.

Final

Academic Integrity Pledge

- Copy the Paragraph Below by Hand
- Sign Your Name Beneath the Handwritten Text
- *Replace* the Text Box with a Photo of Your Handwritten Pledge


This portfolio contains only my work, as submitted on Canvas. All collborations have been explicitly acknowledged, as explained in the syllabus. I understand that a dishonest portfolio is grounds for an "F" grade in the course and referral to CSA.

Zhangian Wu.

Canvas *Homework* Assignment Summary

- Assignments Submitted on Time (List)

All Assignments Submitted on Time.

- Assignments Submitted Late (List)

No late assignments.

- Assignments Submitted Rogue (List & Explain Briefly)

No rogue assignments.

I'm not sure if my HW5 Problem 4 /Quiz 9 are correct.

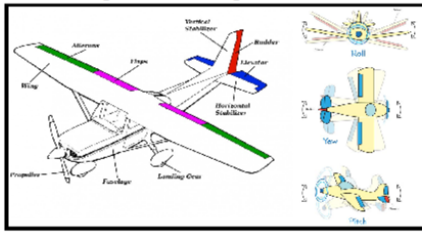
- Assignments Missing (List & Explain Briefly)

No Missing assignments.

HW01

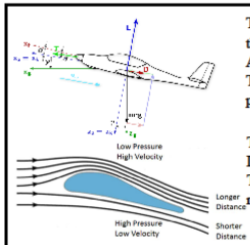
MEAM 543 — Zhanqian Wu

Basic Components of RC planes



Ailerons: Control the roll of the aircraft by altering the lift on each wing.
Elevators: Adjust the pitch of the airplane, allowing it to ascend or descend.
Rudder: Controls the yaw, helping the plane to turn left or right.
Flaps: Increase lift at lower speeds, particularly during takeoff and landing.

Basic Principles of Aircraft



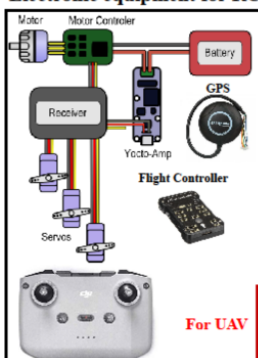
The shape of airfoil is designed so that air travels faster over the top surface than the bottom surface.
According to Bernoulli's Principle, faster-moving air has lower pressure. Therefore, the pressure on the top surface of the wing is less than the pressure on the bottom surface, creating an upward force known as lift.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

The force balance in an airplane involves the balance of four main forces: Lift, Weight, Thrust, and Drag.
Torque balance involves the rotation of the airplane about its three axes: roll, pitch, and yaw.

$$\begin{cases} \text{Lift (L)} = \text{Weight (W)} & \text{(Vertical Force Balance)} \\ \text{Thrust (T)} = \text{Drag (D)} & \text{(Horizontal Force Balance)} \\ \text{Total Moment (M)} = 0 & \text{(Moment Stability)} \end{cases}$$

Electronic equipment for RC aircraft



Transmitter: The handheld device that sends control signals to the aircraft, allowing the pilot to maneuver it remotely.
Receiver: Installed in the aircraft, it receives signals from the transmitter and relays commands to control the servos and motors.
Servos: Small motors that move the control surfaces (like ailerons and rudders) according to the signals received, allowing for precise maneuvering.
Electronic Speed Controller: Regulates power to the electric motor, controlling speed and direction based on input from the receiver.
Battery: Provides the electrical power necessary for the motor, receiver, and other onboard electronics; typically a rechargeable lithium-polymer (LiPo) battery.
Motor: Converts electrical energy from the battery into mechanical energy to spin the propeller, generating thrust for the aircraft.
Flight Controller: An advanced system in some RC aircraft that helps stabilize flight, manage autonomous flying, and integrate various sensors.
GPS Module: Used in advanced RC models for navigation, autonomous flight, and real-time location tracking.

Please Include Photos from Problem #2 & Results from Any Other Problems You Want to Show

HW02

Please Include Your Graph from Problem #1 & Results from Any Other Problems You Want to Show

1 Problem 1 – Airbus Zephyr High-Altitude Pseudo Satellite (HAPS)

The basic specifications of Zephyr 7 is shown in Tab 1

Table 1: Zephyr 7 Aircraft Specifications

Specification	Value
Max Takeoff Weight	53 kg
Cruise Speed	56 km/hr (15.6 m/s)
Ceiling (Maximum Altitude)	21,500 m
Wing Area	32.6 m^2
Wing Span	22.5 m
Airfoil	Eppler E395

E395 Airfoil Information is shown in Fig 1

From the graph, we can see that

- $a_{2D} = \frac{1.0-0.7}{3} = 0.1$
- $c_{L_{max}}$=1.45
- $\alpha_0 = -6.5°$

By looking up "Standard Atmosphere" we can get a property of the air as a function of altitude in Tab 2

Fitting the table using the least squares method, we obtain the equation for the variation of air density with elevation f 25°C as Equ 1

$$\rho(h) = -0.0003h^3 + 0.0059h^2 - 0.1214h + 1.2264 \quad (1)$$

The comparison between the fitted and actual values is shown in Fig. 2. It can be seen that the error value between the fitted and actual values is very small, which means that the fitting is very good.

The formula for calculating the lift is:

$$L = C_l \frac{1}{2} \rho V^2 S \quad (2)$$

where:

- C_l is the lift coefficient.
- $L = mg = 530N$ is the lift force, which is equal to the weight of the aircraft in steady flight.
- ρ is the air density.

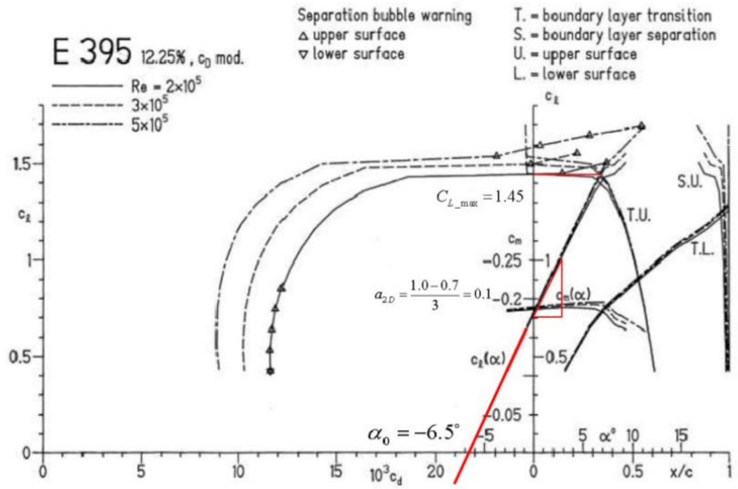


Figure 1: E395 Airfoil Information

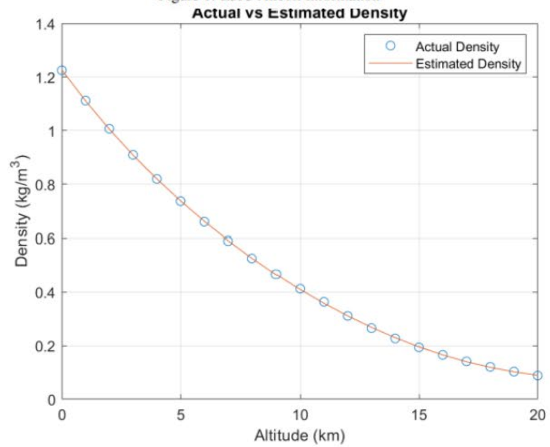


Figure 2: The comparison between the fitted and actual values

HW02

Please Include Your Graph from Problem #1 & Results from Any Other Problems You Want to Show

$$C_L = \frac{32.515}{\rho v^2} \tag{3}$$

Assuming the wing is two-dimensional, we will have the 2D lift curve as follows:

$$\begin{aligned} C_l &= C_{l_0} + a_0\alpha \\ &= a_0\left(\alpha^2 - \alpha_{0L}\right) \end{aligned} \tag{4}$$

Taking the known information into the equation, the equ 3 can be simplified to

$$C_L = C_{l_0} + a_0\alpha = 0.75 + 0.1\alpha = \frac{32.515}{\rho v^2} \tag{5}$$

However, in 3-dimensions, the finite wing have a reduced local α_{eff} while the wing is at a real angle-of-attack α, where $\alpha_{eff} = \alpha - \alpha i$, that is to say, the finite wing has less lift since it has a smaller angle of attack.

we will have the 3D lift curve as follows:

$$C_L = a_0\alpha_{eff} = a_0\left(\alpha - \alpha_i\right) = a_0\left(\ \alpha - \frac{C_L}{\pi ARe_w}\ \right) \tag{6}$$

$$C_L = \frac{a_0\alpha}{1 + \frac{a_0}{\pi ARe_w}} \tag{7}$$

Taking the known information into the equation, the Equ 6 can be simplified to

$$C_L = C_{l_0} + \frac{a_0\alpha}{1 + \frac{a_0}{\pi ARe_w}} = \frac{32.515}{\rho v^2} \tag{8}$$

where

- $AR = \frac{b^2}{s} = 15.529$
- we assume $e_w = 0.85$

Then, the Equ 6 can be simplified to

$$C_L = 0.75 + 0.0976\alpha = \frac{32.515}{\rho v^2} \tag{9}$$

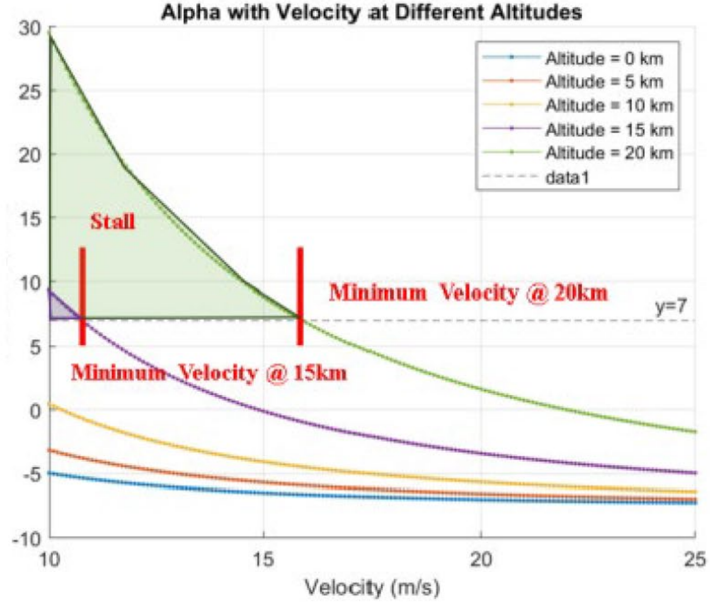


Figure 3: α with Velocity at Different Altitudes

HW03: Include Best Graphs, Analysis, Etc

Please Include Pretty Graphs from Probem #2b & Results from Any Other Problems You Want to Show

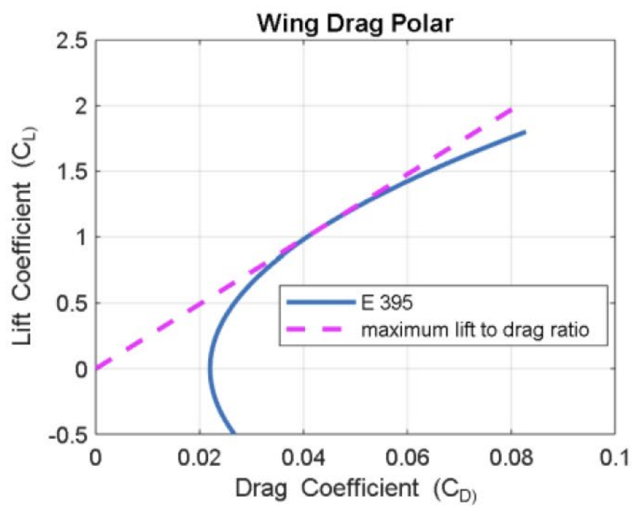


Figure 5: E395 Wing Drag Polar

$$\begin{cases} L = W = \frac{1}{2}\rho C_L V_\infty^2 S \\ T = D = \frac{1}{2}\rho C_D V_\infty^2 S \end{cases} \tag{16}$$

Hence

$$T = D = \frac{1}{2}\rho V_\infty^2 S (C_{D_{adjust}} + r^2 C_L) \tag{17}$$

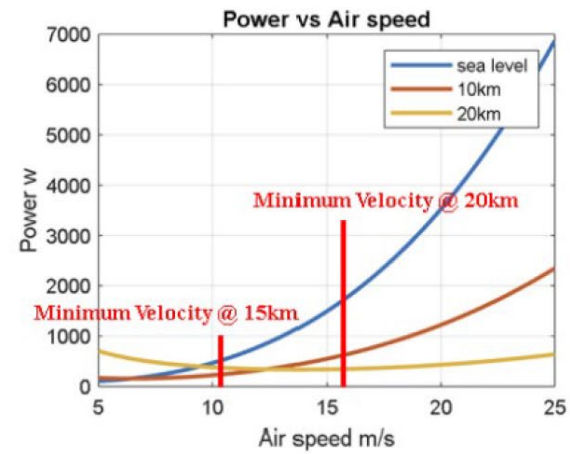
where

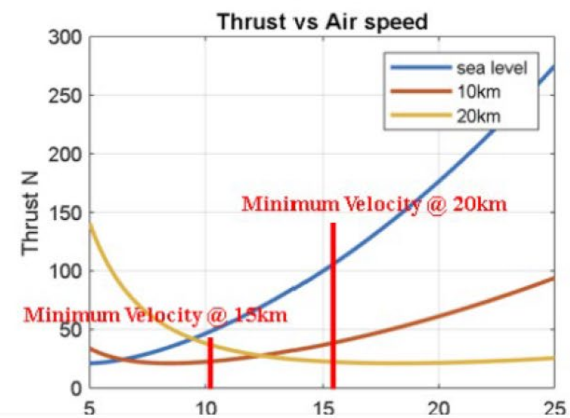
$$C_L = \frac{2w}{\rho V_\infty^2 S} \tag{18}$$

A check of the standard air density table shows that the density of air at sea level, 10km, and 20km is 1.225 kg/m^3, 0.4135 kg/m^3 and 0.08891 kg/m^3.

Finally,we can fraw thrust and power required in level flight vs airspeed at altitudes of 0km (sea level), 10km, and 20km as shown in Figs below.

rom A





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HW04: Include Best Graphs, Analysis, Etc

Please Include Pretty Graphs from Problem #3 & Results from Any Other Problems You Want to Show

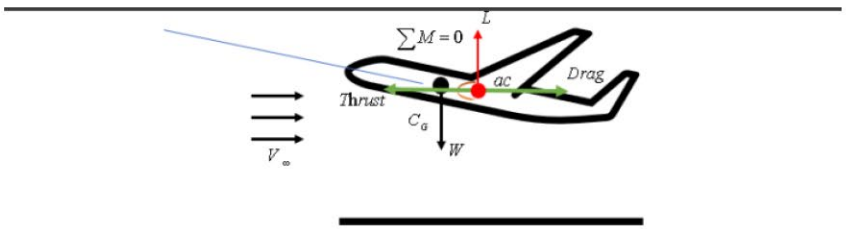

Figure 12: Schematic of the force balance of an airplane

Then, we will calculate minimum thrust required in level flight. As is shown in Fig. 12, the force balance equations can be shown as Equ. 13:

$$\begin{cases} L = W = \frac{1}{2}\rho C_L V_\infty^2 S \\ T = D = \frac{1}{2}\rho C_D V_\infty^2 S \end{cases} \tag{13}$$

Hence

$$V_\infty = \sqrt{\frac{2\omega}{\rho s C_L}} = 9.55m/s \tag{14}$$

Minimum thrust and the corresponding power in level flight is

$$T_{min} = \frac{C_D}{C_L} w = 0.786N = 80.29g \tag{15}$$

$$P_{T_{min}} = T_{min}V = 9.55 \times 0.786 = 7.51W \tag{16}$$

Minimum power and the corresponding thrust in level flight is

$$P = DV_\infty = \frac{1}{2}\rho s \frac{2\omega}{\rho s}^{\frac{3}{2}} \frac{C_D}{C_L^{\frac{3}{2}}} \tag{17}$$

as we can see from the above equation, if we want to achieve minimum power, $\frac{C_L^{1.5}}{C_D}$ should take its maximum value.

We denote C_L and C_D when $\frac{C_L^{1.5}}{C_D}$ takes its maximum value as $C_{L_{pm}}$ and $C_{D_{pm}}$. As we can see from Fig 3, $\frac{C_L^{1.5}}{C_D} = 8.59$, where $C_{L_{pm}} = 1.136$ and $C_{D_{pm}} = 0.141$

$$T_{Pmin} = \frac{C_D}{C_L} w = 0.855N = 87.31g \tag{18}$$

$$V_\infty = \sqrt{\frac{2\omega}{\rho S_{wing} C_{L_{pm}}}} = 6.86m/s \tag{19}$$

$$P_{min} = DV_\infty = \sqrt{2}\rho^{-0.5} s (\frac{\omega}{S_{wing}})^{\frac{3}{2}} \frac{C_{D_{pm}}}{C_{L_{pm}}^{\frac{3}{2}}} = 6.02W \tag{20}$$

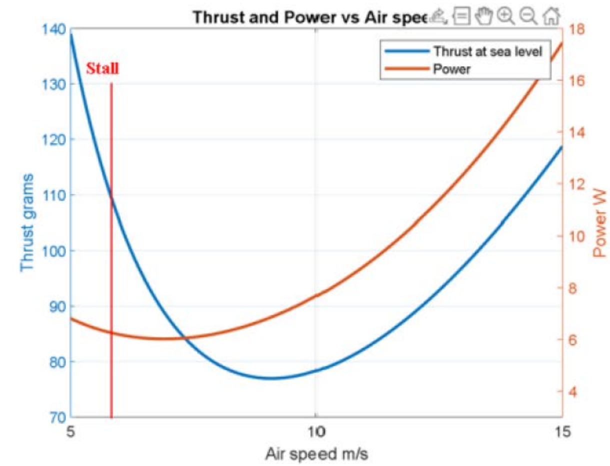

Figure 13: Thrust and Power vs Air speed

Finally, we obtained the result shown in Fig. 13. It's worth noting that we can't fly too slowly; when the wing attack angle is greater than the stall attack angle, the airplane will stall, at which point the speed is:

$$V_{stall} = \sqrt{\frac{2\omega}{\rho S_{wing} C_{L_{stall}}}} = 5.97m/s \tag{21}$$

As is shown in the figure, both thrust and power required by the aircraft decreases sharply as the airspeed increases from the leftmost part of the x-axis, reaching a minimum point before increasing again.

Why Minimum Thrust Speed is Not the Minimum Speed: : Drag consists of two primary components: induced drag and parasitic (or profile) drag. Induced drag is related to the production of lift, and it decreases with an increase in speed. This is because as speed increases, less angle of attack is required to maintain the same amount of lift, leading to a reduction in induced drag. Parasitic drag, on the other hand, increases with the square of the speed and is due to factors such as skin friction and form drag.

The minimum speed for flight is determined by the stall speed, which is the point at which the aircraft can no longer generate enough lift to support its weight. This occurs at a lower speed than the speed for minimum thrust required because, at the stall speed, the aircraft is operating at a very high angle of attack, which is inefficient and results in high induced drag. Thus, the speed for minimum thrust is above the stall speed because it represents a balance between minimizing induced drag and not excessively increasing parasitic drag, rather than the absolute minimum speed at which the aircraft can fly.

Add Extra Page if Needed

HW05: Include Best Graphs, Analysis, Etc

Please Include Graphs Problem #3d and #3f & Results from Any Other Problems You Want to Show

Iterate Over Airspeed Range: For each airspeed in the range v_{inf}, perform the following calculations:

Calculate Lift and Drag Coefficients:

Lift coefficient

$$C_L = \frac{2W}{S\rho v_{\text{inf}}^2} \tag{7}$$

Drag coefficient

$$C_D = C_{D0} + KC_L^2 \tag{8}$$

Solve for RPM: Use the fzero function to find the RPM (n) that balances the forces, by solving the equation:

$$0.5\rho v_{\text{inf}}^2 C_D S = \rho\left(\frac{n}{60}\right)^2 D^4 \mathrm{f}(p_{CT}, \frac{v_{\text{inf}}}{(\frac{n}{80})D}) \tag{9}$$

Calculate Power, Current, and Voltage: Power required by the propeller

$$P = \rho\left(\frac{n}{60}\right)^3 D^5 \mathrm{f}(p_{CP}, \frac{v_{\text{sf}}}{(\frac{n}{60})D}) \tag{10}$$

Electric current

$$I = \frac{P}{K(\frac{n}{60}2\pi)} + i_0 \tag{11}$$

Electric voltage

$$V = K(\frac{n}{60}2\pi) + IR_m \tag{12}$$

Calculate Efficiencies:

Motor efficiency

$$\eta_{\text{motor}} = \frac{P_m}{P_e} \tag{13}$$

Propeller efficiency

$$\eta_{\text{propeller}} = \frac{P_r}{P_m} \tag{14}$$

Total system efficiency

$$r_{\text{Hotal}} = \frac{P_r}{P_e} \tag{15}$$

Maximum possible steady climb rate:

$$P_r = \max(\mathrm{PR}) - \mathrm{PR}; \quad \frac{dh}{dt} = \left(\frac{P_r}{W}\right) \times 60 \tag{16}$$

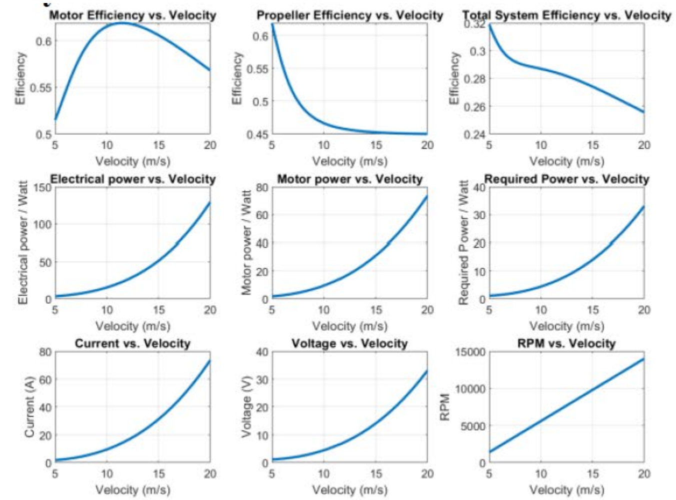

Figure 4: Level-Flight Performance Analysis

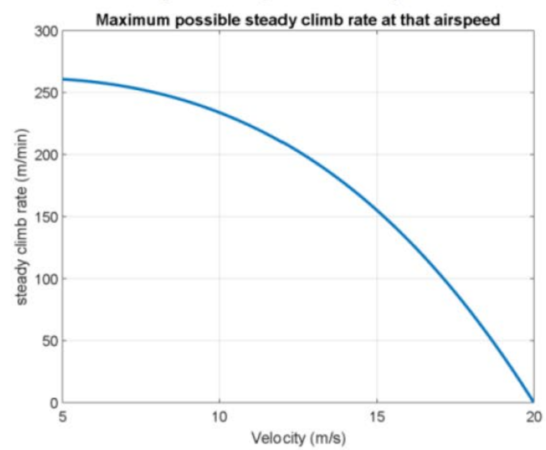

Figure 5: Maximum possible steady climb rate at that airspeed

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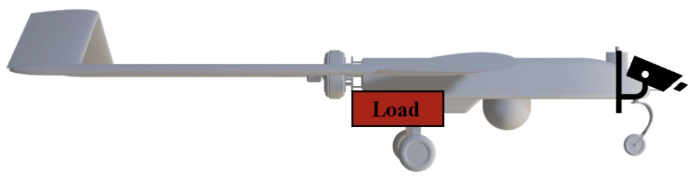
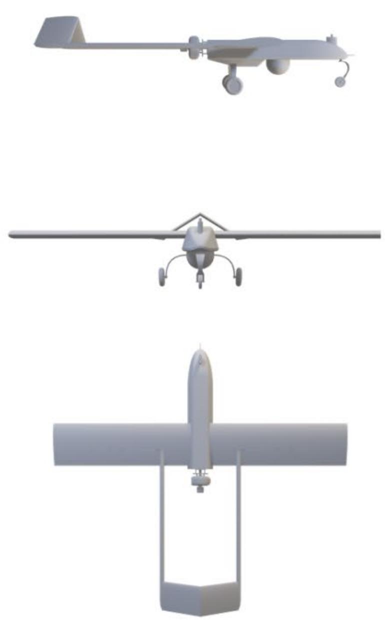
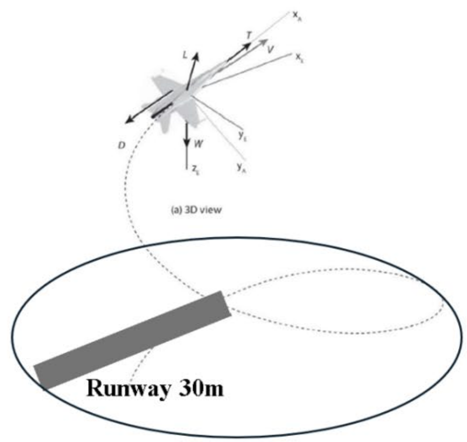
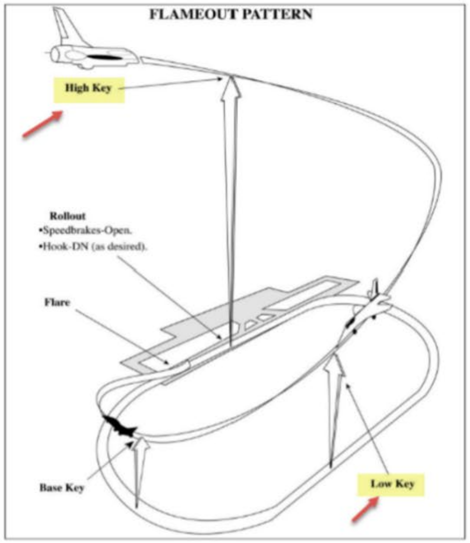
HW06: Include Best Graphs, Analysis, Etc

Please Include Whichever Design You Think is Most Compelling or Interesting









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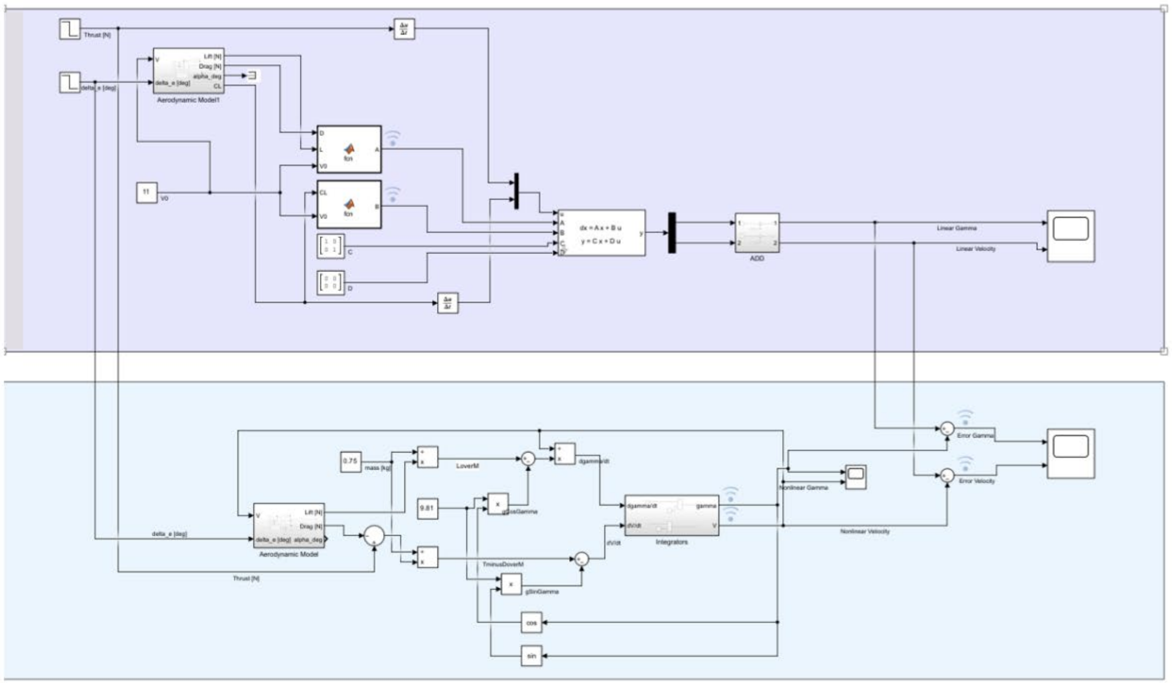
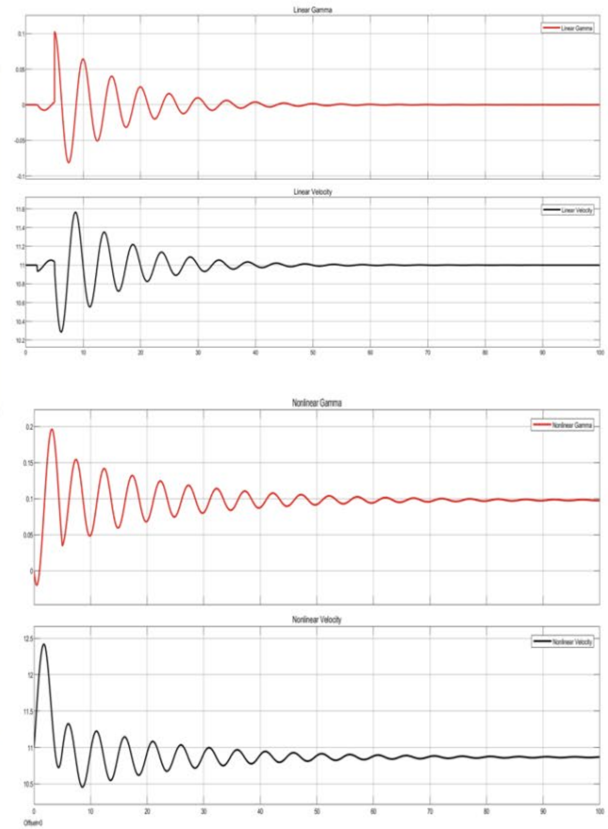
Final

HW07: Include Best Graphs, Analysis, Etc

Please Include Results from Problem #3 & Results from Any Other Problems You Want to Show





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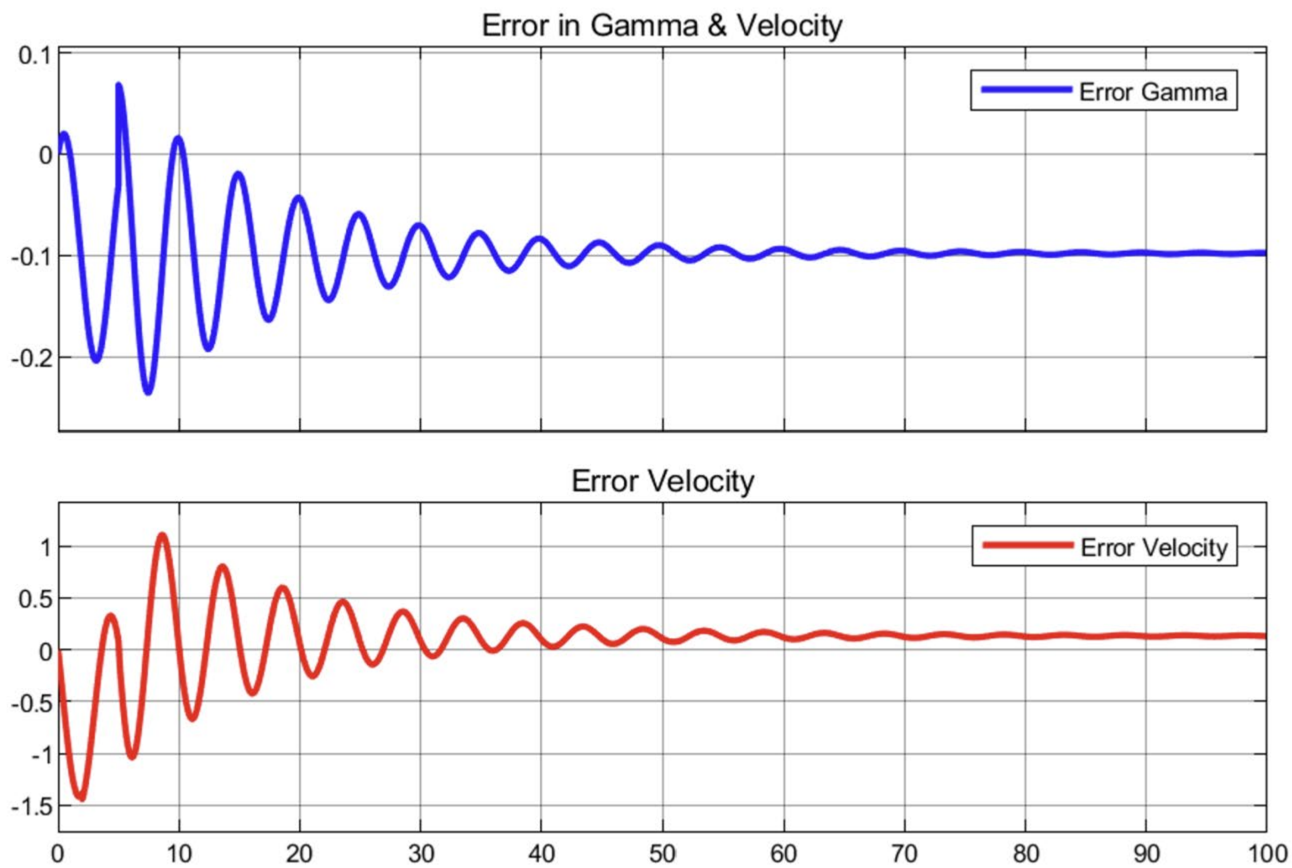
Final

HW07: Include Best Graphs, Analysis, Etc

Please Include Results from Problem #3 & Results from Any Other Problems You Want to Show



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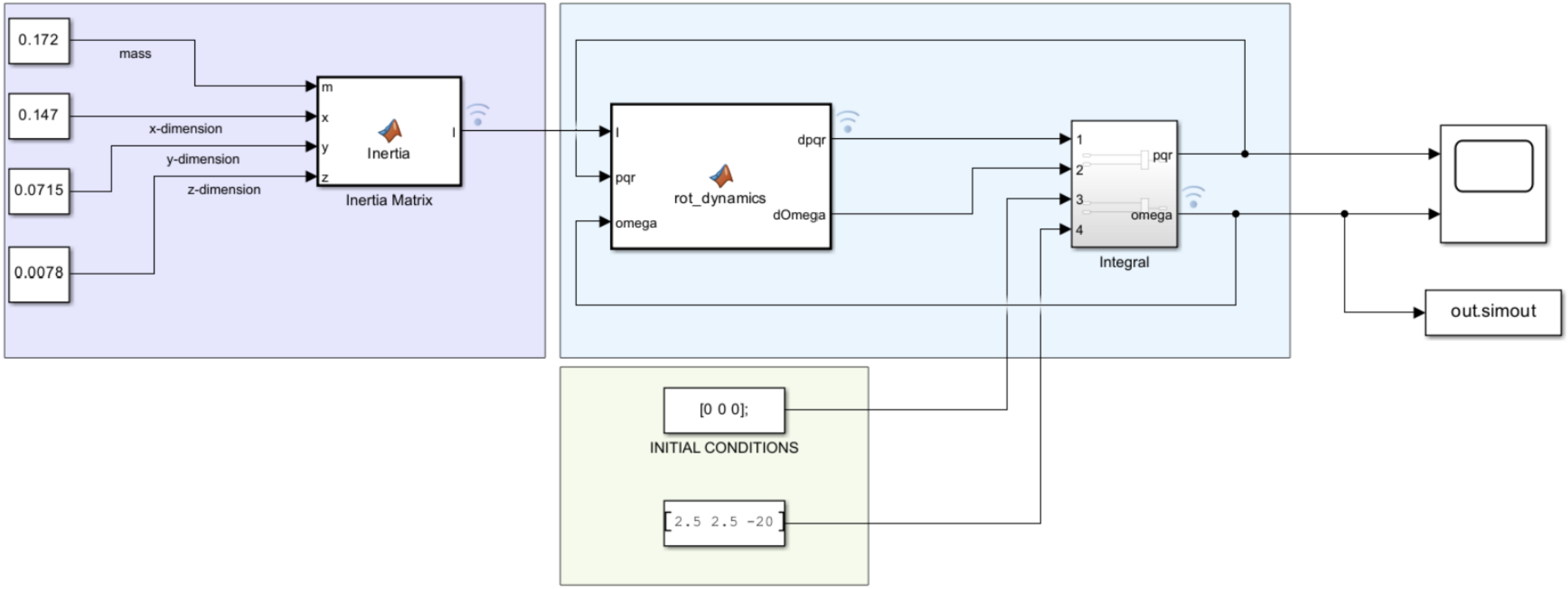
Final

HW08: Include Best Graphs, Analysis, Etc

Please Include the Graphs Comparing Angular Velocities from Simulink and Experiment

[1] Reconstructing the motion of a tossed iPhone | Rotations. (n.d.). Rotations.berkeley.edu. Retrieved April 8, 2024, from .https://rotations.berkeley.edu/reconstructing-the-motion-of-a-tossed-iphone/



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HW08: Include Best Graphs, Analysis, Etc

Please Include the Graphs Comparing Angular Velocities from Simulink and Experiment

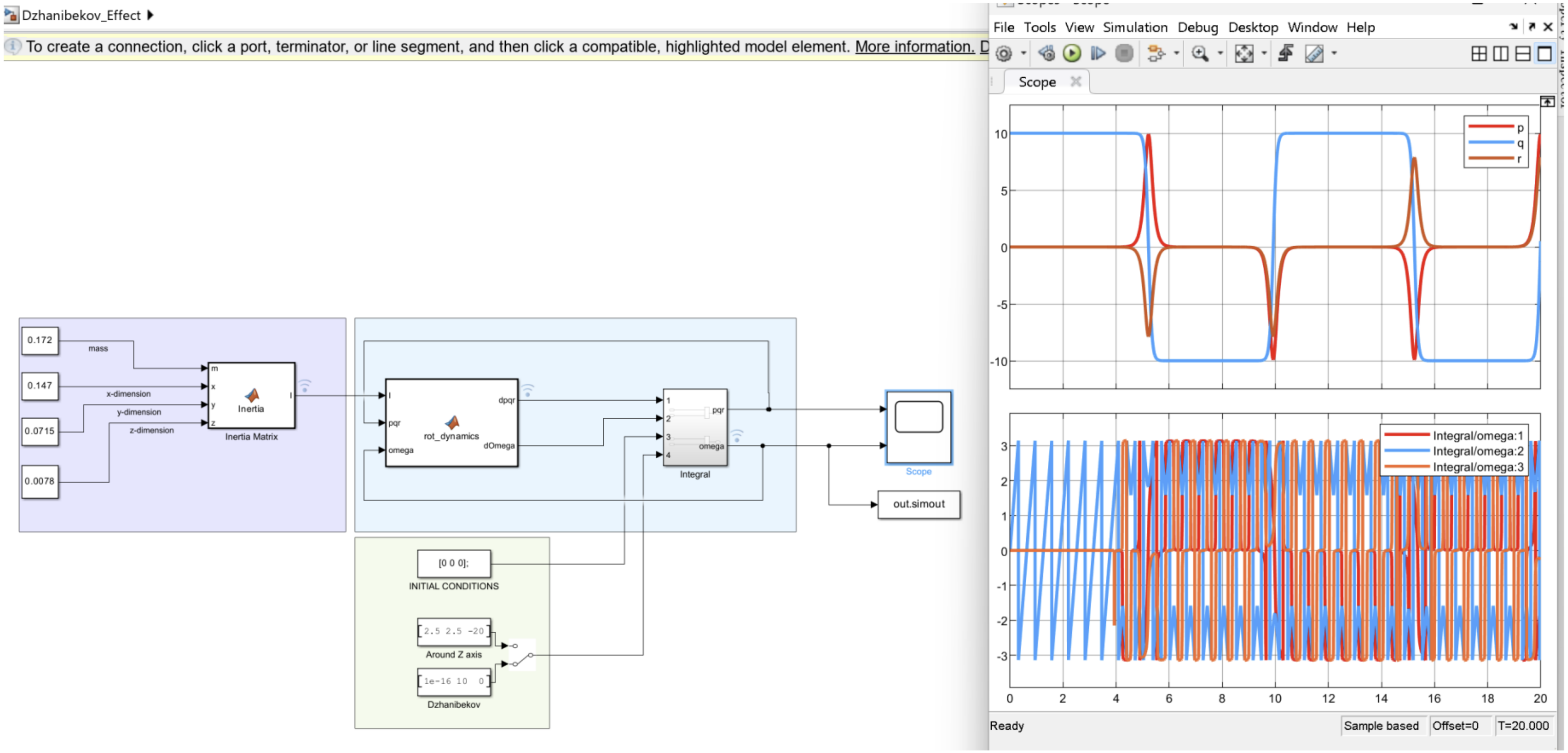

Figure 3: Dzhanibekov Effect Simulation

Add Extra Page if Needed

HW08: Include Best Graphs, Analysis, Etc

Please Include the Graphs Comparing Angular Velocities from Simulink and Experiment

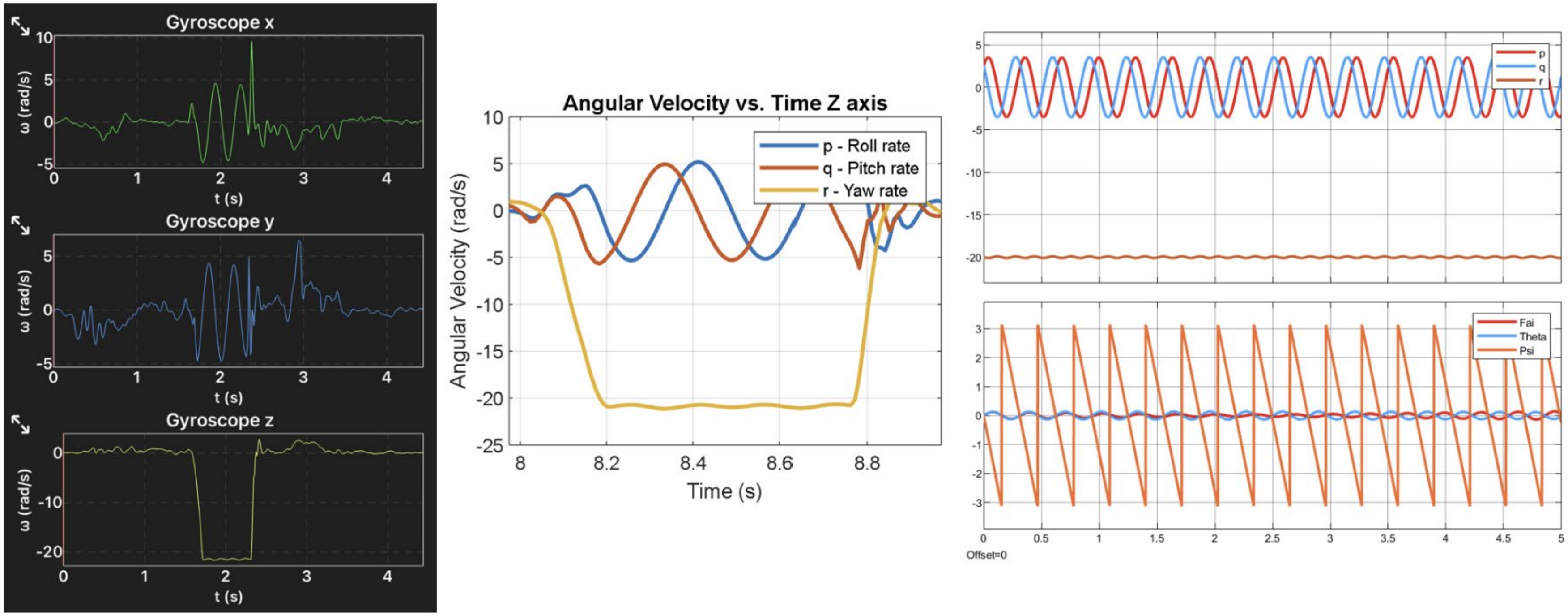

Figure 4: Rotate about Z axis (maximum axis)

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Final

HW08: Include Best Graphs, Analysis, Etc

Please Include the Graphs Comparing Angular Velocities from Simulink and Experiment

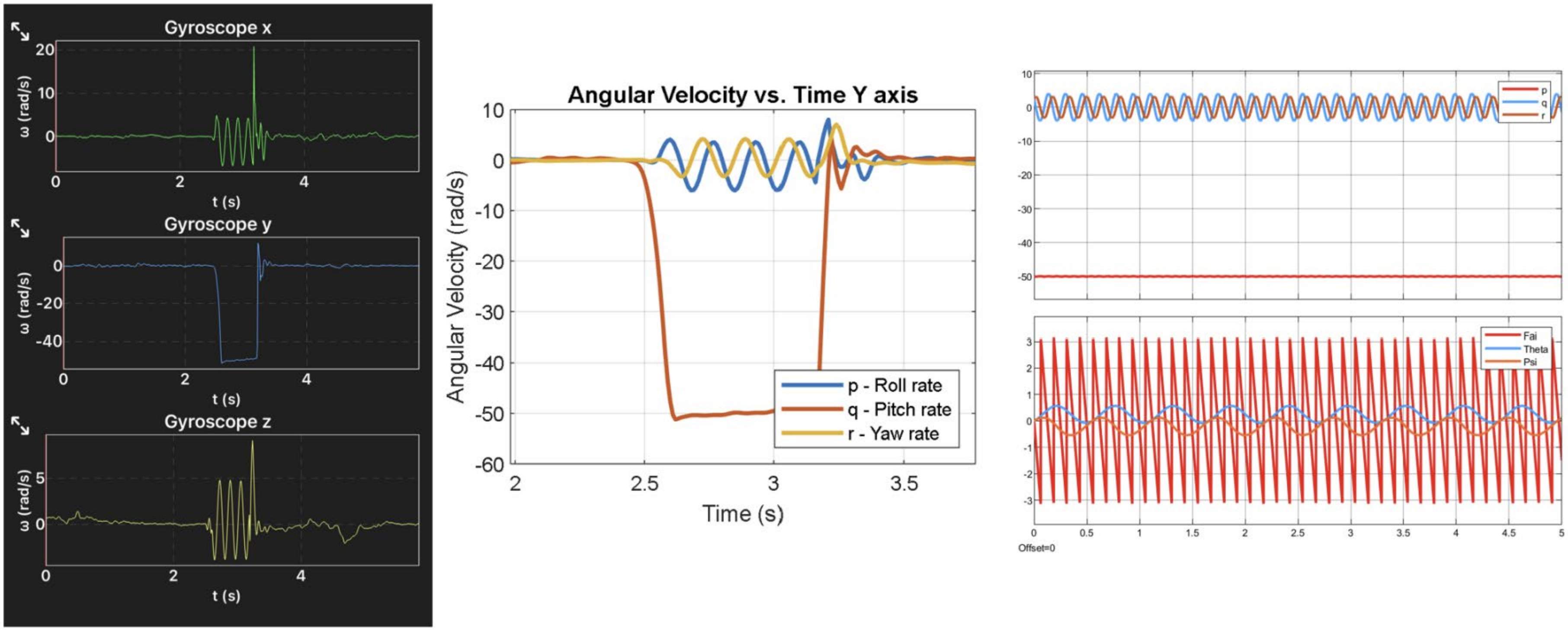

Figure 5: Rotate about y axis(minimum axis)

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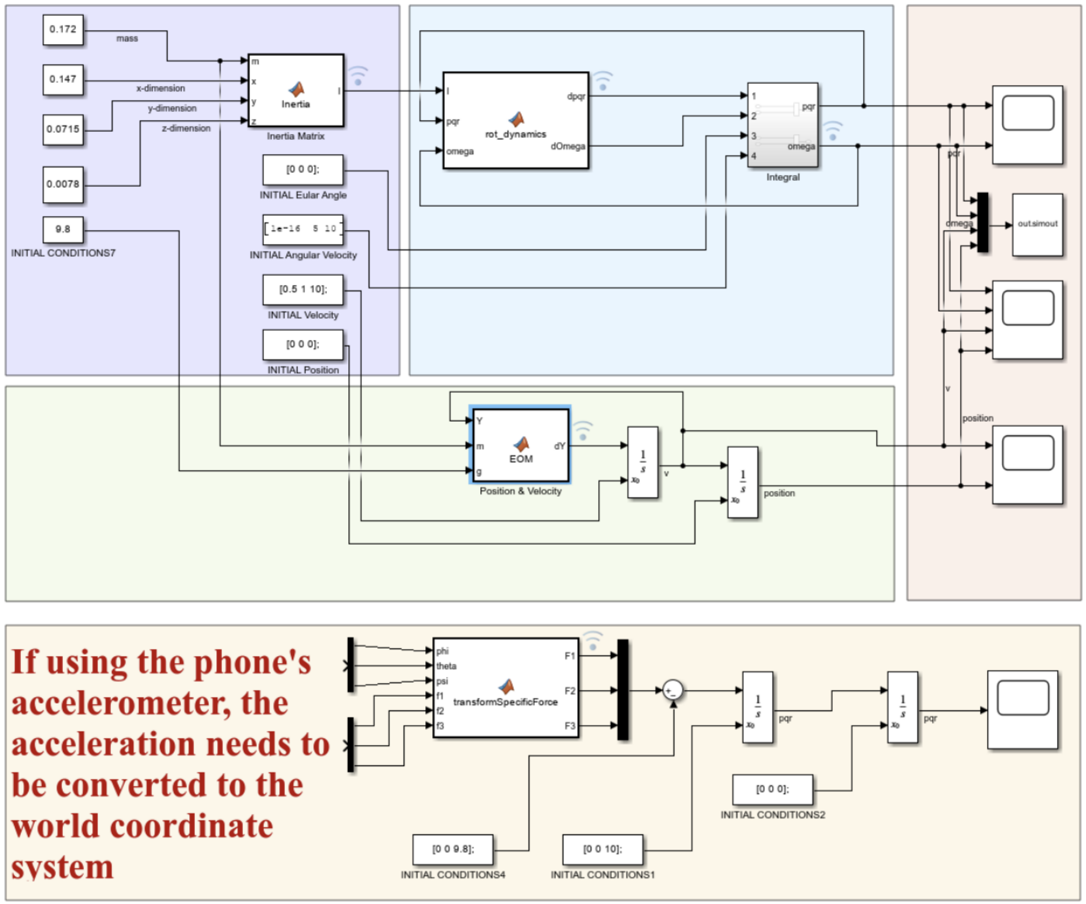
HW09: Include Best Graphs, Analysis, Etc

Please Include from Problem #3 & Results from Any Other Problems You Want to Show



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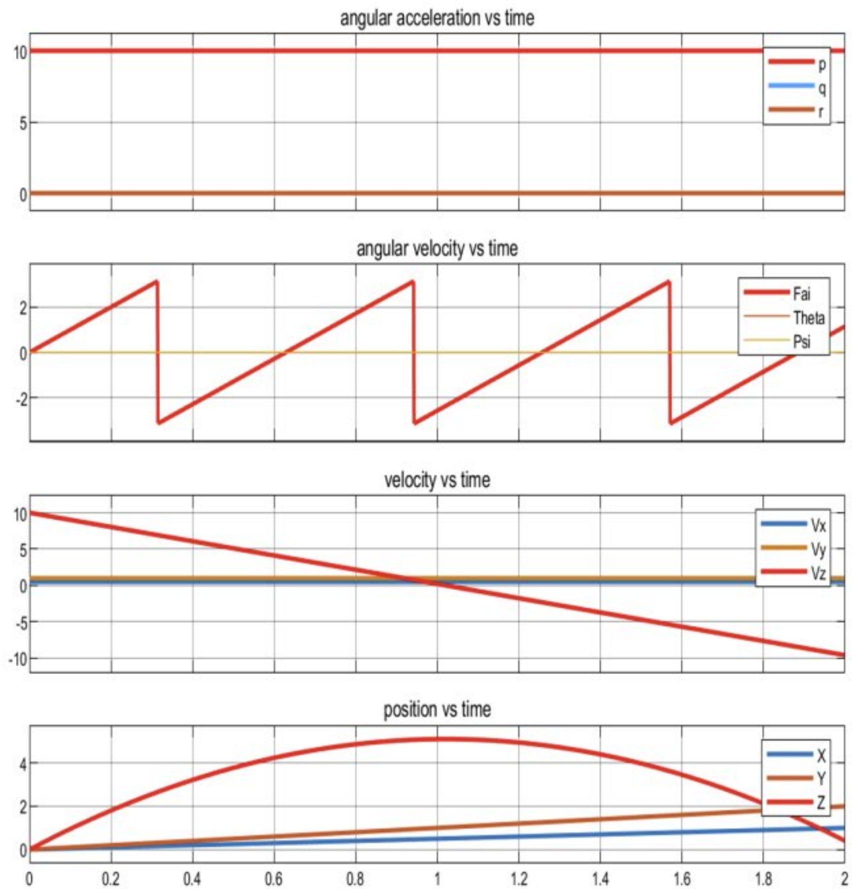
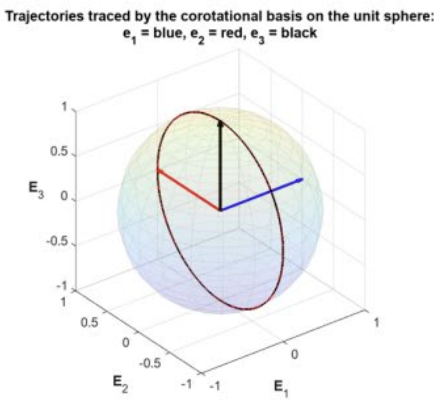
Final

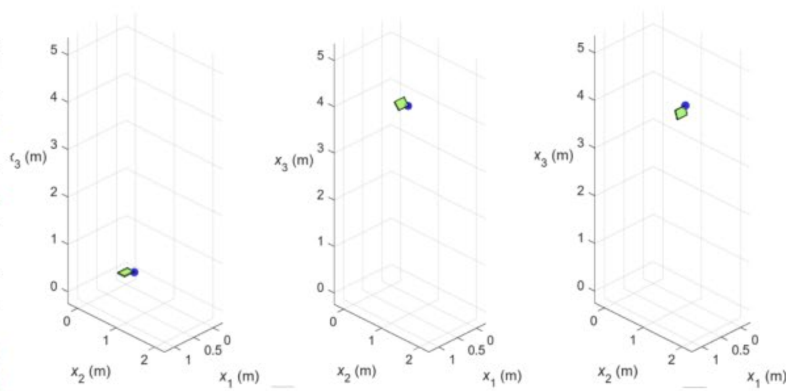
HW09: Include Best Graphs, Analysis, Etc

Please Include from Problem #3 & Results from Any Other Problems You Want to Show

Reference: https://rotations.berkeley.edu/a-book-tossed-in-the-air/







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HW09: Include Best Graphs, Analysis, Etc

Please Include from Problem #3 & Results from Any Other Problems You Want to Show

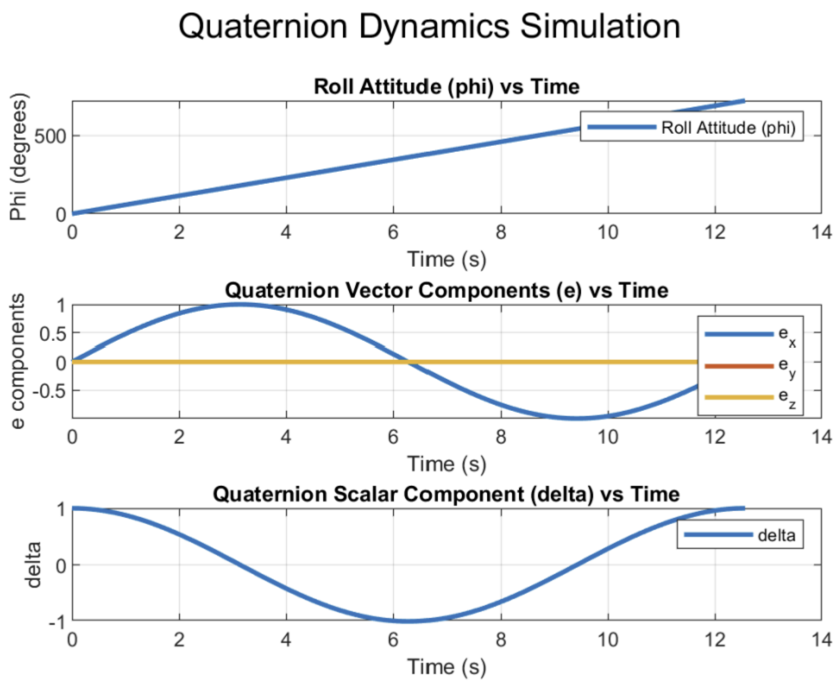

Figure 4: Results of HW9-3-a

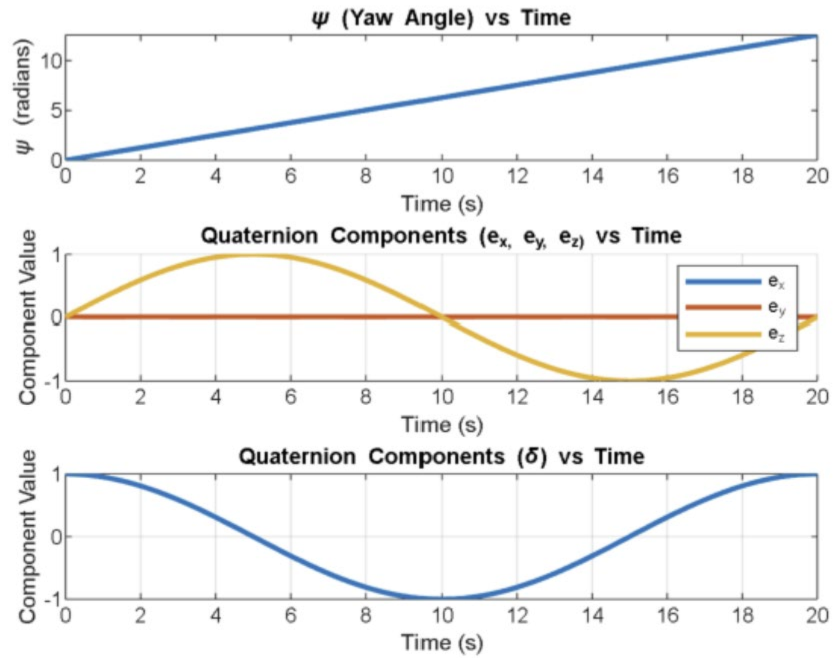

Figure 5: HW9-3-b-result

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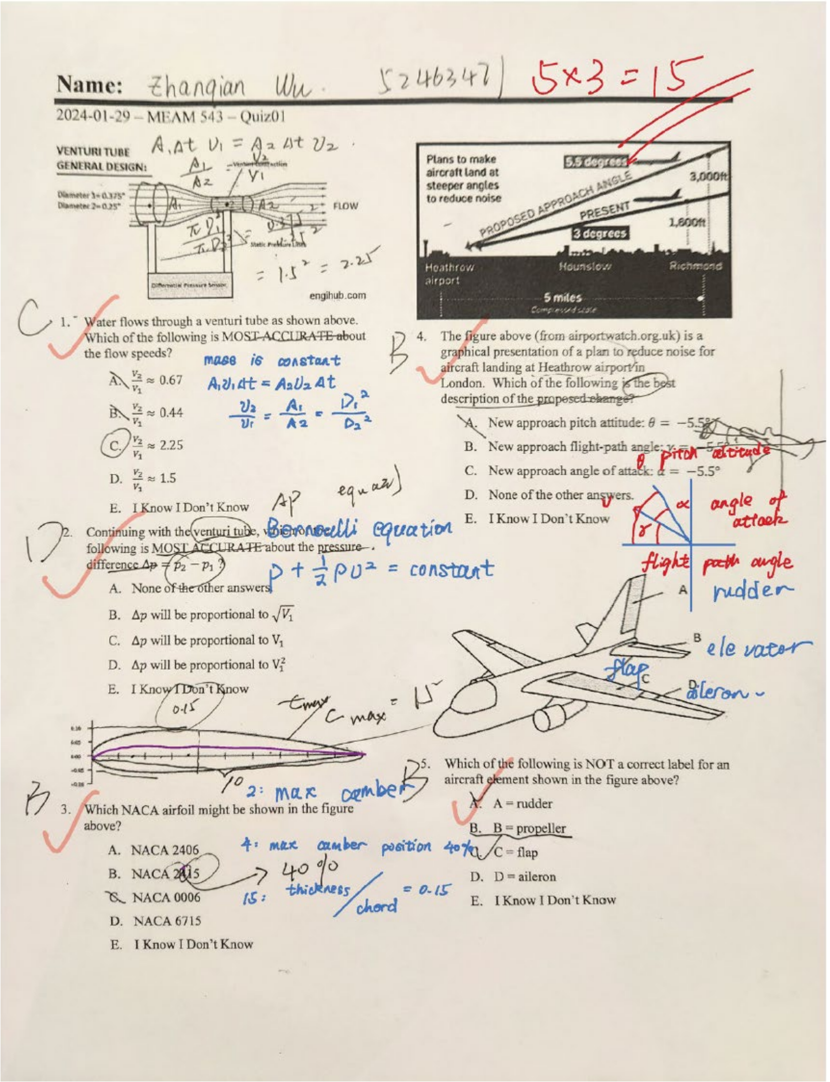
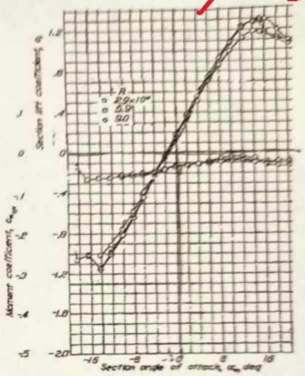
Quiz #1 Photo, Comments, Final Score



Quiz #2 Photo, Comments, Final Score

Name: Zhangian Wu $3\times3 + 2\times1 + 1 = 12 / 15$

2024-02-05 – MEAM 543 – Quiz02



C ✓ 1. Which of the following is most accurate about the procedure for submitting a homework solution after the assignment on canvas has closed, making the homework "rogue"?

A. Email the solution to the professor.

B. On Canvas, click on the assignment, then click on the "go rogue" button in the lower left corner.

C. On Canvas, click on "Grades" then click the assignment, then click on "upload comment" and attach the file.

D. This isn't relevant for me, because if I accept the very loose due dates, I just know I will fall behind and then I won't learn much. So, I'm going to force myself to do the work in a timely manner by asking the professor to give me a zero on any assignment that would be rogue.

E. I Know I Don't Know

D ✓ 2. Which of the following is MOST ACCURATE about "stall" on an airplane wing?

A. Stall limits the maximum value of lift coefficient.

B. Stall is caused by flow separation on the upper surface of the wing.

C. Stall angle depends on Reynolds number.

D. All of the other answers.

E. I Know I Don't Know.

gliding
T = 0

$L = W\cos\gamma$

$T = D + W\sin\gamma$

$-\frac{D}{L} = \tan\gamma$

C ✗ 3. Which of the following is the equation for the flight-path angle of a gliding aircraft (no thrust)?

A. $\tan\gamma = -\frac{D}{L}$

B. $\sec\gamma \approx 0$

C. $\sin\gamma = -\frac{W}{L}$

D. None of the other answers.

E. I Know I Don't Know

C ✓ 4. The figure above shows measurements[1] for lift and moment of an NACA 4-digit airfoil. Which of the following is the MOST REASONABLE inference about the airfoil based on these data?

A. The zero-lift angle is about -2°, so the airfoil has a small positive camber.

B. Stall is gradual, so the airfoil is relatively thick.

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know

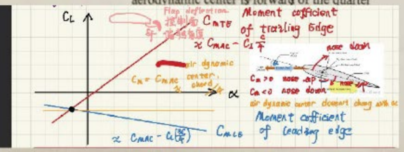
B ✗ 5. Continuing with the figure from the previous problem, which of the following is MOST ACCURATE about the moment data?

A. The moment at zero lift is negative, so the airfoil has negative camber.

B. The moment at the quarter chord (shown in the figure) has positive slope, so the aerodynamic center is forward of the quarter

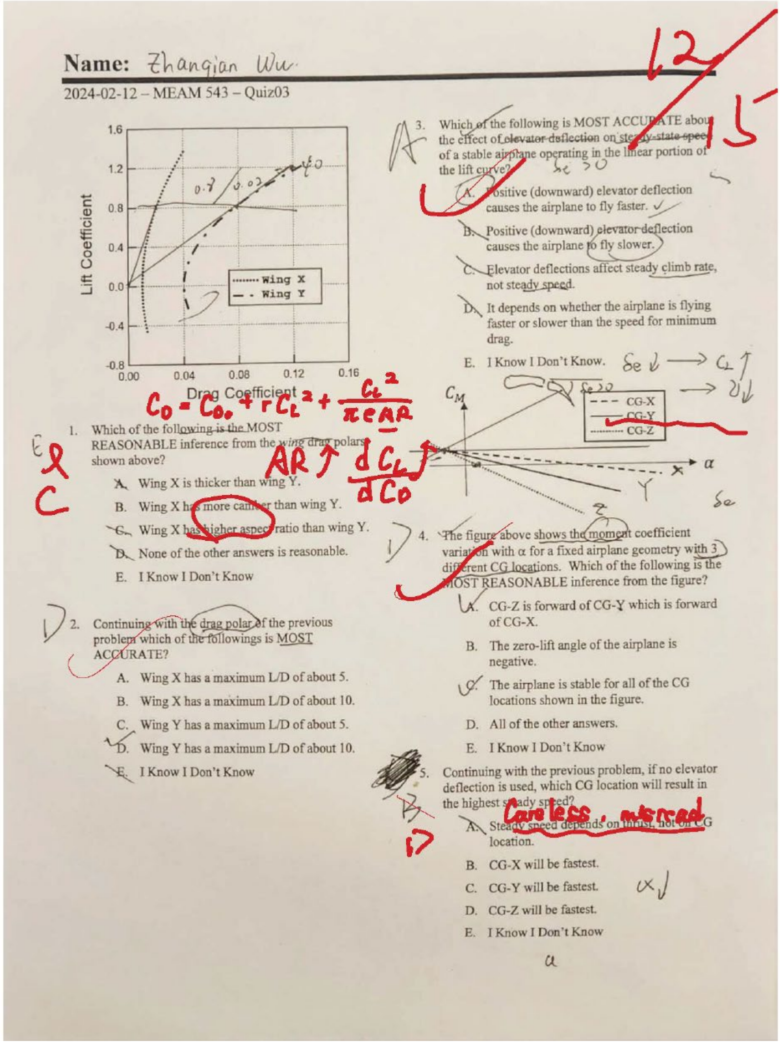
$$\frac{v}{V} = \frac{[\sin(\alpha_0+\phi)+\sin(\alpha_0+\epsilon_{TE})]\,e^{\psi_0}}{\sqrt{(\sinh^2\psi+\sin^2\theta)\left[\left(1-\frac{d\epsilon}{d\phi}\right)^2+\left(\frac{d\psi}{d\phi}\right)^2\right]}}$$

No question about the formula for velocity distribution on an airfoil from conformal mapping

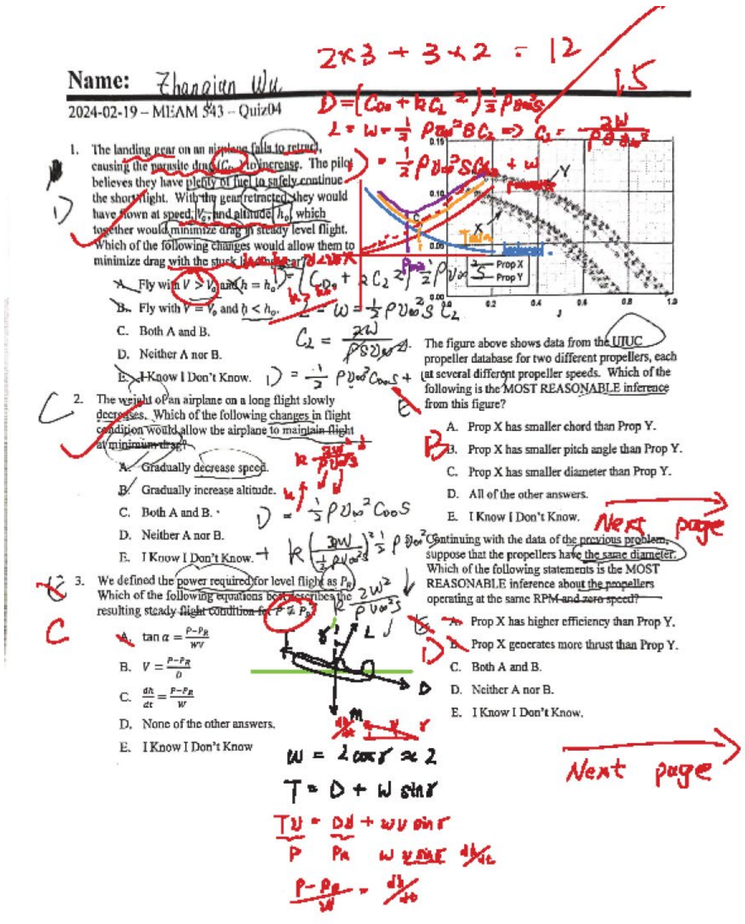


[1] NACA Report 824, *Summary of Airfoil Data*

Quiz #3 Photo, Comments, Final Score



Quiz #4 Photo, Comments, Final Score



$2\times3 + 3\times2 = 12$

1.5

Name: Zhangian Wu

2024-02-19 – MEAM 543 – Quiz04

1. The landing gear on an airplane fails to retract, causing the parasite drag (C_{D_0}) to increase. The pilot believes they have plenty of fuel to safely continue the short flight. With the gear retracted, they would have flown at speed, V_0, and altitude, h_0, which together would minimize drag in steady level flight. Which of the following changes would allow them to minimize drag with the stuck landing gear?
   - A. Fly with $V > V_0$ and $h = h_0$.
   - B. Fly with $V = V_0$ and $h < h_0$.
   - C. Both A and B.
   - D. Neither A nor B.
   - E. I Know I Don't Know.

2. The weight of an airplane on a long flight slowly decreases. Which of the following changes in flight condition would allow the airplane to maintain flight at minimum drag?
   - A. Gradually decrease speed.
   - B. Gradually increase altitude.
   - C. Both A and B.
   - D. Neither A nor B.
   - E. I Know I Don't Know.

3. We defined the power required for level flight as P_R. Which of the following equations best describes the resulting steady flight condition for $P \neq P_R$?
   - A. $\tan\alpha = \frac{P-P_R}{WV}$
   - B. $V = \frac{P-P_R}{D}$
   - C. $\frac{dh}{dt} = \frac{P-P_R}{W}$
   - D. None of the other answers.
   - E. I Know I Don't Know

4. The figure above shows data from the UIUC propeller database for two different propellers, each at several different propeller speeds. Which of the following is the MOST REASONABLE inference from this figure?
   - A. Prop X has smaller chord than Prop Y.
   - B. Prop X has smaller pitch angle than Prop Y.
   - C. Prop X has smaller diameter than Prop Y.
   - D. All of the other answers.
   - E. I Know I Don't Know.

5. Continuing with the data of the previous problem, suppose that the propellers have the same diameter. Which of the following statements is the MOST REASONABLE inference about the propellers operating at the same RPM ~~and zero speed~~?
   - A. Prop X has higher efficiency than Prop Y.
   - B. Prop X generates more thrust than Prop Y.
   - C. Both A and B.
   - D. Neither A nor B.
   - E. I Know I Don't Know.

$D = \left(C_{D_0} + kC_L^2\right)\frac{1}{2}\rho V_\infty^2 S$

$L = W = \frac{1}{2}\rho V_\infty^2 S C_L$

$C_L = \frac{2W}{\rho S V_\infty^2}$

$D = \frac{1}{2}\rho V_\infty^2 C_{D_0} S + k\left(\frac{2W}{\frac{1}{2}\rho V_\infty^2 S}\right)^2\frac{1}{2}\rho V_\infty^2 S$

$W = L\cos\gamma \approx L$

$T = D + W\sin\gamma$

$\frac{TV}{P} = \frac{DV}{P_R} + WV\sin\gamma$

$\frac{P-P_R}{W} = \frac{dh}{dt}$

Next page

Next page

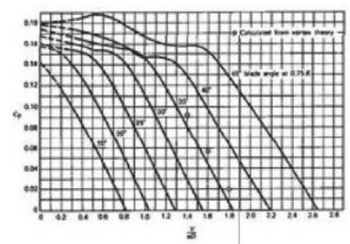


Figure 11.29: Typical propeller thrust curves as a function of advance ratio ($J = u_0/nD$) and blade angle (McCormick, 1979).

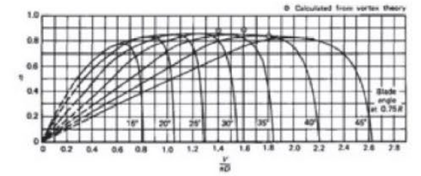


Figure 11.28: Typical propeller efficiency curves as a function of advance ratio ($J = u_0/nD$) and blade angle

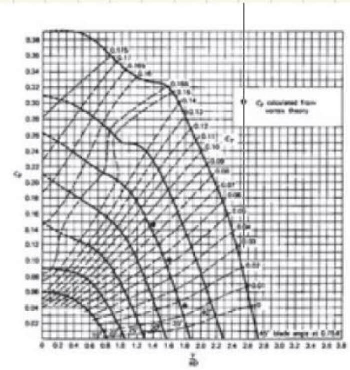
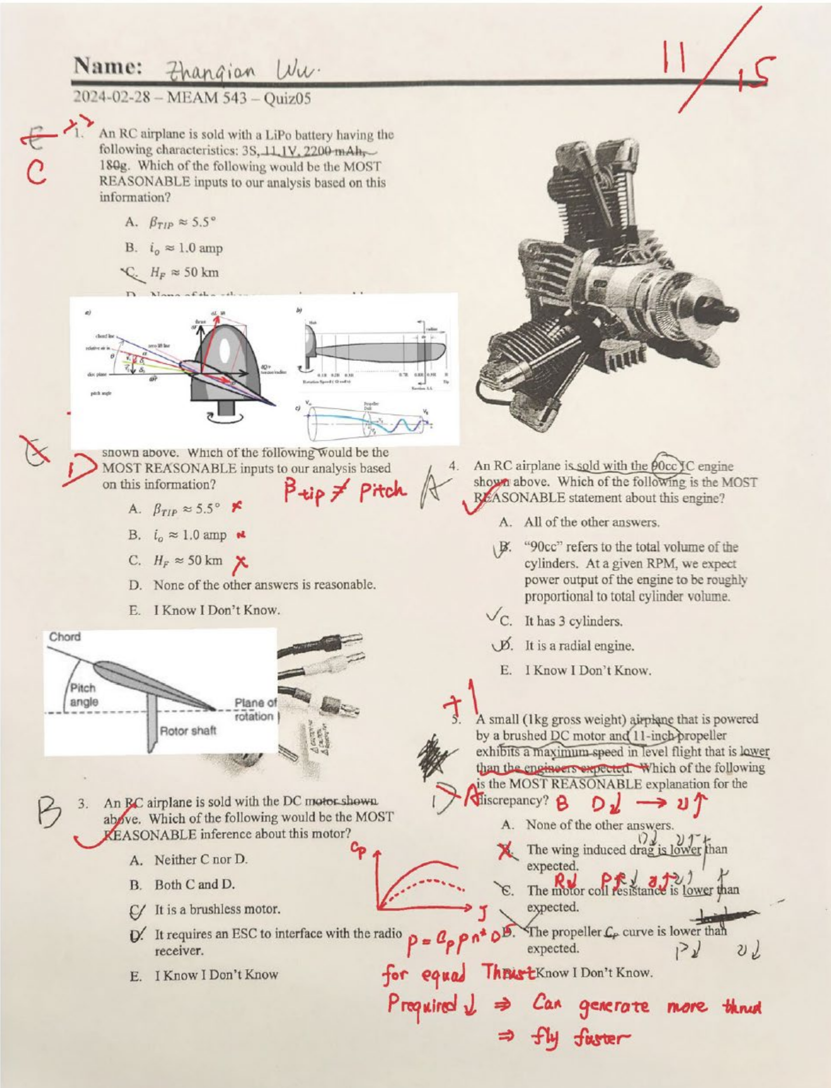


Figure 11.30: Typical propeller power curves as a function of advance ratio ($J = u_0/nD$) and blade angle (McCormick,

Quiz #5 Photo, Comments, Final Score



Name: Zhanqian Wu. 11/15

2024-02-28 – MEAM 543 – Quiz05

1. An RC airplane is sold with a LiPo battery having the following characteristics: 3S, 11.1V, 2200 mAh, 180g. Which of the following would be the MOST REASONABLE inputs to our analysis based on this information?
   - A. $\beta_{TIP} \approx 5.5°$
   - B. $i_o \approx 1.0$ amp
   - C. $H_F \approx 50$ km

+1 C

shown above. Which of the following would be the MOST REASONABLE inputs to our analysis based on this information?

- A. $\beta_{TIP} \approx 5.5°$ ✗
- B. $i_o \approx 1.0$ amp ✗
- C. $H_F \approx 50$ km ✗
- D. None of the other answers is reasonable.
- E. I Know I Don't Know.

D β-tip ≠ Pitch

3. An RC airplane is sold with the DC motor shown above. Which of the following would be the MOST REASONABLE inference about this motor?
   - A. Neither C nor D.
   - B. Both C and D.
   - C. It is a brushless motor.
   - D. It requires an ESC to interface with the radio receiver.
   - E. I Know I Don't Know

B

4. An RC airplane is sold with the 90cc IC engine shown above. Which of the following is the MOST REASONABLE statement about this engine?
   - A. All of the other answers.
   - B. "90cc" refers to the total volume of the cylinders. At a given RPM, we expect power output of the engine to be roughly proportional to total cylinder volume.
   - C. It has 3 cylinders.
   - D. It is a radial engine.
   - E. I Know I Don't Know.

A

+1

5. A small (1kg gross weight) airplane that is powered by a brushed DC motor and 11-inch propeller exhibits a maximum speed in level flight that is lower than the engineers expected. Which of the following is the MOST REASONABLE explanation for the discrepancy? B D↓ → v↑
   - A. None of the other answers.
   - B. The wing induced drag is lower than expected.
   - C. The motor coil resistance is lower than expected. R↓ P↓ v↑
   - D. The propeller C_P curve is lower than expected. P↓ v↓
   - E. I Know I Don't Know.

1) A

C_P vs. J

$P = C_P \rho n^3 D^5$

for equal Thrust

P required ↓ ⇒ Can generate more thrust

⇒ fly faster

Final

Quiz #6 Photo, Comments, Final Score

Name: Zhangian Wu

11/15

2024-03-13 – MEAM 543 – Quiz06

~~E~~ B

1. A young engineer is using a software program to estimate the size of a small UAV. The company has decided to invest in a new structural design which they estimate will reduce the aircraft empty weight fraction, $\frac{W_E}{W_0}$, from 0.6 to 0.5. When the engineer puts this change into the software, but keeps the payload weight the same, the estimated gross weight of the aircraft drops by a factor of 2! Which of the following is the MOST REASONABLE inference about this result?

   A. The fuel weight fraction, $\frac{W_F}{W_0}$, is about 0.3.

   B. The engineer has obviously made a mistake.

   C. The engineer must have accidentally changed the propulsion system from hydrocarbon to electric by mistake.

   D. None of the other answers is reasonable.

   E. I Know I Don't Know.

C

2. We have generally talked about airplanes cruising at the C_L that maximizes $\frac{L}{D}$. Suppose we have an airplane with a propulsion system which burns fuel at a rate that is proportional to thrust. If we want to maximize the *range* of the aircraft, how does this change the optimal C_L for cruise?

   A. We should still fly at the C_L for maximum $\frac{L}{D}$.

   B. We should fly at a higher C_L than the C_L for maximum $\frac{L}{D}$.

   C. We should fly at a lower C_L than the C_L for maximum $\frac{L}{D}$.

   D. It depends on altitude.

   E. I Know I Don't Know.

$\frac{L}{D} = \frac{C_L}{C_{D0} + kC_L^2}$

A 1)

3. Which of the following performance metrics generally increase with increased wing area, for a given drag polar, altitude, and aircraft weight?

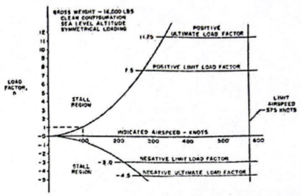
   A. Turn radius

   B. Stall speed

   C. Takeoff distance

   D. None of the other answers

   E. I Know I Don't Know



~~E~~ 1)

4. The figure above shows an example V-n diagram from the Air Training Command Manual (1963). The figure includes aircraft gross weight (16000 pounds) in the top left corner. Which of the following is the MOST REASONABLE conjecture about how the figure would change if the gross weight were increased without changing the altitude or the aerodynamic characteristics of the aircraft?

   A. All of the other answers.

   B. The "positive limit load factor" would decrease.

   C. The stall speed (currently about 100 knots) would decrease.

   D. The corner speed (currently about 300 knots) would decrease.

   E. I Know I Don't Know.

1)

5. Continuing with the figure of the previous question, what does the annotation "clean configuration" mean?

   A. Gear retracted.

   B. Flaps retracted.

   C. Neither A nor B.

   D. Both A and B.

   E. I Know I Don't Know.

T-45 Goshawk has similar weight and max speed

Final

Quiz #7 Photo, Comments, Final Score

Name: Zhangian Wu. 9/15

2024-03-25 – MEAM 543 – Quiz07

$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$
$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma$$

$m\frac{dv}{dt} = T - D - mg\sin\varphi_0$

1. According to the simple flight dynamics equations we wrote in lecture (above), which of the following MUST ~~be true about~~ an airplane that is flying vertically upward for 5 seconds?
   - A. The airplane has zero drag. $v \neq 0$ Drag $\neq 0$
   - B. The airplane has zero lift.
   - ~~C.~~ The airplane has thrust greater than weight. depends on $T-D-mg$
   - ~~D.~~ The airplane is decelerating.
   - E. I Know I Don't Know.

   E A

2. Continuing with the simple ~~flight dynamics~~ equations…While initially descending, an airplane executes a symmetric pull-up maneuver that is a circular arc ~~of radius R.~~ The maneuver is performed at constant thrust. At the end of the maneuver, the airplane is in steady level flight. Which of the following MUST be true during the maneuver?
   - ~~A.~~ $\frac{d\gamma}{dt}$ = constant throughout the maneuver.
   - B. $L > W$ throughout the maneuver.
   - ~~C.~~ $\frac{dV}{dt} = 0$ throughout the maneuver.
   - D. None of the other answers.
   - E. I Know I Don't Know.

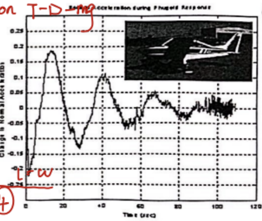
   B D $L = W$ $L < W$ $L > W$

3. Which of the following is an assumption that we used when deriving the flight dynamics equations above?
   - ~~A.~~ None of the other answers.
   - ~~B.~~ The airplane is statically stable (moment coefficient versus angle of attack has negative slope).
   - C. The airplane is flying above the speed for minimum thrust.
   - D. Thrust is aligned with velocity.
   - E. I Know I Don't't Know.

   D

4. Given that $0.7^2 + 2.4^2 = 2.5^2$, which of the following is a solution of
$$\frac{d^2\gamma}{dt^2} + 1.4\frac{d\gamma}{dt} + 6.25\gamma = 0\ ?$$
   - A. $\gamma = 0.1e^{-0.7t}\cos(2.4t)$
   - B. $\gamma = 0.7e^{-2.4t}\cos(0.1t)$
   - C. $\gamma = 2.4e^{-0.1t}\cos(0.7t)$
   - D. None of the other answers
   - E. I Know I Don't Know

   A $ae^{-bt}\cos(ct)$ $-ab\,e^{-bt}\cos(ct)$



5. The figure above shows the change in normal loadfactor (from the nominal value of 1g) during a phugoid evaluation flight test[2]. Which of the following is LEAST ACCURATE?
   - A. Accelerometer data is often noisy due to a variety of sources of vibration in the aircraft.
   - B. The variation in loadfactor implies that the pilot is moving the stick fore and aft to generate more or less lift. Pilot induced oscillation.
   - C. A linear analysis of this motion would produce a complex eigenvalue with a real part that is less than zero. stable mode require real part < 0
   - D. A linear analysis of this motion would produce a complex eigenvalue with imaginary part of approximately 0.25.
   - E. I Know I Don't Know.

   C B Next page

$r^2 + pr + q = 0$ $y'' + py' + qy = 0$

$r_1 \neq r_2$ $y = C_1e^{r_1x} + C_2e^{r_2x}$

$r_1 = r_2$ $y = (C_1 + C_2x)e^{r_1x}$

$r_{1,2} = \alpha \pm i\beta$ $y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$

$$r = \frac{-1.4 \pm \sqrt{1.4^2 - 4\times 6.25}}{2} = -0.7 \pm 2.4i$$

[2] https://old.bd-4.org/kenkopp/kenkopp_flighttest_4.html

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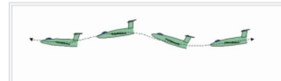
There can be two factors causing the landing mishaps mentioned:

Phugoid: the natural phenomenon of any aircraft.

Pilot Induced Oscillations: the pilot-in-the-loop oscillation of the aircraft while in control of a pilot.

To test the phugoid, trim the aircraft to a calm condition and do not touch the controls for several seconds.

In aviation, a **phugoid** or **fugoid** (/ˈfjuːɡɔɪd/) is an aircraft motion in which the vehicle pitches up and climbs, and then pitches down and descends, accompanied by speeding up and slowing down as it goes "downhill" and "uphill". This is one of the basic flight dynamics modes of an aircraft (others include short period, roll subsidence, dutch roll, and spiral divergence).



A diagrammatic representation of a fixed-wing airplane in phugoid

Detailed description [edit]

The phugoid has a nearly constant angle of attack but varying pitch, caused by a repeated exchange of airspeed and altitude. It can be excited by an elevator singlet (a short, sharp deflection followed by a return to the centered position) resulting in a pitch increase with no change in trim from the cruise condition. As speed decays, the nose drops below the horizon. Speed increases, and the nose climbs above the horizon. Periods can vary from under 30 seconds for light aircraft to minutes for larger aircraft. Microlight aircraft typically show a phugoid period of 15–25 seconds, and it has been suggested[by whom?] that birds and model airplanes show convergence between the phugoid and short period modes. A classical model for the phugoid period can be simplified to about (0.85 × speed in knots) seconds, but this only really works for larger aircraft.[further explanation needed]

Phugoids are often demonstrated to student pilots as an example of the speed stability of the aircraft and the importance of proper trimming. When it occurs, it is considered a nuisance, and in lighter airplanes (typically showing a shorter period) it can be a cause of pilot-induced oscillation.

The phugoid, for moderate amplitude,[1] occurs at an effectively constant angle of attack, although in practice the angle of attack actually varies by a few tenths of a degree. This means that the stalling angle of attack is never exceeded, and it is possible (in the <1g section of the cycle) to fly at speeds below the known stalling speed. Free flight models with badly unstable phugoid typically stall or loop, depending on thrust.[2]

An unstable or divergent phugoid is caused, mainly, by a large difference between the incidence angles of the wing and tail. A stable, decreasing phugoid can be attained by building a smaller stabilizer on a longer tail, or, at the expense of pitch and yaw "static" stability, by shifting the center of gravity to the rear.[why?][citation needed]

Aerodynamically efficient aircraft typically have low phugoid damping.[3]:464

The term "phugoid" was coined by Frederick W. Lanchester, the British aerodynamicist who first characterized the phenomenon. He derived the word from the Greek words φυγή and εἶδος to mean "flight-like" but recognized the diminished appropriateness of the derivation given that φυγή meant flight in the sense of "escape" (as in the word "fugitive") rather than vehicle flight.[4]

Final

Quiz #8 Photo, Comments, Final Score

Name: Zhanqian Wu 9/15

2024A – MEAM 543 – Quiz 8 – 3-April

1. Beginning at $\phi = \theta = \psi = 0$, an airplane rotates for one second with constant angular velocity: $p = \frac{\pi}{2}, q = r = 0$. It subsequently rotates one second with constant angular velocity: $p = r = 0$, $q = \frac{\pi}{2}$. The resulting Euler angles will be…
   - A. $\phi = \frac{\pi}{2}, \theta = \frac{\pi}{2}, \psi = 0$
   - B. $\phi = \frac{\pi}{2}, \theta = 0, \psi = \frac{\pi}{2}$
   - C. $\phi = 0, \theta = \frac{\pi}{2}, \psi = \frac{\pi}{2}$
   - D. None of the other answers.
   - E. I Know I Don't Know.

around body's x axis

$R_y(90°)R_x(90°)$

C

2. If $\phi = -\frac{\pi}{4}$ and $\theta = \psi = 0$, which of the following is correct when $\phi = \theta = 0$ and $\psi > 0$?
   - A. $p > 0$
   - B. $q > 0$
   - C. $r > 0$
   - D. All of the other answers.
   - E. I Know I Don't Know.

body's z axis

Euler's angle → world frame

p,q,r vehicle frame

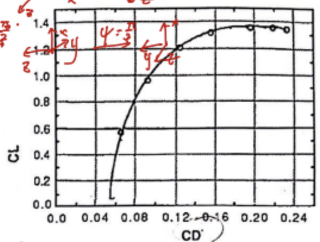
C

3. Which of the following is NOT a 3D rotation matrix?
   - A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
   - B. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
   - C. $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   - D. All of the other answers. (In other words, none of the other answers is a 3D rotation matrix.)
   - E. I Know I Don't Know.

- orthogonal. it's inverse is equal to transpose
- determinant must be +1
- columns & rows have a length of 1

B

B Not orthogonal & determinant ≠ +1



4. Which of the following is the MOST REASONABLE statement about the trimmed aircraft drag polar shown above?
   - A. Because $C_D \approx 0.95$ when $C_L \approx 1.0$, a reasonable value of K in our drag polar equation is 0.95
   - B. The wing is stalled for $C_L \gtrsim 1.3$.
   - C. Both A and B.
   - D. Neither A nor B.
   - E. I Know I Don't Know.

5. Continuing with the aircraft drag polar from the previous question…which of the following is the MOST REASONABLE expected consequence of reducing the span of the main wing, while keeping the main wing area constant?
   - A. The drag coefficient would decrease at all values of C_L.
   - B. The drag coefficient would increase mostly at high values of C_L.
   - C. The drag coefficient would increase mostly at low values of C_L.
   - D. None of the other answers. (Wingspan does not affect drag coefficient in our simple aero model.)
   - E. I Know I Don't Know.

$C_D = C_{D_0} + kC_L^2$

$k = \frac{1}{\pi AR e}$

$\frac{b^2}{S}$

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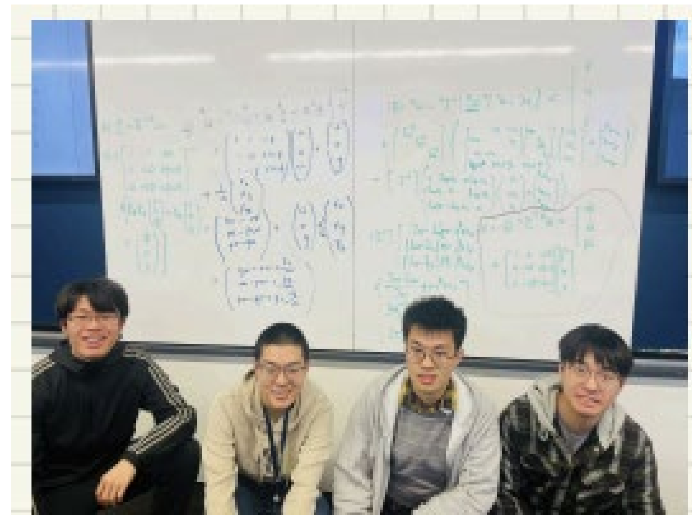


Final

Quiz #9 Photo, Comments, Final Score

$\frac{12}{15}$

Name: Zhanqian Wu.

2024A – MEAM 543 – Quiz 9 – 17-April

1. Which rotation matrix corresponds to $\phi = 0, \theta = 0, \psi = \pi/3$?

   general rotation Z − Y − X order

   A. $\begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

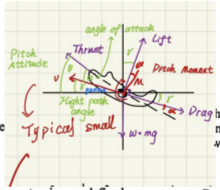
   $R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -s\phi \\ 0 & s\phi & \cos\phi \end{pmatrix}$

   B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$

   C. $\begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & -1/2 \end{bmatrix}$

   D. None of the other answe

   E. I Know I Don't Know.



2. Which of the following is the quaternion, as defined in class, that corresponds to $\phi = 0, \theta = 0, \psi = \pi/3$?

   $\mu = (1, 0, 0)$ $\theta = \phi = \frac{\pi}{3}$

   A. None of the other answers. $(e_x, e_y, e_z) = (\mu_x, \mu_y, \mu_z)\sin(\frac{\theta}{2})$

   B. $\underline{q} = \begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}^T$

   C. $\underline{q} = (\pi/3 \quad 0 \quad 0 \quad 1)^T$ $S = \cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{2}$

   out of order

   D. $\underline{q} = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}^T$ $(\frac{1}{2}, 0, 0)$ $S = \frac{\sqrt{3}}{2}$

   E. I Know I Don't Know.

3. For flight at zero sideslip, which of the following is the correct relationship between the aerodynamic body-axis X ~~force and the lift~~ and drag forces?

   A. $X = -D + L \tan\alpha$

   B. $X = -D \cot\alpha$

   $X = -D\cos\alpha + L\sin\alpha$

   None of the other answers.

   I Know I Don't Know.

hich of the following is a state in the 12-state namic model of aircraft dynamics that we veloped in class?

   All of the other answers.

   B. Pitch attitude, θ

   C. Angle of attack, α

   D. Flight-path angle, γ

   E. I Know I Don't Know.

   position $(x, y, z)_E$
   velocity (u, v, w)
   attitude (φ, θ, ψ) roll pitch yaw
   rate (p, q, r)

5. If we write the dynamic model that we developed in class as $\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u})$, which of the following is true about the derivatives of $\underline{f}$?

   A. $\frac{\partial \underline{f}}{\partial w} = \underline{0}$

   B. $\frac{\partial \underline{f}}{\partial \delta_e} = \underline{0}$

   C. $\frac{\partial \underline{f}}{\partial q} = \underline{0}$

   D. $\frac{\partial \underline{f}}{\partial \psi} = \underline{0}$

   E. I Know I Don't Know.

In this section, we develop the small-disturbance equations for longitudinal motions in standard state-variable form. Recall that the linearized equations describing small longitudinal perturbations from a longitudinal equilibrium state can be written

$$\left[\frac{d}{dt} - X_u\right] u + g_0 \cos\Theta_0 \theta - X_w w = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T$$
$$-Z_u u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right] w - [u_0 + Z_q] q + g_0 \sin\Theta_0 \theta = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T \quad (5.37)$$
$$-M_u u - \left[M_{\dot{w}}\frac{d}{dt} + M_w\right] w + \left[\frac{d}{dt} - M_q\right] q = M_{\delta_e}\delta_e + M_{\delta_T}\delta_T$$

If we introduce the longitudinal state variable vector

$$\mathbf{x} = [u \quad w \quad q \quad \theta]^T \quad (5.38)$$

and the longitudinal control vector

$$\eta = [\delta_e \quad \delta_T]^T \quad (5.39)$$

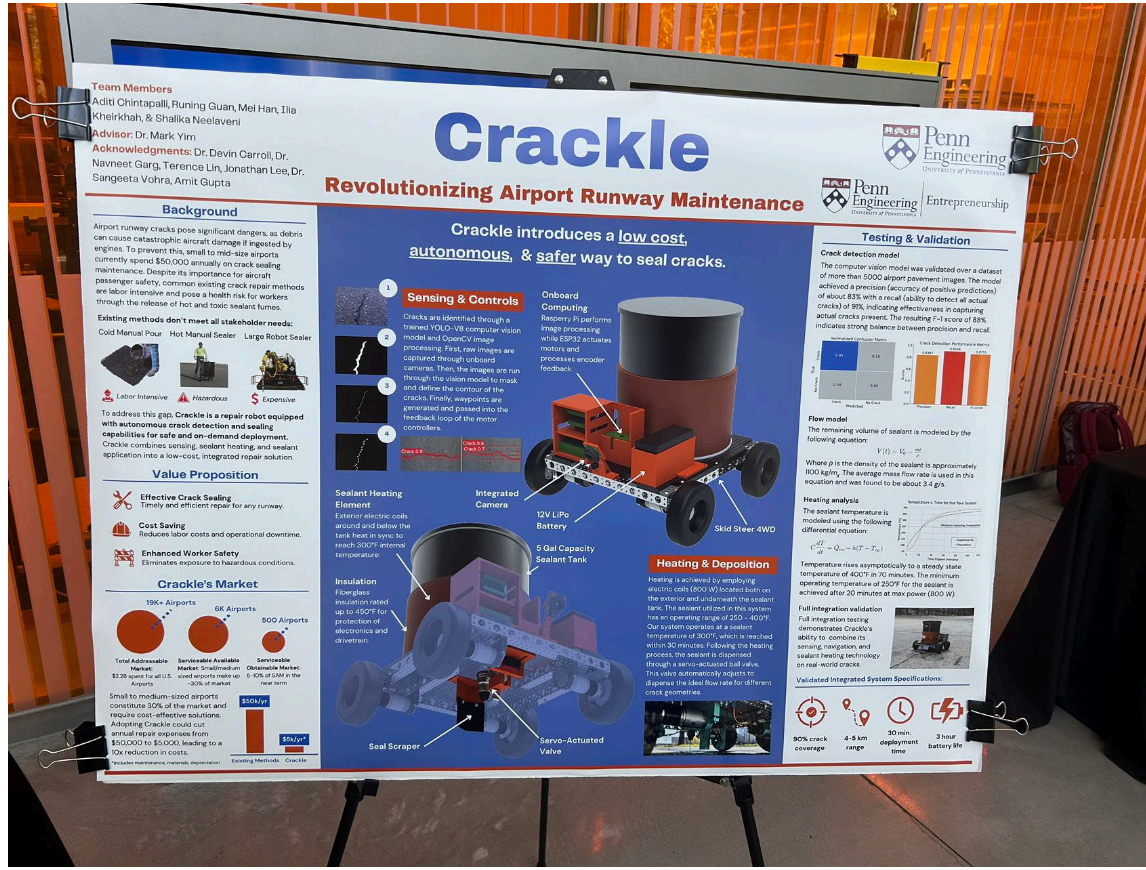
these equations are equivalent to the system of first-order equations

$$\mathbf{I}_n \dot{\mathbf{x}} = \mathbf{A}_n \mathbf{x} + \mathbf{B}_n \eta \quad (5.40)$$

where $\dot{\mathbf{x}}$ represents the time derivative of the state vector $\mathbf{x}$, and the matrices appearing in this equation are

$$\mathbf{A}_n = \begin{pmatrix} X_u & X_w & 0 & -g_0\cos\Theta_0 \\ Z_u & Z_w & u_0 + Z_q & -g_0\sin\Theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\mathbf{I}_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - Z_{\dot{w}} & 0 & 0 \\ 0 & -M_{\dot{w}} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{B}_n = \begin{pmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} & M_{\delta_T} \\ 0 & 0 \end{pmatrix} \quad (5.41)$$

MEAM Senior Design



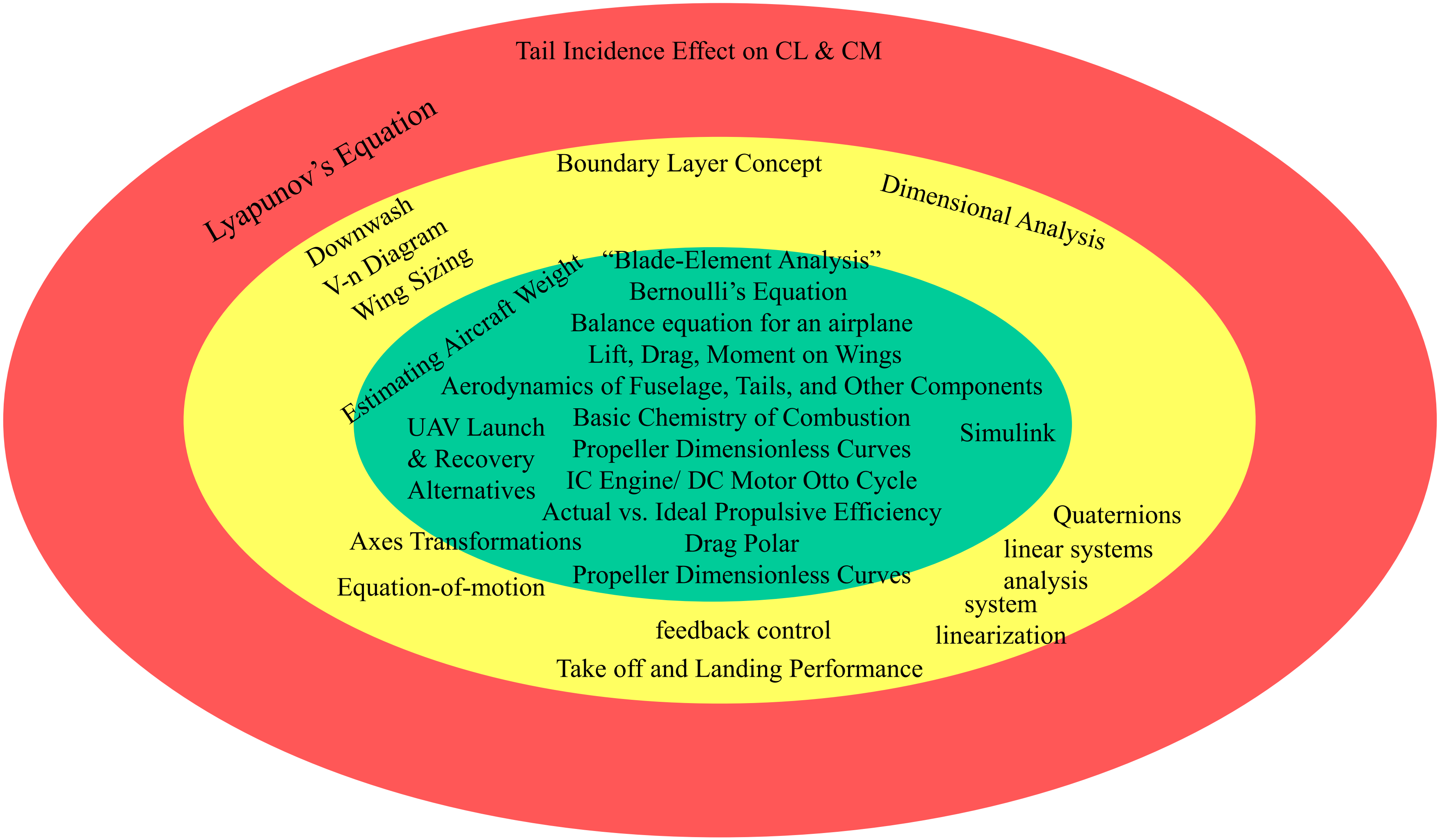
Crackle is presented as a low-cost, autonomous, and safer way to seal cracks, addressing issues with current runway crack sealing methods being labor-intensive, hazardous, and expensive.

The team's value proposition highlights effective crack sealing, cost savings, and enhanced worker safety.

The device has a sealant heating mechanism that operates at 400°F to ensure proper sealing, and an integrated camera for inspection.

Final (Update)

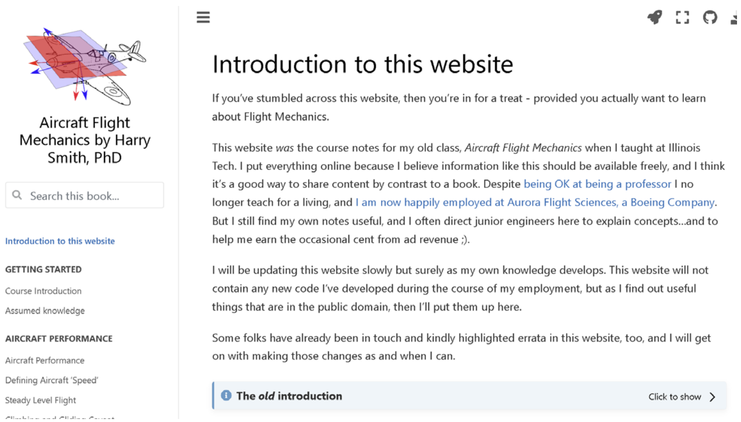
Concept Map (Understand; Confused; Lost)



Final

Bonus: Flight Dynamics Simulation Model

Please Include Anything You Want About the Flight Dynamics Simulation Model


Aircraft Flight Mechanics by Harry Smith, PhD

Search this book...

Introduction to this website

GETTING STARTED

Course Introduction

Assumed knowledge

AIRCRAFT PERFORMANCE

Aircraft Performance

Defining Aircraft 'Speed'

Steady Level Flight

Introduction to this website

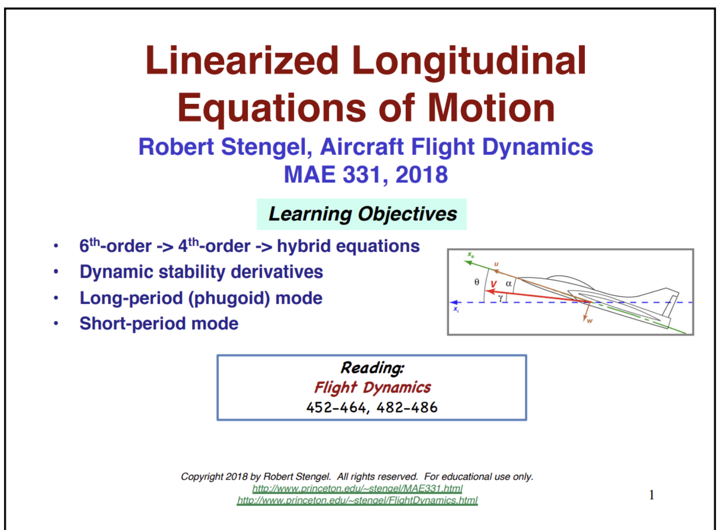
If you've stumbled across this website, then you're in for a treat - provided you actually want to learn about Flight Mechanics.

This website *was* the course notes for my old class, *Aircraft Flight Mechanics* when I taught at Illinois Tech. I put everything online because I believe information like this should be available freely, and I think it's a good way to share content by contrast to a book. Despite being OK at being a professor I no longer teach for a living, and I am now happily employed at Aurora Flight Sciences, a Boeing Company. But I still find my own notes useful, and I often direct junior engineers here to explain concepts...and to help me earn the occasional cent from ad revenue ;).

I will be updating this website slowly but surely as my own knowledge develops. This website will not contain any new code I've developed during the course of my employment, but as I find out useful things that are in the public domain, then I'll put them up here.

Some folks have already been in touch and kindly highlighted errata in this website, too, and I will get on with making those changes as and when I can.

The *old* introduction Click to show


Linearized Longitudinal Equations of Motion

Robert Stengel, Aircraft Flight Dynamics MAE 331, 2018

Learning Objectives

- 6^{th}-order -> 4^{th}-order -> hybrid equations
- Dynamic stability derivatives
- Long-period (phugoid) mode
- Short-period mode

Reading:
Flight Dynamics
452-464, 482-486

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http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

1

It's not enough to just listen to lectures in class/learn from the board, I think there are many similar lessons and resources online that can help us better understand the

[1]https://aircraftflightmechanics.com/NotesIntroduction.html#
[2]http://www.stengel.mycpanel.princeton.edu/MAE331Lecture14.pdf

MEAM 543 HW1

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1 Problem 1 – UAV Database

This project introduces a comprehensive solution for autonomous pick-and-place operations using the Franka Panda robotic arm. The system incorporates advanced features such as collision detection, path planning, and dynamic block manipulation. The collision detection algorithm ensures real-time monitoring and response, crucial for safeguarding the robot and its environment. A hybrid path planning approach, integrating Rapidly-exploring Random Trees (RRT) and linear interpolation, showcases superior efficiency. The system demonstrates success in stacking both static and dynamic blocks, with physical tests revealing reliable performance, despite occasional mismatches. Valuable lessons learned from challenges encountered during a competition underscore the importance of designing robust, adaptable systems for real-world applications.

2 Problem 2 – How Does an RC Airplane Work?

2.1 Selfie

2.2 What was the most difficult or confusing part?

Understanding Aerodynamics: The principles of flight and how control surfaces affect the airplane's movement can be complex. The relationship between lift, drag, thrust, and weight is fundamental to understanding how an airplane flies.

Control Surfaces and Their Functions: The ailerons, elevator, rudder, and flaps each play a specific role in controlling the aircraft. It can be confusing to remember which control surface affects which axis of flight (roll, pitch, and yaw).

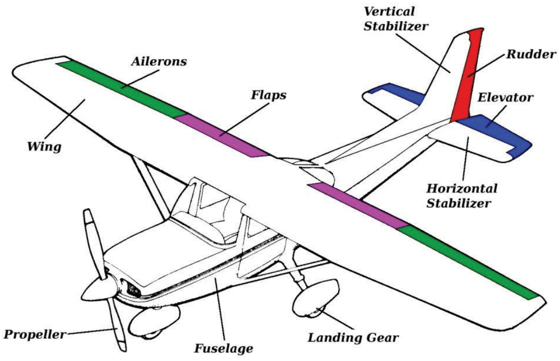
Radio Transmission: How the transmitter sends signals to the receiver to control the airplane, including the use of different channels for different functions, might be hard to grasp.

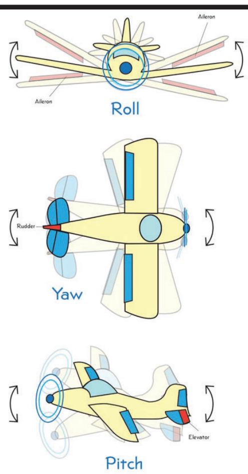
Electronics and Wiring: There are many components and cable harnesses in the system, and how to connect them makes a beginner's head spin!

2.3 What else do you want to know?

- How FPV systems are integrated into RC airplanes, allowing pilots to fly from the cockpit view.
- The rules and best practices for flying RC airplanes safely to avoid accidents or violations of airspace regulations.
- How different wing shapes and airfoil designs affect flight characteristics.

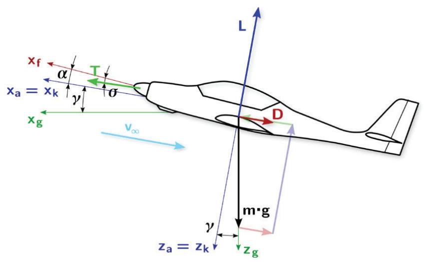
Basic Components of RC planes

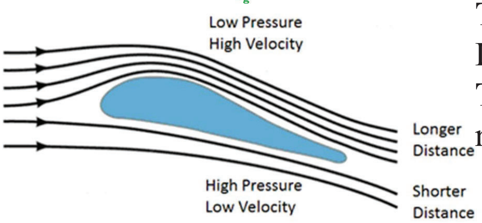




Ailerons: Control the roll of the aircraft by altering the lift on each wing.
Elevators: Adjust the pitch of the airplane, allowing it to ascend or descend.
Rudder: Controls the yaw, helping the plane to turn left or right.
Flaps: Increase lift at lower speeds, particularly during takeoff and landing.

Basic Principles of Aircraft





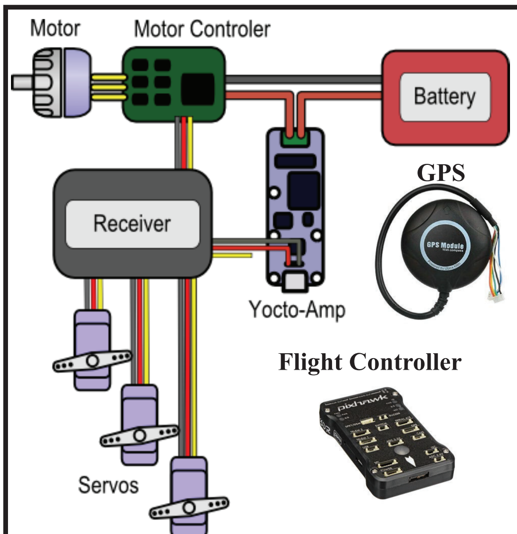
The shape of airfoil is designed so that air travels faster over the top surface than the bottom surface.
According to Bernoulli's Principle, faster-moving air has lower pressure. Therefore, the pressure on the top surface of the wing is less than the pressure on the bottom surface, creating an upward force known as lift.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

The force balance in an airplane involves the balance of four main forces: Lift, Weight, Thrust, and Drag.
Torque balance involves the rotation of the airplane about its three axes: roll, pitch, and yaw.

$$\begin{cases} \text{Lift (L)} = \text{Weight (W)} & \text{(Vertical Force Balance)} \\ \text{Thrust (T)} = \text{Drag (D)} & \text{(Horizontal Force Balance)} \\ \text{Total Moment (M)} = 0 & \text{(Moment Stability)} \end{cases}$$

Electronic equipment for RC aircraft



For UAV

Transmitter: The handheld device that sends control signals to the aircraft, allowing the pilot to maneuver it remotely.
Receiver: Installed in the aircraft, it receives signals from the transmitter and relays commands to control the servos and motors.
Servos: Small motors that move the control surfaces (like ailerons and rudders) according to the signals received, allowing for precise maneuvering.
Electronic Speed Controller: Regulates power to the electric motor, controlling speed and direction based on input from the receiver.
Battery: Provides the electrical power necessary for the motor, receiver, and other onboard electronics; typically a rechargeable lithium-polymer (LiPo) battery.
Motor: Converts electrical energy from the battery into mechanical energy to spin the propeller, generating thrust for the aircraft.
Flight Controller: An advanced system in some RC aircraft that helps stabilize flight, manage autonomous flying, and integrate various sensors.
GPS Module: Used in advanced RC models for navigation, autonomous flight, and real-time location tracking.

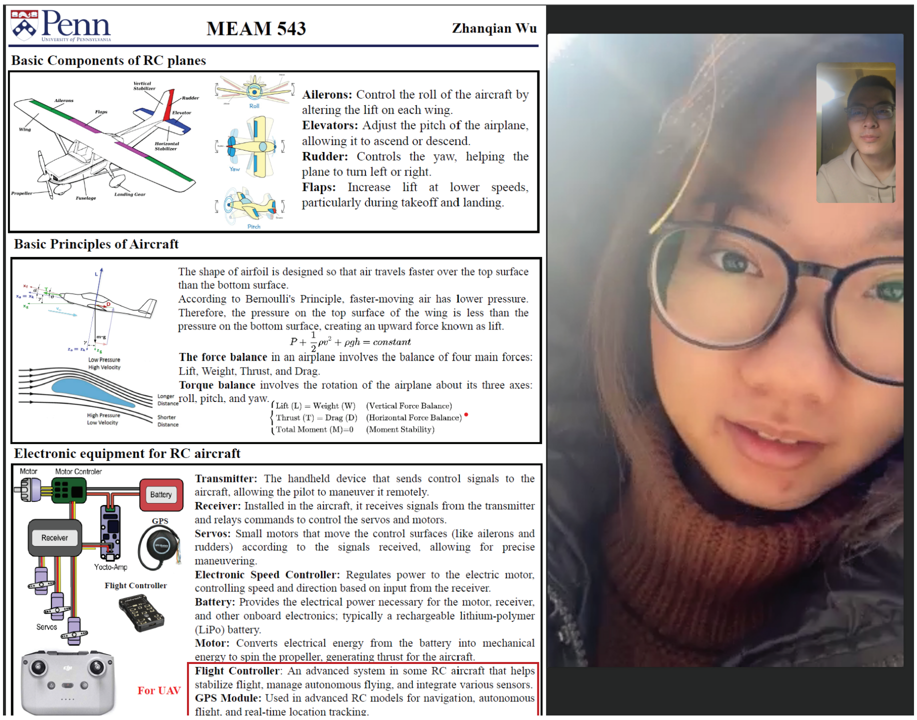


Figure 1: Selfie

3 Problem 3 – Eigenvalue Eigenvector Review

Consider the matrix

$$A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix} \tag{1}$$

To find the eigenvalues of a matrix, we need to solve the characteristic equation, which is derived from the matrix equation:

$$\det(A - \lambda I) = 0 \tag{2}$$

where det is the determinant, A is the given matrix, λ is the eigenvalue, and I is the identity matrix of the same size as A.

Hence, the eigenfunction is:

$$-\lambda^3 + 4\lambda^2 + 27\lambda - 90 = 0 \tag{3}$$

where we have three solutions:

$$\lambda_1 = -5, \lambda_2 = 3, \lambda_3 = 6, \tag{4}$$

For each eigenvalue λ, solve $(A - \lambda I)q = 0$ to find the corresponding eigenvector q.

For example, given the eigenvalue $\lambda_1 = -5$

$$(A - \lambda I)q = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 6 & 2 \\ 4 & 2 & 10 \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{12} \\ q_{13} \end{bmatrix} = 0 \tag{5}$$

Hence,

$$\underline{q}_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \tag{6}$$

Similarly, we can obtain the eigenvalues and the eigenvector of the matrix A as:

$$\begin{aligned} \lambda_1 &= -5, \underline{q}_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \\ \lambda_2 &= 3, \;\; \underline{q}_2 = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \\ \lambda_3 &= 6, \underline{q}_3 = \begin{pmatrix} 1 \\ 6 \\ 16 \end{pmatrix} \end{aligned} \tag{7}$$

Suppose we have a linear system of ODEs expressed using vector-matrix notation:

$$\frac{dx}{dt} = A\underline{x} \tag{8}$$

Express the solution to the system in the form

$$\underline{x} = c_1 e^{\lambda_1 t}\underline{q}_1 + c_2 e^{\lambda_2 t}\underline{q}_2 + c_3 e^{\lambda_3 t}\underline{q}_3 \tag{9}$$

where the constants, $c_k$are found to satisfy the initial condition

$$\underline{x}(0) = \begin{pmatrix} 11 \\ 3 \\ -11 \end{pmatrix} \tag{10}$$

From the above process it is known that

$$\underline{x}_0 = c_1\underline{q}_1 + c_2\underline{q}_2 + c_3\underline{q}_3 \tag{11}$$

where

$$\begin{aligned} \lambda_1 &= -5, \underline{q}_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \\ \lambda_2 &= 3, \;\; \underline{q}_2 = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \\ \lambda_3 &= 6, \underline{q}_3 = \begin{pmatrix} 1 \\ 6 \\ 16 \end{pmatrix} \end{aligned} \tag{12}$$

which equals to:

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & 3 & 6 \\ -1 & 1 & 16 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0 \tag{13}$$

Solving the equation, we get

$$c_1 = -5, c_2 = 3, c_3 = 6, \tag{14}$$

Suppose we have a linear system of ODEs expressed using vector-matrix notation:

$$\frac{d\underline{x}}{dt} = A\underline{x} \tag{15}$$

Express the solution to the system in the form:

$$\underline{x} = c_1 e^{\lambda_1 t}\underline{q}_1 + c_2 e^{\lambda_2 t}\underline{q}_2 + c_3 e^{\lambda_3 t}\underline{q}_3 \tag{16}$$

where the constants, c_k , are found to satisfy the initial condition:

$$\underline{x}(0) = \begin{pmatrix} 11 \\ 3 \\ -11 \end{pmatrix} \tag{17}$$

From above conditions, we can know that

$$c_1 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 6 \\ 16 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \\ -11 \end{pmatrix} \tag{18}$$

solving the above equation, we can get

$$(c_1, c_2, c_3) = \begin{pmatrix} \frac{11}{2} & -\frac{1}{6} & -\frac{1}{3} \end{pmatrix} \tag{19}$$

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[1] How do remote controlled airplanes work? (n.d.). Www.youtube.com. Retrieved January 30, 2024, from .https://www.youtube.com/watch?v=rFPhnfet_t4

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[3] RC Electronics for Noobs. (2016). [YouTube Video]. In YouTube. https://www.youtube.com/watch?v=j61Q3e8AFR4

MEAM 543 HW2

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1 Problem 1 – Airbus Zephyr High-Altitude Pseudo Satellite (HAPS)

The basic specifications of Zephyr 7 is shown in Tab 1

Table 1: Zephyr 7 Aircraft Specifications

Specification	**Value**
Max Takeoff Weight	53 kg
Cruise Speed	56 km/hr (15.6 m/s)
Ceiling (Maximum Altitude)	21,500 m
Wing Area	32.6 m^2
Wing Span	22.5 m
Airfoil	Eppler E395

E395 Airfoil Information is shown in Fig 1

From the graph, we can see that

- $a_{2D} = \frac{1.0-0.7}{3} = 0.1$
- $c_{L_{max}=1.45}$
- $\alpha_0 = -6.5°$

By looking up "Standard Atmosphere" we can get a property of the air as a function of altitude in Tab 2

Fitting the table using the least squares method, we obtain the equation for the variation of air density with elevation for 25°C as Equ 1

$$\rho(h) = -0.0003h^3 + 0.0059h^2 - 0.1214h + 1.2264 \tag{1}$$

The comparison between the fitted and actual values is shown in Fig. 2. It can be seen that the error value between the fitted and actual values is very small, which means that the fitting is very good.

The formula for calculating the lift is:

$$L = C_l \frac{1}{2}\rho V^2 S \tag{2}$$

where:

- C_l is the lift coefficient.
- $L = mg = 530N$ is the lift force, which is equal to the weight of the aircraft in steady flight.
- ρ is the air density.

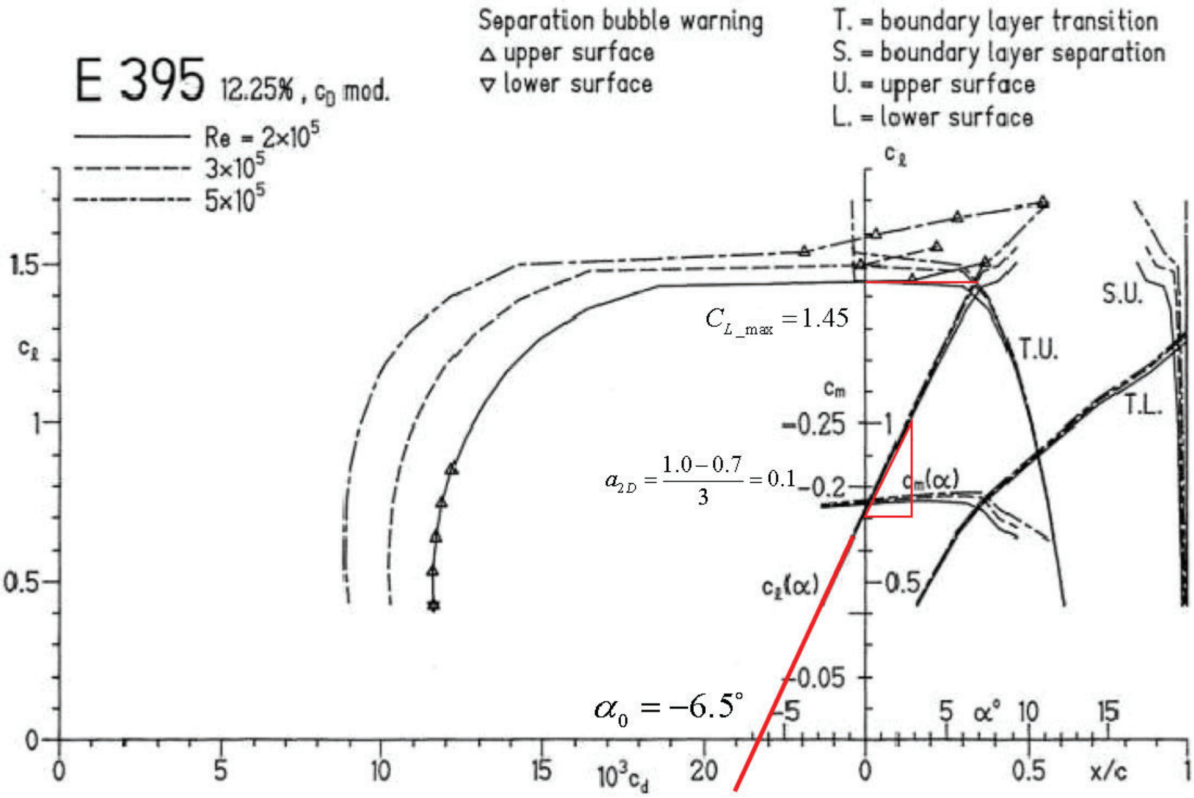

Figure 1: E395 Airfoil Information

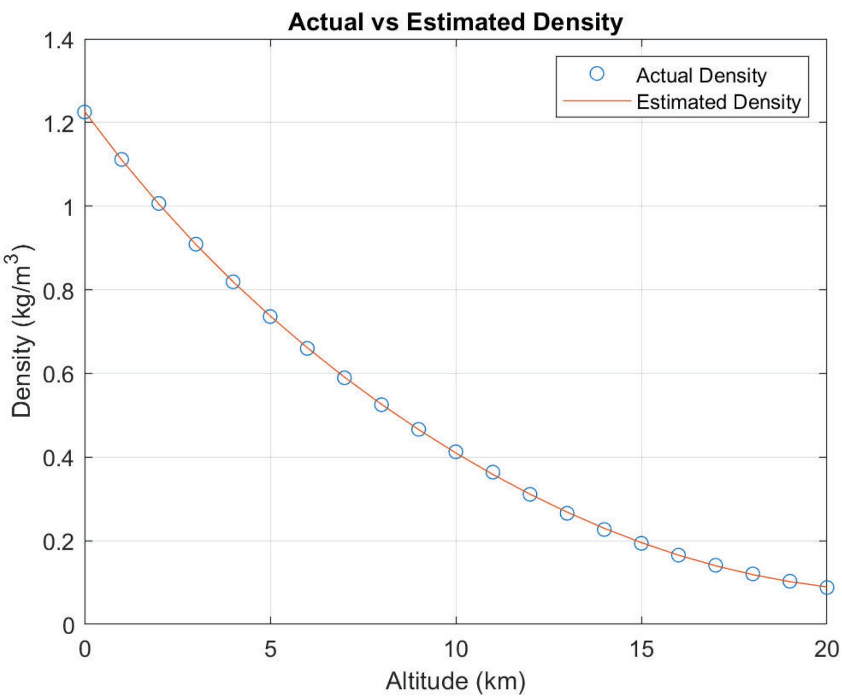

Figure 2: The comparison between the fitted and actual values

- V is the true airspeed of the aircraft.
- $S = 32.6m^2$ is the wing area.

Taking the known information into the equation, the formula can be simplified to

$$C_L = \frac{32.515}{\rho v^2} \tag{3}$$

Assuming the wing is two-dimensional, we will have the 2D lift curve as follows:

$$\begin{aligned} C_l &= C_{l_0} + a_0\alpha \\ &= a_0\left(\alpha^2 - \alpha_{0L}\right) \end{aligned} \tag{4}$$

Taking the known information into the equation, the equ 3 can be simplified to

$$C_L = C_{l_0} + a_0\alpha = 0.75 + 0.1\alpha = \frac{32.515}{\rho v^2} \tag{5}$$

However, in 3-dimensions, the finite wing have a reduced local α_{eff} while the wing is at a real angle-of-attack α, where $\alpha_{eff} = \alpha - \alpha i$, that is to say, the finite wing has less lift since it has a smaller angle of attack.

we will have the 3D lift curve as follows:

$$C_L = a_0\alpha_{eff} = a_0\left(\alpha - \alpha_i\right) = a_0\left(\ \alpha - \frac{C_L}{\pi ARe_w}\ \right) \tag{6}$$

$$C_L = \frac{a_0\alpha}{1 + \frac{a_0}{\pi ARe_w}} \tag{7}$$

Taking the known information into the equation, the Equ 6 can be simplified to

$$C_L = C_{l_0} + \frac{a_0\alpha}{1 + \frac{a_0}{\pi ARe_w}} = \frac{32.515}{\rho v^2} \tag{8}$$

where

- $AR = \frac{b^2}{s} = 15.529$
- we assume $e_w = 0.85$

Then, the Equ 6 can be simplified to

$$C_L = 0.75 + 0.0976\alpha = \frac{32.515}{\rho v^2} \tag{9}$$

Finally, we plot the relationship between α and velocity in Fig 3. The graph displays the relationship between the angle of attack (α), and velocity at different altitudes: sea level (0 km), 10 km, and 20 km. As altitude increases, the angle of attack required at a given velocity also increases, suggesting that higher altitudes—with their reduced air density—diminish the dynamic pressure effect, thereby necessitating a larger angle of attack to generate the same lift. From Figure 1, we also know that the E395 Airfoil's α_{stall} is about 7°, so when α reaches 7°, the airplane goes into stall, so the airplane can't fly too slowly, and the graph also show that the airplane's minimum flight speed rises as the altitude increases.

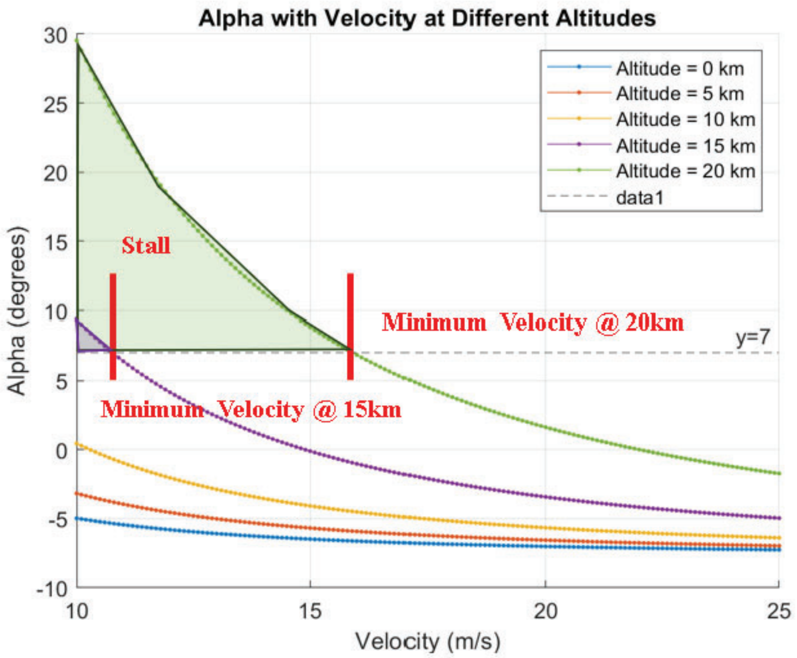

Figure 3: α with Velocity at Different Altitudes

Table 2: Atmospheric Properties as a Function of Altitude

Altitude (km)	Temperature (K)	Pressure (Pa)	Density (kg/m³)	Speed of Sound (m/s)	Viscosity (Pa.s)
0.00000	288.150	101325	1.22500	340.294	0.0000181206
1.00000	281.650	89874.6	1.11164	336.434	0.0000177943
2.00000	275.150	79495.2	1.00649	332.529	0.0000174645
3.00000	268.650	70108.5	0.909122	328.578	0.0000171311
4.00000	262.150	61640.2	0.819129	324.579	0.0000167940
5.00000	255.650	54019.9	0.736116	320.529	0.0000164531
6.00000	249.150	47181.0	0.659697	316.428	0.0000161084
7.00000	242.650	41060.7	0.589501	312.274	0.0000157596
8.00000	236.150	35599.8	0.525168	308.063	0.0000154068
9.00000	229.650	30742.5	0.466348	303.793	0.0000150498
10.0000	223.150	26436.3	0.412707	299.463	0.0000146884
11.0000	216.650	22632.1	0.363918	295.070	0.0000143226
12.0000	216.650	19330.4	0.310828	295.070	0.0000143226
13.0000	216.650	16510.4	0.265483	295.070	0.0000143226
14.0000	216.650	14101.8	0.226753	295.070	0.0000143226
15.0000	216.650	12044.6	0.193674	295.070	0.0000143226
16.0000	216.650	10287.5	0.165420	295.070	0.0000143226
17.0000	216.650	8786.68	0.141288	295.070	0.0000143226
18.0000	216.650	7504.84	0.120676	295.070	0.0000143226
19.0000	216.650	6410.01	0.103071	295.070	0.0000143226
20.0000	216.650	5474.89	0.0880349	295.070	0.0000143226

2 Problem 2 – Parrot Disco Flying Wing Drone

2.1 What is the lift coefficient of the wing at the specified minimum speed?

The lift of the wing is related to the velocity by the following equation

$$L = mg = C_l \frac{1}{2} \rho V^2 S \tag{10}$$

where:

- m=0.75kg
- g=10 m/s^2
- $\rho = 1.225 kh/m^3$
- $s = 0.38 m^2$
- $v = 5.55 m/s$

Taking in the known information, we can get

$$C_{Lmax} = \frac{2mg}{\rho v^2 s} = 1.044 \tag{11}$$

2.2 What is the lift coefficient of the wing at the specified minimum speed?

This value is quite low for a maximum lift coefficient, which is typically higher for most aircraft to sustain level flight at minimum speed.

several factors could be contributing to this observation:

Air Density: The value of air density used (1.225 kg/m^3) is a standard at sea level. If the drone is flying at a higher altitude where the air is less dense, the lift coefficient calculated at sea level would not be accurate.

Weight Estimation: If the actual weight is less than the assumed weight, the lift coefficient required for level flight would also be lower.

This might suggest that the actual minimum flying speed of the drone could be lower than 20 km/hr, allowing for a higher lift coefficient in practice.

2.3 How does "washout" reduce the achievable maximum lift coefficient?

Washout is an aerodynamic design feature used in wing construction where the angle of attack decreases progressively from the root of the wing to the tip. This means that the wing root will stall before the wing tip. The primary purpose is to improve flight stability and control by ensuring that the ailerons remain effective even as the stall begins at the root.

Washout also affects the wing's lift generation capacity ($C_L max$). Washout ensures that not all parts of the wing are operating at their optimal lift-generating angle simultaneously. While the root may be close to or at its maximum lift coefficient, the tips are operating at a lower angle of attack, producing less lift. This distribution results in a lower overall maximum lift coefficient for the entire wing compared to if the wing had a uniform angle of attack along its span.

2.4 What other factors might limit the minimum speed?

Wing Design and Airfoil Shape: Airfoil shape and wing design will affect its lift-generating capabilities. If the wing has been optimized for higher-speed flight, it may not generate as much lift at lower speeds.

Surface Roughness: Surface roughness affects lift by influencing the boundary layer's behavior over the wing. A smooth surface allows for a more streamlined flow of air, reducing drag and promoting efficient lift generation.

Flaps or slats: Flaps extend the wing surface and change its camber, increasing C_L and allowing the aircraft to fly slower. Slats modify the wing's leading edge to improve airflow at high angles of attack, also allowing for slower flight.

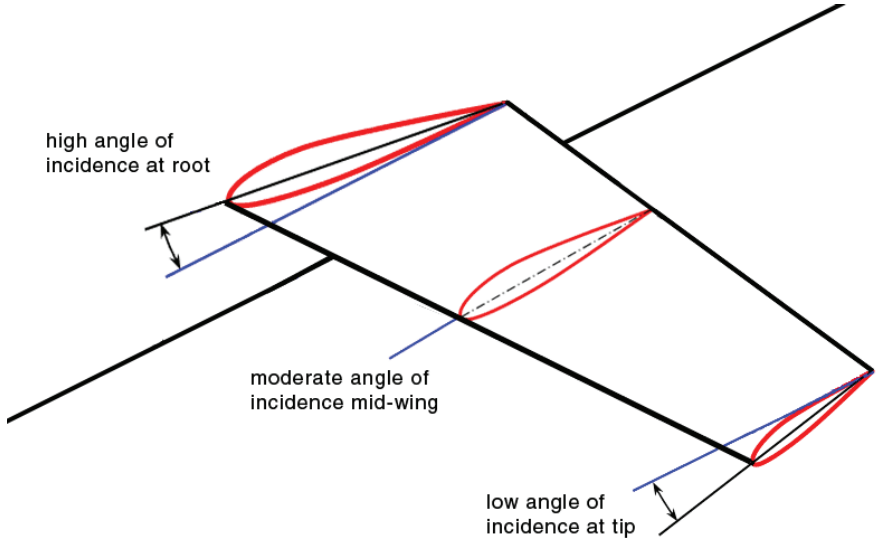

Figure 4: washout

Altitude: Altitude impacts an aircraft's minimum flying speed due to changes in air density. As altitude increases, the air becomes less dense, which means that for an aircraft to generate the same amount of lift, it must either increase its speed or have a higher angle of attack.

3 Problem 3 – An Unorthodox Rectangular Flying Wing

For the given airplane, we can list the force balance equation and moment balance equation as follows:

$$\sum F = 0 \tag{12}$$

$$\sum M = 0 \tag{13}$$

For most airplanes, we can balance the moments by adjusting the elevator. But for a given airplane, we need to balance the moments by adjusting the center of gravity position. The schematic is shown in Fig 5.

The pitching moment can be calculated by the following equation:

$$M = C_{mac}\frac{1}{2}\rho V^2 Sc \tag{14}$$

where:

- M is the generated moment (N·m).
- ρ is the air density (kg/m³).
- V is the flight velocity relative to the air (m/s).
- S is the wing area (m²).
- c is the chord length (m).

C_{mac} refers to the coefficient of moment about the aerodynamic center of an airfoil. A positive C_{mac} indicates nose to pitch up , while a negative C_{mac} indicates a nose-down pitching tendency.

Checking the table shows that NACA 2415 aerofoil has a $C_{mac} = -0.05$.

A detailed discussion of Eq. 13 gives us

$$\Delta XW = M_{ac} \tag{15}$$

$$\begin{cases} \Delta XW = C_{mac}\frac{1}{2}\rho V^2 Sc \\ W = C_L\frac{1}{2}\rho V^2 S \end{cases} \tag{16}$$

Simplifying the above system of equations, we get

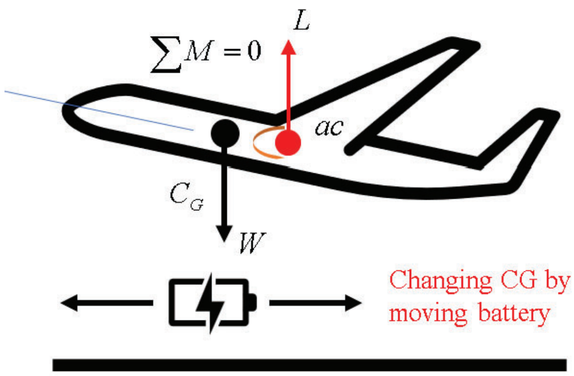

Figure 5: An Unorthodox Rectangular Flying Wing

$$\frac{\Delta X}{C} = \frac{C_{mac}}{C_L} \tag{17}$$

where ΔX denotes the distance between air dynamic center and center of gravity.

$$\frac{X_{ac} - X_{CG}}{C} = \frac{C_{mac}}{C_L} \tag{18}$$

Considering that X_{ac} is generally at $1/4$ of the chord length

$$\frac{X_{CG}}{C} = 0.25 - \frac{C_{mac}}{C_L} \tag{19}$$

Thus the range of CG is

$$|X_{CGmax} - X_{CGmin}| = c|\frac{C_{mac}}{C_{Lmin}} - \frac{C_{mac}}{C_{Lmax}}| \tag{20}$$

Bringing the data into Equ. 16,

$$(C_{Lmin}, C_{Lmax}) = (0.06, 1.02) \tag{21}$$

Finally,

$$Range = |X_{CGmax} - X_{CGmin}| = 0.882c \tag{22}$$

MEAM 543 HW3

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1 Problem 1 — Parrot Disco Flying Wing Drone

1.1 (a)

First, we do a simple estimation of the drag on a wing:

$$C_D = c_{do} + rC_L^2 + \frac{C_L^2}{\pi e AR} \tag{1}$$

where:

- C_{d0} is the zero-lift drag coefficient of the airfoil.
- rC_L^2 represents the increase in airfoil (2D) drag with lift
- $AR$is the wing aspect ratio. Long slender wings have lower induced drag.
- $e = 0.9$ is the span efficiency factor.

As we can see from Fig 1,

- $C_{d0} = 0.01$
- $r = C_{d_{c_l=1}} - C_{d0} = 0.008$
- $AR = \frac{b^2}{s} = 3.48$
- $e = 0.9$

Bringing the data into the Equ. 14, we shall get:

$$C_D = 0.01 + 0.1096C_L^2 \tag{2}$$

Then, we plot wing drag polar in Fig. 2.

1.2 (b)

Achieving maximum lift-to-drag ratio means the slope of the maximum tangent line in the graph. As is shown in Fig. 2, the maximum lift-to-drag ratio(slope of the maximum tangent line) is 15.098. where $C_L = 0.3081$ and $C_D = 0.0204$.

Then, we will calculate minimum thrust required in level flight. As is shown in Fig. 3, the force balance equations can be shown as Equ. 16:

$$\begin{cases} L = W = \frac{1}{2}\rho C_L V_\infty^2 S \\ T = D = \frac{1}{2}\rho C_D V_\infty^2 S \end{cases} \tag{3}$$

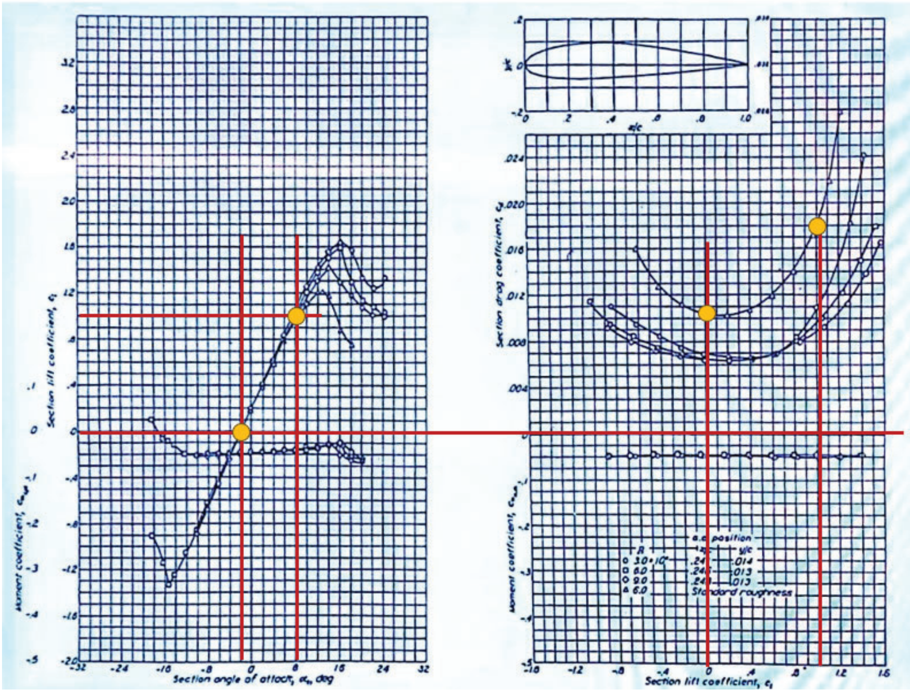

Figure 1: NACA 2415 Airfoil basic parameters

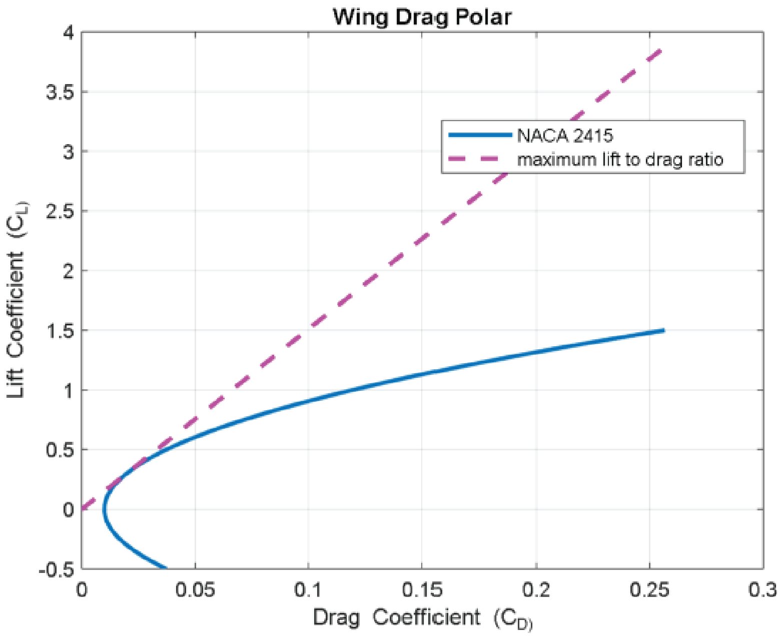

Figure 2: Wing Drag Polar

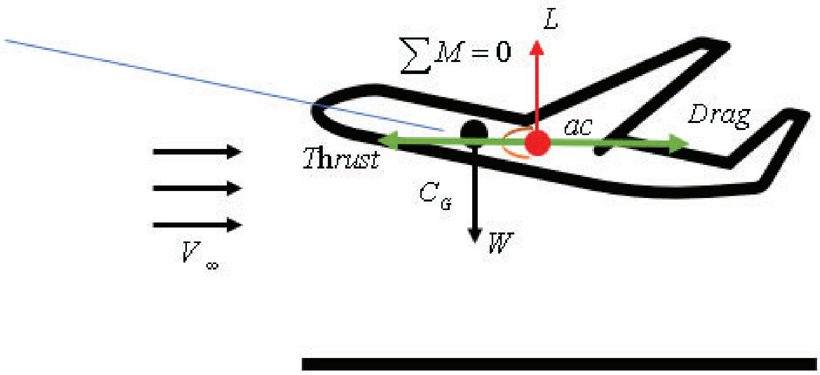

Figure 3: Schematic of the force balance of an airplane

Hence

$$V_\infty = \sqrt{\frac{2\omega}{\rho s C_L}} \tag{4}$$

Minimum thrust and the corresponding power in level flight is

$$T_{min} = \frac{C_D}{C_L} w = 0.487N \tag{5}$$

$$P_{T_{min}} = T_{min} V = 10.127 \times 0.487 = 4.84W \tag{6}$$

Minimum power and the corresponding thrust in level flight is

$$T_{min} = \frac{C_D}{C_L} w = 0.56N \tag{7}$$

$$P = DV_\infty = \frac{1}{2}\rho s \frac{2\omega}{\rho s}^{\frac{3}{2}} \frac{C_D}{C_L^{\frac{3}{2}}} \tag{8}$$

as we can see from the above equation, if we want to achieve minimum power, $\frac{C_L^{1.5}}{C_D}$ should take its maximum value.

We denote C_L and C_D when $\frac{C_L^{1.5}}{C_D}$ takes its maximum value as $C_{L_{pm}}$ and $C_{D_{pm}}$. As we can see from Fig 1, $\frac{C_L^{1.5}}{C_D} = 9.46$, where $C_{L_{pm}} = 0.530$ and $C_{D_{pm}} = 0.041$

$$V_\infty = \sqrt{\frac{2\omega}{\rho S_{wing} C_{L_{pm}}}} = 7.72m/s \tag{9}$$

$$P_{min} = DV_\infty = \sqrt{2}\rho^{-0.5} s(\frac{\omega}{S_{wing}})^{\frac{3}{2}} \frac{C_{D_{pm}}}{C_{L_{pm}}^{\frac{3}{2}}} = 4.32W \tag{10}$$

1.3 (c)

Energy Stored in the Battery (E): measured in watt-hours (Wh) or joules (J). It represents the total amount of energy the battery can store.

$$\text{Energy stored in the battery} = 11.1 \times 2.7 \times 3600 = 107892J \tag{11}$$

Power Drawn from the Battery (P): the rate at which the battery delivers energy to the system, measured in watts (W).

$$\text{Power drawn from the battery} = P_{min}Endurance_{max} = 4.32 \times 45 \times 60 = 11664J \tag{12}$$

$$E = \frac{\text{Power drawn from the battery}}{\text{Energy stored in the battery}} = 10.8\% \tag{13}$$

2 Problem 2 — Airbus Zephyr High-Altitude Pseudo Satellite (HAPS)

2.1 (a)

First, we do a simple estimation of the drag on a wing:

$$C_D = c_{do} + rC_L^2 + \frac{C_L^2}{\pi e AR} \tag{14}$$

where:

- C_{d0} is the zero-lift drag coefficient of the airfoil.
- rC_L^2 represents the increase in airfoil (2D) drag with lift
- $AR$is the wing aspect ratio. Long slender wings have lower induced drag.
- $e = 0.9$ is the span efficiency factor.

As we can see from Fig 4,

- $C_{d0} = 0.02$
- $r = C_{d_{c_l=1}} - C_{d0} = -0.004$
- $AR = \frac{b^2}{s} = 15.53$
- $e = 0.9$

Bringing the data into the Equ. 14, and add 20 counts to the zero-lift drag coefficient of the wing, we shall get:

$$C_D = 0.022 + 0.0188C_L^2 \tag{15}$$

Then, we make a wing drag polar as is shown in Fig .5.

2.2 (b)

Then, we will calculate thrust required in level flight. As is shown in Fig. 3, the force balance equations can be shown as Equ. 16:

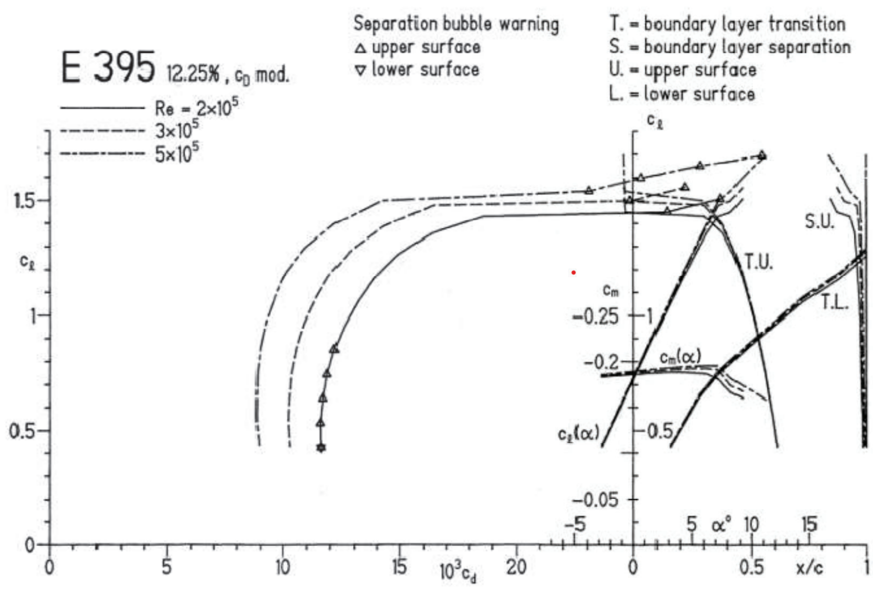

Figure 4: E395 Airfoil basic parameters

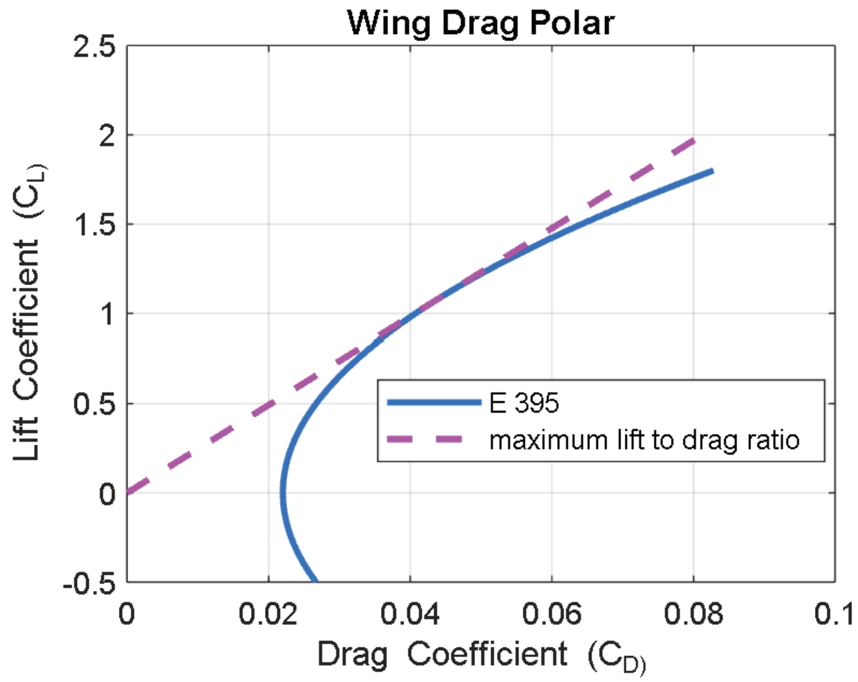

Figure 5: E395 Wing Drag Polar

$$\begin{cases} L = W = \frac{1}{2}\rho C_L V_\infty^2 S \\ T = D = \frac{1}{2}\rho C_D V_\infty^2 S \end{cases} \tag{16}$$

Hence

$$T = D = \frac{1}{2}\rho V_\infty^2 S (C_{D_{adjust}} + r^2 C_L) \tag{17}$$

where

$$C_L = \frac{2w}{\rho V_\infty^2 S} \tag{18}$$

A check of the standard air density table shows that the density of air at sea level, 10km, and 20km is 1.225 kg/m^3, 0.4135 kg/m^3 and 0.08891 kg/m^3.

Finally,we can fraw thrust and power required in level flight vs airspeed at altitudes of 0km (sea level), 10km, and 20km as shown in Figs below.

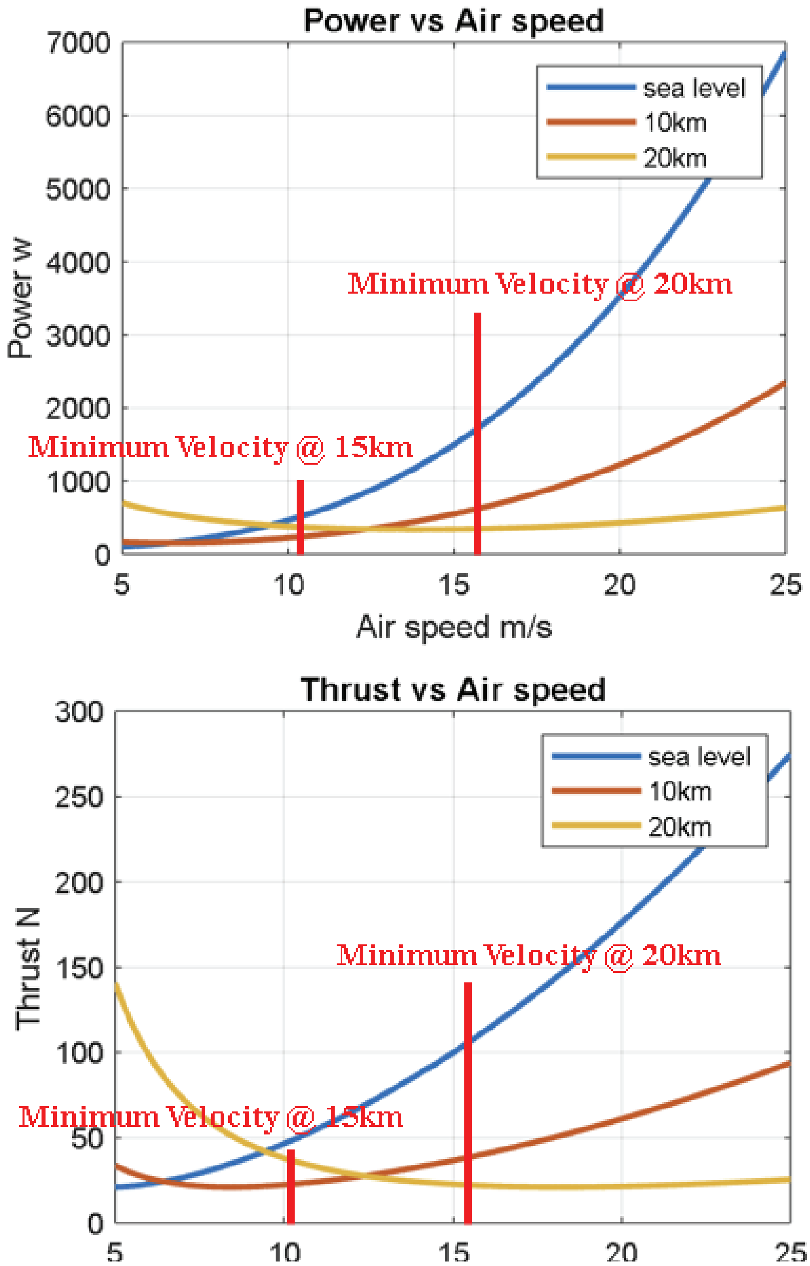

Figure 6: Enter Caption

It is important to note that airspeed cannot be reduced indefinitely. As airspeed decreases, the angle of approach of the wing increases, and when the angle of approach reaches a certain value, a stall will occur. Therefore, we need to maintain a certain cruise speed to ensure that we do not stall. The minimum flight speeds at different altitudes are marked on the graph with red lines.

Thrust vs Airspeed At sea level (blue curve): Thrust increases significantly with airspeed, suggesting that to maintain level flight at higher speeds, substantially more thrust is required. This is consistent with the higher air density at sea level, which provides more molecules to generate lift and thrust but also creates more drag.

At 10 km (orange curve) and 20 km (yellow curve) altitudes: The initial decrease in thrust requirement with increasing airspeed can be attributed to a reduction in drag as the aircraft transitions through different flight regimes. As airspeed continues to increase, the thrust requirement begins to rise, albeit at a much lower rate than at sea level. The reduced rate of increase at higher altitudes could be due to the lower air density reducing both lift and drag forces.

Power vs Airspeed At sea level (blue curve): Power requirement increases sharply with airspeed, which is expected as power to overcome drag increases with the cube of velocity. More power is needed to maintain level flight as speed increases due to higher drag forces.

At 10 km (orange curve) and 20 km (yellow curve) altitudes: The power required at higher altitudes starts lower and increases with airspeed, but the slope of the curve is less steep compared to sea level. The initial lower power requirements can be explained by the thinner air at higher altitudes, which generates less drag. However, as airspeed increases, the power required to maintain level flight also increases, though the total power needed is still less than at sea level due to the reduced air density.

2.3 (c)

First, we need to calculate the energy produced by the solar cell. Solar Power Generated per m^2 is

$$P_{generated} = S_{wing} \times S \times \eta_{solar} = 6.52kw \tag{19}$$

where

- Solar Irradiance (S): Given as 1 kW/m².
- Efficiency of Solar Panels (η_{solar}): 20%.
- Wing area (S_{wing}): 32.6 m^2.

Since only 1/3 of the generated power is used directly for flight and 2/3 needs to be stored, the average power available over a 24-hour period for flight (P_{use}) is:

$$P_{use} = \frac{P_{\text{gener ated}}}{3} = 2.17kw \tag{20}$$

Next we will consider the loss of power, $\eta_{propulsion}$ denotes the overall propulsion system efficiency as , which accounts for the efficiency of converting electrical energy into mechanical energy (including losses in the motor, propeller, and other components).

We assume

Motor Efficiency: efficiency at which the electric motor converts electrical energy from the battery into mechanical energy. Modern electric motors can be highly efficient, often exceeding 90% under optimal conditions.

Propeller Efficiency: refers to how effectively the propeller converts the mechanical energy it receives from the motor into thrust. The efficiency of a propeller can vary widely based on its design, operating conditions, and speed. It's not uncommon for propeller efficiencies to range from 70% to 85%.

The energy required for continuous flight during sunlight and night-time hours can be described as

$$P_{flight} = P_{use}\eta_{propulsion} = 1.47kw \tag{21}$$

Calculations show that to remain in continuous flight, an airplane must consume a maximum of 1.47 kw of energy.

We plot this limitation in Fig. 7 and find its intersection with the airspeed energy curve, and by considering the stall speed limitation, we finally obtain the velocity range.

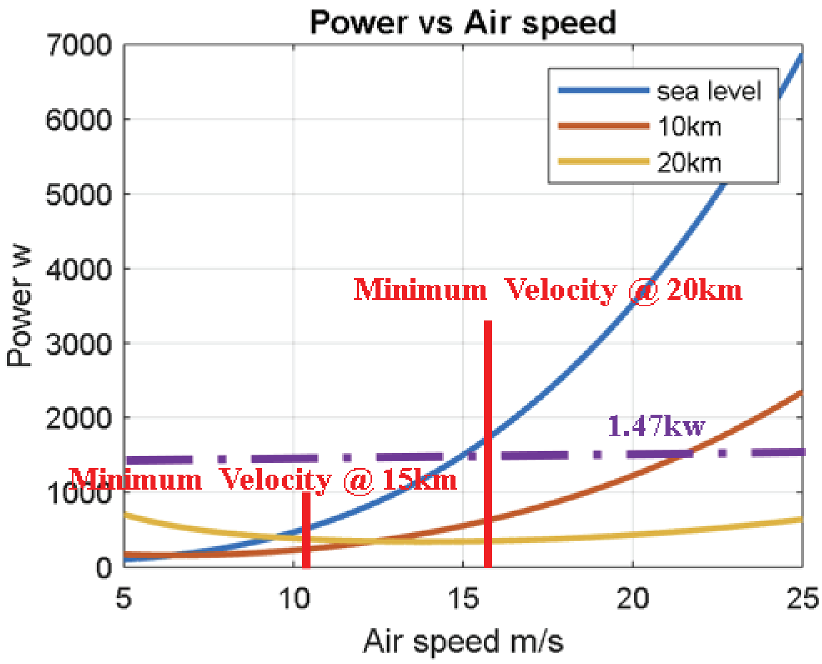

Figure 7: at which you think the airplane could fly indefinitely

- Velocity range at sea level: $4.9(stallvelocity) - 15.2m/s(powerlimit)$
- Velocity range at 10km: $8.1(stallvelocity) - 21.6m/s(powerlimit)$
- Velocity range at 20km: $15.4(stallvelocity) - 34.2m/s(powerlimit, Ifthemaximumspeedcanbeachieved)$

3 Problem 3 — Falling Bowling Ball

The differential equations are Newton's law and vertical kinematics:

$$\begin{aligned}\frac{dV}{dt} &= g - \frac{1}{m}D \\ \frac{dh}{dt} &= -V\end{aligned} \tag{22}$$

To calculate drag, we use following equations:

$$D = \frac{1}{2}\rho v_\infty^2 C_D S^2 \tag{23}$$

$$C_D = \frac{24}{\mathrm{Re}} + \frac{2.6\left(\frac{\mathrm{Re}}{5.0}\right)}{1+\left(\frac{\mathrm{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\mathrm{Re}}{10^6}\right)}{1+\left(\frac{\mathrm{Re}}{10^6}\right)} \tag{24}$$

$$Re = \frac{\rho v d}{\mu} \tag{25}$$

where:

- ρ denotes the air density
- $\mu = 1.81e-5$ denotes the dynamic viscosity of air
- $d = 0.218$ denotes diameter of a standard bowling ball (m)

Calculating the above system of differential equations, the results are obtained as shown in Fig. 9. Fig. 9 (a) shows the velocity increases quickly at first and then levels off, approaching a horizontal asymptote. This behavior is characteristic of an object accelerating under gravity and then reaching terminal velocity, where the force of gravity is balanced by the drag force, resulting in no further acceleration. As the altitude drops, the air density begins to rise, the Reynolds

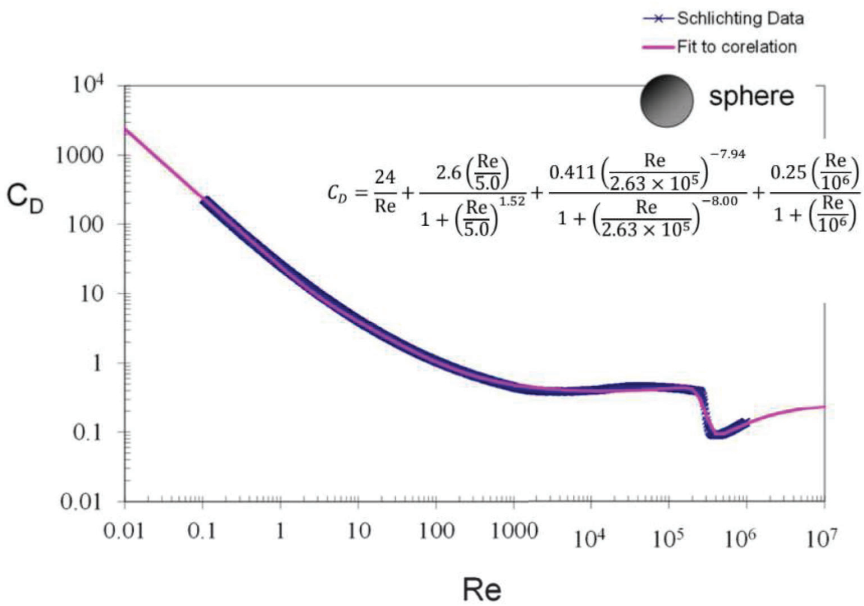

Figure 8: The relationship between CD and Re

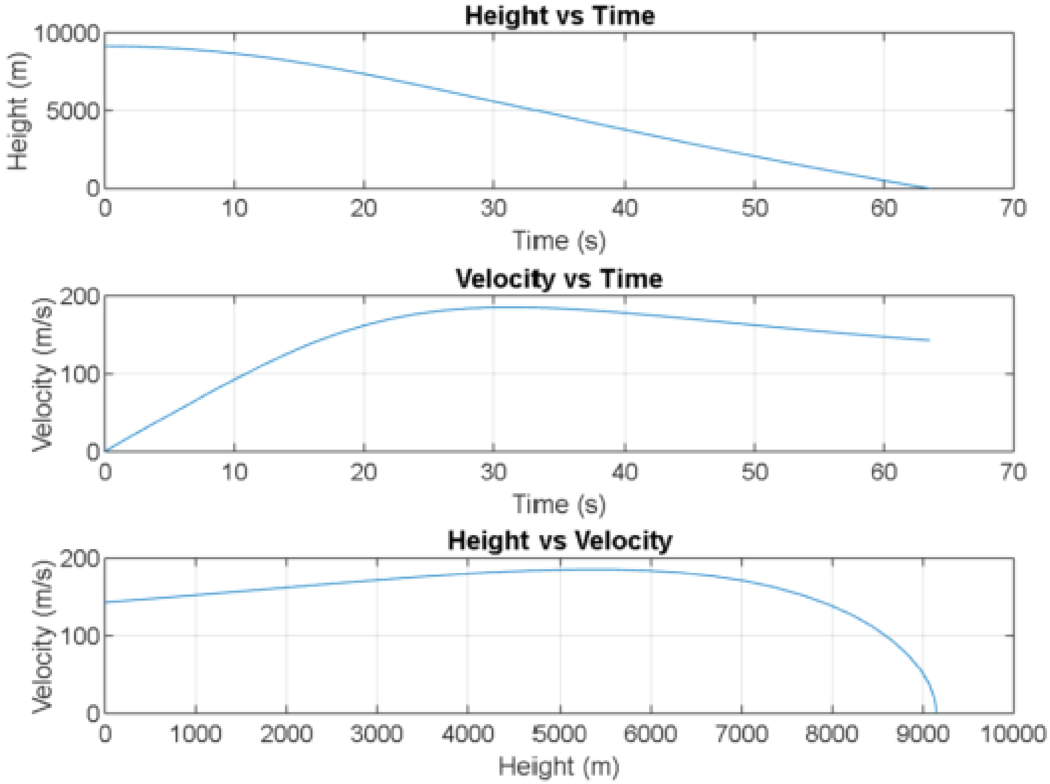

Figure 9: speed of the ball vary with altitude

number rises, the drag rises, and the speed drops slightly. Fig. 9 (b) shows velocity on the y-axis and height on the x-axis. The velocity starts at zero , increases to a peak, and then decreases back. This indicates that as the object falls and height decreases, its velocity increases up to a peak, where the force of gravity is balanced by the drag force. As the altitude drops, the air density begins to rise, the Reynolds number rises, the drag rises, and the speed drops slightly.

```
function ball_fall
% Constants
    g = 9.81; % Acceleration due to gravity (m/s^2)
    m = 7.26; % Mass of the bowling ball (kg) - approximate
    d = 0.218; % Diameter of a standard bowling ball (m)
    mu = 1.81e-5; % Dynamic viscosity of air at °15C in ·Pas

    % Initial conditions
    h0 = 30000 * 0.3048; % Convert initial altitude from feet to meters
    v0 = 0.0001; % Initial velocity (m/s)
    % Time span
    tspan = [0 80];

    % Given altitude and density data
    altitude = [0:20]; % Altitude in km
    density = [1.22500, 1.11164, 1.00649, 0.909122, 0.819129, 0.736116, 0.659697, ...
               0.589501, 0.525168, 0.466348, 0.412707, 0.363918, 0.310828, 0.265483, ...
               0.226753, 0.193674, 0.165420, 0.141288, 0.120676, 0.103071, 0.0880349];

    % Fit a polynomial to the data
    p = polyfit(altitude, density, 5); % Change the degree of the polynomial as needed
```

```
    options = odeset('Events', @events); % Configure the ODE solver to stop when h = 0

    [T, Y] = ode45(@(t, y) model(t, y, m, g, d, p ,mu), tspan, [h0; v0], options);


    figure;
    subplot(3,1,1);
    plot(T, Y(:,1));
    title('Height vs Time');
    xlabel('Time (s)');
    ylabel('Height (m)');
    grid on

    subplot(3,1,2);
    plot(T, Y(:,2));
    title('Velocity vs Time');
    xlabel('Time (s)');
    ylabel('Velocity (m/s)');
    grid on


    subplot(3,1,3);
    plot(Y(:,1), Y(:,2));
    title('Height vs Velocity');
    xlabel('Height (m)');
    ylabel('Velocity (m/s)');
    grid on

end

function dydt = model(t, y, m, g, d, p ,mu)
    h = y(1);
    v = y(2);

    rho = polyval(p, h/1000);

    Re = calculateReynoldsNumber(rho, v, d, mu);
    Cd = calculateDragCoefficient(Re);

    D= (1/2)*Cd*v^2*rho*pi*(d/2)^2;

    dvdt = g - D/m;
    dhdt = -v;

    dydt = [dhdt; dvdt];
end


function Re = calculateReynoldsNumber(rho, V, d, mu)
    Re = (rho * V * d) / mu;
end


function Cd = calculateDragCoefficient(Re)
    % Calculate the drag coefficient Cd based on the given Reynolds number Re.
    term1 = 24.0 / Re;
    term2 = (2.6 * (Re / 5.0)) / (1 + (Re / 5.0)^1.52);
    term3 = (0.411 * (Re / 2.63e5)^-7.94) / (1 + (Re / 2.63e5)^-8.00);
```

```
    term4 = (0.25 * (Re / 1e6)) / (1 + (Re / 1e6));
    Cd = term1 + term2 + term3 + term4;
end

function [value,isterminal,direction] = events(t,y)
    value = y(1);      % Detect when height (h) is 0
    isterminal = 1;    % Stop the integration
    direction = -1;    % The zero can be approached from above only
end
```

References

[1] NACA 2415 (n2415-il). (n.d.). Airfoiltools.com. .http://airfoiltools.com/airfoil/details?airfoil=n2415-il

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MEAM 543 HW4

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1 Problem 1

1.1 (a) Dimension

First of all, we have to determine the dimensions of each part of the aircraft, although some of the key parameters are given in the table, we are still missing some parameters, for this reason, we rasterize the image as in Fig .1, where each grid represents 0.88inch. The dimension is shown in Tab .1.

1.2 (b) Equations

After determining the dimensions of the airplane, we can list the equations for the force balance and moment balance of the airplane. We calculate the lift of the airplane, which can be seen from Fig. 2. The lift of the airplane has two components, namely the lift of the wing and the lift of the tail, where the lift of the wing can be calculated using the following equations: where:

- L is the total lift, which is the sum of the wing lift and the tail lift.
- ρ_∞ is the air density in the free stream (far field).
- V_∞ is the velocity of the aircraft relative to the air in the free stream.
- S is the reference area (wing area).
- C_L is the lift coefficient, a dimensionless number that represents the lift generated by the entire aircraft per unit area and dynamic pressure.
- D is the total drag, composed of wing drag, fuselage drag, miscellaneous drag, and tail drag.
- $C_D$is the drag coefficient.
- M is the total pitching moment.
- M_w and M_f are the moments due to the wing and fuselage, respectively.

Table 1: Dimensions of the aircraft

	Parameter	Value		Parameter	Value
0	A	2.2	**0**	Wing Span	47
1	B	24.2	**1**	Wing Chord	7.5
2	C	5.7	**2**	Tail Span	16
3	D	6.2	**3**	Tail Chord	5
4	E	2.1	**4**	Elevator Chord	2
5	F	5.7	**5**	L_t	18
6	G	1.9	**6**	Thickness	0.25
7	H	8.1	**7**		
8	I	5.3	**8**		

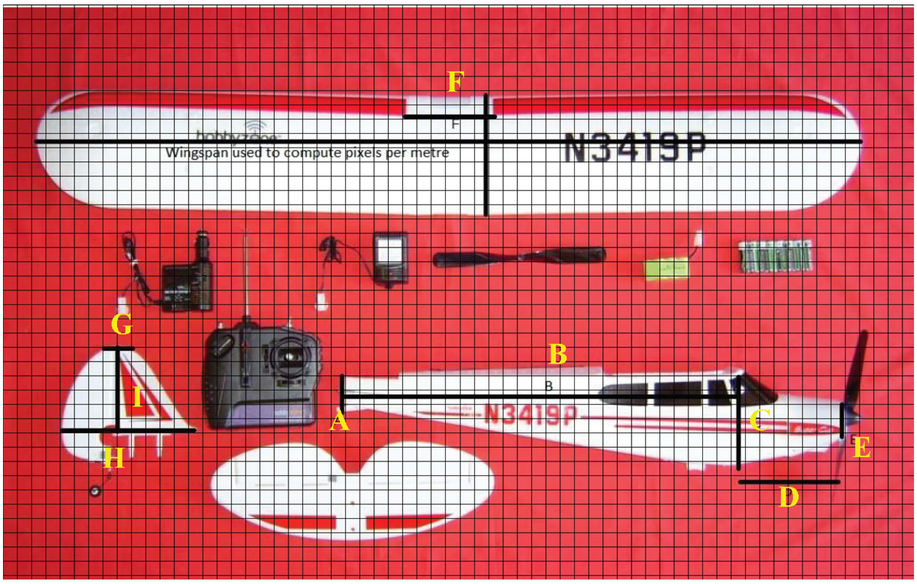

Figure 1: Dimensions of the aircraft

- $X_{cg}$is the position of the center of gravity.
- x_{acw} is the aerodynamic center of the wing.
- l_t is the distance from the aircraft's center of gravity to the tail
- c is the mean aerodynamic chord of the wing.
- $C_{MW}$is the moment coefficient about the wing's aerodynamic center.
- V is the tail volume ratio, a measure of the tail's effectiveness in producing a moment.
- h is the dimensionless distance of the center of gravity from the wing's leading edge, expressed as a fraction of the mean aerodynamic chord.
- h_{acw} is the dimensionless aerodynamic center location.
- η_h is the efficiency factor of the horizontal tail.
- V_H is the horizontal tail volume coefficient.
- C_{Lt} is the lift coefficient of the tail.
- C_M is the overall moment coefficient of the aircraft.

$$M_{ac} = M_w + M_f \tag{1}$$

$$\text{L} = L_w + L_t = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \tag{2}$$

$$\mathcal{D} = D_w + D_f + D_{misc} + D_t = \frac{1}{2}\rho_\infty V_\infty^2 S C_D \tag{3}$$

$$M = M_w + M_f + (x_{cg} - x_{ac_w})L_w - l_t L_t \tag{4}$$

$$M = \frac{1}{2}\rho_\infty V_\infty^2 cS \left(C_{M_W} + \frac{2\mathcal{V}}{c\mathcal{S}}\alpha + (h - h_{\alpha\mathcal{C}_W})C_{L_W} - \eta_h V_H C_{L_t} \right) = \frac{1}{2}\rho_\infty V_{\mathcal{O}}^2 c\mathcal{S} C_M \tag{5}$$

1.3 (c) Wing Lift, Drag and Moment

2D lift:

$$L_w = \frac{1}{2}\rho v_\infty^2 S_w C_L w_{2D} \tag{6}$$

3D lift:

However, in 3-dimensions, the finite wing have a reduced local α_{eff} while the wing is at a real angle-of-attack α, where $\alpha_{eff} = \alpha - \alpha i$, that is to say, the finite wing has less lift since it has a smaller angle of attack. we will have the 3D lift curve as follows:

$$C_L = C_{l0} + \frac{a_0\alpha}{1 + \frac{a_0}{\pi ARe_w}} \tag{7}$$

Drag

The drag coefficient of the wing can be described by the following equation:

$$C_{D_w} = \frac{D_w}{\frac{1}{2}\rho_\infty V_\infty^2 S_w} = c_{do_w} + r_w C_{L_w}^2 + \frac{C_{L_w}^2}{\pi e_w AR} \tag{8}$$

From Fig. 3 , we can also see that the Reynolds number affects the Cd0, so we need to determine the Cd0 from the Reynolds number. however, only 4 cases are given in the figure, and in reality, we may enter many different cases. Therefore, we have polynomially interpolated the data in the figure with the following code:

```
% Given data points
Rec = [6e5, 9e4, 6e4, 3e4];
Cd0 = [0.0115, 0.07, 0.065, 0.06];

p_degree = 2;

% Find polynomial coefficients
p_coeff = polyfit(Rec, Cd0, p_degree);

% Define a function that takes Rec and returns an estimated Cd0
% This uses the coefficients from the polyfit
cd0_from_rec = @(Rec_input) polyval(p_coeff, Rec_input);

Rec_input = 7e-4; % input value of Rec
Cd0_output = cd0_from_rec(Rec_input); % Get the corresponding Cd0

% Display the result
fprintf('For Rec = %.1e, the estimated Cd0 is %.4f\n', Rec_input, Cd0_output);

% If you want to plot the fit over a range of Rec values:
Rec_range = linspace(min(Rec), max(Rec), 100);
Cd0_fit = polyval(p_coeff, Rec_range);

% Plotting the original data and the fit
figure;
plot(Rec, Cd0, 'o', Rec_range, Cd0_fit, '-');
xlabel('Reynolds number (Rec)');
ylabel('Drag coefficient (Cd0)');
title('Polynomial fit of Cd_0 vs. Rec');
legend('Data points', 'Polynomial fit');
grid on;
```

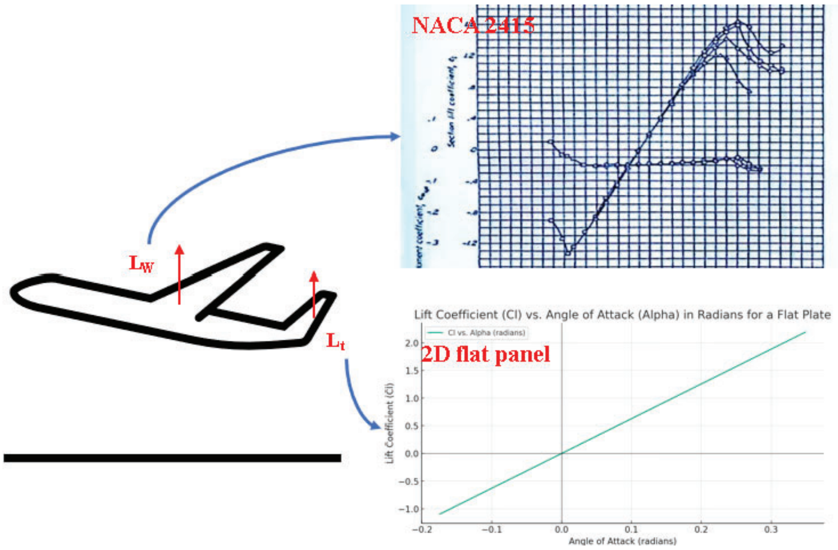

Figure 2: Schematic diagram of the lift components of an airplane

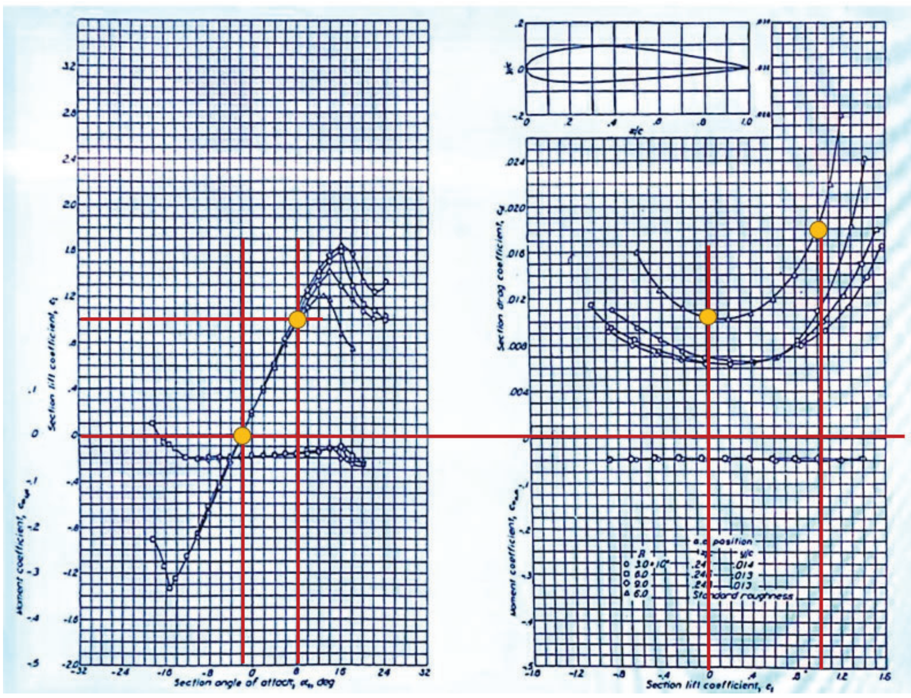

Figure 3: NACA 2415 Airfoil basic parameters

Moment

The moment coefficient of the wing we can get from Fig. 3. In this paper, we consider that C_M = -0.05.

1.4 (d) Tail Lift, Drag and Moment

Lift

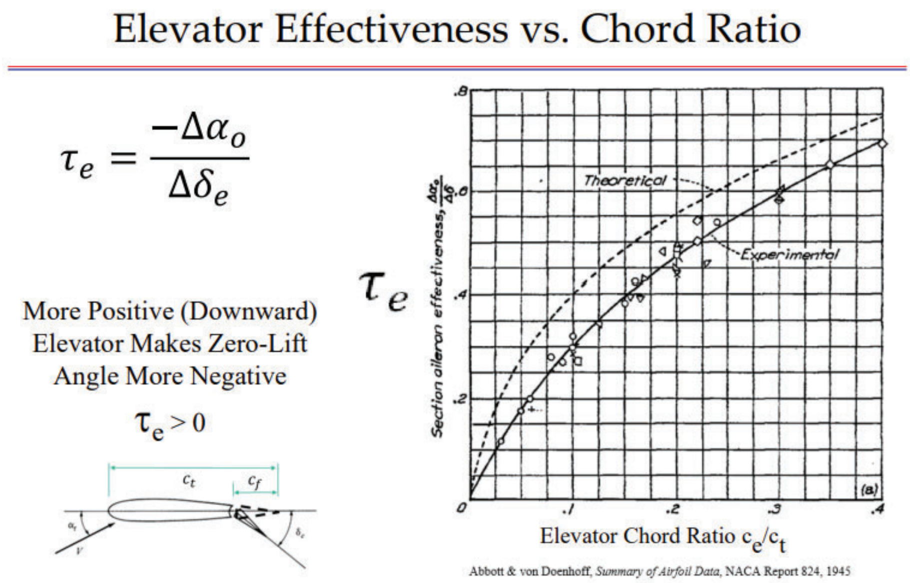

Figure 4: Elevator Effectiveness vs chord ratio

In this problem, we consider the tail to be a 0.25 inch flat plate, so we can use Thin airfoil theory to calculate tail lift. In aerodynamics, for a simple two-dimensional flat plate, the lift coefficient C_L with the attack angle α can be expressed by the following equation:

$$L_t = \frac{1}{2}\rho_\infty V_\infty^2 \eta_h C_{L_t} = \frac{1}{2}\rho_\infty V_\infty^2 \eta_h a_t \left[\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\alpha - \epsilon_o + i_h + \tau_e \delta_e\right] \tag{9}$$

where:

- $\frac{\partial \epsilon}{\partial \alpha}$ A factor accounting for the downwash effect on the tail. This term effectively reduces the angle of attack experienced by the tail due to the airflow being deflected downwards by the wing.
- ϵ_o: The downwash angle at zero lift. This is the angle by which the airflow is deflected downwards due to the presence of the wing, even when the aircraft is not generating lift.
- i_h: The incidence angle of the tail relative to the fuselage. It's a fixed geometric angle set during the design of the aircraft.
- τ_e: The effectiveness factor of the elevator, which modifies the lift generated by the tail when the elevator is deflected.
- δ_e: The elevator deflection angle, with positive values indicating upwards deflection.

We can find the corresponding values in the Fig. 5 and 4.

Drag

We consider both horizontal and vertical tail as 2D streamlined bodies as is shown in Fig. 6.

Moment

$$M_t = -l_t L_t = C_{L_W} - \eta_h V_H C_{L_t} \tag{10}$$

1.5 (e) Fuselage

Drag

We consider fuselage as 3D streamlined bodies as is shown in Fig. 7.

Moment

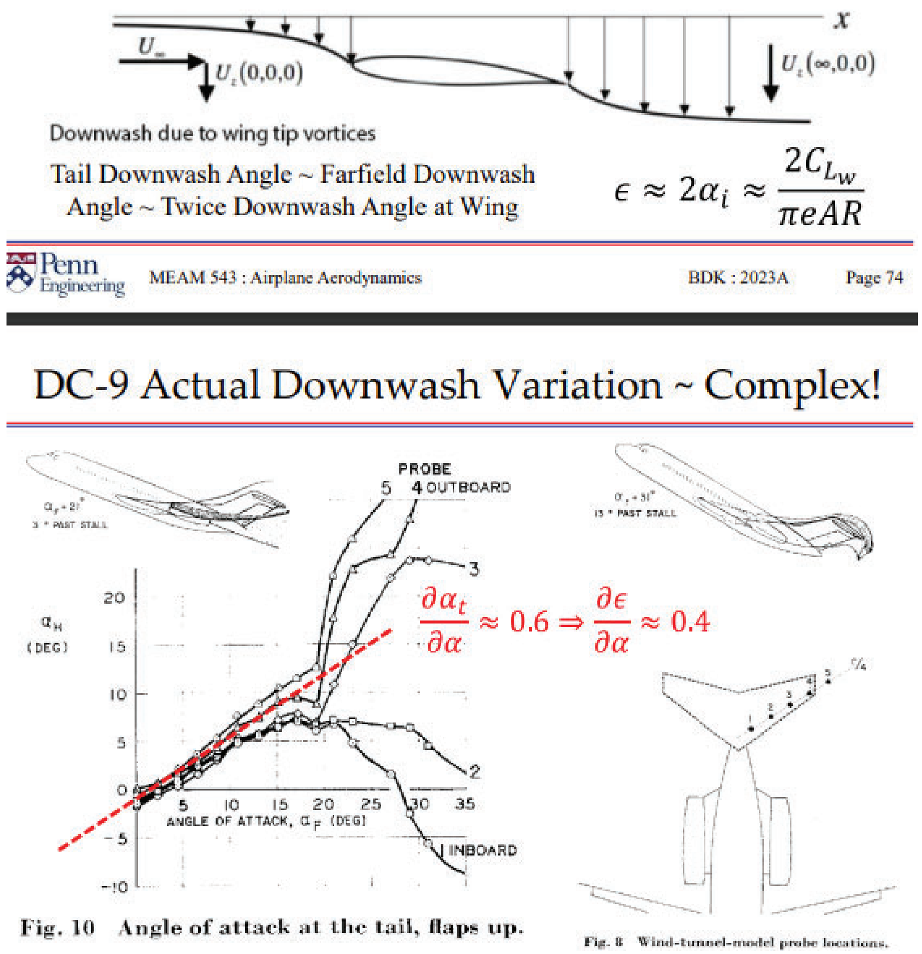

Figure 5: Downwash Variation

$$M_f = \frac{2\mathcal{V}}{c\mathcal{S}}\alpha \tag{11}$$

1.6 (f) Miscellaneous

We also consider miscellaneous as 3D streamlined bodies as is shown in Fig. 7.

1.7 (g) code integration

At this point, we have completed all the pieces of the puzzle, and we have consolidated the above process into the following matlab code:

```
function [CM, CL, CD] = super_cub_aero(alpha, del_e, Re_c, h)
    % super_cub_aero: Aerodynamic analysis of the Super Cub aircraft
    % This function calculates the aerodynamic coefficients CM, CL, and CD
    % for the Super Cub aircraft based on input conditions.
    %
    % Input arguments:
    % alpha_deg - Airplane angle of attack (degrees)
    % del_e_deg - Elevator deflection angle (degrees)
    % Re_c - Reynolds number based on wing chord
```

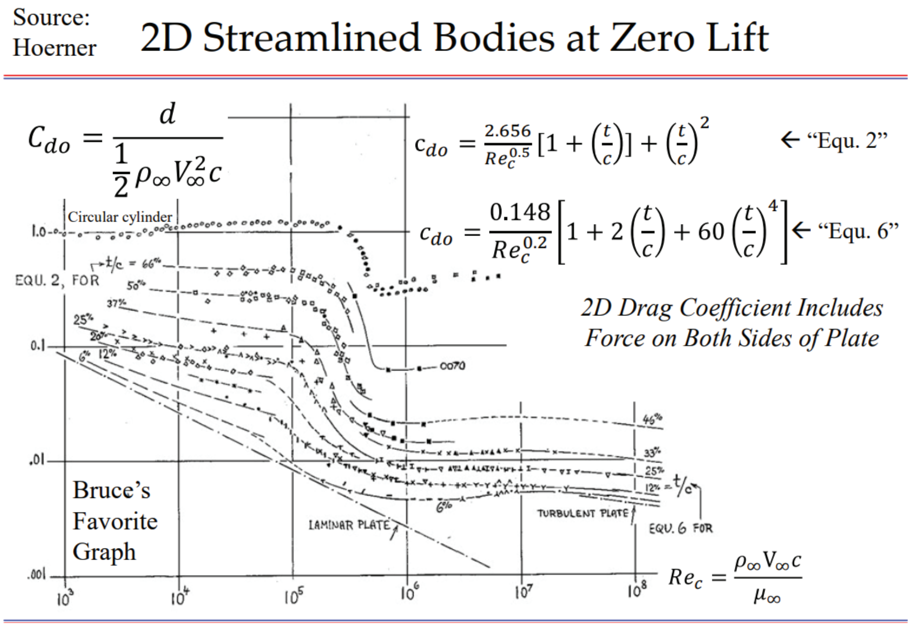

Figure 6: 2D streamlined bodies

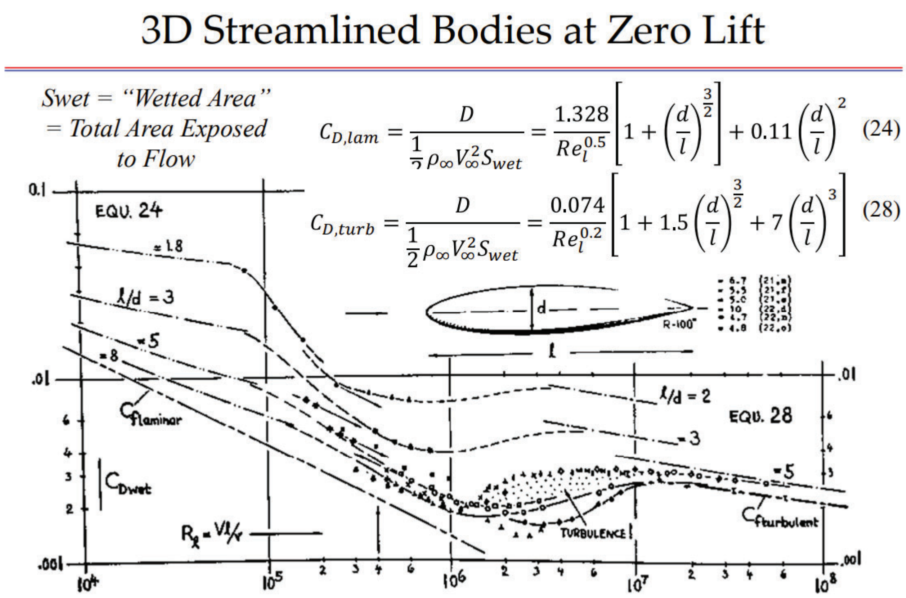

Figure 7: 3D Streamlined Bodies

```
% h - Dimensionless position of the airplane CG


% Constants
inch2m = 0.0254;

%  Determine Cd0 based on Rec
Rec = [6e5, 9e4, 6e4, 3e4];
Cd0 = [0.022, 0.14, 0.13, 0.12];           % Cd0 from NACA 2415 dataset
p_degree = 2;
p_coeff = polyfit(Rec, Cd0, p_degree);     % Find polynomial coefficients
cd0_from_rec = @(Re_c) polyval(p_coeff, Re_c);
CD0_w = cd0_from_rec(Re_c);                % Get the corresponding Cd0_w

% WING
b = 47*inch2m;    % Wingspan
c = 7.5*inch2m;   % Wing chord [m]
S = b*c;          % Wing reference area [m^2]
AR_w = b^2/S;     % Wing aspect ratio
CL0_w = 0.2;     % Lift coefficient at zero angle of attack
alpha_L0 = -2;    % Zero-lift angle of attack (degrees)
a_2d_w = 0.1744;
r_w=0.013;
e=0.7;

% Tail
tail_c = 5*inch2m;           % Tail chord [m]
ele_c = 2*inch2m;            % Elevator chord [m]
tail_b = 16*inch2m;          % Tailspan [m]
S_tail = tail_b*tail_c;      % Tail reference area [m^2]
S_ele = tail_b*ele_c;        % Elevator reference area [m^2]
AR_t = tail_b^2/S_tail;      % Tail aspect ratio
AR_e = tail_b^2/S_ele;       % Elevator aspect ratio
Eta_t = 0.9;                 % Tail efficiency
lt = 18*inch2m;              % Distance Between Wing & Tail [m]
thickness =0.25*inch2m;      % Thickness [m]
vH = lt*(S_tail)/(S*c);      % Tail Volume [m^3]
t_c_tail = thickness/tail_c; % Tail thickness-to-chord ratio
a_t= 0.096;
Incidence =0;
epsilon_alpha=2*(a_2d_w)/pi/e/AR_w;
Tao_e=0.7;

Ver_tail_c = 5.0* inch2m;   % Vertical Tail chord [m]
t_c_Ver= thickness/Ver_tail_c; % Vertical Tail thickness-to-chord ratio
S_Ver = 5* 5.3*inch2m^2;    % Vertical Tail reference area [m^2]

% Fuselage
d = 5.7*inch2m;
l = 30.8*inch2m;
S_Fuse = (5.7*inch2m)*l;     % Fuselage reference area [m^2]
d_l= d/l;                    % diameter/thickness to length
v = 1e-5;                 % Fuselage Volume [m^3]

% Miscellaneous



% Main wing lift, drag, and moment calculation
```

```
    CL_w = CL0_w+(alpha)*a_2d_w/(1+(a_2d_w)/pi/e/AR_w);   % Lift coefficient of the wing
    CD_w = CD0_w + r_w*(CL_w^2) + (CL_w^2)/ (pi * AR_w * e); % Drag coefficient of the wing
    CM_w = -0.2;                                           % Moment coefficient of the wing

    % Tail lift, drag, and moment calculation
    CL_t = a_t*(alpha-0-epsilon_alpha*alpha+Incidence+Tao_e*del_e) ; % Lift coefficient of the tail
    CD_t = (0.148 / (Re_c*tail_c/c)^0.2) * (1 + 2*(t_c_tail) + 60*(t_c_tail)^4);   % Drag coefficient of the tail
    CM_t = Eta_t*vH*CL_t;


    % Vertical tail drag
    CD_v = (0.148 / (Re_c*Ver_tail_c/c)^0.2) * (1 + 2*(t_c_Ver) + 60*(t_c_Ver)^4);

    % Fuselage, and miscellaneous drag contributions
    CD_fuselage = (0.074 / (Re_c*l/c)^0.2) * (1 + 1.5*(d_l)^1.5 + 7*(d_l)^3); % Fuselage drag coefficient
    CM_fuselage = alpha*(2*v)/(c*S);

    % Miscellaneous
    CD_misc =  0.1*(0.074 / (Re_c*l/c)^0.2) * (1 + 1.5*(d_l)^1.5 + 7*(d_l)^3); % Miscellaneous drag coefficient

    % Summation of forces and moments
    CL = (CL_w + CL_t * Eta_t * (S_tail/S) );      % Total lift coefficient
    CD = (CD_w + CD_t *(S_tail/S)+CD_v *(S_Ver/S)+ ...
        CD_fuselage *(S_Fuse/S) + CD_misc) ; % Total drag coefficient
    CM = (CM_w + CM_fuselage + CL_w*(h-0.25)- CM_t) ; % Total moment coefficient

end
```

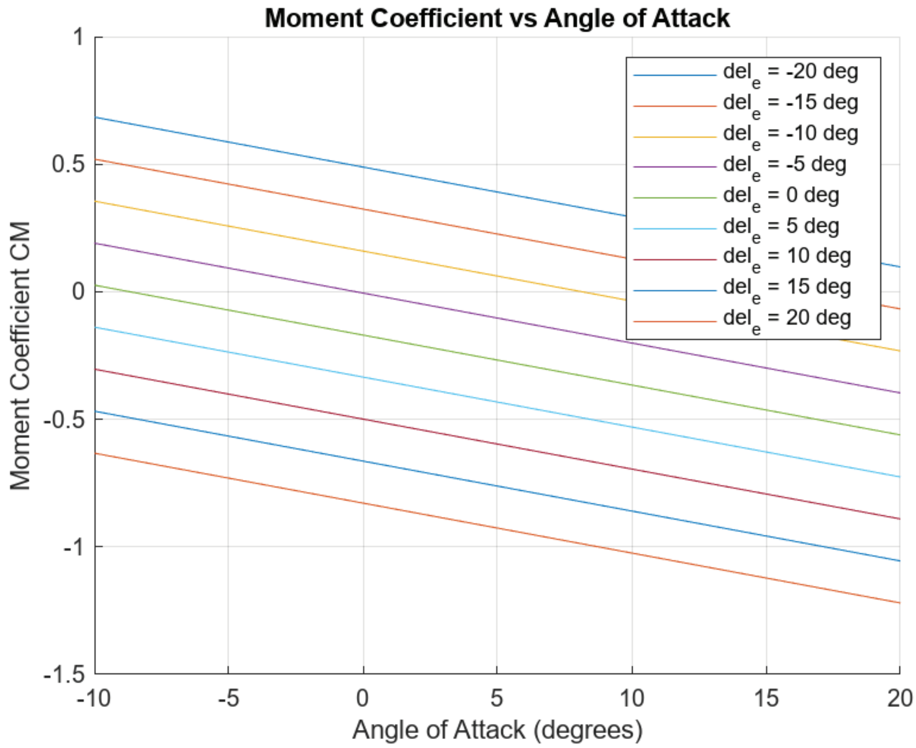

Figure 8: Moment Coefficient vs Angle of Attack

1.8 (h) Evalution

(a) Moment coefficient versus angle of attack, for various elevator angles.

(b) Lift coefficient versus angle of attack, using the same elevator angles

(c) An untrimmed drag polar—that is lift coefficient versus drag coefficient, for the same elevator angles

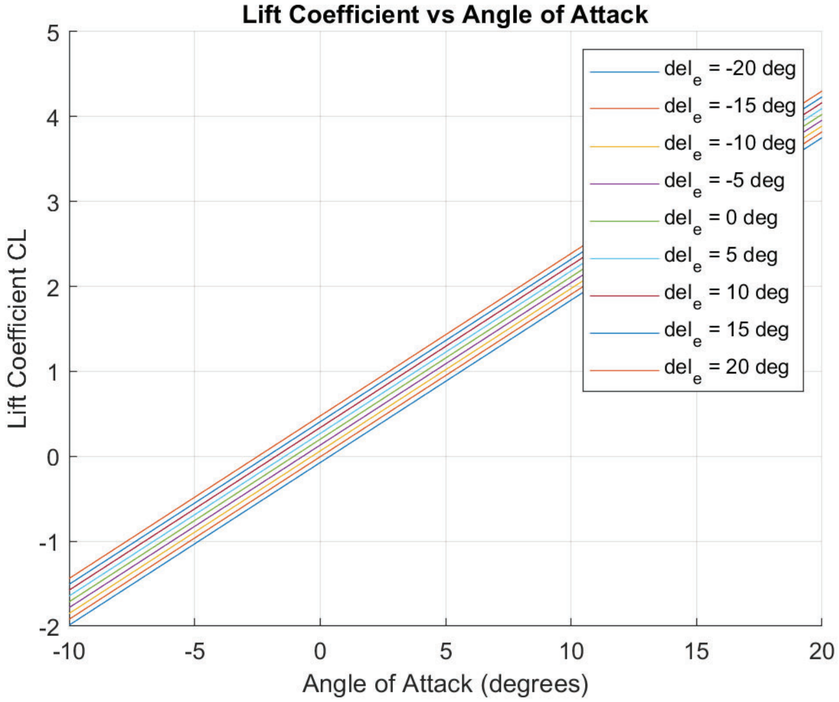

Figure 9: Lift coefficient versus angle of attack

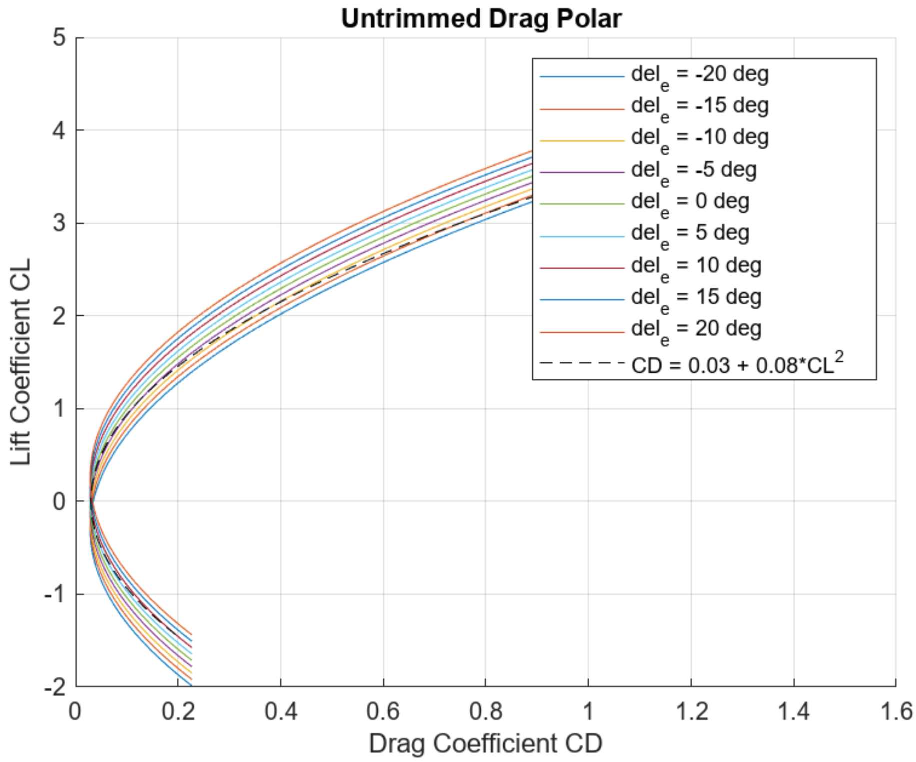

Figure 10: An untrimmed drag polar

2 Problem 2

Create a trimmed drag polar and determine the best-fit constants to the parabolic drag polar

$$C_D = C_{Do} + KC_L^2 \tag{12}$$

To do this, we need to find the elevator angle that results in zero moment at each value of angle of attack. We can do this using the function we wrote in the previous problem.

```
clc
clear all

% Define the range of angle of attack from -5 to 20 degrees
alpha_range = -5:0.5:20;
% Assumed Reynolds number
Re_c_test = 6e5;
% Assumed non-dimensional position of the aircraft center of gravity
h_test = 0.4;

% Initialize data storage as one-dimensional arrays
CM_data = zeros(1, length(alpha_range));
CL_data = zeros(1, length(alpha_range));
CD_data = zeros(1, length(alpha_range));

% Loop over the angle of attack range
for j = 1:length(alpha_range)
    alpha = alpha_range(j);
    % Define the function handle for CM as a function of elevator deflection
    f_CM = @(x) super_cub_aero(alpha, x, 6E5, 0.4);
    % Find the trim elevator deflection angle using fsolve
    del_e_deg_trim = fsolve(f_CM, 0);
    % Calculate the aerodynamic coefficients at the trim condition
    [CM, CL, CD] = super_cub_aero(alpha, del_e_deg_trim, 6E5, 0.4);
    CM_data(j) = CM;
    CL_data(j) = CL;
    CD_data(j) = CD;
end

% Plot the drag polar
plot(CD_data, CL_data, 'o','DisplayName', 'Trimmed Drag Polar')
hold on;
% Create a fine range of CL values for plotting the comparison line
CL_plot = linspace(min(CL_data), max(CL_data), 100);
% Comparison line equation
CD_plot = 0.03 + 0.08 * CL_plot.^2;
% Plot the comparison line
plot(CD_plot, CL_plot, 'k--', 'DisplayName', 'CD = 0.03 + 0.08*CL^2');


title('Trimmed Drag Polar with Comparison to Reference Curve');
xlabel('Drag Coefficient CD');
ylabel('Lift Coefficient CL');
legend('show');
grid on;
hold on;

% Fit options for custom equation
ft = fittype('a + b*x^2', 'independent', 'x', 'dependent', 'y');
opts = fitoptions('Method', 'NonlinearLeastSquares');
```

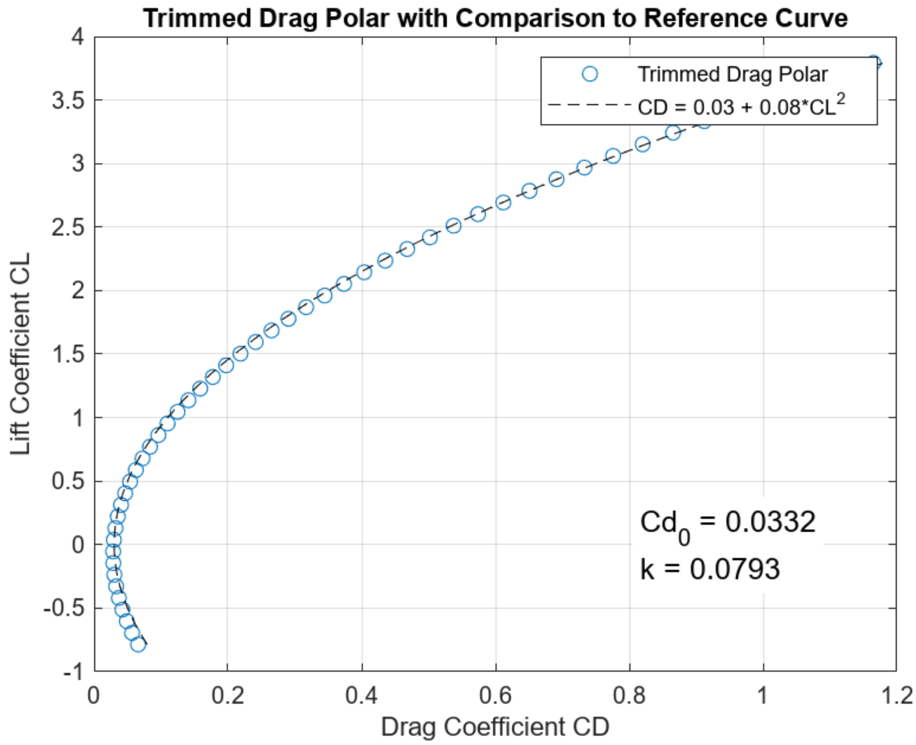

Figure 11: Trimmed Drag Polar

```
opts.StartPoint = [0.03, 0.08]; % Initial guess for [Cd0, k]

% Ensure CL_data and CD_data are column vectors for the fit function
CL_data_col = CL_data(:);
CD_data_col = CD_data(:);

% Perform the fit
[fitresult, gof] = fit(CL_data_col, CD_data_col, ft, opts);

% Extract the fitted parameters Cd0 and k
Cd0 = fitresult.a;
k = fitresult.b;

% Display the results
fprintf('Cd0: %f\n', Cd0);
fprintf('k: %f\n', k);

% Display the results on the graph
str = sprintf('Cd_0 = %.4f\nk = %.4f', Cd0, k);
text(0.7*max(CD_data), 0, str, 'FontSize', 12, 'BackgroundColor', 'white');

hold off
```

3 Problem 3

Achieving maximum lift-to-drag ratio means the slope of the maximum tangent line in the graph. As is shown in Fig. 11, the maximum lift-to-drag ratio(slope of the maximum tangent line) is 9.340. where $C_L = 0.587$ and $C_D = 0.0665$.

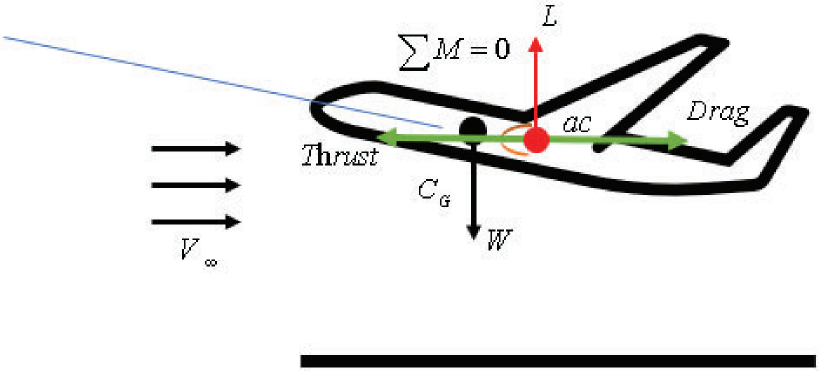

Figure 12: Schematic of the force balance of an airplane

Then, we will calculate minimum thrust required in level flight. As is shown in Fig. 12, the force balance equations can be shown as Equ. 13:

$$\begin{cases} L = W = \frac{1}{2}\rho C_L V_\infty^2 S \\ T = D = \frac{1}{2}\rho C_D V_\infty^2 S \end{cases} \tag{13}$$

Hence

$$V_\infty = \sqrt{\frac{2\omega}{\rho s C_L}} = 9.55m/s \tag{14}$$

Minimum thrust and the corresponding power in level flight is

$$T_{min} = \frac{C_D}{C_L} w = 0.786N = 80.29g \tag{15}$$

$$P_{T_{min}} = T_{min} V = 9.55 \times 0.786 = 7.51W \tag{16}$$

Minimum power and the corresponding thrust in level flight is

$$P = DV_\infty = \frac{1}{2}\rho s \frac{2\omega}{\rho s}^{\frac{3}{2}} \frac{C_D}{C_L^{\frac{3}{2}}} \tag{17}$$

as we can see from the above equation, if we want to achieve minimum power, $\frac{C_L^{1.5}}{C_D}$ should take its maximum value.

We denote C_L and C_D when $\frac{C_L^{1.5}}{C_D}$ takes its maximum value as $C_{L_{pm}}$ and $C_{D_{pm}}$. As we can see from Fig 3, $\frac{C_L^{1.5}}{C_D} = 8.59$, where $C_{L_{pm}} = 1.136$ and $C_{D_{pm}} = 0.141$

$$T_{Pmin} = \frac{C_D}{C_L} w = 0.855N = 87.31g \tag{18}$$

$$V_\infty = \sqrt{\frac{2\omega}{\rho S_{wing} C_{L_{pm}}}} = 6.86m/s \tag{19}$$

$$P_{min} = DV_\infty = \sqrt{2}\rho^{-0.5} s (\frac{\omega}{S_{wing}})^{\frac{3}{2}} \frac{C_{D_{pm}}}{C_{L_{pm}}^{\frac{3}{2}}} = 6.02W \tag{20}$$

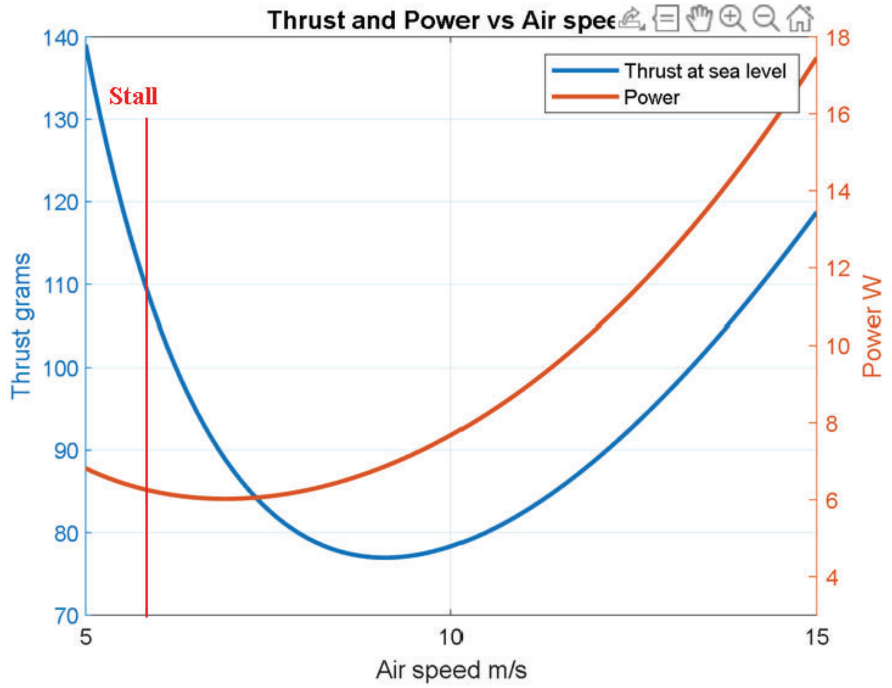

Figure 13: Thrust and Power vs Air speed

Finally, we obtained the result shown in Fig. 13. It's worth noting that we can't fly too slowly; when the wing attack angle is greater than the stall attack angle, the airplane will stall, at which point the speed is:

$$V_{stall} = \sqrt{\frac{2\omega}{\rho S_{wing} C_{L_{stall}}}} = 5.97m/s \tag{21}$$

As is shown in the figure, both thrust and power required by the aircraft decreases sharply as the airspeed increases from the leftmost part of the x-axis, reaching a minimum point before increasing again.

Why Minimum Thrust Speed is Not the Minimum Speed: : Drag consists of two primary components: induced drag and parasitic (or profile) drag. Induced drag is related to the production of lift, and it decreases with an increase in speed. This is because as speed increases, less angle of attack is required to maintain the same amount of lift, leading to a reduction in induced drag. Parasitic drag, on the other hand, increases with the square of the speed and is due to factors such as skin friction and form drag.

The minimum speed for flight is determined by the stall speed, which is the point at which the aircraft can no longer generate enough lift to support its weight. This occurs at a lower speed than the speed for minimum thrust required because, at the stall speed, the aircraft is operating at a very high angle of attack, which is inefficient and results in high induced drag. Thus, the speed for minimum thrust is above the stall speed because it represents a balance between minimizing induced drag and not excessively increasing parasitic drag, rather than the absolute minimum speed at which the aircraft can fly.

4 Appendix

```

clc
clear all
% Define the test range and step size
alpha_range = -10:0.5:20; % Angle of attack range from -5 degrees to 20 degrees
```

```
del_e_range = -20:5:20; % Elevator deflection angle range from -10 degrees to 10 degrees
Re_c_test = 6e5;       % Reynolds number
h_test = 0.4;          % non-dimensional position of the aircraft center of gravity

% Initialize data storage
CM_data = zeros(length(del_e_range), length(alpha_range));
CL_data = zeros(length(del_e_range), length(alpha_range));
CD_data = zeros(length(del_e_range), length(alpha_range));

% Loop to calculate coefficients for each elevator deflection and angle of attack
    combination
for i = 1:length(del_e_range)
    del_e = del_e_range(i);
    for j = 1:length(alpha_range)
        alpha = alpha_range(j);
        [CM, CL, CD] = super_cub_aero(alpha, del_e, Re_c_test, h_test);
        CM_data(i,j) = CM;
        CL_data(i,j) = CL;
        CD_data(i,j) = CD;
    end
end

% Plotting graphs
% Moment coefficient versus angle of attack plot
figure;
hold on;
for i = 1:length(del_e_range)
    plot(alpha_range, CM_data(i,:), 'DisplayName', ['del_e = ' num2str(del_e_range(i)) '
        deg']);
end
title('Moment Coefficient vs Angle of Attack');
xlabel('Angle of Attack (degrees)');
ylabel('Moment Coefficient CM');
legend('show');
grid on;
hold off;

% Lift coefficient versus angle of attack plot
figure;
hold on;
for i = 1:length(del_e_range)
    plot(alpha_range, CL_data(i,:), 'DisplayName', ['del_e = ' num2str(del_e_range(i)) '
        deg']);
end
title('Lift Coefficient vs Angle of Attack');
xlabel('Angle of Attack (degrees)');
ylabel('Lift Coefficient CL');
legend('show');
grid on;
hold off;



% Untrimmed drag polar plot (lift coefficient against drag coefficient)
figure;
hold on;
for i = 1:length(del_e_range)
    plot(CD_data(i,:), CL_data(i,:), 'DisplayName', ['del_e = ' num2str(del_e_range(i))
        ' deg']);
```

```
end
title('Untrimmed Drag Polar');
xlabel('Drag Coefficient CD');
ylabel('Lift Coefficient CL');
legend('show');
grid on;
hold on;

% Create a fine range of CL values for plotting the comparison line
CL_plot = linspace(min(CL_data(i,:)), max(CL_data(i,:)), 100);
% Comparison line equation
CD_plot = 0.03 + 0.08 * CL_plot.^2;
% Plot the comparison line
plot(CD_plot, CL_plot, 'k--', 'DisplayName', 'CD = 0.03 + 0.08*CL^2');

% Labeling the plot
title('Untrimmed Drag Polar ');
xlabel('Drag Coefficient CD');
ylabel('Lift Coefficient CL');
legend('show');
grid on;
hold off;




clc
clear all

% Define the range of angle of attack from -5 to 20 degrees
alpha_range = -5:0.5:20;
% Assumed Reynolds number
Re_c_test = 6e5;
% Assumed non-dimensional position of the aircraft center of gravity
h_test = 0.4;

% Initialize data storage as one-dimensional arrays
CM_data = zeros(1, length(alpha_range));
CL_data = zeros(1, length(alpha_range));
CD_data = zeros(1, length(alpha_range));

% Loop over the angle of attack range
for j = 1:length(alpha_range)
    alpha = alpha_range(j);
    % Define the function handle for CM as a function of elevator deflection
    f_CM = @(x) super_cub_aero(alpha, x, 6E5, 0.4);
    % Find the trim elevator deflection angle using fsolve
    del_e_deg_trim = fsolve(f_CM, 0);
    % Calculate the aerodynamic coefficients at the trim condition
    [CM, CL, CD] = super_cub_aero(alpha, del_e_deg_trim, 6E5, 0.4);
    CM_data(j) = CM;
    CL_data(j) = CL;
    CD_data(j) = CD;
end

% Plot the drag polar
plot(CD_data, CL_data, 'o','DisplayName', 'Trimmed Drag Polar')
hold on;
% Create a fine range of CL values for plotting the comparison line
```

```
CL_plot = linspace(min(CL_data), max(CL_data), 100);
% Comparison line equation
CD_plot = 0.03 + 0.08 * CL_plot.^2;
% Plot the comparison line
plot(CD_plot, CL_plot, 'k--', 'DisplayName', 'CD = 0.03 + 0.08*CL^2');


title('Trimmed Drag Polar with Comparison to Reference Curve');
xlabel('Drag Coefficient CD');
ylabel('Lift Coefficient CL');
legend('show');
grid on;
hold on;

% Fit options for custom equation
ft = fittype('a + b*x^2', 'independent', 'x', 'dependent', 'y');
opts = fitoptions('Method', 'NonlinearLeastSquares');
opts.StartPoint = [0.03, 0.08]; % Initial guess for [Cd0, k]

% Ensure CL_data and CD_data are column vectors for the fit function
CL_data_col = CL_data(:);
CD_data_col = CD_data(:);

% Perform the fit
[fitresult, gof] = fit(CL_data_col, CD_data_col, ft, opts);

% Extract the fitted parameters Cd0 and k
Cd0 = fitresult.a;
k = fitresult.b;

% Display the results
fprintf('Cd0: %f\n', Cd0);
fprintf('k: %f\n', k);

% Display the results on the graph
str = sprintf('Cd_0 = %.4f\nk = %.4f', Cd0, k);
text(0.7*max(CD_data), 0, str, 'FontSize', 12, 'BackgroundColor', 'white');

hold off

% Calculate L/D ratio
LD_ratio = CL_data ./ CD_data;
max(LD_ratio)
[max_LD, max_LD_index] = max(LD_ratio);
% Find corresponding CL and CD at maximum L/D
CL_max_LD = CL_data(1,max_LD_index)
CD_max_LD = CD_data(1,max_LD_index)

% Filter for CL > 0
positive_CL_indices = CL_data > 0;
CL_positive = CL_data(positive_CL_indices);
CD_positive = CD_data(positive_CL_indices);

% Calculate L/D ratio for the positive CL part
LD_ratio_15 = CL_positive .^ 1.5 ./ CD_positive;
[max_LD, max_LD_index] = max(LD_ratio_15);

% Find corresponding CL and CD at maximum L/D
CL_max_LD = CL_positive(max_LD_index);
```

```
CD_max_LD = CD_positive(max_LD_index);

% Display the results
fprintf('Maximum L/D ratio is: %.2f\n', max_LD);
fprintf('Corresponding CL at max L/D is: %.4f\n', CL_max_LD);
fprintf('Corresponding CD at max L/D is: %.4f\n', CD_max_LD);


% Constants
CD0_adjusted = 0.0332; % Adjusted for fuselage and empennage
e = 0.85; % Oswald efficiency factor, assuming a typical value
wing_area = 0.224; % m^2
wing_span = 1.1938; % m
AR = wing_span^2 / wing_area; % Aspect ratio
Weight = 0.75;

% Air densities at different altitudes (kg/m^3)
rho_sea_level = 1.225;

rho=[rho_sea_level];
% Airspeed for cruise (m/s)
V_cruise = linspace(5,15); % Convert from 56 km/hr to m/s

% Thrust required (T) and Power required (P) at sea level, 10km, and 20km altitudes
T = 1000/9.8*0.5 .* rho .* V_cruise.^2 .* wing_area .*
    (CD0_adjusted+0.0793.*(2.*Weight.*9.8./(rho.*wing_area.* V_cruise.^2)).^2);
P = T .* V_cruise*9.8/1000;

% Create a new figure
figure;

% Select the left y-axis for the Thrust data
yyaxis left;
plot(V_cruise, T(1,:), 'LineWidth', 2); % Thrust vs Air speed
ylabel('Thrust grams'); % Label for the left y-axis

% Set properties that apply to the whole plot
xlabel('Air speed m/s'); % X-axis label applies to both datasets
title('Thrust and Power vs Air speed'); % Title for the plot
grid on; % Turn the grid on

% Now switch to the right y-axis for the Power data
yyaxis right;

plot(V_cruise, P(1,:), 'LineWidth', 2); % Power vs Air speed
ylabel('Power W'); % Label for the right y-axis
ylim([3 18]);
% Add a legend to distinguish between the two datasets
legend('Thrust at sea level', 'Power');


function [CM, CL, CD] = super_cub_aero(alpha, del_e, Re_c, h)
    % super_cub_aero: Aerodynamic analysis of the Super Cub aircraft
    % This function calculates the aerodynamic coefficients CM, CL, and CD
    % for the Super Cub aircraft based on input conditions.
    %
    % Input arguments:
    % alpha_deg - Airplane angle of attack (degrees)
    % del_e_deg - Elevator deflection angle (degrees)
```

```
% Re_c - Reynolds number based on wing chord
% h - Dimensionless position of the airplane CG


% Constants
inch2m = 0.0254;

%  Determine Cd0 based on Rec
Rec = [6e5, 9e4, 6e4, 3e4];
Cd0 = [0.022, 0.14, 0.13, 0.12];           % Cd0 from NACA 2415 dataset
p_degree = 2;
p_coeff = polyfit(Rec, Cd0, p_degree);     % Find polynomial coefficients
cd0_from_rec = @(Re_c) polyval(p_coeff, Re_c);
CD0_w = cd0_from_rec(Re_c);                % Get the corresponding Cd0_w

% WING
b = 47*inch2m;    % Wingspan
c = 7.5*inch2m;   % Wing chord [m]
S = b*c;          % Wing reference area [m^2]
AR_w = b^2/S;     % Wing aspect ratio
CL0_w = 0.2;     % Lift coefficient at zero angle of attack
alpha_L0 = -2;    % Zero-lift angle of attack (degrees)
a_2d_w = 0.1744;
r_w=0.013;
e=0.7;

% Tail
tail_c = 5*inch2m;           % Tail chord [m]
ele_c = 2*inch2m;            % Elevator chord [m]
tail_b = 16*inch2m;          % Tailspan [m]
S_tail = tail_b*tail_c;      % Tail reference area [m^2]
S_ele = tail_b*ele_c;        % Elevator reference area [m^2]
AR_t = tail_b^2/S_tail;      % Tail aspect ratio
AR_e = tail_b^2/S_ele;       % Elevator aspect ratio
Eta_t = 0.9;                 % Tail efficiency
lt = 18*inch2m;              % Distance Between Wing & Tail [m]
thickness =0.25*inch2m;      % Thickness [m]
vH = lt*(S_tail)/(S*c);      % Tail Volume [m^3]
t_c_tail = thickness/tail_c; % Tail thickness-to-chord ratio
a_t= 0.096;
Incidence =0;
epsilon_alpha=2*(a_2d_w)/pi/e/AR_w;
Tao_e=0.7;

Ver_tail_c = 5.0* inch2m;   % Vertical Tail chord [m]
t_c_Ver= thickness/Ver_tail_c; % Vertical Tail thickness-to-chord ratio
S_Ver = 5* 5.3*inch2m^2;    % Vertical Tail reference area [m^2]

% Fuselage
d = 5.7*inch2m;
l = 30.8*inch2m;
S_Fuse = (5.7*inch2m)*l;     % Fuselage reference area [m^2]
d_l= d/l;                    % diameter/thickness to length
v = 1e-5;                 % Fuselage Volume [m^3]

% Miscellaneous
```

```
    % Main wing lift, drag, and moment calculation
    CL_w = CL0_w+(alpha)*a_2d_w/(1+(a_2d_w)/pi/e/AR_w);   % Lift coefficient of the wing
    CD_w = CD0_w + r_w*(CL_w^2) + (CL_w^2)/ (pi * AR_w * e); % Drag coefficient of the wing
    CM_w = -0.2;                                            % Moment coefficient of the wing

    % Tail lift, drag, and moment calculation
    CL_t = a_t*(alpha-0-epsilon_alpha*alpha+Incidence+Tao_e*del_e) ; % Lift coefficient of the tail
    CD_t = (0.148 / (Re_c*tail_c/c)^0.2) * (1 + 2*(t_c_tail) + 60*(t_c_tail)^4);   % Drag coefficient of the tail
    CM_t = Eta_t*vH*CL_t;


    % Vertical tail drag
    CD_v = (0.148 / (Re_c*Ver_tail_c/c)^0.2) * (1 + 2*(t_c_Ver) + 60*(t_c_Ver)^4);

    % Fuselage, and miscellaneous drag contributions
    CD_fuselage = (0.074 / (Re_c*l/c)^0.2) * (1 + 1.5*(d_l)^1.5 + 7*(d_l)^3); % Fuselage drag coefficient
    CM_fuselage = alpha*(2*v)/(c*S);

    % Miscellaneous
    CD_misc =  0.1*(0.074 / (Re_c*l/c)^0.2) * (1 + 1.5*(d_l)^1.5 + 7*(d_l)^3); % Miscellaneous drag coefficient

    % Summation of forces and moments
    CL = (CL_w + CL_t * Eta_t * (S_tail/S) );      % Total lift coefficient
    CD = (CD_w + CD_t *(S_tail/S)+CD_v *(S_Ver/S)+ ...
        CD_fuselage *(S_Fuse/S) + CD_misc) ; % Total drag coefficient
    CM = (CM_w + CM_fuselage + CL_w*(h-0.25)- CM_t) ; % Total moment coefficient

end
```

References

[1] NACA 2415 (n2415-il). (n.d.). Airfoiltools.com. .http://airfoiltools.com/airfoil/details?airfoil=n2415-il

[2] EPPLER 395 AIRFOIL (e395-il). (n.d.). Airfoiltools.com. Retrieved February 16, 2024, from .http://airfoiltools.com/airfoil/details?airfoil=e395-il

[3] Efficiency of Aircraft Engine: Unlocking Performance Potential. (2023, September 3). Saabaircraft.com. .https://saabaircraft.com/efficiency/

MEAM 543 HW5

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1 Problem 1 — Hobbyzone Super Cub Propeller Performance

1.1 (a)

```

% Sample data
J = [0.096, 0.121, 0.143, 0.167, 0.191, 0.217, 0.241, 0.266, 0.291, 0.311, 0.339, 0.362,
    0.383, 0.406, 0.429, 0.453, 0.476];
CT = [0.1195, 0.1190, 0.1180, 0.1171, 0.1162, 0.1150, 0.1132, 0.1111, 0.1093, 0.1068,
    0.1042, 0.1015, 0.0988, 0.0961, 0.0929, 0.0897, 0.0863];
CP = [0.0583, 0.0590, 0.0595, 0.0601, 0.0607, 0.0614, 0.0617, 0.0621, 0.0625, 0.0624,
    0.0626, 0.0626, 0.0623, 0.0622, 0.0618, 0.0613, 0.0607];
ETA = [0.197, 0.244, 0.284, 0.326, 0.365, 0.407, 0.442, 0.476, 0.508, 0.533, 0.564,
    0.587, 0.607, 0.627, 0.645, 0.662, 0.678];

% Polynomial degree, for example, let's use 2nd degree (quadratic) polynomial
degree = 2;

% Fit polynomial to CT vs J
p_CT = polyfit(J, CT, degree);

% Fit polynomial to CP vs J
p_CP = polyfit(J, CP, degree);

% Fit polynomial to ETA vs J
p_ETA = polyfit(J, ETA, degree);
```

1.2 (b)

From previous Hw4, we know that $V_{stall} = 5.97m/s$, $V_{minT} = 9.55m/s$

T produced by a propeller is given by:

$$T = C_T \rho n^2 D^4 \tag{1}$$

where:

- C_T is the thrust coefficient, which is a dimensionless number describing the efficiency of the propeller.
- ρ is the air density (in kilograms per cubic meter).
- n is the propeller rotational speed in revolutions per second.

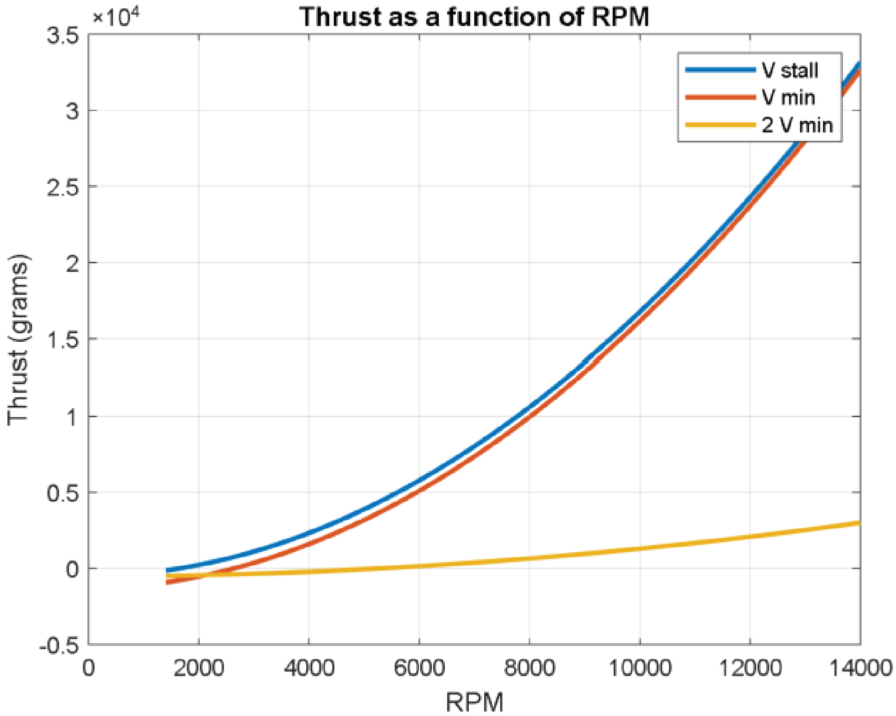


Figure 1: Thrust as a function of RPM

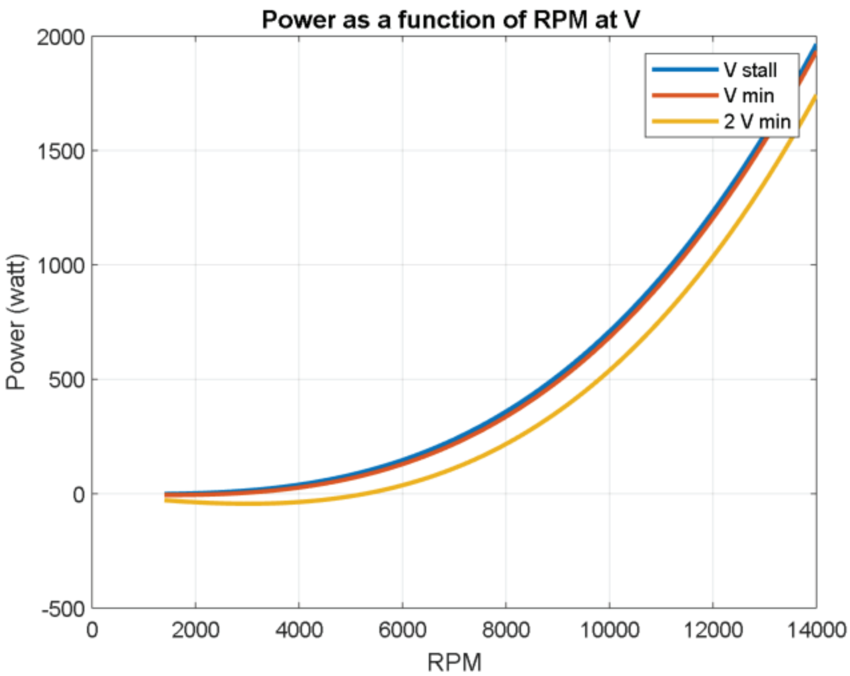


Figure 2: Power as a function of RPM at V

- D is the diameter of the propeller in meters.

P produced by a propeller is given by:

$$P = C_T \rho n^3 D^5 \tag{2}$$

where:

- C_P is the power coefficient, which is a dimensionless number describing the efficiency of the propeller.
- ρ is the air density (in kilograms per cubic meter).
- n is the propeller rotational speed in revolutions per second.
- D is the diameter of the propeller in meters.

2 Problem 2 — Hobbyzone Super Cub Propulsion System Operating Points

480 DC motor model has 3 empirical constants, and a good fit to the data described here can be obtained with these values:

- Motor constant: k_m = 4.2 [N-mm/amp] (note: this is 0.0042 Nm/amp)
- Coils Resistance: R_m = 0.25 [ohms]
- Zero-load current: i_o = 0.6 [amp]

The motor equation is:

$$Q = ki - Q_0 \tag{3}$$

$$e = kw + iR \tag{4}$$

where

- Q is the motor torque,
- e is the applied voltage,
- i is the current flowing through the motor,
- R is the resistance of the motor windings,
- k is a constant that relates to the motor's construction and the magnetic field strength
- Q_0 is the no-load torque (the torque necessary to overcome friction and other losses in the motor when no external load is applied).
- w represents the angular velocity (speed) of the motor.

Mechanical power produced by the motor as a function of propeller rotational speed is:

$$P_{\text{motor}} = K_t \cdot \left(\frac{\text{RPM} \cdot \pi}{30}\right) \cdot \left(e_{\text{max}} - K_t \cdot \left(\frac{\text{RPM} \cdot \pi}{30}\right)\right) / R_m - K_t \cdot \left(\frac{\text{RPM} \cdot \pi}{30}\right) \cdot i_0 \tag{5}$$

Propeller power as a function of propeller rotational speed is:

$$P = C_T \rho n^3 D^5 \tag{6}$$

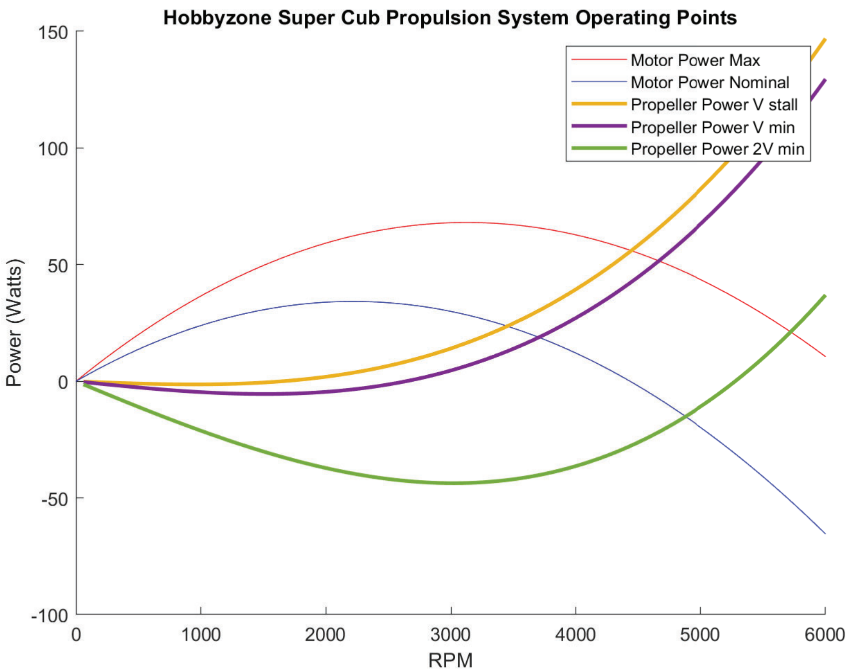


Figure 3: Hobbyzone Super Cub Propulsion System Operating Points

3 Problem 3 — Level-Flight Performance Analysis

Iterate Over Airspeed Range: For each airspeed in the range v_{inf}, perform the following calculations:

Calculate Lift and Drag Coefficients:

Lift coefficient

$$C_L = \frac{2W}{S\rho v_{\text{inf}}^2} \tag{7}$$

Drag coefficient

$$C_D = C_{D0} + KC_L^2 \tag{8}$$

Solve for RPM: Use the fzero function to find the RPM (n) that balances the forces, by solving the equation:

$$0.5\rho v_{\text{inf}}^2 C_D S = \rho \left(\frac{n}{60}\right)^2 D^4 \mathrm{f}(p_{CT}, \frac{v_{\text{inf}}}{(\frac{n}{80})D}) \tag{9}$$

Calculate Power, Current, and Voltage: Power required by the propeller

$$P = \rho \left(\frac{n}{60}\right)^3 D^5 \mathrm{f}(p_{CP}, \frac{v_{\text{sf}}}{(\frac{n}{60})D}) \tag{10}$$

Electric current

$$I = \frac{P}{K(\frac{n}{60}2\pi)} + i_0 \tag{11}$$

Electric voltage

$$V = K(\frac{n}{60}2\pi) + IR_m \tag{12}$$

Calculate Efficiencies:

Motor efficiency

$$\eta_{\text{motor}} = \frac{P_m}{P_\epsilon} \tag{13}$$

Propeller efficiency

$$\eta_{\text{propeller}} = \frac{P_r}{P_m} \tag{14}$$

Total system efficiency

$$r_{\text{Hotal}} = \frac{P_{\text{r}}}{P_{\text{e}}} \tag{15}$$

Maximum possible steady climb rate:

$$P_r = \max(\text{PR}) - \text{PR}; \quad \frac{dh}{dt} = \left(\frac{P_r}{W}\right) \times 60 \tag{16}$$

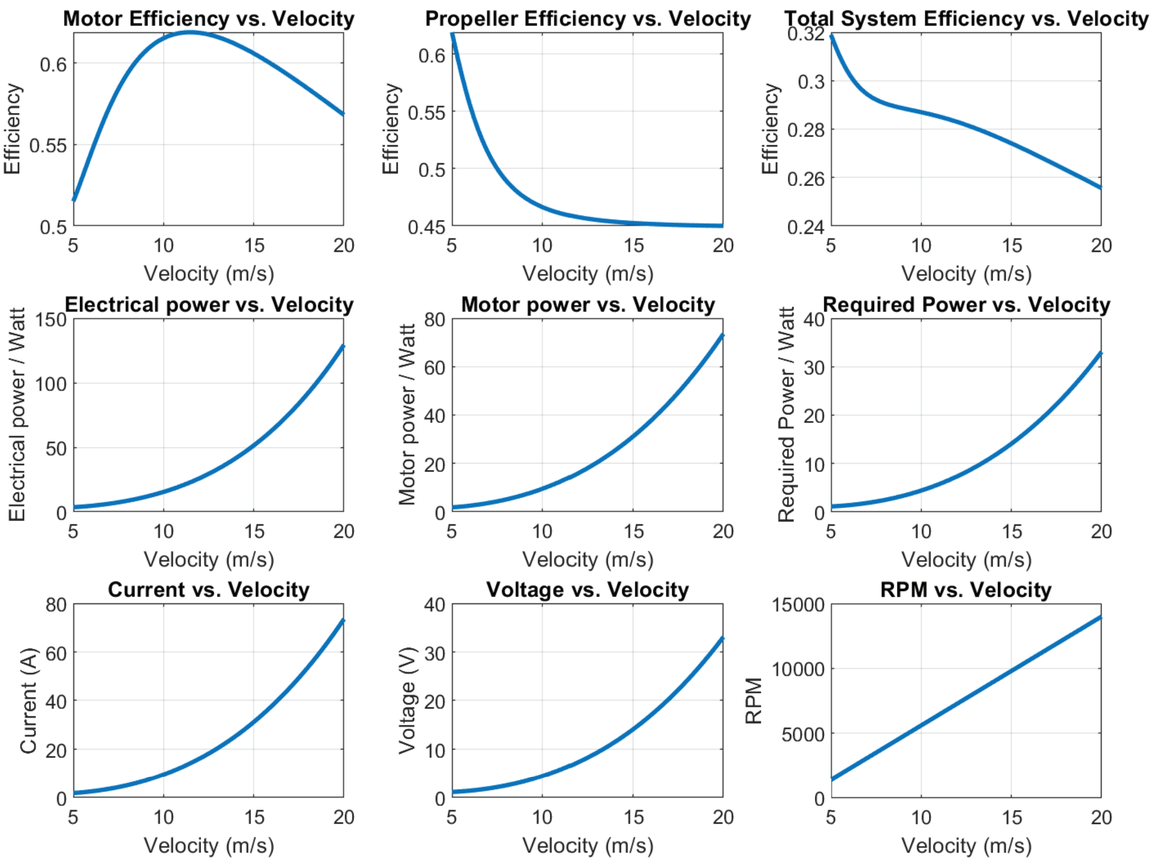


Figure 4: Level-Flight Performance Analysis

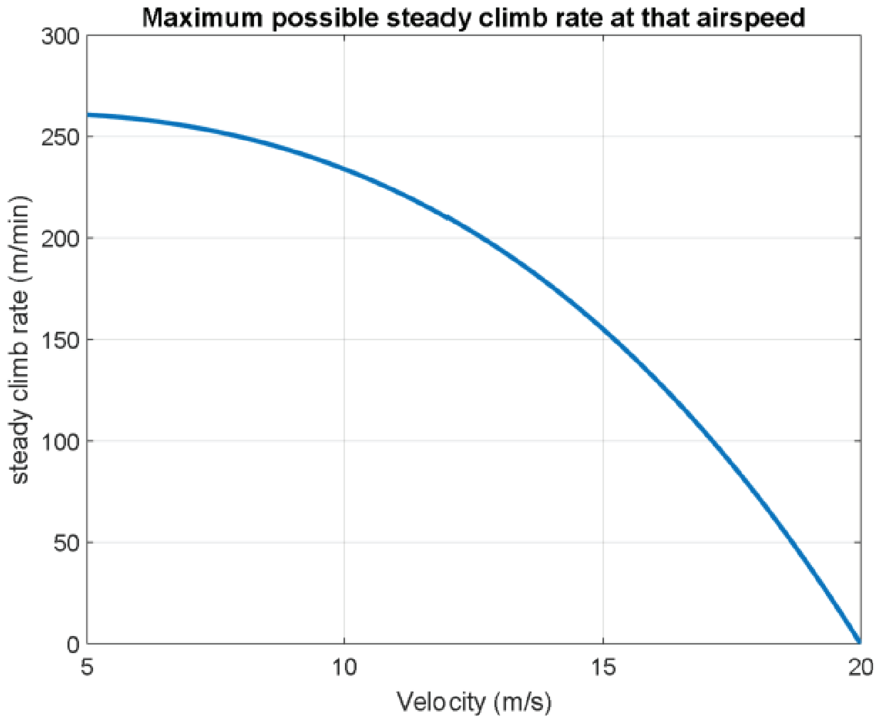


Figure 5: Maximum possible steady climb rate at that airspeed

4 Problem 4 — Why the Gearbox?

A comparison of the efficiency of the motor with and without the gearbox is shown in Fig. 6. As you can see from the graph, without the gearbox, the propeller always works in the lower efficiency range. The presence of a gearbox between the motor and the propeller allows for a mechanical advantage, enabling the motor to operate within an efficient RPM range while delivering the required propeller speed for optimal thrust. Directly driving the propeller might not always be feasible or efficient, especially if the motor's most efficient RPM range does not align with the propeller's operational requirements.

To quantitatively evaluate the choice of gear ratio, including the 3:1 choice versus a hypothetical 1:1 ratio, we can consider a few critical parameters and calculations:

We calculate the efficiency of the whole system based on the aforementioned code by changing the gear ratio as Fig. 7. The graph effectively communicates where the system operates most efficiently. The x-axis ranges from 1 to 5, which indicates different gearbox ratio settings. The y-axis represents total system efficiency, ranging from approximately $27\% to 34\%$. A prominent feature of the graph is a blue curve that peaks at a gearbox ratio of about 2.25. This peak, circled in red, represents the point of maximum efficiency for the system, which is annotated directly on the graph.

This is slightly different from the 1:3 given in the question, first of all this is because, at different speeds, the optimal gear ratio is not the same. Secondly, the propeller data we chose may be different from the actual value. However, overall, it is reasonable to choose a 1:3 reduction ratio to effectively improve the efficiency of the system.

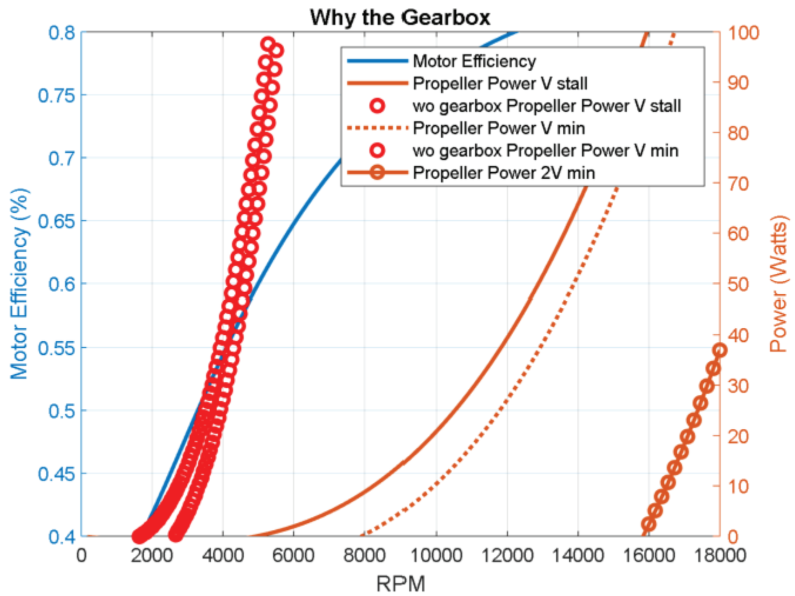


Figure 6: Comparison of efficiency with and without gearbox motors

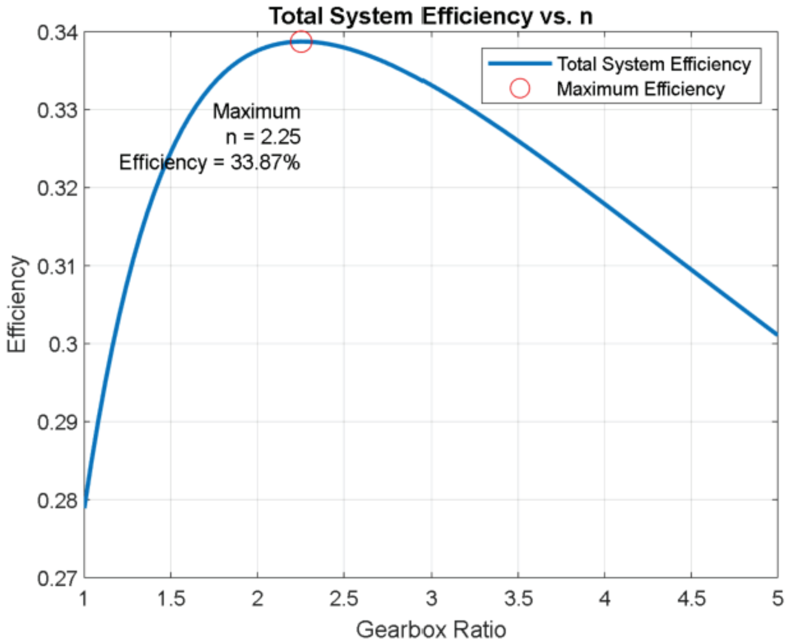


Figure 7: Total System Efficiency vs. Gearbox ratio

References

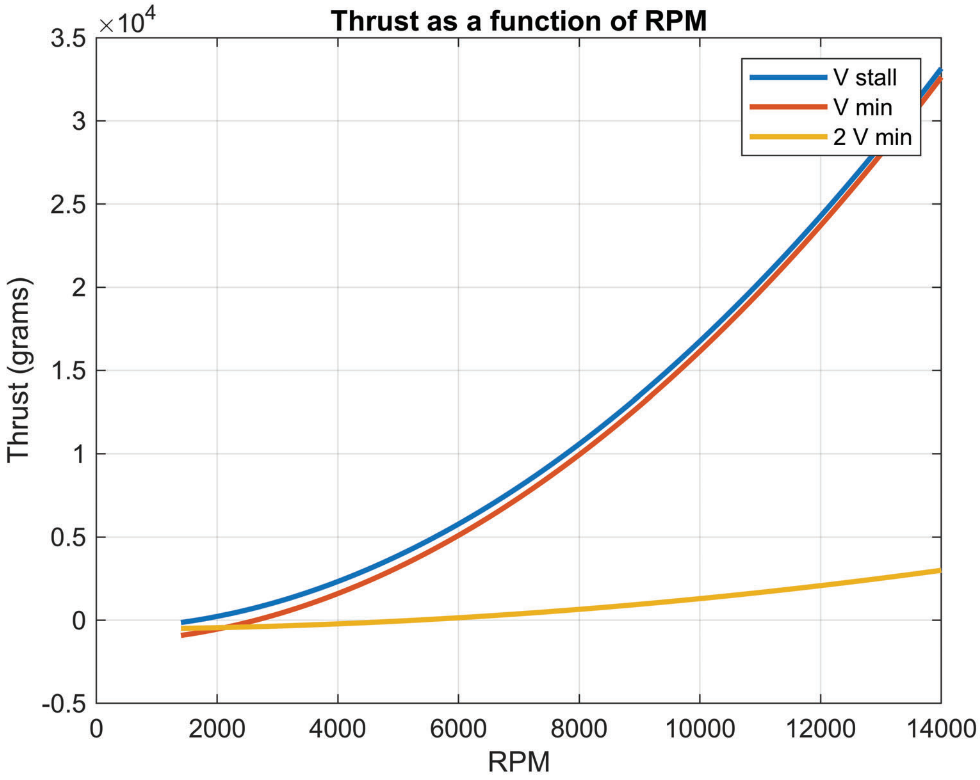
[1] NACA 2415 (n2415-il). (n.d.). Airfoiltools.com. .http://airfoiltools.com/airfoil/details?airfoil=n2415-il

[2] EPPLER 395 AIRFOIL (e395-il). (n.d.). Airfoiltools.com. Retrieved February 16, 2024, from .http://airfoiltools.com/airfoil/details?airfoil=e395-il

[3] Efficiency of Aircraft Engine: Unlocking Performance Potential. (2023, September 3). Saabaircraft.com. .https://saabaircraft.com/efficiency/

```
% Sample data
J_data = [0.096, 0.121, 0.143, 0.167, 0.191, 0.217, 0.241, 0.266, 0.291, 0.311,
0.339, 0.362, 0.383, 0.406, 0.429, 0.453, 0.476];
CT_data = [0.1195, 0.1190, 0.1180, 0.1171, 0.1162, 0.1150, 0.1132, 0.1111, 0.1093,
0.1068, 0.1042, 0.1015, 0.0988, 0.0961, 0.0929, 0.0897, 0.0863];
CP_data = [0.0583, 0.0590, 0.0595, 0.0601, 0.0607, 0.0614, 0.0617, 0.0621, 0.0625,
0.0624, 0.0626, 0.0626, 0.0623, 0.0622, 0.0618, 0.0613, 0.0607];
ETA_data = [0.197, 0.244, 0.284, 0.326, 0.365, 0.407, 0.442, 0.476, 0.508, 0.533,
0.564, 0.587, 0.607, 0.627, 0.645, 0.662, 0.678];

% Polynomial degree, for example, let's use 2nd degree (quadratic) polynomial
degree = 2;

% Fit polynomial to CT vs J
p_CT = polyfit(J_data, CT_data, degree);
% Fit polynomial to CP vs J
p_CP = polyfit(J_data, CP_data, degree);
% Fit polynomial to ETA vs J
p_ETA = polyfit(J_data, ETA_data, degree);


RPM = linspace(1400, 14000, 100); %RPM range
inch2m = 0.0254;
D = 10*inch2m;

V=5.97;
J= V./((RPM/60)*D);
rho=1.225;
T= polyval(p_CT, J)*1.225*D^4.*(RPM/60).^2*1000;
% Thrust vs RPM²
figure;
plot(RPM, T, 'LineWidth', 2); % Adjust the line width as desired
xlabel('RPM');
title('Thrust as a function of RPM');
grid on;
ylabel('Thrust (grams)');
hold on

V=9.55;
J= V./((RPM/60)*D);
rho=1.225;
T= polyval(p_CT, J)*1.225*D^4.*(RPM/60).^2*1000;
hold on
plot(RPM, T, 'LineWidth', 2); % Adjust the line width as desired


V=9.55*2;
J= V./((RPM/60)*D);
rho=1.225;
T= polyval(p_CT, J)*1.225*D^4.*(RPM/60).^2*1000/9.8;
```

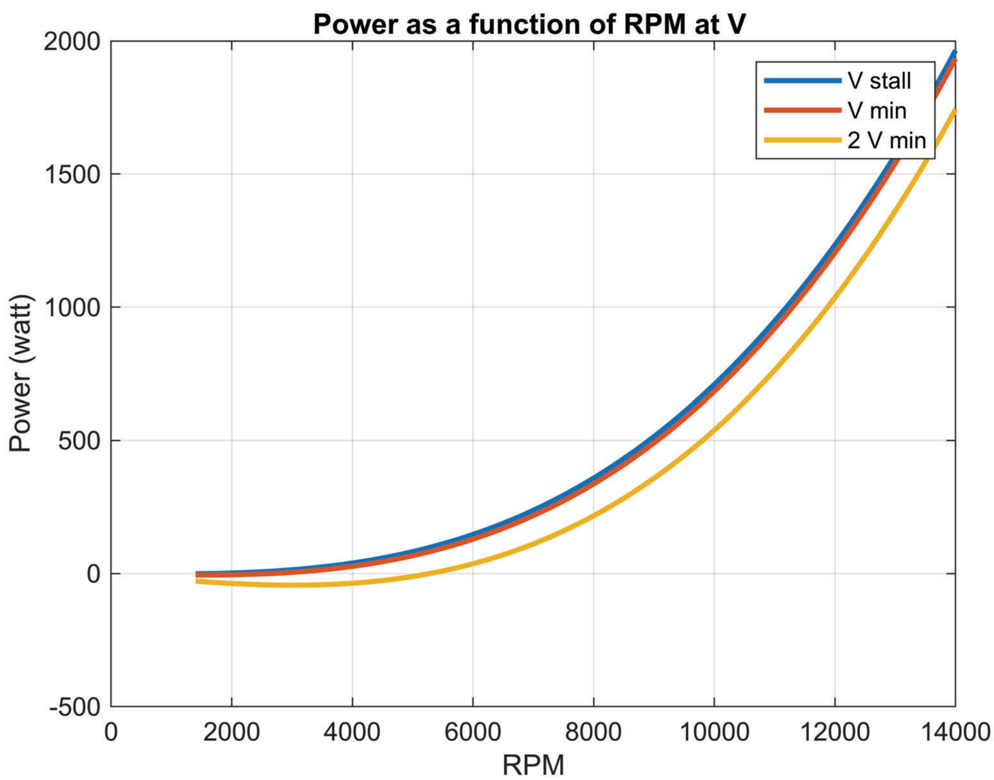
```
hold on
plot(RPM, T, 'LineWidth', 2); % Adjust the line width as desired
legend("V stall","V min","2 V min")
hold off
```




```
V=5.97;
J= V./((RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(RPM/60).^3;
% Thrust vs RPM²
figure;
plot(RPM, P, 'LineWidth', 2); % Adjust the line width as desired
xlabel('RPM');
title('Power as a function of RPM at V ');
grid on;
ylabel('Power (watt)');
hold on

V=9.55;
J= V./((RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(RPM/60).^3
```

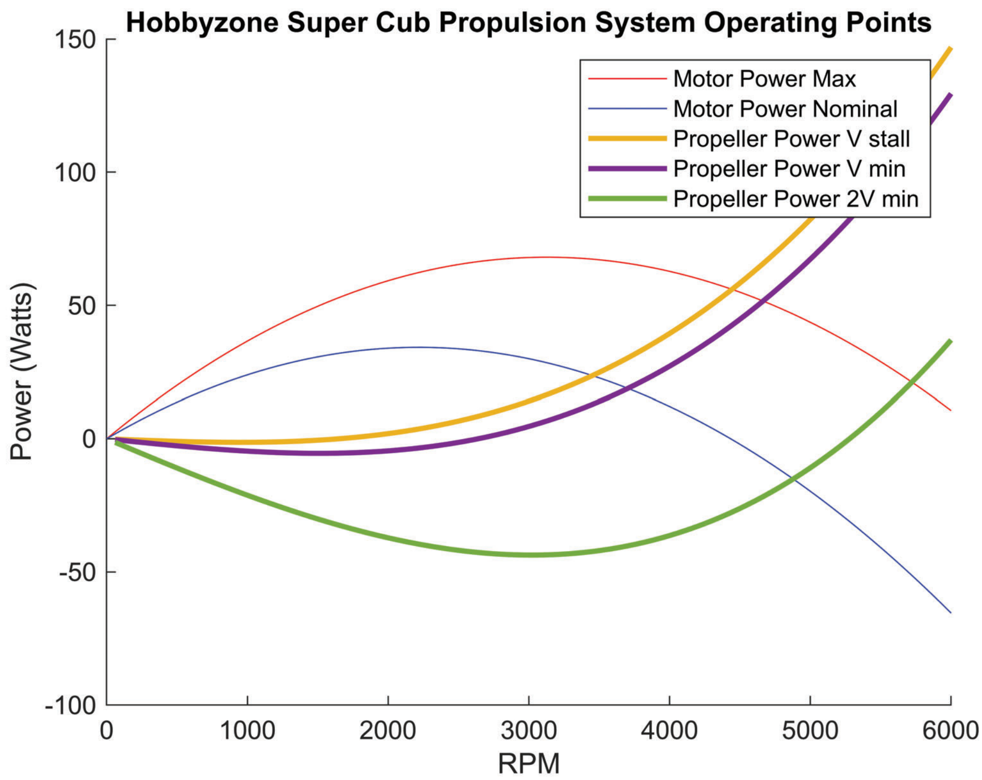
```
P = 1×100
10^3 ×
   -0.0054   -0.0055   -0.0054   -0.0052   -0.0049   -0.0044   -0.0038   -0.0030 ···
```

```
hold on
plot(RPM, P, 'LineWidth', 2); % Adjust the line width as desired


V=9.55*2;
J= V./((RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(RPM/60).^3;
hold on
plot(RPM, P, 'LineWidth', 2); % Adjust the line width as desired
legend("V stall","V min","2 V min")
hold off
```



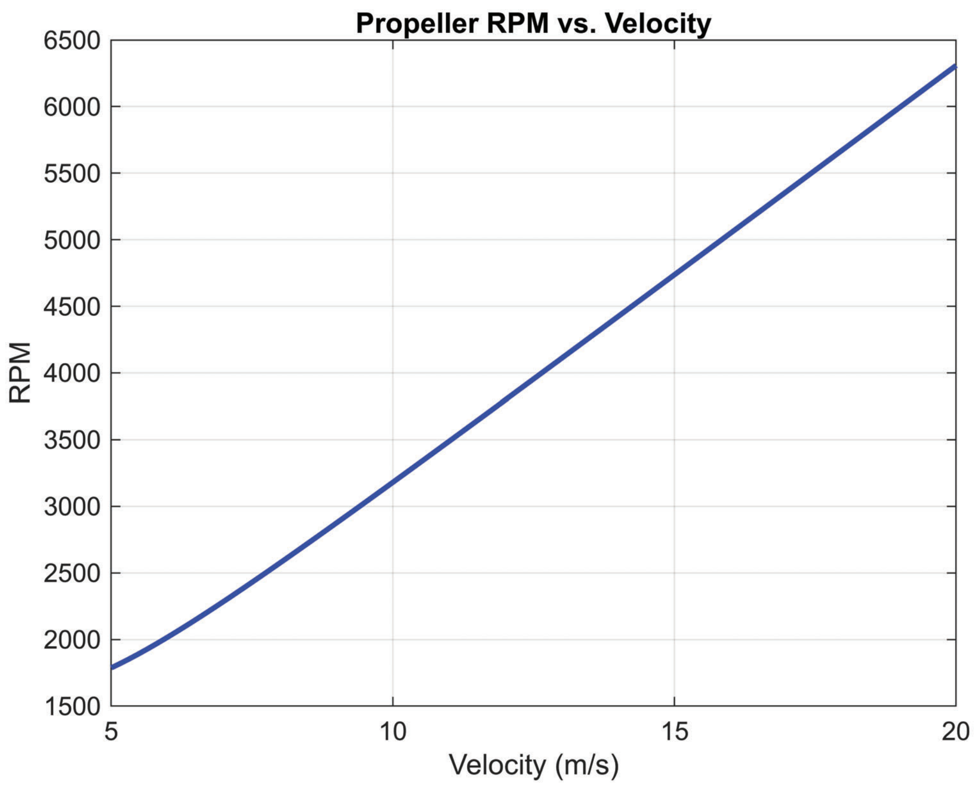

```
% Constants
inch2m = 0.0254;
D = 10 * inch2m; % Propeller diameter in meters
rho = 1.225; % Air density in kg/m^3
K_t = 0.0042; % Motor constant in Nm/A
R_m = 0.25; % Coils resistance in ohms
i_0 = 0.6; % Zero-load current in amps

% Applied voltages for max and nominal power settings (example values)
e_max = 8.4; % Max power setting voltage in volts
e_nominal = 6; % Nominal power setting voltage in volts
```

```
% RPM range - replace with your actual RPM range
max_pro_RPM=6000;
P_RPM = linspace(0, max_pro_RPM, 100); % Example RPM range

% Calculate motor power for max and nominal settings
P_motor_max = K_t * (3*P_RPM * pi / 30).*(e_max-K_t * (3*P_RPM * pi / 30))/R_m- K_t
* (3*P_RPM * pi / 30)*i_0;
P_motor_nominal = K_t * (3*P_RPM * pi / 30).*(e_nominal-K_t * (3*P_RPM * pi / 30))/
R_m- K_t * (3*P_RPM * pi / 30)*i_0;

% Adding to the plot
figure;
hold on;
plot(P_RPM, P_motor_max, '-r', 'DisplayName', 'Motor Power Max');
plot(P_RPM, P_motor_nominal, '-b', 'DisplayName', 'Motor Power Nominal');

hold on
V= 5.97;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM, P, 'LineWidth', 2,'DisplayName', 'Propeller Power V stall');

hold on
V= 9.55;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM, P, 'LineWidth', 2,'DisplayName', 'Propeller Power V min');

hold on
V= 2*9.55;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM, P, 'LineWidth', 2,'DisplayName', 'Propeller Power 2V min');

legend show;
xlabel('RPM');
ylabel('Power (Watts)');
title('Hobbyzone Super Cub Propulsion System Operating Points');
hold off
```




```
% Define known parameters
v_inf_range = linspace(5,20,100); % Velocity range from 5 to 20 m/s
max_pro_RPM = 6000;
rho = 1.225; % Air density in kg/m^3
S = 0.224; % Wing area in square meters
W = 0.75 * 9.8; % Weight in Newtons, assuming 0.75 kg mass
CD0 = 0.03; % Zero-lift drag coefficient
K = 0.008; % Induced drag factor


% Preallocate array for solved RPMs for each velocity
solved_RPMs = zeros(size(v_inf_range));

% Loop over velocity range to solve for RPM at each velocity
for idx = 1:length(v_inf_range)
    v_inf = v_inf_range(idx); % Current velocity

    % Calculate lift and drag coefficients
    CL = (2 * W) / (S * rho * v_inf^2);
    CD = CD0 + K * CL^2;

    % Define the equation to find RPM that balances forces
    fun = @(n) 0.5 * rho * v_inf^2 * CD * S - rho * (n/60)^2 * D^4 * polyval(p_CT,
v_inf / ((n/60) * D));
```

```
    % Solve for RPM using fzero
    n_initial_guess = 8000; % Initial guess for RPM
    solved_RPMs(idx) = fzero(fun, n_initial_guess);
    % Output the solved RPMs

    %fprintf('Velocity: %d m/s, Solved RPM: %.2f\n', v_inf, solved_RPMs(idx));

end

% Plotting RPM vs. Velocity
figure;
plot(v_inf_range, solved_RPMs, 'b-', 'LineWidth', 2);
xlabel('Velocity (m/s)');
ylabel('RPM');
title('Propeller RPM vs. Velocity');
grid on;
```

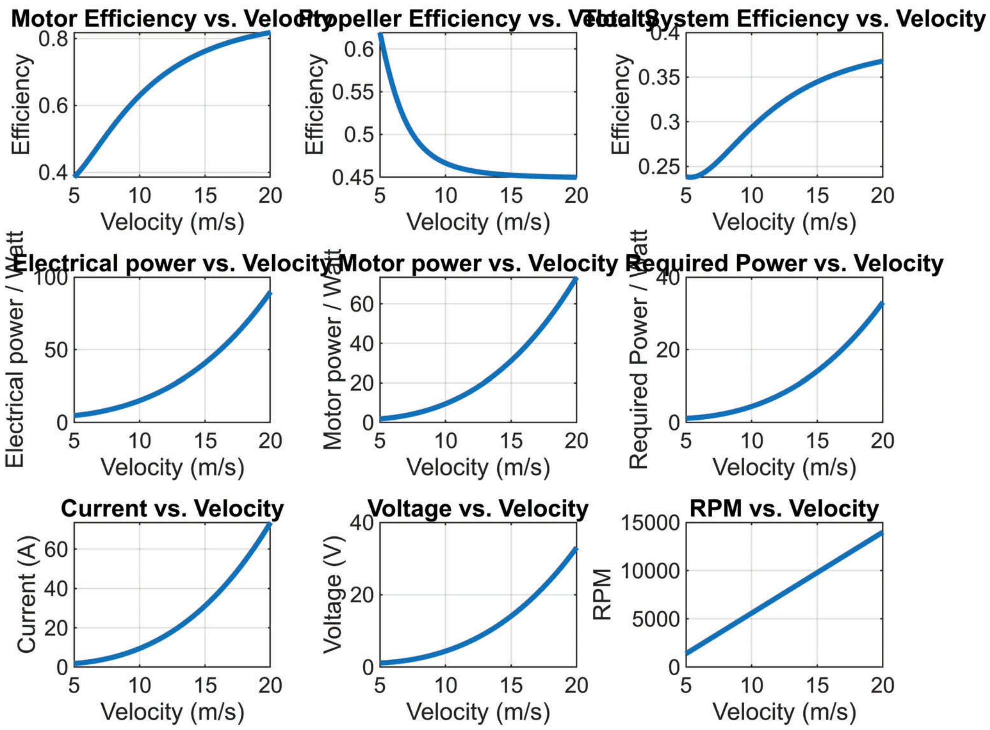



```
Motor_n = 3*n;
K_t = 0.0042; % Motor constant in Nm/A
R_m = 0.25; % Coils resistance in ohms
i_0 = 0.6; % Zero-load current in amps


omega = (2 * pi * Motor_n) / 60;
E = K * omega;
```

```
V = I * R_m + E;

% Preallocate array for solved RPMs for each velocity
solved_RPMs = zeros(size(v_inf_range));
motor_efficiencies = zeros(size(v_inf_range));
propeller_efficiencies = zeros(size(v_inf_range));
total_system_efficiencies = zeros(size(v_inf_range));
currents = zeros(size(v_inf_range));
voltages = zeros(size(v_inf_range));
PE = zeros(size(v_inf_range));
PM = zeros(size(v_inf_range));
PR = zeros(size(v_inf_range));

% Loop over velocity range to solve for RPM at each velocity
for idx = 1:length(v_inf_range)
    v_inf = v_inf_range(idx); % Current velocity

    % Calculate lift and drag coefficients
    CL = (2 * W) / (S * rho * v_inf^2);
    CD = CD0 + K * CL^2;

    % Define the equation to find RPM that balances forces
    fun = @(n) 0.5 * rho * v_inf^2 * CD * S - rho * (n/60)^2 * D^4 * polyval(p_CT,
v_inf / ((n/60) * D));

    % Solve for RPM using fzero
    n_initial_guess = 8000; % Initial guess for RPM
    solved_RPMs(idx) = fzero(fun, n_initial_guess);
    % Output the solved RPMs
    P = rho * (solved_RPMs(idx)/60)^3 * D^5 * polyval(p_CP, v_inf /
((solved_RPMs(idx)/60) * D));
    I= P/(K*(solved_RPMs(idx)*3/60*6.28))+i_0;
    V= K*(solved_RPMs(idx)*3/60*6.28) + I*R_m;

    currents(idx)=I;
    voltages(idx)=V;

    P_e= V*I;
    P_m = P;
    P_r= v_inf*0.5 * rho * v_inf^2 * CD * S;

    PE(idx)=P_e;
    PM(idx)=P_m;
    PR(idx)=P_r;

    motor_efficiencies(idx) = P_m / P_e;
    propeller_efficiencies(idx) = P_r / P_m;
    total_system_efficiencies(idx) = P_r / P_e;

    %fprintf('Velocity: %d m/s, Solved RPM: %.2f\n', v_inf, solved_RPMs(idx));
```

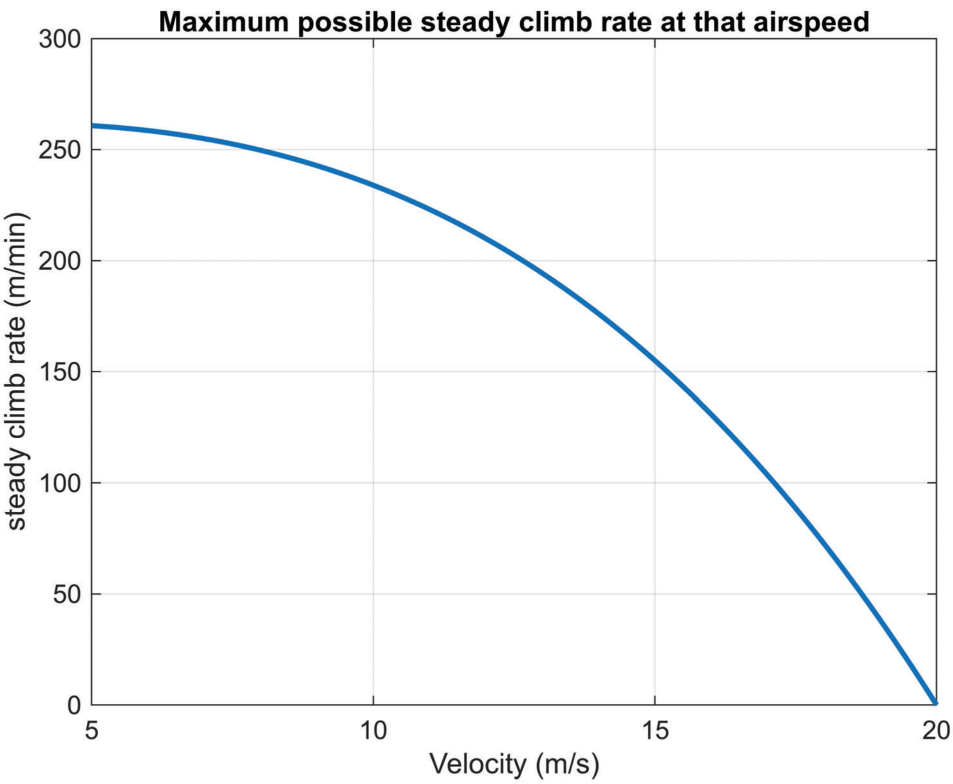
```
end



figure;
subplot(3,3,1);
plot(v_inf_range, motor_efficiencies, 'LineWidth', 2);
title('Motor Efficiency vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Efficiency');
grid on

subplot(3,3,2);
plot(v_inf_range, propeller_efficiencies, 'LineWidth', 2);
title('Propeller Efficiency vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Efficiency');
grid on

subplot(3,3,3);
plot(v_inf_range, total_system_efficiencies, 'LineWidth', 2);
title('Total System Efficiency vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Efficiency');
grid on

subplot(3,3,4);
plot(v_inf_range, PE, 'LineWidth', 2);
title('Electrical power vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Electrical power / Watt');
grid on

subplot(3,3,5);
plot(v_inf_range, PM, 'LineWidth', 2);
title('Motor power vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Motor power / Watt');
grid on

subplot(3,3,6);
plot(v_inf_range, PR, 'LineWidth', 2);
title('Required Power vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Required Power / Watt');
grid on

subplot(3,3,7);
plot(v_inf_range, PM, 'LineWidth', 2);
```

```
title('Current vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Current (A)');
grid on

subplot(3,3,8);
plot(v_inf_range, PR, 'LineWidth', 2);
title('Voltage vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('Voltage (V)');
grid on

subplot(3,3,9);
plot(v_inf_range, RPM, 'LineWidth', 2);
title('RPM vs. Velocity');
xlabel('Velocity (m/s)');
ylabel('RPM');
grid on
```



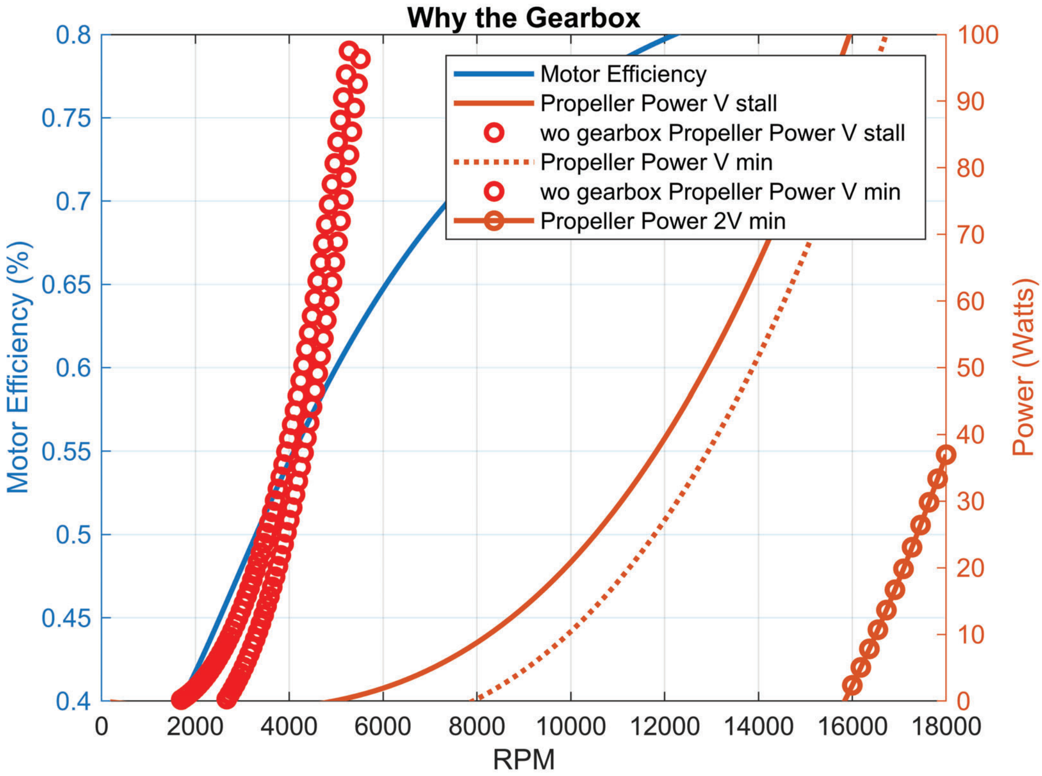

```
figure

P_r=max(PR)-PR;

dh_dt=(P_r/W)*60;
plot(v_inf_range, dh_dt, 'LineWidth', 2);
title('Maximum possible steady climb rate at that airspeed');
xlabel('Velocity (m/s)');
```

```
ylabel('steady climb rate (m/min)');
grid on
```




```
% Constants
inch2m = 0.0254;
D = 10 * inch2m; % Propeller diameter in meters
rho = 1.225; % Air density in kg/m^3
K_t = 0.0042; % Motor constant in Nm/A
R_m = 0.25; % Coils resistance in ohms
i_0 = 0.6; % Zero-load current in amps

% Applied voltages for max and nominal power settings (example values)
e_max = 8.4; % Max power setting voltage in volts
e_nominal = 6; % Nominal power setting voltage in volts

% RPM range - replace with your actual RPM range
max_pro_RPM=6000;
P_RPM = linspace(0, max_pro_RPM, 100); % Example RPM range

% Calculate motor power for max and nominal settings
P_motor_max = K_t * (3*P_RPM * pi / 30).*(e_max-K_t * (3*P_RPM * pi / 30))/R_m- K_t
* (3*P_RPM * pi / 30)*i_0;
P_motor_nominal = K_t * (3*P_RPM * pi / 30).*(e_nominal-K_t * (3*P_RPM * pi / 30))/
R_m- K_t * (3*P_RPM * pi / 30)*i_0;
```

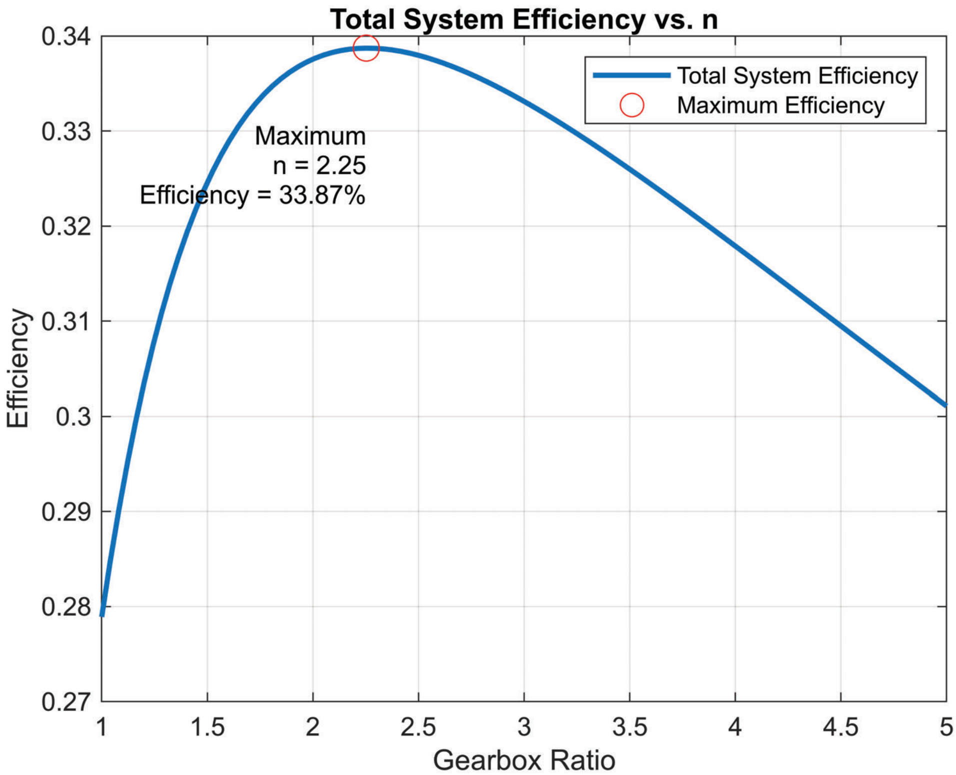
```
% Adding to the plot
figure;

yyaxis left; % Activate left y-axis for motor efficiencies
plot(RPM, motor_efficiencies, 'LineWidth', 2,'DisplayName', 'Motor Efficiency  ');
ylim([0.4 0.8])
ylabel('Motor Efficiency (%)');

yyaxis right; % Activate right y-axis for motor power

hold on
V= 5.97;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM*3, P, 'LineWidth', 2,'DisplayName', 'Propeller Power V stall');
hold on
plot(P_RPM, P, 'ro','LineWidth', 2,'DisplayName', 'wo gearbox Propeller Power V
stall');

hold on
V= 9.55;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM*3, P, 'LineWidth', 2,'DisplayName', 'Propeller Power V min');
hold on
plot(P_RPM, P, 'ro','LineWidth', 2,'DisplayName', 'wo gearbox Propeller Power V
min');

hold on
V= 2*9.55;
J= V./((P_RPM/60)*D);
rho=1.225;
P= polyval(p_CT, J)*1.225*D^5.*(P_RPM / 60).^3;
plot(P_RPM*3, P, 'LineWidth', 2,'DisplayName', 'Propeller Power 2V min');
ylim([0,100])

legend show;
xlabel('RPM');
ylabel('Power (Watts)');
title('Why the Gearbox');
grid on
hold off
```




```
K_t = 0.0042; % Motor constant in Nm/A
R_m = 0.25; % Coils resistance in ohms
i_0 = 0.6; % Zero-load current in amps
n_range=linspace(1,5,100);

omega = (2 * pi * Motor_n) / 60;
E = K * omega;
V = I * R_m + E;

% Preallocate array for solved RPMs for each velocity
solved_RPMs = zeros(size(n_range));
motor_efficiencies = zeros(size(n_range));
propeller_efficiencies = zeros(size(n_range));
total_system_efficiencies = zeros(size(n_range));
currents = zeros(size(n_range));
voltages = zeros(size(n_range));
PE = zeros(size(n_range));
PM = zeros(size(n_range));
PR = zeros(size(n_range));

% Loop over velocity range to solve for RPM at each velocity
for idx = 1:length(n_range)
    v_inf = 13.5; % Current velocity
```

```
    % Calculate lift and drag coefficients
    CL = (2 * W) / (S * rho * v_inf^2);
    CD = CD0 + K * CL^2;

    % Define the equation to find RPM that balances forces
    fun = @(n) 0.5 * rho * v_inf^2 * CD * S - rho * (n/60)^2 * D^4 * polyval(p_CT,
v_inf / ((n/60) * D));

    % Solve for RPM using fzero
    n_initial_guess = 8000; % Initial guess for RPM
    solved_RPMs(idx) = fzero(fun, n_initial_guess);
    % Output the solved RPMs
    P = rho * (solved_RPMs(idx)/60)^3 * D^5 * polyval(p_CP, v_inf /
((solved_RPMs(idx)/60) * D));
    I= P/(K*(solved_RPMs(idx)*n_range(idx)/60*6.28))+i_0;
    V= K*(solved_RPMs(idx)*n_range(idx)/60*6.28) + I*R_m;

    currents(idx)=I;
    voltages(idx)=V;

    P_e= V*I;
    P_m = P;
    P_r= v_inf*0.5 * rho * v_inf^2 * CD * S;

    PE(idx)=P_e;
    PM(idx)=P_m;
    PR(idx)=P_r;

    motor_efficiencies(idx) = P_m / P_e;
    propeller_efficiencies(idx) = P_r / P_m;
    total_system_efficiencies(idx) = P_r / P_e;

    %fprintf('Velocity: %d m/s, Solved RPM: %.2f\n', v_inf, solved_RPMs(idx));

end

figure
% Assuming n_range and total_system_efficiencies are already defined
plot(n_range, total_system_efficiencies, 'LineWidth', 2, 'DisplayName', 'Total
System Efficiency');
xlabel('Gearbox Ratio');
ylabel('Efficiency');
title('Total System Efficiency vs. n');
legend('show');
grid on;
hold on;

% Find and annotate the maximum efficiency
[maxEfficiency, maxIdx] = max(total_system_efficiencies);
maxN = n_range(maxIdx);
```

```
% Plotting the maximum point
plot(maxN, maxEfficiency, 'ro', 'MarkerSize', 10, 'DisplayName', 'Maximum
Efficiency');

% Annotating the maximum point
text(maxN, maxEfficiency*0.95, sprintf('Maximum\nn = %0.2f\nEfficiency = %0.2f%%',
maxN, maxEfficiency*100), 'VerticalAlignment', 'bottom', 'HorizontalAlignment',
'right');

hold off;
grid on
```



MEAM 543 HW6

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1 Problem 1 — Highway Pursuit UAV Preliminary Sizing

1.1 (a)

I think the ALBATROSS UAV might be a suitable alternative. From what's already publicly available online. https://store.appliedaeronautics.com/albatross-uav-rtf/.It meets all the requirements.

Table 1: UAV Specifications

Specification	Value
Wingspan	3000MM or 9.8FT
MTOW	10 KG or 22 LBS
Available Payload	2KG - 4.4 KG (battery dependent)
Endurance	4 HRS (varies based on setup)
Range	100 KM+
Cruise Speed	18 M/S or 40 MPH
Max Level Speed	40 M/S or 90 MPH
Takeoff	50 FT (varied surfaces)
Glide Ratio (L/D)	28:1 - 30:1
Center of Gravity	85 MM - 95 MM from root leading edge

Figure 1: ALBATROSS UAV

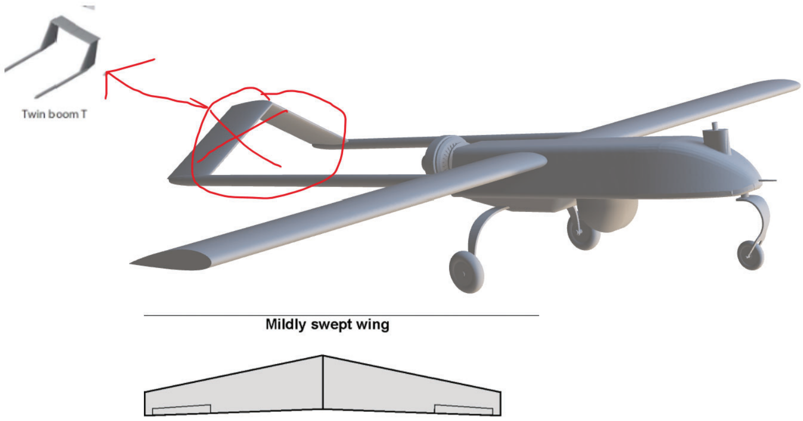

Figure 2: a sketch of proposed confguration

1.2 (b)

The proposed aircraft features a twin-boom configuration, which is characterized by two distinct tail booms extending backward from the main fuselage. This setup allows for a clear space between the booms where we insatll the propeller and engine.

Additionally, the aircraft has a mildly swept wing design, which offers a compromise between high-speed aerodynamics and structural efficiency.

The advantages of a twin-boom layout include improved stability and control, as well as the ability to accommodate a variety of payloads or propulsion systems in the space between the booms. The mildly swept wings provide better performance at high speeds compared to straight wings, while not requiring the structural complexity and weight of highly swept wings.

1.3 (c)

We use the NACA 4415 wing and we can find the drag polar of the wing from Egidijus Pakalnis's paper `https://www.researchgate.net/publication/330087372_Analysis_of_calculation_results_of_lift_and_drag_forces_for_several_wings_using_nonlinear_section_data`. Where we can find the drag polar of a 2D wing. It should be noticed that

$$\mathrm{D} = D_w + D_f + D_{misc} + D_t = \frac{1}{2}\rho_\infty V_\infty^2 S C_D \tag{1}$$

We can consider the fuselage, tail, and miscellaneous as 3D/2D Streamlined Bodies, and obtain the drag from Bruce's Favorite Graph.

Finally, the drag polar equation is:

$$C_D = 0.02 + 0.09 * C_L^2 \tag{2}$$

1.4 (d)

$$\frac{W_F}{W_o} = \frac{R}{H_F} \cdot \frac{1}{\eta_{TOT}} \cdot \frac{1}{(L/D)} \tag{3}$$

$\frac{W_F}{W_o}$ is the fuel weight fraction, where R: Stands for range, the distance an aircraft can fly on a certain amount of fuel. H_f: This is the fuel consumption per unit time (fuel flow), which when integrated over the flight duration gives you

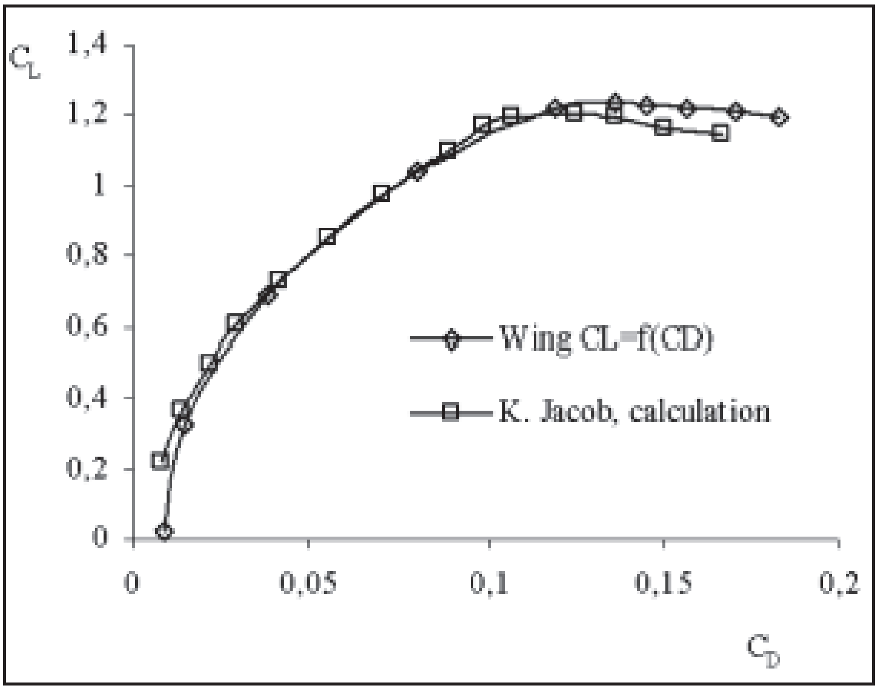

Figure 3: Lift-drag characteristics for a NACA 4415 rectangular wing with AR=6.2 at Re=2.1x106. Experimental results were used as 2D section data `https://www.researchgate.net/publication/330087372_Analysis_of_calculation_results_of_lift_and_drag_forces_for_several_wings_using_nonlinear_section_data`

total fuel consumed. η_{TOT}: This represents the total efficiency of the propulsion system. L/D: The inverse of the lift-to-drag ratio.

In the earliest stages of design, we thought $H_f = 0.5Mj/km$,$R = 30miles$ and $L/D = 10$,we assume $\frac{w_e}{w_o} = 0.55$ from Fig 4.

$$W_o = \frac{W_p}{1 - \frac{W_e}{W_o} - \frac{W_f}{W_o}} \tag{4}$$

Hence, $W_p = 1.5kg$,$W_o = 4.3kg$,$W_f = 0.4kg$ and ,$W_e = 2.4kg$

The drag polar gives insight into the amount of drag the aircraft will experience at different lift coefficients. Since drag directly affects the amount of power—and thus energy—needed to maintain flight, the drag polar can be used to estimate fuel or battery consumption over the course of a mission. A more favorable drag polar (lower CD for a given C L) will reduce the amount of energy needed. Since the total energy requirement is affected by the drag characteristics, the drag polar directly influences the weight of fuel or batteries needed. This impacts the gross weight calculations, as the energy weight is a component of the gross weight. A more efficient drag polar means less energy weight, which might also allow for more payload or longer range for the same total weight.

1.5 (e)

Achieving maximum lift-to-drag ratio means the slope of the maximum tangent line in the graph. As is shown in Fig. 15 , the maximum lift-to-drag ratio(slope of the maximum tangent line) is 11.78. where CL = 0.4712 and CD = 0.04.

Then we can calculate the wing area using the following equation

$$L = W = \frac{1}{2}\rho C_L V_\infty^2 S \tag{5}$$

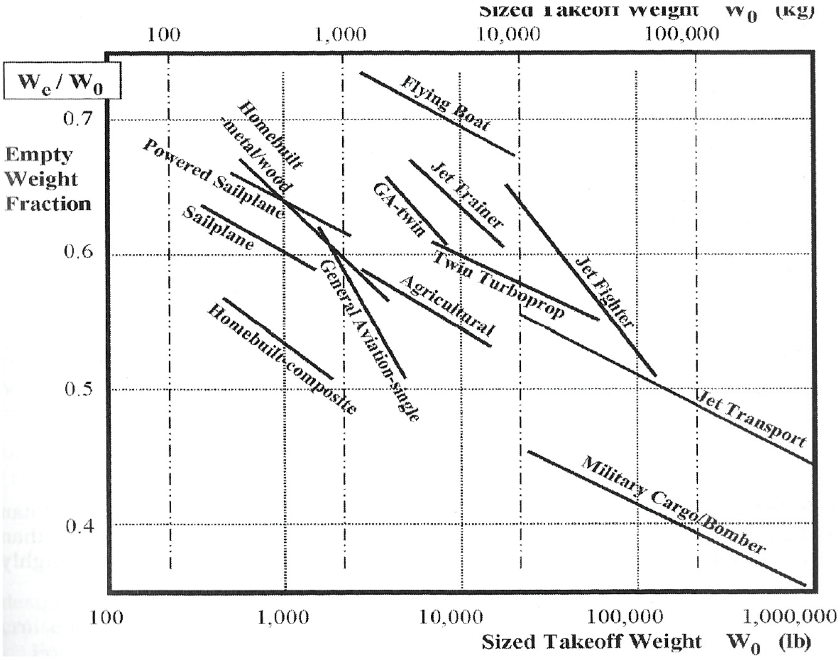

Figure 4: Empty Weight Fraction

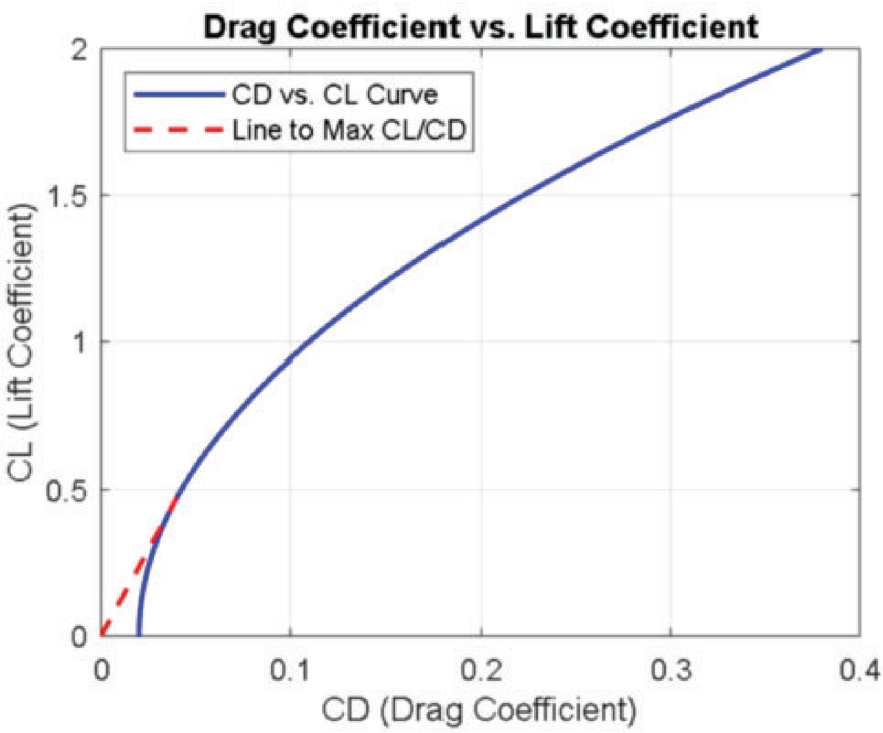

Figure 5: Drag polar

Table 2: Flap deflection vs attack angle

	Attack Angle	**Flap Deflection**
0	8	60
1	10	55
2	12	50

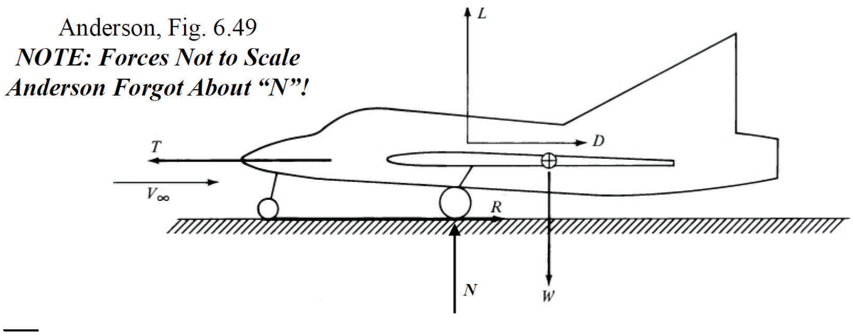

Figure 6: Forces During Landing

where, we can obtain $s = 0.072m^2$

1.6 (e)

Suppose that the airplane lands at a speed corresponding to a lift coefficient that is 90% of the maximum lift coefficient for the airfoil.

$$C_{L_Landing} = 0.424 \tag{6}$$

Then we can calculate the speed at landing using the following equation

$$L = W = \frac{1}{2}\rho C_L V_{Landing}^2 S \tag{7}$$

where $V_{Landing} = 48m/s = 173km/h$, which is a super high speed.

And after that, we can quickly obtain the desired acceleration using the following equation:

$$s_L = \frac{W}{S}\frac{1.3}{\rho C_{L,MAX} a_{DECEL}} \tag{8}$$

In this case, the required acceleration is -2.5g, which is clearly greater than the design requirement, so we need to add flaps.

The potential for a vehicle to topple nose-down upon landing when the deceleration exceeds 0.4g can be explained through a free-body diagram and understanding the forces and moments acting on the aircraft during landing shown in Fig. 6. The main landing gear is typically located behind the CG. The force due to deceleration (which is proportional to the mass of the aircraft multiplied by the deceleration) acts horizontally at the CG. This force creates a tipping moment because it has a vertical lever arm from the point where the main wheels contact the ground. During deceleration, if the force exerted by braking (which acts through the center of gravity) is large enough, it can create a moment about the main landing gear that could tip the aircraft forward.

If we want to reduce the acceleration to -0.4g, at which point $C_L = 2.61$, From the flap deflection vs attack angle graph 7 we can see the relationship between the angle of approach and the flap angle.

As we can see, the angle of approach required in our design for a safe landing is very close to the stall angle of approach, although it meets the design requirements, which is not a very good design. We could have added brakes, deceleration plates or counterthrusting to achieve the design requirements.

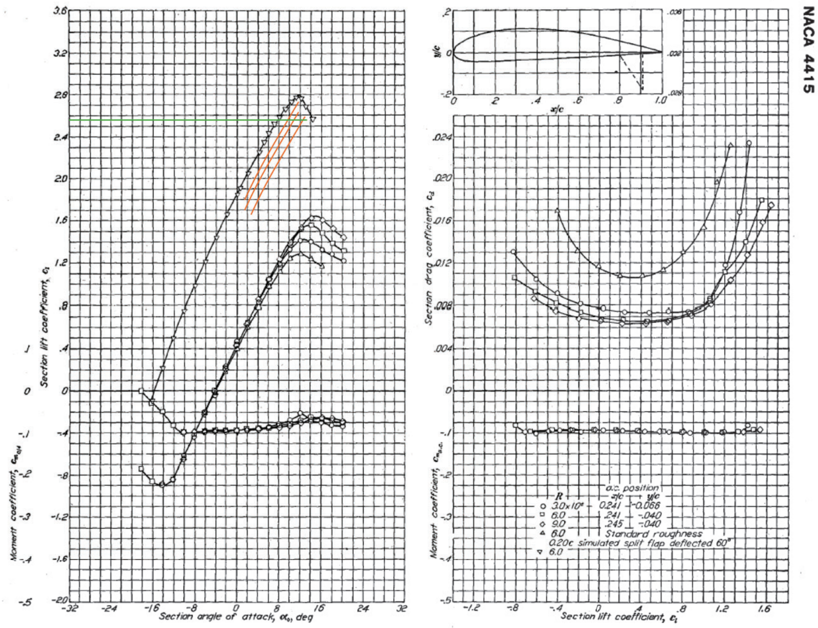

Figure 7: flap deflection vs attack angle

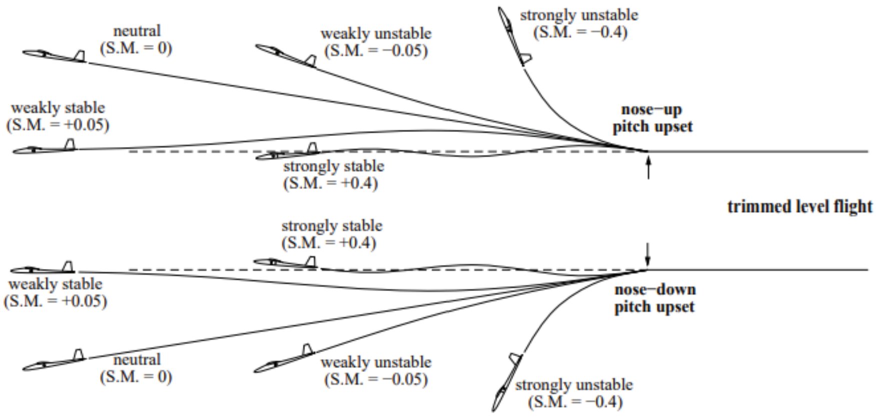

Figure 8: Natural aircraft responses to a pitch disturbance, for different amounts of pitch stability. [4]

1.7 (g)

An aircraft's horizontal tail size and position, and the CG position are the dominant factors controlling the aircraft's pitch stability, which is the tendency to automatically maintain an angle of attack and airspeed. The basic effects of moving the CG position are:

- Decrease xcg/c (move CG fwd.): increased stability; more resistance to α and V changes.
- Increase xcg/c (move CG back): decreased stability; less resistance to α and V changes.

There is one particular CG position which gives neutral stability, which is called the Neutral Point (NP). Figure 8 demonstrates how an airplane reacts to a pitch disturbance across various values of Static Margin (S.M.). The airplane exhibits unstable behavior when the S.M. is negative, indicating that the Center of Gravity (CG) is positioned aft of the Neutral Point (NP). Given that pitch instability significantly complicates or outright prevents effective aircraft control, the position of the NP is deemed a practical limit for the aft CG placement.

In general, the small positive S.M. suggested by the following rule is the ideal situation [4].

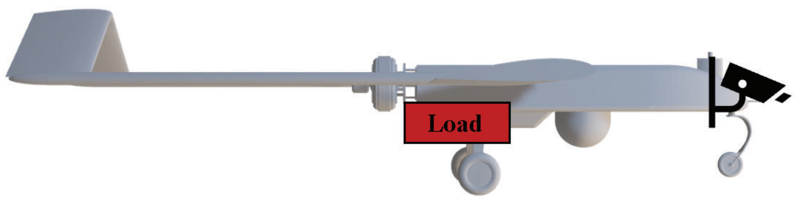

Figure 9: Load distribution of aircraft

$$S.M. \equiv \frac{x_{\text{np}} - x_{\text{cg}}}{c} = +0.05 \ldots + 0.15 \tag{9}$$

Therefore, we will place the loads as shown in Fig9 to ensure that the S.M. is within this range.

A measure of this tail effectiveness is the horizontal tail volume coefficient:

$$\mathrm{V_h} \equiv \frac{S_{\text{h}} \ell_{\text{h}}}{Sc} \tag{10}$$

A well-behaved aircraft typically has a Vh which falls in the range 0.3-0.6 [4].

Table 3: Tail Specifications

Specification	Detail
V_h	0.4
Tail area	$0.0072 m^2$
L_h	0.8m

The V_h also directly affects the NP location xnp, which can be estimated by the following expression.

$$\frac{x_{\text{np}}}{c} \simeq \frac{1}{4} + \frac{1 + 2/AR}{1 + 2/AR_{\text{h}}} \left(1 - \frac{4}{AR + 2}\right) V_{\text{h}} \tag{11}$$

Considerations for Operation:

CG Shifts: The ability to operate without certain payload components (like a camera) without adversely affecting the CG depends on the initial design balance. If removing a component shifts the CG too far forward or aft, it may exceed the horizontal tail's control authority, requiring design modifications. Therefore, we do not consider this approach

Balance and Trim: In practical terms, you might need to add ballast or redistribute other components to maintain a suitable CG position if parts of the payload are removed.

1.8 (h)

First, we need to choose a propeller that can generate the required thrust at cruise speed. you have selected a propeller based on these requirements.Due to the high cruising speed of the airplane, we need to use a high-speed propeller. We chose the APC SPORT 10X10 propeller The data is as follows `https://m-selig.ae.illinois.edu/props/volume-1/plots/apcsp_10x10_ct.png`

Solving the following system of equations using matlab yields that the rotational speed of the propeller at this point is 183 rps (10980 rpm), which we think is a reasonable value.

$$J = \frac{V}{nD} \quad C_T = \frac{T}{\rho n^2 D^4} \tag{12}$$

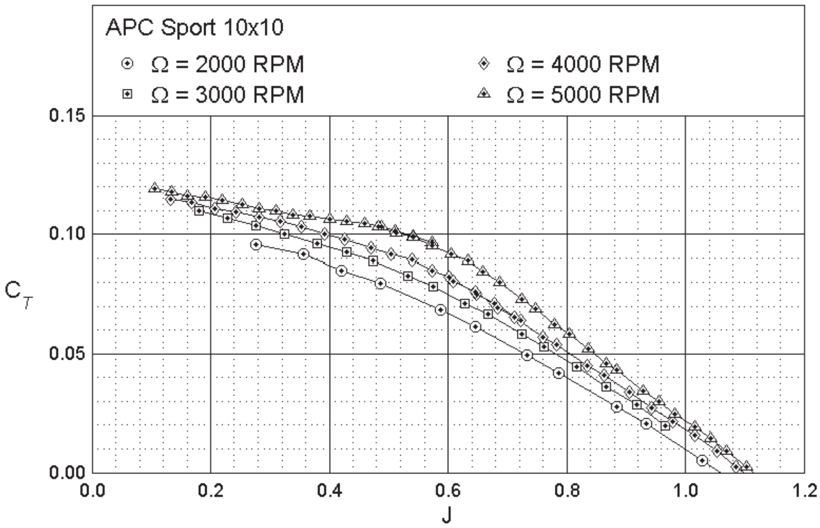

Figure 10: APC Sport 10x10 CT

The power required to spin the propeller can be estimated using the following equation:

$$P = T \times V \div \eta_P \tag{13}$$

Where $\eta_P = 0.65$ as is shown in Fig. 11, $P = 219.7w$

We chose Sunnysky 2220-1100KV brushless motor, as you can see from the website test data `http://en.rcsunnysky.com/x-fixedwingseries/53.html`

At 14.8v the motor can produce up to 423w of power, the motor can be used with 11.1/14.8v and because of the high RPM's we need we use a 14.8v battery.From the test data, we can see that at a power of 219w, the current is about 15A.

After determining the current draw and the required voltage, we can select a battery that meets these electrical specifications. The mission requires 1/3 hour of cruise time and the motor draws 15 A, the battery would need a minimum capacity of 5 Ah. We finally chose the 4S2P Battery Pack as shown in the picture 13 `https://voltaplex.com/4s2p-14.4v-5ah-li-ion-18650-battery-pack-samsung-25r5-cuboid`

Table 4: 14.4V Lithium-Ion Battery Pack Specifications

Specification	Detail
Nominal Voltage	14.4V
Configuration	4S2P
Cell Type	Samsung 25R5 (INR18650-25R5) cylindrical cells
Capacity	5Ah
Max Continuous Discharge	40A
Weight	344g (the assumptions $W_f = 400g$)
Voltage Per Cell Assumption	3.6V (not inflated 3.7V)

1.9 (i)

Finally, we give the specification details of the UAV.

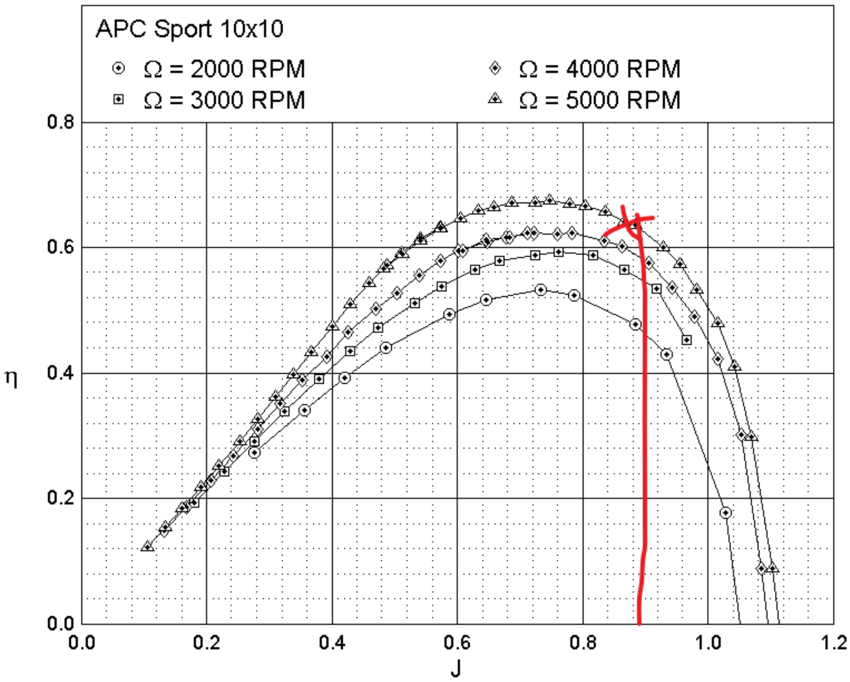

Figure 11: APC SPORT 10X10 ETA

Figure 12: Sunnysky 2220-1100KV brushless motor

Figure 13: 4S2P Battery Pack with Samsung 25R5 Cells, 5Ah, 40A

Figure 14: Three views of UAV

Table 5: UAV Specifications

Specification	Detail
Length	1.2m
Wing span	0.9m
Empty Weight	2.4kg
Load Weight	1.5kg
Max Fuel Weigh	0.4kg
Cruising Speed	144km/h
Cruise time	20min
Airfoil	NACA 4415
MOTOR	2220-1100kV
Propeller	APC Sport 10x10
Battery	4S2P Battery Pack with Samsung 25R5 Cells, 5Ah, 40A

2 Problem 2: "Stadium UAV" Preliminary Design

I particularly liked the airplane design in question 1, so I continued to use the layout from question 1 in the main design.

2.1 (a)

We began by making a series of assumptions about the airplane.

Table 6: Assumptions about the airplane

Variable	Value/Formula	Explanation
`V_crusing`	16.6m/s (60km/h)	Cruising speed
`R`	66km	Range (Round trips with redundant)
`HF`	0.4 MJ/kg	Heating value
`ETA_TOT`	0.20	Overall efficiency of the propulsion system
`T_W`	0.10	Thrust-to-weight ratio (crusing)
`Wp`	1kg	Weight of the payload
`We_Wo`	0.483	Empty weight fraction
`Wf_Wo`	0.08	Fuel weight fraction of the aircraft
`W_o`	2.23kg	Takeoff weight of the aircraft
`rho`	0.9573 kg/m^3	Air density at altitude in 2500m

2.2 (b)

With the above data, we can calculate the wing area as:

$$S_{Wing} = W_o/(0.5 \cdot rho \cdot V_{crusing}^2 \cdot CL_{max}) \tag{14}$$

We are going to use the same NACA 4415 wing as in the first question, and we want cruise speed to correspond to maximum (L/D).

Table 7: Assumptions about the wing

Variable	Value/Formula
`Type`	NACA4415
s_{wing}	$0.0417m^2$
`Wing span`	0.9m
`Aspect ratio`	19.42
CD_0	0.02
`L-D MAX`	11.78
CL_{CRUISE}	0.412
CD_{CRUISE}	0.04

We obtained the above data through continuous iteration, and although there is still some error with the assumptions in (a), we believe that the design is reasonable due to the small error.

2.3 (c)

During the design process, we found that it would take a lot of power to get the airplane to take off and ascend in a straight line, which is almost impossible to achieve. Therefore, we use the following takeoff process as in Fig 16, where the airplane takes off on a 30m runway and then hovers until it reaches a predetermined altitude.

We calculate the thrust required to climb using the following equation

$$T_{\text{max}} = W_o \left(\frac{V_{\text{climb}}}{V} \right) + 0.5 \times \rho \times V^2 \times S_{\text{Wing}} \times CD_0 + \frac{2 \times K \times W_o}{\rho \times V^2 \times S_{\text{Wing}} \times \cos^2(\mu)} = 17.46N \tag{15}$$

- W_o represents the total aircraft weight.

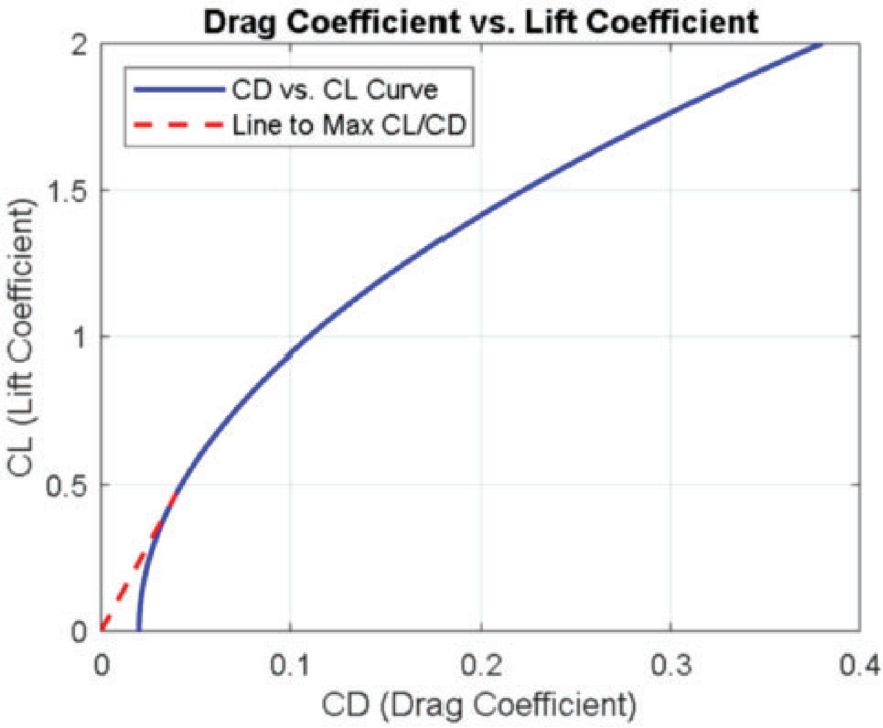

Figure 15: Drag polar

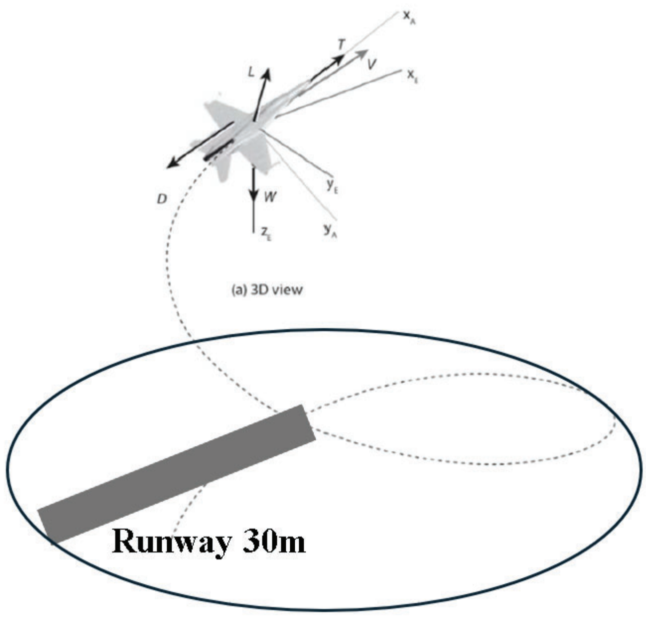

Figure 16: Sketch of the airplane takeoff process

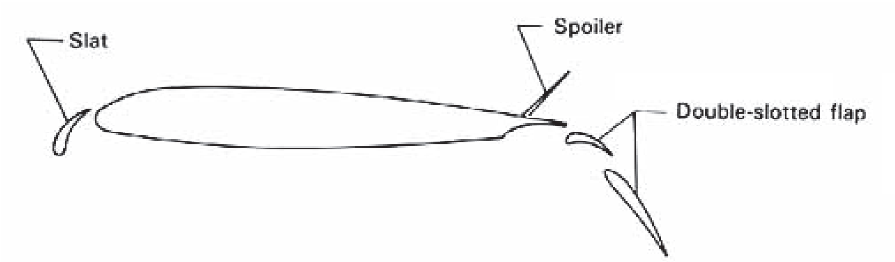

Figure 17: double-slotted flaps

- V_{climb} is the velocity during climb, which is 20% above the stall speed for safety.
- V is the cruising speed of the aircraft.
- S_{Wing} is the wing area.
- CD_0 is the parasite drag coefficient, representing the drag in a zero lift situation.
- K is aspect ratio.
- mu represents the bank angle coefficient, affecting the turn performance and induced drag during climbs.

At this point, if we don't use flaps, the required runway length is

$$s_{LO} \approx \frac{\frac{1}{2}mV_{LO}^2}{T} = \frac{\frac{1}{2}\frac{W}{g}\left(\frac{1.44(2W)}{\rho SC_{L,MAX}}\right)}{T} = \left(\frac{1.44}{\rho gC_{L,MAX}}\right)\frac{W}{T/S} = 197.4m \tag{16}$$

This clearly does not meet the requirements, so we must add flaps. To take off in 30m, we need a lift coefficient of 2.8.

A quick google search tells me after deploying flaps, the lift coefficient can significantly increase. Depending on the wing design and the type of flaps, the maximum lift coefficient C_{Lmax}can usually reach:

- For simple flaps (such as plain flaps), C_{Lmax}may increase to about 1.2 to 1.5.
- For split flaps or single-slotted flaps, C_{Lmax} may range from 1.5 to 1.9.
- For double-slotted flaps, C_{Lmax} can achieve between 2.0 to 2.8.
- For triple-slotted flaps or more complex designs, in some extreme cases, C_{Lmax} may even exceed 3.0.

As a result, we will choose double-slotted flaps as is shown in Fig. 17.

At this point, if we use flaps, the required runway length is

$$s_{LO} \approx \frac{\frac{1}{2}mV_{LO}^2}{T} = \frac{\frac{1}{2}\frac{W}{g}\left(\frac{1.44(2W)}{\rho SC_{L,MAX}}\right)}{T} = \left(\frac{1.44}{\rho gC_{L,MAX}}\right)\frac{W}{T/S} = 29.04m \tag{17}$$

We also need to check the radius of rotation to make sure the plane doesn't fly out of the defined area.

$$R = \frac{2n\left(W/S\right)}{\sqrt{n^2-1}g\rho_{\infty C_L}} = 22.67m \tag{18}$$

- R is the turn radius, the distance from the center of the turn to the aircraft's path, given in meters.
- n is the load factor, which is the ratio of the lift to the aircraft's weight.
- W is the weight of the aircraft.
- S is the wing area of the aircraft.
- g is the acceleration due to gravity.
- ρ is the air density at the altitude of the aircraft.

- CL is the lift coefficient at the specific condition of the turn.

The result is given as R=22.67 meters, which suggests that under the given conditions, the aircraft would have a turn radius of 22.67 meters.

In summary, the design meets the requirements

2.4 (d)

We have two specific metrics, maximum thrust 17.28N when taking off and thrust 1.928N at cruise, and we need to meet the maximum thrust while making the cruise as efficient as possible.

```
% Parameters
C_T = 0.1;   % Assumed thrust coefficient
rho_cruse = 0.9573; % Standard atmospheric density at sea level, in kilograms per cubic
    meter
D = 10 * 0.0254; % Propeller diameter in meters
V_cruse = 16.6; % Cruising speed in m/s
T = 1.92;      % Thrust in Newtons

a = -0.16;
b = 0.18;

% Define the equation to solve for n
equation = @(n) a * (V_cruse / (n * D)) + b - T / (rho_cruse * n^2 * D^4);

% Initial guess for n
initialGuess = 100;

% Solve for n using the fzero function
n = fzero(equation, initialGuess);

% Calculate J and C_T with the found n
J = V_cruse / (n * D);
C_T_calculated = a * J + b;

% Display the results
fprintf('Found rotational speed n: %.4f rev/s\n', n);
fprintf('Advance Ratio J: %.4f\n', J);
fprintf('Thrust Coefficient C_T: %.4f\n', C_T_calculated);

n_RPM = n * 60


% Parameters
C_T = 0.1;   % Assumed thrust coefficient
rho_cruse = 1.225; % Standard atmospheric density at sea level, in kilograms per cubic
    meter
D = 10 * 0.0254; % Propeller diameter in meters
V_cruse = 6.64; % Cruising speed in m/s
T = 17.28;      % Thrust in Newtons

a = -0.16;
b = 0.18;

% Define the equation to solve for n
equation = @(n) a * (V_cruse / (n * D)) + b - T / (rho_cruse * n^2 * D^4);

% Initial guess for n
initialGuess = 100;
```

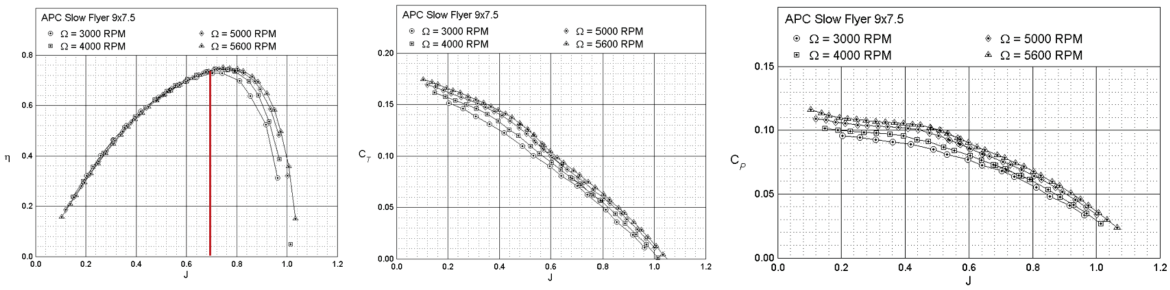

Figure 18: APCSF 9X7.5 properller

```

% Solve for n using the fzero function
n = fzero(equation, initialGuess);

% Calculate J and C_T with the found n
J = V_cruse / (n * D);
C_T_calculated = a * J + b;

% Display the results
fprintf('Found rotational speed n: %.4f rev/s\n', n);
fprintf('Advance Ratio J: %.4f\n', J);
fprintf('Thrust Coefficient C_T: %.4f\n', C_T_calculated);

n_RPM = n * 60
```

We wrote a solution program in matlab and brought in data from the UIUC propeller database to find the optimal solution. **database** `https://m-selig.ae.illinois.edu/props/volume-2/propDB-volume-2.html`

Finally, we choose the APCSF 9X7.5 propeller as is shown in Fig 18 and the result of the solution is as follows.

Table 8: Calculated Propeller Characteristics

Parameter	Maximum thrust	Cruise thrust
Advance Ratio (J)	0.7395	0.1751
Thrust Coefficient (C_T)	0.0617	0.1520
Power Coefficient (C_P)	0.1100	0.0800
Rotational Speed (RPM)	5302.9	8959.4
η	0.7250	0.2550

It is important to note that the airplane cruises right in the propeller's maximum efficiency zone, so we believe that the design is reasonable.

Power produced by a propeller is given by:

$$P = C_T \rho n^3 D^5 \tag{19}$$

So we can choose X2216 1100KV brushless motor with 11.1V battery. `http://en.rcsunnysky.com/x-fixedwingseries/1102.html`

2.5 (e)

We shall use "Five-Leg Approach Procedure" shown in Fig 20, which is a standardized procedure used in aviation to safely guide aircraft during the final phase of flight as they prepare to land at an airport.

Figure 19: X2216 MOTOR

1. Downwind Leg The aircraft flies parallel to the landing runway but in the opposite direction of the landing. It's typically at a controlled altitude and speed, preparing for the approach.

2. Base Leg Turning from the downwind leg, the aircraft begins the base leg by flying perpendicular to the runway. This leg involves further descent and speed reduction in preparation for final approach.

3. Intermediate/Final Approach Fix This isn't always considered a separate leg but is an important transition point. The aircraft adjusts its path to align with the final approach course towards the runway.

4. Final Approach The aircraft is now aligned with the runway, continuing its descent. The pilots focus on maintaining the correct glide path, speed, and alignment for landing.

5. Missed Approach Point If for some reason the aircraft cannot safely land (due to weather, traffic, or other issues), the pilots execute a missed approach procedure. This involves climbing to a specified altitude and following instructions to circle around and either try to land again or divert to another airport.

If we don't use any device other than flaps, the landing distance is 46.71m, which can satisfy the design requirements. But in addition, we can add brakes and counterthrust to further shorten the landing distance.

$$s_L = \frac{W}{S}\frac{1.3}{\rho C_{L,MAX} a_{DECEL}} = 46.71m \tag{20}$$

2.6 (f)

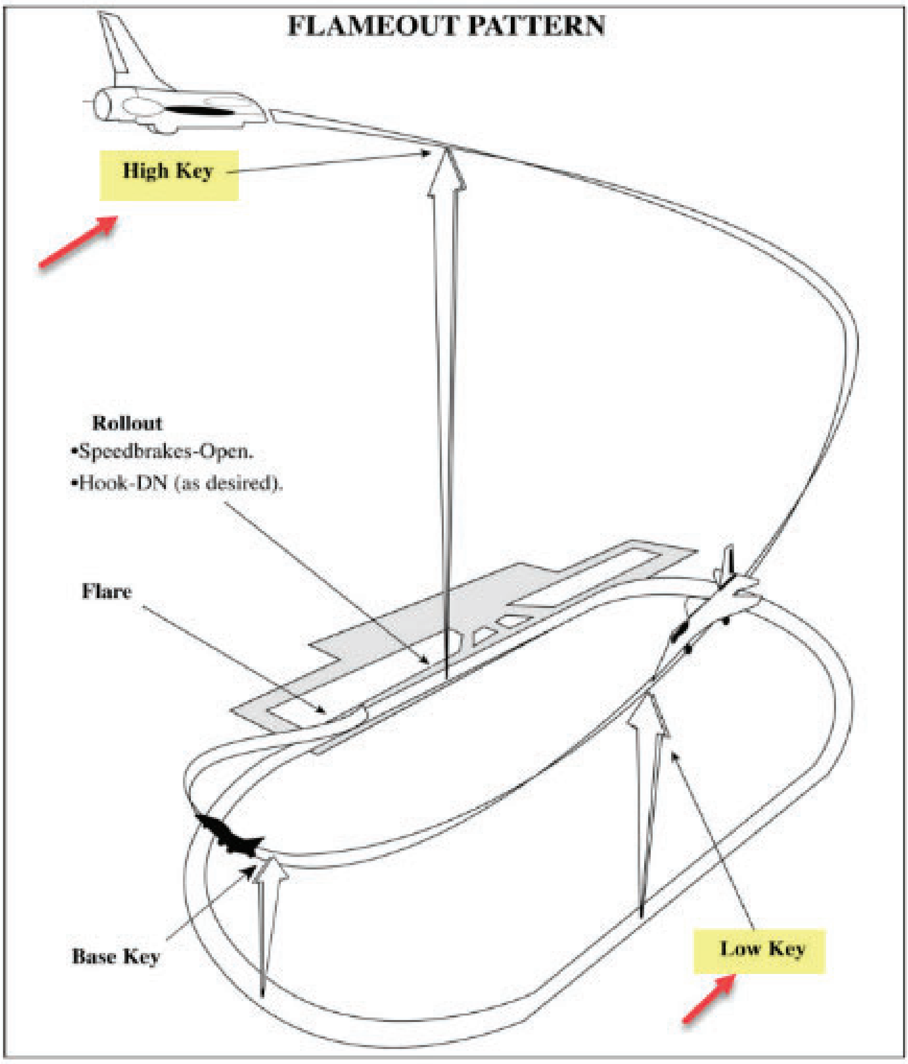

Figure 20: aircraft approach process

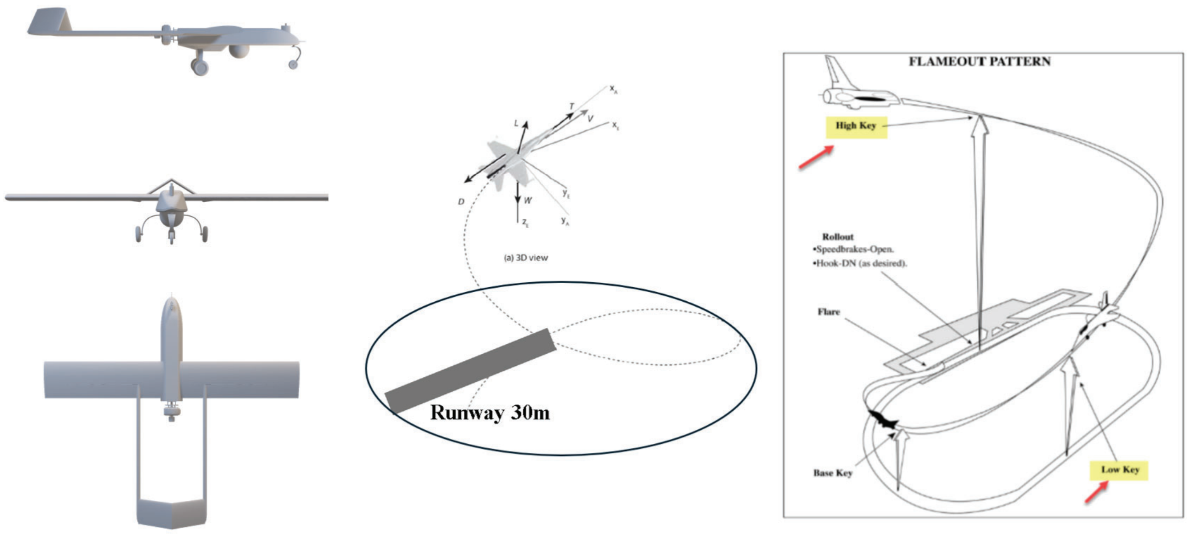

Figure 21: Enter Caption

References

[1] NACA 2415 (n2415-il). (n.d.). Airfoiltools.com. .http://airfoiltools.com/airfoil/details?airfoil=n2415-il

[2] EPPLER 395 AIRFOIL (e395-il). (n.d.). Airfoiltools.com. Retrieved February 16, 2024, from .http://airfoiltools.com/airfoil/details?airfoil=e395-il

[3] Efficiency of Aircraft Engine: Unlocking Performance Potential. (2023, September 3). Saabaircraft.com. .https://saabaircraft.com/efficiency/

[4] Lab 11 Notes -Basic Aircraft Design Rules. (n.d.). Retrieved March 21, 2024, from .https://web.mit.edu/16.unified/www/SPRING/systems/Lab_Notes/desrules.pdf

```
V_crusing = 16.6; % Cruising speed in meters per second (approximately 60 km/h)
R = V_crusing * 0.5 * 60 * 60 * 2.2; % Range formula calculation
HF = 0.4e6; % Heating value of fuel
ETA_TOT = 0.20; % Total engine efficiency
L_D = 10.78; % Lift-to-drag ratio
T_W = 1 / L_D % Thrust-to-weight ratio
Wp = 1; % Payload weight
We_Wo = 0.45 + 0.35 * (T_W); % Empty weight fraction of the total weight
Wf_Wo = (R / HF) * (1 / ETA_TOT) * 1 / (L_D); % Fuel weight fraction of the total weight
W_o = Wp / (1 - Wf_Wo - We_Wo); % Total aircraft weight

1 - We_Wo - Wf_Wo - Wp / W_o; % Calculating remaining weight fraction

rho = 0.9573; % Air density
CL_max = 0.412; % Maximum lift coefficient
CD_max = 0.04; % Maximum drag coefficient
CD_0 = 0.02; % Parasite drag coefficient
S_Wing = W_o / (0.5 * rho * V_crusing^2 * CL_max); % Wing area calculation

mu = 0.9; % Bank angle coefficient for turn performance
V = V_crusing; % Cruising speed
V_climb = 0.5 * V_crusing; % Climb speed, set as 20% higher than stall speed for safety
b = 0.9; % Efficiency factor
K = b^2 / S_Wing; % Induced drag factor
g = 9.81; % Gravity acceleration

% Maximum thrust required calculation
T_max = W_o * (V_climb / V) + 0.5 * 1.225 * V^2 * S_Wing * CD_0 + 2 * K * W_o / (1.225 *
    V^2 * S_Wing * cos(mu)^2)

% Takeoff distance calculation
S_LO = (1.44 / (1.225 * g * CL_max)) * (W_o * g / S_Wing) / (T_max / (W_o * g))

% Alternative takeoff distance calculation with specific parameters
S_LO = (1.44 / (1.225 * g * 2.8)) * (W_o * g / S_Wing) / (T_max / (W_o * g))

n_max = 4; % Maximum load factor

% Radius of turn calculation
R = 2 * n_max * (W_o / S_Wing) / (sqrt(n_max^2 - 1) * g * 1.225 * CL_max)

T_cruse = T_W * W_o * g; % Thrust required at cruise

% Landing distance calculation
S_landing = (W_o * g / S_Wing) * (1.3 / (1.225 * 2.5 * g));

% Parameters
C_T = 0.1;   % Assumed thrust coefficient
rho_cruse = 0.9573; % Standard atmospheric density at sea level, in kilograms per cubic
    meter
D = 10 * 0.0254; % Propeller diameter in meters
V_cruse = 16.6; % Cruising speed in m/s
T = 1.92;      % Thrust in Newtons

a = -0.16;
b = 0.18;

% Define the equation to solve for n
```

```
equation = @(n) a * (V_cruse / (n * D)) + b - T / (rho_cruse * n^2 * D^4);

% Initial guess for n
initialGuess = 100;

% Solve for n using the fzero function
n = fzero(equation, initialGuess);

% Calculate J and C_T with the found n
J = V_cruse / (n * D);
C_T_calculated = a * J + b;

% Display the results
fprintf('Found rotational speed n: %.4f rev/s\n', n);
fprintf('Advance Ratio J: %.4f\n', J);
fprintf('Thrust Coefficient C_T: %.4f\n', C_T_calculated);

n_RPM = n * 60


% Parameters
C_T = 0.1;   % Assumed thrust coefficient
rho_cruse = 1.225; % Standard atmospheric density at sea level, in kilograms per cubic
    meter
D = 10 * 0.0254; % Propeller diameter in meters
V_cruse = 6.64; % Cruising speed in m/s
T = 17.28;      % Thrust in Newtons

a = -0.16;
b = 0.18;

% Define the equation to solve for n
equation = @(n) a * (V_cruse / (n * D)) + b - T / (rho_cruse * n^2 * D^4);

% Initial guess for n
initialGuess = 100;

% Solve for n using the fzero function
n = fzero(equation, initialGuess);

% Calculate J and C_T with the found n
J = V_cruse / (n * D);
C_T_calculated = a * J + b;

% Display the results
fprintf('Found rotational speed n: %.4f rev/s\n', n);
fprintf('Advance Ratio J: %.4f\n', J);
fprintf('Thrust Coefficient C_T: %.4f\n', C_T_calculated);

n_RPM = n * 60
```

MEAM 543 HW7

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1 Problem 1 — Fun with 3D Rotation Matrices

VIDEO: https://drive.google.com/file/d/1Y9p2oNfrEswj8lCbituSOReNe2VWw1hk/view?usp=sharing

1.1 (a)

Flight vehicle makes the following sequence of rotations:

- i. about the vehicle Y axis by 20 degrees
- ii. about the vehicle X axis by -30 degrees
- iii. about the vehicle Y axis by -20 degrees
- iv. about the vehicle X axis by 30 degrees

What are the Euler angles that describe the final orientation? Explain!

For rotation about the Y axis by θ degrees:

$$R_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \tag{1}$$

For rotation about the X axis by θ degrees:

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \tag{2}$$

The final orientation of the vehicle can be found by multiplying the rotation matrices in the order the rotations are applied:

$$R_{final} = R_x(30) \cdot R_y(-20) \cdot R_x(-30) \cdot R_y(20) \tag{3}$$

```
% Define the rotation angles in radians
theta1 = deg2rad(20); % Rotation about Y
theta2 = deg2rad(-30); % Rotation about X
theta3 = deg2rad(-20); % Rotation about Y
theta4 = deg2rad(30); % Rotation about X

% Define the rotation matrices
Rx = @(theta) [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
```

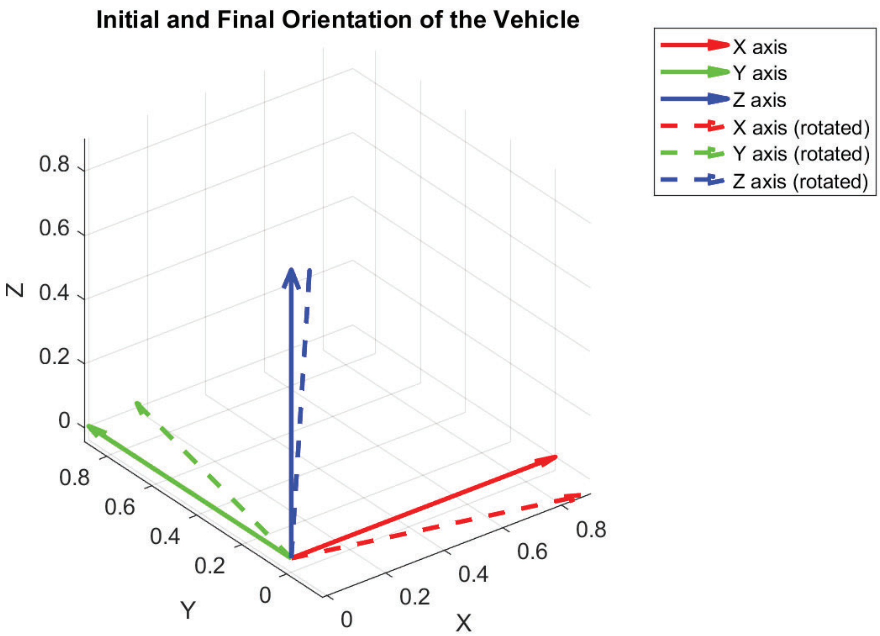


Figure 1: Initial and Final Orientation of the Vehicle(a)

```
Ry = @(theta) [cos(theta) 0 sin(theta); 0 1 0; -sin(theta) 0 cos(theta)];

% Calculate the combined rotation matrix
R_final = Ry(theta3) * Rx(theta4) * Ry(theta1) * Rx(theta2);

% Extract Euler angles from the final rotation matrix
% Assuming the final orientation can be described by ZYX (yaw-pitch-roll) Euler angles
yaw = atan2(R_final(2,1), R_final(1,1));
pitch = atan2(-R_final(3,1), sqrt(R_final(3,2)^2 + R_final(3,3)^2));
roll = atan2(R_final(3,2), R_final(3,3));

% Convert Euler angles to degrees
yaw_deg = rad2deg(yaw);
pitch_deg = rad2deg(pitch);
roll_deg = rad2deg(roll);

% Display the Euler angles
fprintf('Yaw: %.2f degrees\n', yaw_deg);
fprintf('Pitch: %.2f degrees\n', pitch_deg);
fprintf('Roll: %.2f degrees\n', roll_deg);
```

$$\begin{gathered}\text{Yaw: -9.78 degrees}\\ \text{Pitch: -2.76 degrees}\\ \text{Roll: 1.49 degrees}\end{gathered}\tag{4}$$

1.2 (b)

For the second question, we have a flight vehicle making the following sequence of rotations:

- About the vehicle Y axis by 90 degrees

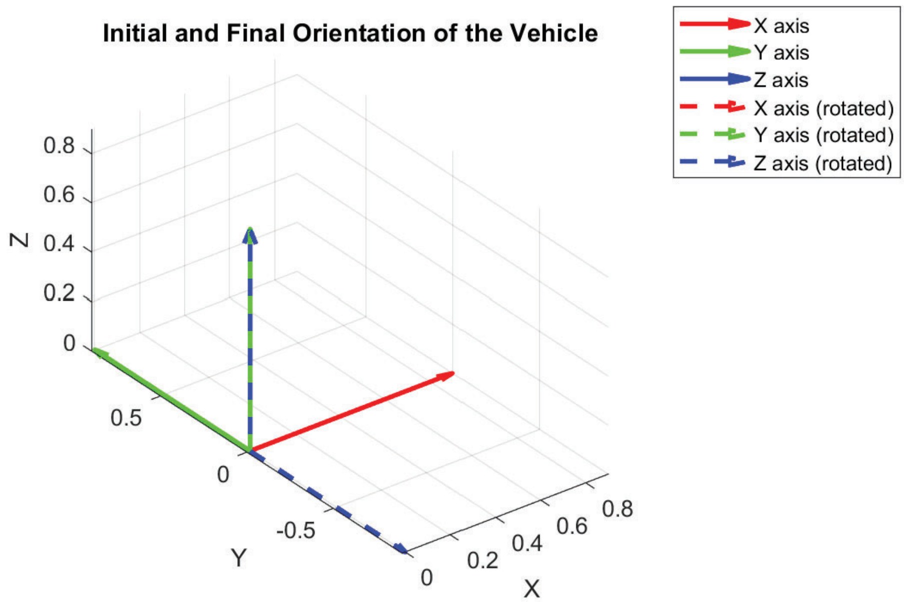


Figure 2: Initial and Final Orientation of the Vehicle(b)

- About the vehicle X axis by 90 degrees
- About the vehicle Z axis by -90 degrees

To return the vehicle to its original orientation

$$R_{b\_final} = R_z(-90) \cdot R_x(90) \cdot R_y(90) \tag{5}$$

The resultant rotation matrix for the second sequence of rotations is:

$$R_{b\_final} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \tag{6}$$

$$\begin{gathered}\text{Euler angles (yaw,pitch,roll)in degrees:} \\ \text{Yaw: 0.00000,Pitch: 0.000000,Roll: 90.000000}\end{gathered} \tag{7}$$

in this case, we know the rotations and their sequences, so we can apply the inverse rotations in the reverse order to return to the original state:

Rotate about the Z-axis by 90 degrees (undoing the -90-degree rotation). Rotate about the X-axis by -90 degrees (undoing the 90-degree rotation). Rotate about the Y-axis by -90 degrees (undoing the 90-degree rotation).

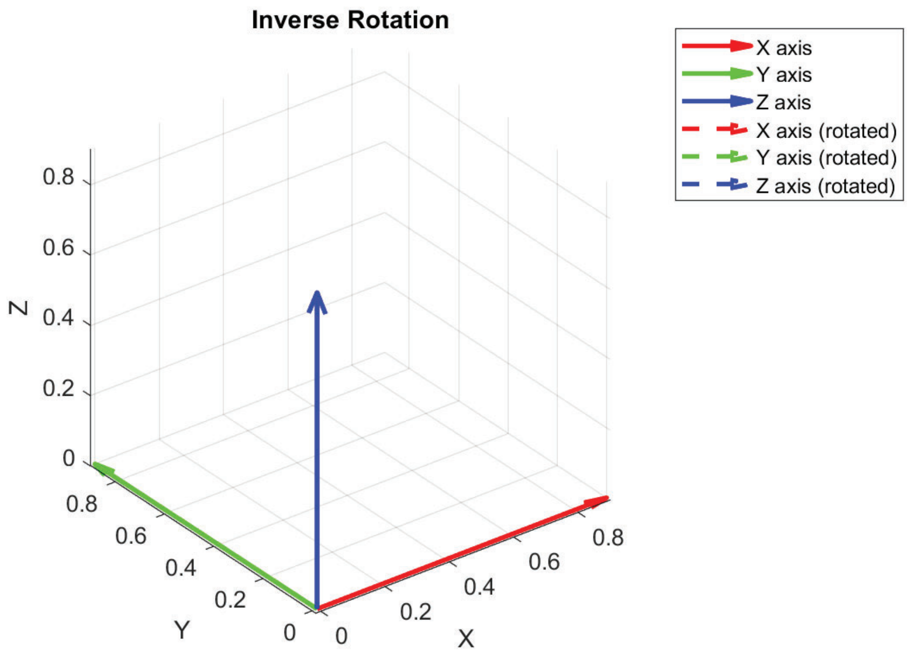


Figure 3: Inverse Rotation

2 Problem 2 — Combined Constant Magnitude Rotation and Translation

The angular velocity vector $\cdot\vec{\omega} = r\hat{\hat{k}}$ points along the positive 'z' axis if we consider a right-handed coordinate system, and 'r' represents the rate of rotation.

For translational motion, there are two different scenarios described:

a. The translational velocity vector $\cdot\vec{V} = V\hat{i}$ is pointing in the direction of the positive 'x' axis.

b. The translational velocity vector $\vec{V} = V\hat{j}$ is pointing in the direction of the positive 'y' axis.

Let's denote θ as the yaw angle (rotation about the 'z' axis),

Case (a): Since the yaw rate is constant, θ will vary linearly with time, hence $\boldsymbol{\theta}(t) = \boldsymbol{r}t + \theta_0$

The translational velocity in the 'x' direction, Vx, will remain constant because there is no force applied in the 'x' or 'y' directions, so $V_x(t) = V$.

There is no translational velocity in the 'y' direction initially, so $V_y(t) = 0$

Case (b): Similarly, $\boldsymbol{\theta}(t) = \boldsymbol{r}t + \theta_0$. There is no translational velocity in the 'x' direction initially, so $V_x(t) = 0$ at all times, assuming no lateral forces are acting on the vehicle. The translational velocity in the 'y' direction, Vy, will remain constant for the same reasons as Vx in case (a), so $V_y(t) = V$.

Static Force Equilibrium: A vehicle is in static force equilibrium if the total external forces acting on it sum to zero. In both cases (a) and (b), assuming there are no other forces acting on the vehicle (like aerodynamic drag, gravity if in flight, or lateral forces), the vehicle would be in static force equilibrium because the translational motion is at a constant velocity (Newton's First Law).

Static Moment Equilibrium: A vehicle is in static moment equilibrium if the total moments (or torques) acting on it sum to zero. Given that the vehicle is undergoing a constant yaw rotation, and if there are no external moments being applied to counteract this rotation, the vehicle would not be in static moment equilibrium. However, this is a bit counterintuitive because we're given a constant angular velocity, which might imply that after the initial moment to start the rotation, no additional moment is required to maintain it (assuming no angular resistance). Thus, in the

Problem 2

a) $\vec{\omega} = \gamma \cdot \frac{d\theta}{dt}$

$d\frac{{}^{E}v}{dt} = 0 \Rightarrow \theta(t) = \gamma t \, \hat{k}_B$

${}^{E}v(t) = v\hat{i}_B$

${}^{B}v(t) = \begin{bmatrix} v\cos(\gamma t) \\ -v\sin(\gamma t) \\ 0 \end{bmatrix}$

---

b) $\theta(t) = vt \, \hat{k}_B$

${}^{B}v(t) = v\hat{i}_B$

${}^{E}v(t) = \begin{bmatrix} v\cos(\gamma t) \\ v\sin(\gamma t) \\ 0 \end{bmatrix}$

absence of any resistive moments, the vehicle might be seen as being in a rotational equilibrium of sorts, but this would not typically be considered "static" moment equilibrium.

Intuitively, if no additional forces or moments are introduced, and assuming the vehicle is in space or another environment without resistive forces like air drag, both cases (a) and (b) might be considered in a sort of dynamic equilibrium where the state of motion remains constant.

3 Problem 3 — Linearized Phugoid

In the starting state, the errors of the two systems are large and then converge gradually.

The system state equation in the context of control theory and linear systems is often expressed as:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{8}$$

where:

- x(t) is the state vector of the system at time t,
- A is the system matrix,
- B is the input matrix,
- u(t) is the input vector of the system at time

Relationship to Eigenvalues

Stability: For an autonomous system (where u(t)=0), the stability of the system can be determined from the eigenvalues of A. If all eigenvalues have negative real parts, the system is stable; that is, the system's state x(t) will eventually converge to zero as t approaches infinity. If any eigenvalue has a positive real part, the system is unstable.

Mode Shapes and Frequencies: The eigenvectors associated with the eigenvalues of A represent the mode shapes of the system, and the eigenvalues themselves determine the natural frequencies and damping ratios of these modes. The real part of an eigenvalue indicates the rate of exponential decay or growth of the mode (damping), while the imaginary part represents the oscillation frequency.

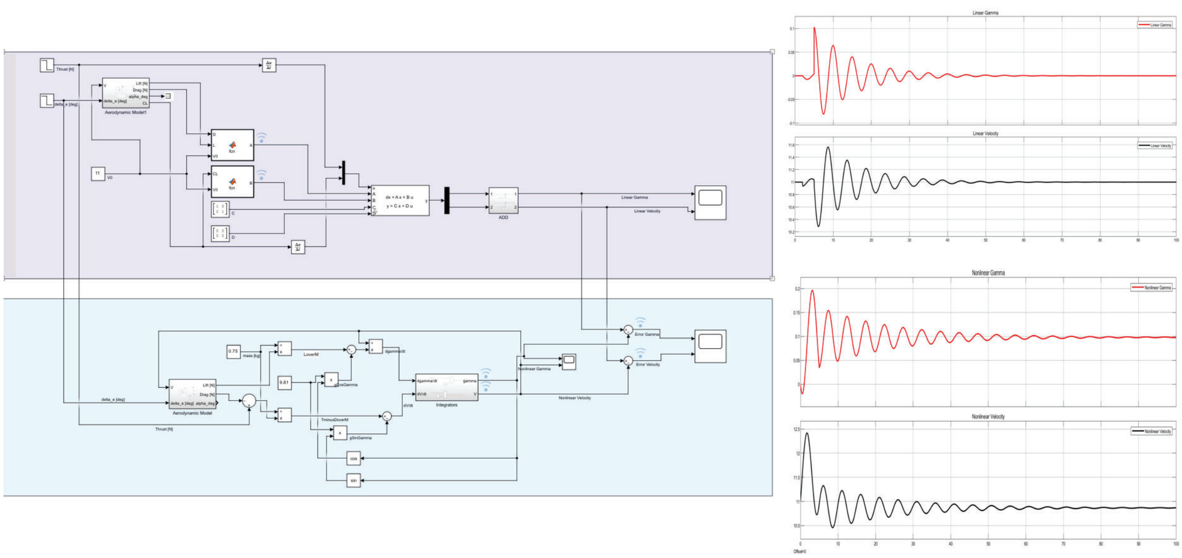


Figure 4: Linearized and nonlinear Phugoid models

$$\frac{d\Delta V}{dt} = \frac{1}{m}\left\{T_0 + \Delta T - \frac{1}{2}\rho S\left[C_{D_0} + k\left(C_{L_0}^2 + 2C_{L_0}\Delta C_L\right)\right]\left(V_0^2 + 2V_0\Delta V\right) - mg\Delta\gamma\right\}$$

$$= \frac{1}{m}\left\{T_0 + \Delta T - \frac{1}{2}\rho S\left(C_{D_0}V_0^2 + 2V_0 C_{D_0}\Delta V + kC_{L_0}^2 V_0^2 + 2kC_{L_0}^2 V_0\Delta V + 2kC_{L_0}V_0^2\Delta C_L\right\} - g\Delta\gamma$$

Using trim condition. $C_{L_0} = \frac{2mg}{\rho S V_0^2}$ $\quad T_0 = \frac{1}{2}\rho S C_{D_0}V_0^2 + \frac{1}{2}\rho S k C_{L_0}^2 V_0^2$

$$= \frac{1}{m}\left\{\Delta T - \rho S V_0 C_{D_0}\Delta V - \cancel{\tfrac{1}{2}\rho S k C_{L_0}^2 V_0^2} - \rho S k C_{L_0}^2 V_0 \Delta V - \rho S k C_{L_0} V_0^2 \Delta C_L\right\} - g\Delta\gamma$$

$$= \overset{a_{11}}{-\frac{\rho S V_0}{m}\left(C_{D_0} + kC_{L_0}^2\right)\Delta V} \overset{a_{12}}{- g\Delta\gamma} + \overset{b_{11}}{\frac{1}{m}\Delta T} \overset{b_{12}}{- \frac{1}{m}\rho S k C_{L_0} V_0^2 \Delta C_L}$$

Hence, we can conclude

$a_{11} = -\frac{\rho S V_0}{m} C_{L_0}\left(\frac{D}{L}\right)_0 = -\left(\frac{D}{L}\right)_0 \frac{2g}{V_0}$

$a_{12} = -g$

$b_{11} = 1/m$

$b_{12} = -\frac{\rho S k C_{L_0} V_0^2}{m} = -\frac{\cancel{\rho S k V_0^2}}{\cancel{m}}\frac{2mg}{\cancel{\rho S V_0^2}} = -2gk$

$$\frac{d\Delta\gamma}{dt} = f\left\{\frac{1}{2}\rho S\left(C_{L_0} + \Delta C_L\right)\left(V_0 + \Delta V\right)^2 - mg\underbrace{\cos(\Delta\gamma)}_{1}\right\}$$

where $f = 1/m(V_0 + \Delta V)$

$$= \frac{\rho S}{2m}\left(C_{L_0} + \Delta C_L\right)\left(V_0 + \Delta V\right) - \frac{g}{V_0 + \Delta V}$$

$$= \frac{\rho S}{2m} C_{L_0} V_0 + \overset{a_{21}}{\frac{\rho S}{2m} C_{L_0}\Delta V} + \overset{b_{22}}{\frac{\rho S}{2m} V_0 \Delta C_L} - \frac{g}{V_0 + \Delta V}$$

$$\frac{\rho S}{2m}\frac{2mg}{\rho S V_0^2} C_{L_0} V_0 = \frac{g}{V_0} - \frac{g}{V_0 + \Delta V} \approx 0 \qquad C_{L_0} = \frac{2mg}{\rho S V_0^2}$$

Hence, we can conclude

$a_{22} = b_{21} = 0$

$a_{21} = \frac{\rho S}{\cancel{2m}} \cdot \frac{\cancel{2m}g}{\rho S V_0^2} = \frac{2g}{V_0^2}$

$b_{22} = \frac{\rho S}{2m} V_0 = \frac{g}{V_0^2 C_{L_0}}$

$$\frac{2mg}{\rho S V_0^2} = C_{L_0} \qquad \frac{g}{V_0^2 C_{L_0}} = \frac{\rho S}{2m}$$

Finally

$$A = \begin{bmatrix} -\left(\frac{D}{L}\right)_0 \frac{2g}{V_0} & -g \\ 2g/V_0^2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{m} & -2kg \\ 0 & \frac{g}{V_0 C_{L_0}} \end{bmatrix}$$

1. **Matrix Representation:** Consider a 2×2 matrix A represented as:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   where $a, b, c,$ and d are the elements of the matrix.

2. **Solve the Characteristic Equation:** Finding the eigenvalues of matrix A involves solving the characteristic equation $\det(A - \lambda I) = 0$, where I is the identity matrix and λ represents the eigenvalues of A. This equation essentially says that the determinant of the matrix $A - \lambda I$ equals zero.
   For a 2×2 matrix, the characteristic equation can be expressed as:

$$\det\left(\begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix}\right) = 0$$

   which simplifies to:

$$(a - \lambda)(d - \lambda) - bc = 0$$

3. **Solve the Quadratic Equation:** After expanding and rearranging the above equation, we obtain a quadratic equation in λ:

$$\lambda^2 - (a + d)\lambda + (ad - bc) = 0$$

   Using the quadratic formula $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a, b,$ and c are the coefficients of the quadratic equation, we can find the eigenvalues λ. Here, the coefficients are:
   - $a = 1$ (since the coefficient of the quadratic term is 1)
   - $b = -(a + d)$
   - $c = ad - bc$

   Therefore, the eigenvalues λ can be calculated using the formula:

$$\lambda = \frac{(a + d) \pm \sqrt{(a + d)^2 - 4(ad - bc)}}{2}$$

$$A = \begin{bmatrix} -\left(\frac{D}{L}\right)_0 \frac{2g}{V_0} & -g \\ \frac{2g}{V_0^2} & 0 \end{bmatrix}$$

Solving for the characteristic equation $\det(A - \lambda I) = 0$

$$\det\left(\begin{pmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{pmatrix}\right) = 0$$

$$\left(-\left(\frac{D}{L}\right)_0 \frac{2g}{V_0} - \lambda\right)(0-\lambda) + \frac{2g^2}{V_0^2} = 0$$

$$\boxed{\lambda^2 + \frac{2g}{V_0}\left(\frac{D}{L}\right)_0 \lambda + 2\left(\frac{g}{V_0}\right)^2 = 0}$$

solving the above equation, the roots are eigenvalue of A

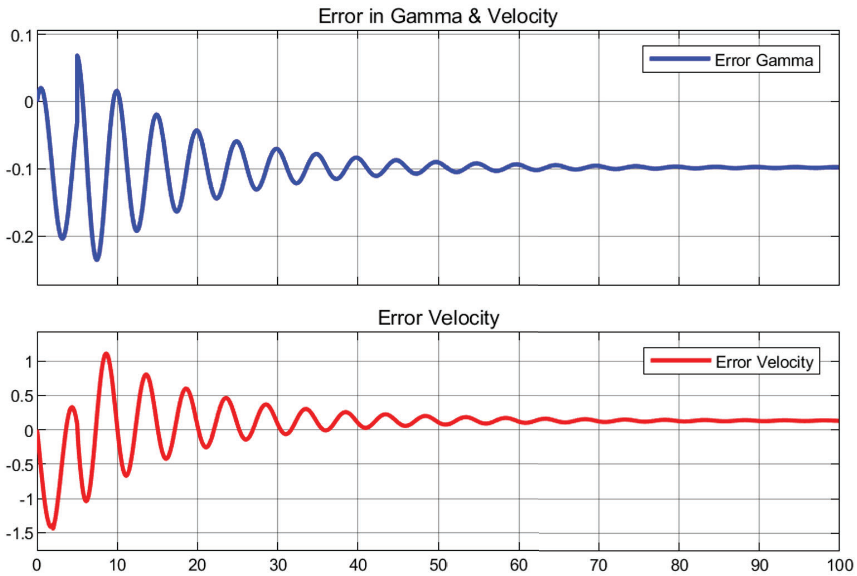


Figure 5: Errors in Gamma and Velocity

References

[1] NACA 2415 (n2415-il). (n.d.). Airfoiltools.com. .http://airfoiltools.com/airfoil/details?airfoil=n2415-il

[2] EPPLER 395 AIRFOIL (e395-il). (n.d.). Airfoiltools.com. Retrieved February 16, 2024, from .http://airfoiltools.com/airfoil/details?airfoil=e395-il

[3] Efficiency of Aircraft Engine: Unlocking Performance Potential. (2023, September 3). Saabaircraft.com. .https://saabaircraft.com/efficiency/

```
% Apply the rotations in the sequence given
% i. About the vehicle Y axis by 20 degrees
R_y1 = rotation_matrix_y(20);

% ii. About the vehicle X axis by -30 degrees
R_x1 = rotation_matrix_x(-30);

% iii. About the vehicle Y axis by -20 degrees
R_y2 = rotation_matrix_y(-20);

% iv. About the vehicle X axis by 30 degrees
R_x2 = rotation_matrix_x(30);

% Calculate the composite rotation matrix
R_final = R_x2 * R_y2 * R_x1 * R_y1;

% Extract the Euler angles from the final rotation matrix
% Assuming the 'ZYZ' rotation sequence for Euler angles
theta = asin(R_final(3,1)); % Pitch
if cos(theta) ~= 0
    psi = atan2(R_final(3,2)/cos(theta), R_final(3,3)/cos(theta)); % Yaw
    phi = atan2(R_final(2,1)/cos(theta), R_final(1,1)/cos(theta)); % Roll
else
    % Gimbal lock condition
    psi = 0; % Yaw is undefined
    phi = atan2(-R_final(2,3), R_final(2,2)); % Roll
end

% Convert to degrees
euler_angles = rad2deg([phi, theta, psi]);

% Display the Euler angles
disp('Euler angles in degrees:');
```

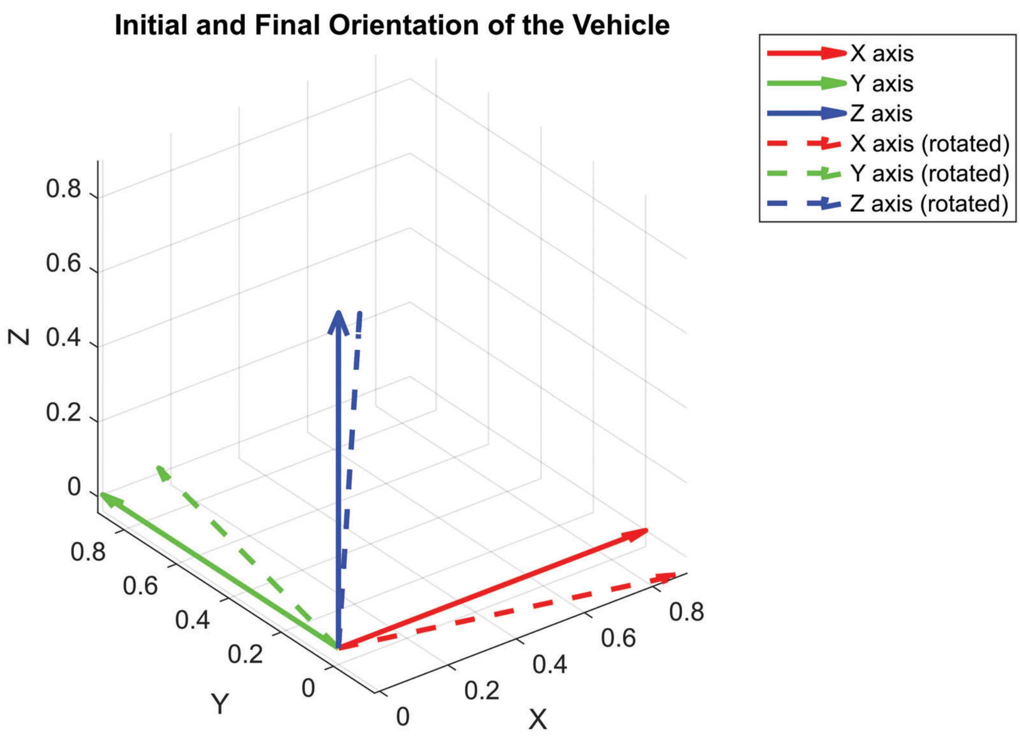
```
Euler angles in degrees:
```

```
disp(euler_angles);
```

```
   -9.7777   -2.7636    1.4981
```

```
% Define rotation angles in radians
theta1 = deg2rad(20); % Rotate around Y-axis
theta2 = deg2rad(-30); % Rotate around X-axis
theta3 = deg2rad(-20); % Rotate around Y-axis
theta4 = deg2rad(30); % Rotate around X-axis

% Define rotation matrices
```

```
Rx = @(theta) [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
Ry = @(theta) [cos(theta) 0 sin(theta); 0 1 0; -sin(theta) 0 cos(theta)];

% Calculate the final rotation matrix
R_final = Rx(theta4) * Ry(theta3) * Rx(theta2) * Ry(theta1);

% Base vectors (X, Y, Z axes)
vectors = eye(3);

% Apply the rotation
rotated_vectors = R_final * vectors;

% Plot the original coordinate system
quiver3(0, 0, 0, vectors(1,1), vectors(2,1), vectors(3,1), 'r', 'LineWidth', 2);
hold on;
quiver3(0, 0, 0, vectors(1,2), vectors(2,2), vectors(3,2), 'g', 'LineWidth', 2);
quiver3(0, 0, 0, vectors(1,3), vectors(2,3), vectors(3,3), 'b', 'LineWidth', 2);

% Plot the rotated coordinate system
quiver3(0, 0, 0, rotated_vectors(1,1), rotated_vectors(2,1), rotated_vectors(3,1),
'r--', 'LineWidth', 2);
quiver3(0, 0, 0, rotated_vectors(1,2), rotated_vectors(2,2), rotated_vectors(3,2),
'g--', 'LineWidth', 2);
quiver3(0, 0, 0, rotated_vectors(1,3), rotated_vectors(2,3), rotated_vectors(3,3),
'b--', 'LineWidth', 2);

% Set graph properties for better observation
axis equal;
grid on;
xlabel('X');
ylabel('Y');
zlabel('Z');
title('Initial and Final Orientation of the Vehicle');
legend('X axis', 'Y axis', 'Z axis', 'X axis (rotated)', 'Y axis (rotated)', 'Z
axis (rotated)');
hold off;
```




```
% Define the rotation angles in degrees
thetaY = 90;
thetaX = 90;
thetaZ = -90;

% Convert degrees to radians for calculations
thetaY_rad = deg2rad(thetaY);
thetaX_rad = deg2rad(thetaX);
thetaZ_rad = deg2rad(thetaZ);

% Define the rotation matrices
Ry = [cos(thetaY_rad) 0 sin(thetaY_rad); 0 1 0; -sin(thetaY_rad) 0 cos(thetaY_rad)];
Rx = [1 0 0; 0 cos(thetaX_rad) -sin(thetaX_rad); 0 sin(thetaX_rad) cos(thetaX_rad)];
Rz = [cos(thetaZ_rad) -sin(thetaZ_rad) 0; sin(thetaZ_rad) cos(thetaZ_rad) 0; 0 0 1];

% Calculate the final rotation matrix
R_final = Rz * Rx * Ry;

% Display the final rotation matrix
disp('Final rotation matrix:');
```

```
Final rotation matrix:
```

```
disp(R_final);
```

```
    1.0000    0.0000         0
         0    0.0000   -1.0000
   -0.0000    1.0000    0.0000
```

```
% Derive Euler angles from the final rotation matrix
% Assuming ZYX rotation sequence for Euler angles
yaw = atan2(R_final(2,1), R_final(1,1));
pitch = atan2(-R_final(3,1), sqrt(R_final(3,2)^2 + R_final(3,3)^2));
roll = atan2(R_final(3,2), R_final(3,3));

% Convert radians back to degrees for readability
yaw_deg = rad2deg(yaw);
pitch_deg = rad2deg(pitch);
roll_deg = rad2deg(roll);

% Display the Euler angles
disp('Euler angles (yaw, pitch, roll) in degrees:');
```

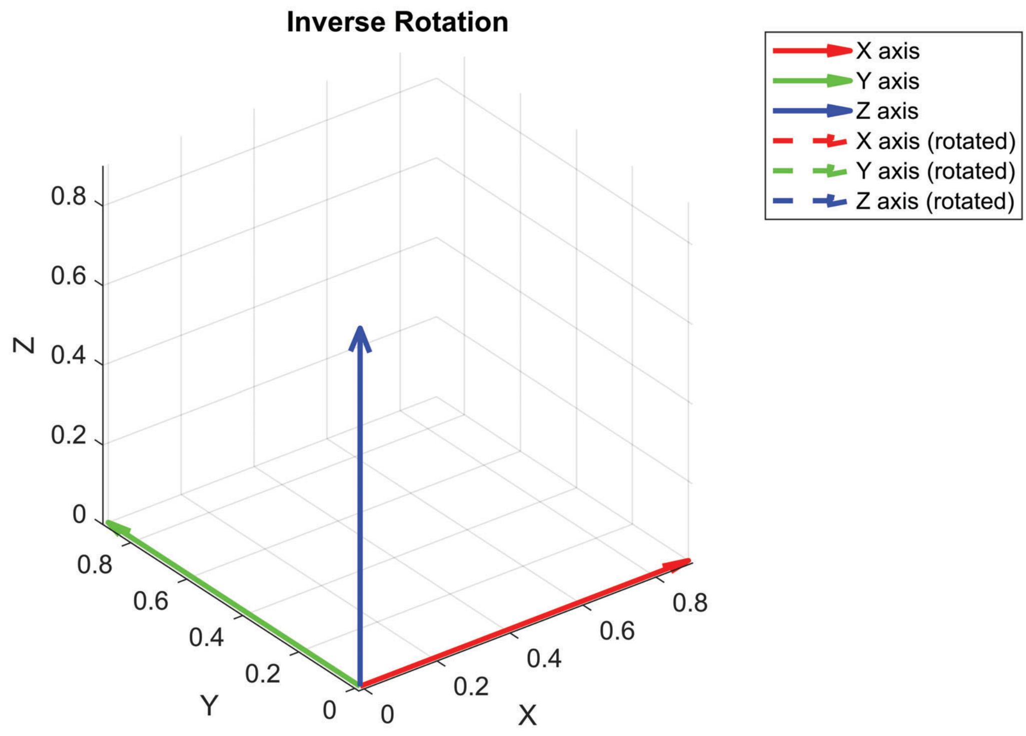
```
Euler angles (yaw, pitch, roll) in degrees:
```

```
fprintf('Yaw: %f, Pitch: %f, Roll: %f\n', yaw_deg, pitch_deg, roll_deg);
```

```
Yaw: 0.000000, Pitch: 0.000000, Roll: 90.000000
```

```
% Define rotation angles in radians
thetaY = deg2rad(90); % Rotate around Y-axis
thetaX = deg2rad(90); % Rotate around X-axis
thetaZ = deg2rad(-90); % Rotate around Z-axis

% Define rotation matrices
Rx = @(theta) [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
Ry = @(theta) [cos(theta) 0 sin(theta); 0 1 0; -sin(theta) 0 cos(theta)];
Rz = @(theta) [cos(theta) -sin(theta) 0; sin(theta) cos(theta) 0; 0 0 1];

% Calculate the final rotation matrix
R_final = Ry(-thetaY)* Rx(-thetaX)*Rz(-thetaZ)*Rz(thetaZ) * Rx(thetaX) * Ry(thetaY);

% Base vectors (X, Y, Z axes)
vectors = eye(3);

% Apply the rotation
rotated_vectors = R_final * vectors;

% Plot the original coordinate system
quiver3(0, 0, 0, vectors(1,1), vectors(2,1), vectors(3,1), 'r', 'LineWidth', 2);
hold on;
quiver3(0, 0, 0, vectors(1,2), vectors(2,2), vectors(3,2), 'g', 'LineWidth', 2);
quiver3(0, 0, 0, vectors(1,3), vectors(2,3), vectors(3,3), 'b', 'LineWidth', 2);
```

```
% Plot the rotated coordinate system
quiver3(0, 0, 0, rotated_vectors(1,1), rotated_vectors(2,1), rotated_vectors(3,1),
'r--', 'LineWidth', 2);
quiver3(0, 0, 0, rotated_vectors(1,2), rotated_vectors(2,2), rotated_vectors(3,2),
'g--', 'LineWidth', 2);
quiver3(0, 0, 0, rotated_vectors(1,3), rotated_vectors(2,3), rotated_vectors(3,3),
'b--', 'LineWidth', 2);

% Set graph properties for better observation
axis equal;
grid on;
xlabel('X');
ylabel('Y');
zlabel('Z');
title('Inverse Rotation');
legend('X axis', 'Y axis', 'Z axis', 'X axis (rotated)', 'Y axis (rotated)', 'Z
axis (rotated)');
hold off;
```




```
% Define the rotation matrices for rotations about the x, y, and z axes
```

```
function R = rotation_matrix_x(angle)
    rad = deg2rad(angle);
    R = [1, 0, 0; 0, cos(rad), -sin(rad); 0, sin(rad), cos(rad)];
```

```
end

function R = rotation_matrix_y(angle)
    rad = deg2rad(angle);
    R = [cos(rad), 0, sin(rad); 0, 1, 0; -sin(rad), 0, cos(rad)];
end
```

MEAM 543 HW8

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1 Problem 1 — Dzhanibekov Effect

VIDEO: https://drive.google.com/file/d/1ZaiI5ov84McLa0MPPT1uXqyhoGMyAM25/view?usp=sharing

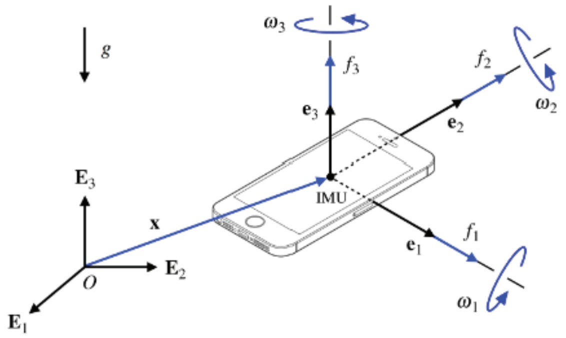
SIM VIDEO: https://drive.google.com/file/d/1G0smCujaa6HouBx24LB4RsbD-hcGzuu_/view?usp=sharing

1.1 (a)

The Dzhanibekov Effect, also known as the tennis racket theorem, is a fascinating phenomenon in classical mechanics that becomes evident under specific conditions, especially in the absence of external torques.

It can be explained by examining the behavior of an object with three distinct moments of inertia as it rotates in space. A tennis racket, with its long handle and broad head, serves as a classic example, having:

- A large moment of inertia when rotating about the axis through the handle (long axis),
- An intermediate moment of inertia when rotating about the axis that lies in the plane of the racket and points from the handle to the top of the head (intermediate axis), and
- A small moment of inertia when rotating about the axis perpendicular to the plane of the racket (short axis).



In the Fig 1, we see a smartphone with three axes labeled: X, Y, and Z, corresponding to the phone's width, height, and depth.

- X-axis (green): This axis typically has the intermediate moment of inertia.
- Y-axis (blue): is the axis with the minimum moment of inertia.
- Z-axis (red): this axis have the maximum moment of inertia.

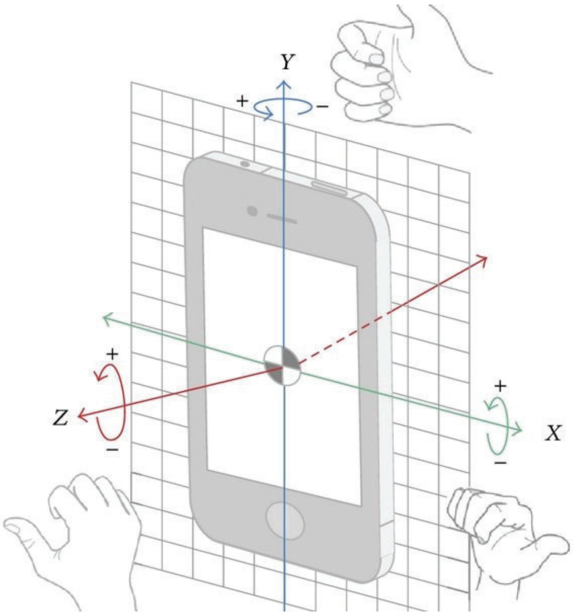


Figure 1: The gyroscope measures rotation around the x-y-and z axes

According to the conservation of angular momentum, if you spin the racket about its intermediate axis with a certain angular velocity and then perturb it slightly, the racket will undergo a spontaneous and surprising flip, rotating about the other two principal axes. This occurs despite the total angular momentum of the system remaining constant, highlighting the instability when rotating about the axis with the intermediate moment of inertia.

The equations are:

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned} \tag{1}$$

where:

- $I_{1\ 3}$ are the moments of inertia about the three principal axes,
- $\omega_{1\ 3}$ are the angular velocities about the three principal axes,
- $\dot{\omega}_{1\ 3}$ are the time derivatives of the angular velocities.

$$I \cdot \dot{\omega} = -\omega \times (I \cdot \omega) \tag{2}$$

The relationship between the rates of change of the Euler angles and the angular velocity vector [p,q,r] is as follows:

$$\begin{aligned} &\dot{\phi} = p + (q\sin(\phi) + r\cos(\phi))\tan(\theta) \text{ (roll rate,} \\ &\dot{\theta} = q\cos(\phi) - r\sin(\phi)\text{(pitch rate)} \\ &\dot{\psi} = (q\sin(\phi) + r\cos(\phi))\sec(\theta) \text{ (yaw rate)} \end{aligned} \tag{3}$$

1.2 (b) Simulink Model

1.3 (c) Experiment

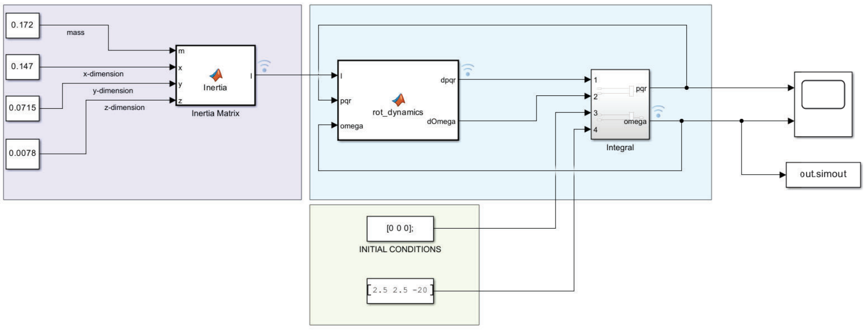


Figure 2: Simulink Model

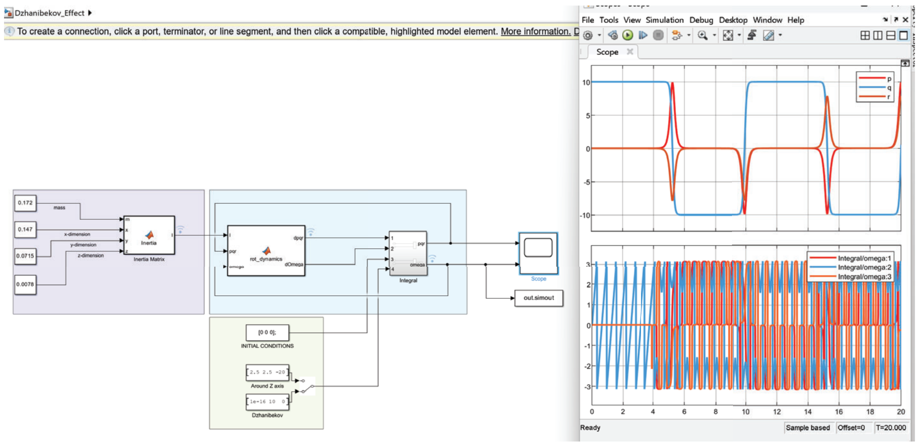


Figure 3: Dzhanibekov Effect Simulation

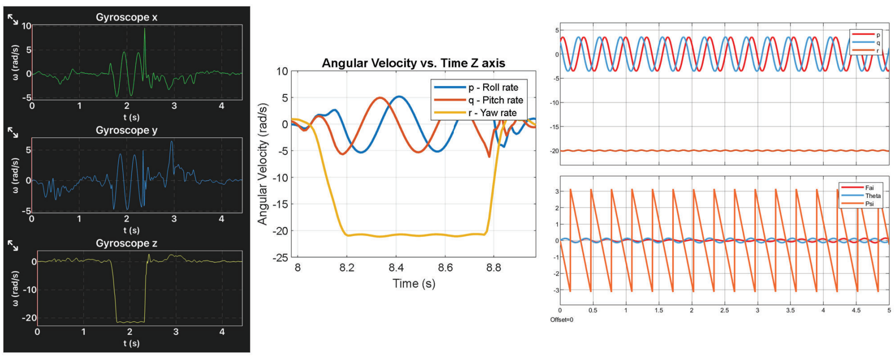


Figure 4: Rotate about Z axis (maximum axis)

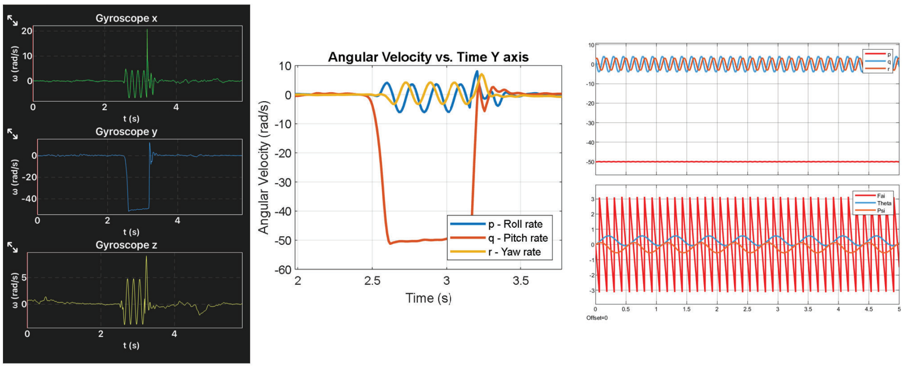


Figure 5: Rotate about y axis(minimum axis)

References

[1] Reconstructing the motion of a tossed iPhone | Rotations. (n.d.). Rotations.berkeley.edu. Retrieved April 8, 2024, from .`https://rotations.berkeley.edu/reconstructing-the-motion-of-a-tossed-iphone/`

MEAM 543 HW9

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1 Problem 1: More Dynamics

VIDEO: https://drive.google.com/file/d/1ZaiI5ov84McLa0MPPT1uXqyhoGMyAM25/view?usp=sharing

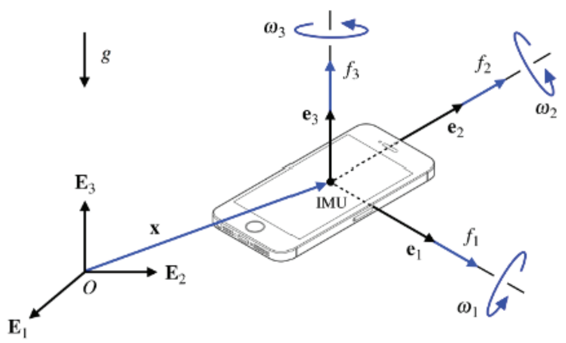
SIM VIDEO: https://drive.google.com/file/d/1n9s7bPu093Qnjz-xe_AZRQ8kzCgaL8GJ/view?usp=sharing

1.1 (a)

The Dzhanibekov Effect, also known as the tennis racket theorem, is a fascinating phenomenon in classical mechanics that becomes evident under specific conditions, especially in the absence of external torques.

It can be explained by examining the behavior of an object with three distinct moments of inertia as it rotates in space. A tennis racket, with its long handle and broad head, serves as a classic example, having:

- A large moment of inertia when rotating about the axis through the handle (long axis),
- An intermediate moment of inertia when rotating about the axis that lies in the plane of the racket and points from the handle to the top of the head (intermediate axis), and
- A small moment of inertia when rotating about the axis perpendicular to the plane of the racket (short axis).



In the Fig 1, we see a smartphone with three axes labeled: X, Y, and Z, corresponding to the phone's width, height, and depth.

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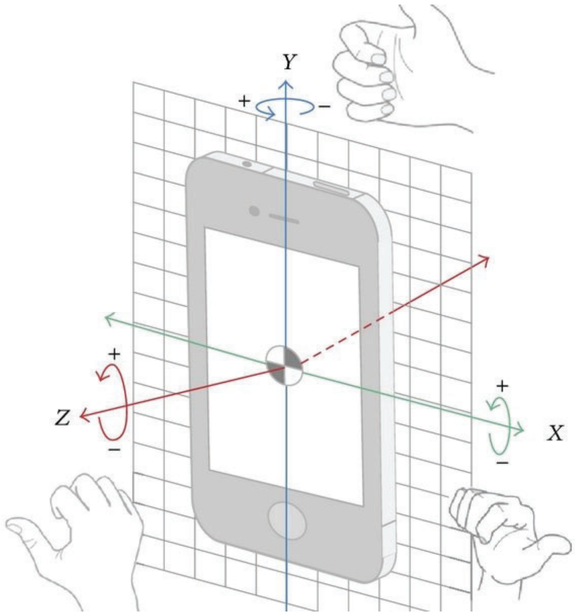


Figure 1: The gyroscope measures rotation around the x-y-and z axes

According to the conservation of angular momentum, if you spin the racket about its intermediate axis with a certain angular velocity and then perturb it slightly, the racket will undergo a spontaneous and surprising flip, rotating about the other two principal axes. This occurs despite the total angular momentum of the system remaining constant, highlighting the instability when rotating about the axis with the intermediate moment of inertia.

The equations are:

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned} \tag{1}$$

where:

- $I_{1\ 3}$ are the moments of inertia about the three principal axes,
- $\omega_{1\ 3}$ are the angular velocities about the three principal axes,
- $\dot{\omega}_{1\ 3}$ are the time derivatives of the angular velocities.

$$I \cdot \dot{\omega} = -\omega \times (I \cdot \omega) \tag{2}$$

The relationship between the rates of change of the Euler angles and the angular velocity vector [p,q,r] is as follows:

$$\begin{aligned} &\dot{\phi} = p + (q\sin(\phi) + r\cos(\phi))\tan(\theta) \text{ (roll rate,} \\ &\dot{\theta} = q\cos(\phi) - r\sin(\phi) \text{(pitch rate)} \\ &\dot{\psi} = (q\sin(\phi) + r\cos(\phi))\sec(\theta) \text{ (yaw rate)} \end{aligned} \tag{3}$$

Unlike HW8, we would like to include translational motion in HW9: velocity and displacement.

We can obtain the equations of motion governing the translation of the mass center $\bar{X}$ and the phone's orientation by applying balances of linear and angular momenta, respectively.

Neglecting the effect of air drag, the only force exerted on the book after being tossed into the air is the weight $-mg\mathbf{k}$ acting at the center of mass, and so a balance of linear momentum yields the system of equations

$$
\begin{aligned}
m\ddot{x}_1 &= 0, \\
m\ddot{x}_2 &= 0, \\
m\ddot{x}_3 &= -mg,
\end{aligned} \tag{4}
$$

Lastly, by introducing the state vector

$$
y = [\dot{x}_1, \dot{x}_2, \dot{x}_3, x_1, x_2, x_3, \omega_1, \omega_2, \omega_3, \psi, \theta, \phi]^T \tag{5}
$$

we can express the phone's mass center $\overline{\mathbf{x}}(t)$ and the orientation in the first-order matrix-vector form

$$
\mathrm{M}(t, \mathrm{y})\dot{\mathrm{y}} = \mathrm{f}(t, \mathrm{y}) \tag{6}
$$

$$
\mathsf{M}(t, y) = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin(\theta) & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(\theta)\sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(\theta)\cos(\phi) & -\sin(\phi) & 0
\end{bmatrix}, \tag{7}
$$

$$
\mathbf{f}(t, \mathbf{y}) = \begin{bmatrix}
0 \\ 0 \\ -mg \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ -(\lambda_3 - \lambda_2)\omega_2\omega_3 \\ -(\lambda_1 - \lambda_3)\omega_1\omega_3 \\ -(\lambda_2 - \lambda_1)\omega_1\omega_2 \\ \omega_1 \\ \omega_2 \\ \omega_3
\end{bmatrix}. \tag{8}
$$

1.2 (b) Simulink Model

The simulation model is shown below, where the acceleration data can be chosen (1) calculated using the initial conditions of simulation (2) reading using the cell phone acceleration sensor.

Note that if a cell phone accelerometer is used, the linear acceleration should be transformed from body frame to earth frame. For a 3-2-1 set of Euler angles, this is accomplished as follows:

$$
\begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} \tag{9}
$$

It should be noted that the following experimental results are from simulink simulation models, and the data from the cell phone sensors are only used as a comparison verification

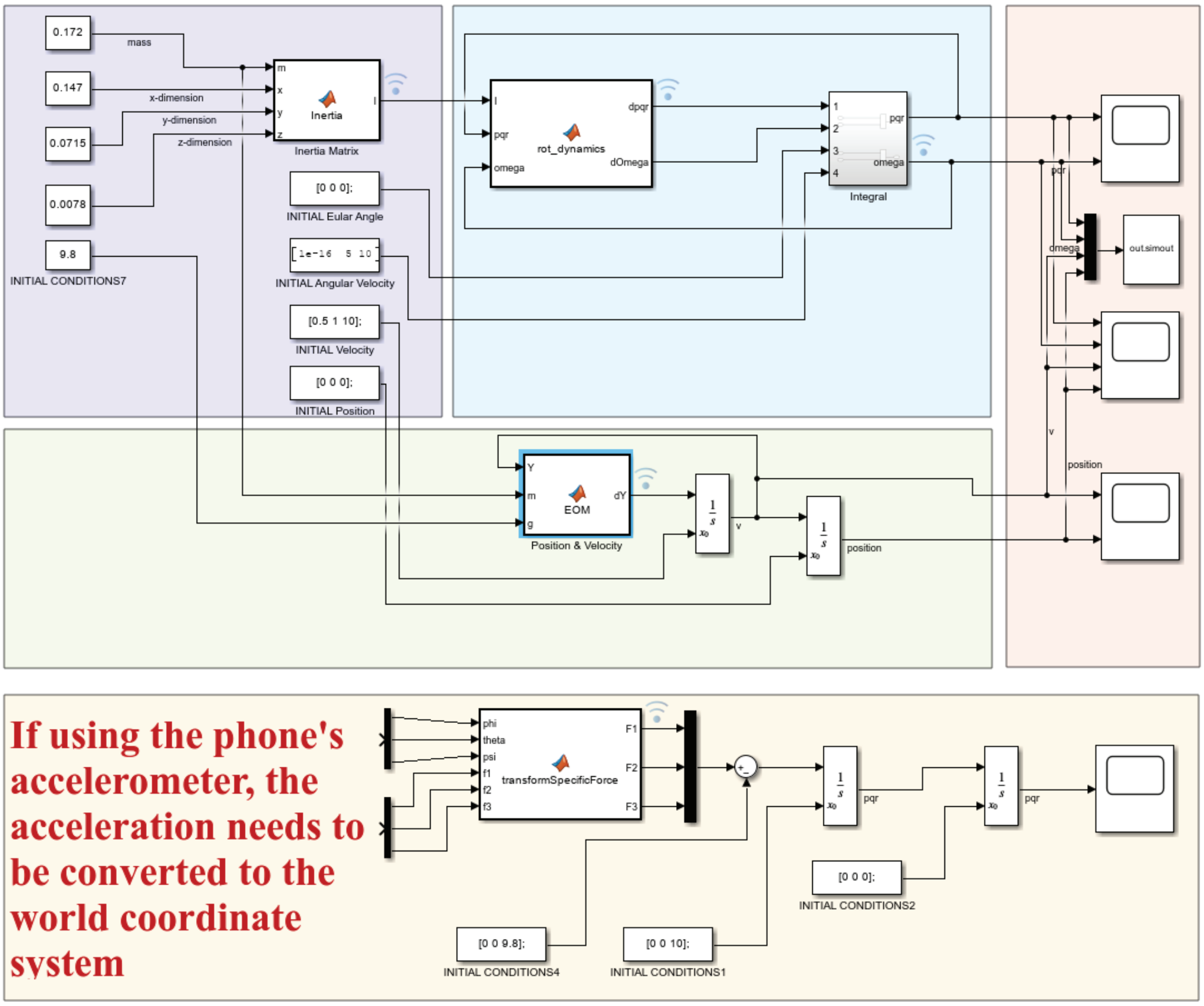


Figure 2: Simulation Model for HW9

1.3 (c) Results

To verify that our calculations are accurate, we visualize the data. This part of the code references the following URL `https://rotations.berkeley.edu/a-book-tossed-in-the-air/`

Our initial conditions are: $\omega_1 = 10, V_x = 0.5, V_y = 1, V_z = 10$ all variables not mentioned are 0.

SIM VIDEO: `https://drive.google.com/file/d/1n9s7bPu093Qnjz-xe_AZRQ8kzCgaL8GJ/view?usp=sharing`

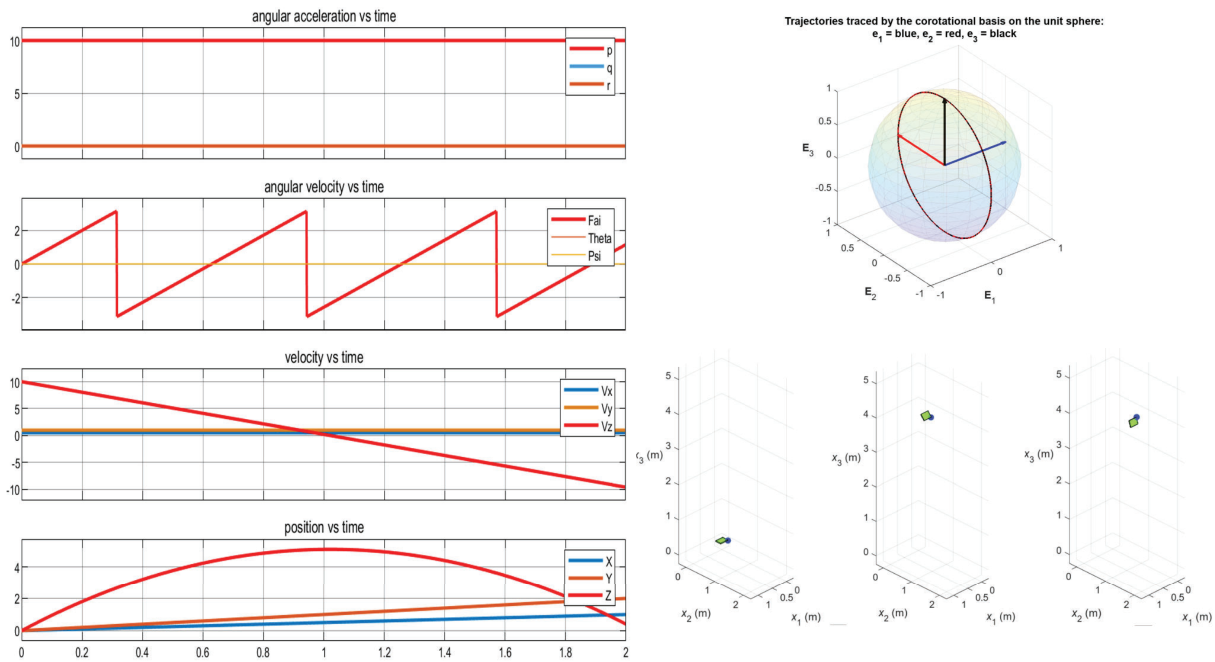


Figure 3: Simulation Results

2 Problem 2: Linearized Dzhanibekov Equations

The intermediate axis theorem explains that rotation about the axis with the intermediate moment of inertia (I_2) leads to unstable rotation.

The eigenvalues computed for rotation about the x (smallest inertia) and z (largest inertia) axes show purely imaginary components indicating stable (oscillatory) behavior.

- Dzhanibekov Equations can be written as

$$\begin{bmatrix}\dot{\omega}_1\\ \dot{\omega}_2\\ \dot{\omega}_3\end{bmatrix} = \begin{bmatrix}(I_2-I_3)/I_1\ \omega_2\omega_3\\ (I_3-I_1)/I_2\ \omega_1\omega_3\\ (I_1-I_2)/I_3\ \omega_1\omega_2\end{bmatrix}$$

- Equilibrium state

$$\begin{bmatrix}\dot{\omega}_1\\ \dot{\omega}_2\\ \dot{\omega}_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

assuming the body rotates steadily around one axis and denoting rotation rate as Ω, we have

$$\begin{cases} ① \ \omega_1 = \Omega,\ \omega_2=\omega_3=0 & (\text{around } x \text{ axis}) \\ ② \ \omega_1=\omega_3=0,\ \omega_2=\Omega & (\text{around } y \text{ axis}) \\ ③ \ \omega_2=\omega_1=0,\ \omega_3=\Omega & (\text{around } z \text{ axis}) \end{cases}$$

① around x axis

$$A_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{I_3-I_1}{I_2}\Omega \\ 0 & \frac{I_1-I_2}{I_3}\Omega & 0 \end{bmatrix}$$

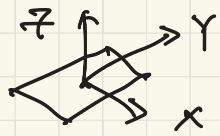
② around y axis

$$A_y = \begin{bmatrix} 0 & 0 & \frac{I_2-I_3}{I_1}\Omega \\ 0 & 0 & 0 \\ \frac{I_1-I_2}{I_3}\Omega & 0 & 0 \end{bmatrix}$$

③ around z axis

$$A_z = \begin{bmatrix} 0 & \frac{I_2-I_3}{I_1}\Omega & 0 \\ \frac{I_3-I_1}{I_2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Delta\dot{x} = A\Delta x$$



$I_3 > I_1$

$I_3 > I_1 > I_2$

- Eigenvalue Analysis

eigenvalues can be found by solving

$$\det(A-\lambda I) = 0$$

① for A_x

$$\det\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & \frac{I_3-I_1}{I_2}\Omega \\ 0 & \frac{I_1-I_2}{I_3}\Omega & -\lambda \end{pmatrix} = -\lambda\left(\lambda^2 - \frac{(I_1-I_2)(I_3-I_1)}{I_2 I_3}\Omega^2\right) = 0$$

$$\lambda = 0,\ \pm\sqrt{\frac{(I_1-I_2)(I_3-I_1)}{I_2 I_3}\Omega^2}$$

② for A_y

$$\det(A_y - I\lambda) = \det\begin{pmatrix} -\lambda & 0 & \frac{I_2-I_3}{I_1}\Omega \\ 0 & -\lambda & 0 \\ \frac{I_1-I_2}{I_3}\Omega & 0 & -\lambda \end{pmatrix}$$

$$= -\lambda^3 + \frac{(I_2-I_3)(I_1-I_2)}{I_1 I_3}\Omega^2\lambda = 0$$

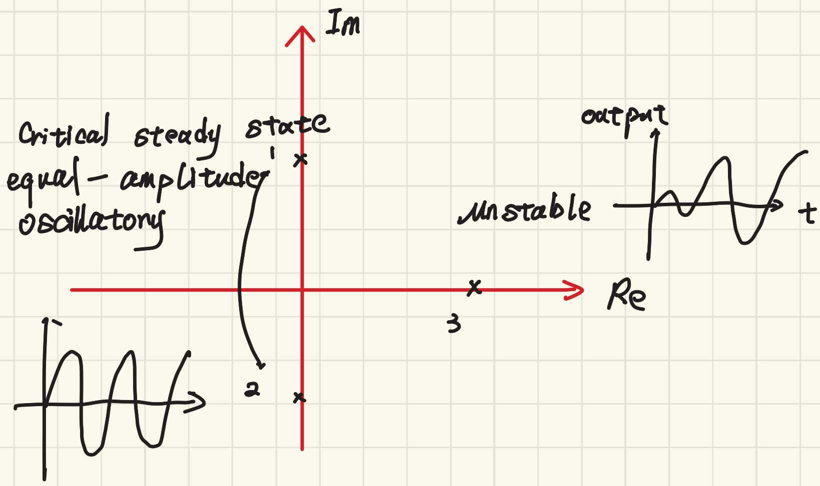
$$\lambda_1 = 0,\ \pm\sqrt{\frac{(I_3-I_2)(I_2-I_1)}{I_1 I_3}\Omega^2}$$

③ for A_z

$$\det(A_z - I\lambda) = \det\begin{pmatrix} -\lambda & \frac{I_2-I_3}{I_1}\Omega & 0 \\ \frac{I_3-I_1}{I_2}\Omega & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix}$$

$$= -\lambda^3 + \frac{(I_2-I_3)(I_3-I_1)}{I_1 I_2}\Omega^2\lambda = 0$$

$$\lambda = 0,\ \pm i\sqrt{\frac{(I_3-I_2)(I_3-I_1)}{I_1 I_2}\Omega^2}$$



3 Problem 3: Quaternion Examples and Intuition

3.1 (a)

When an aircraft has only a positive roll rate (rotation about its longitudinal axis, the x-axis in body coordinates) with zero pitch and yaw rate.

Given:

- The roll rate $\omega_x$is constant.
- The pitch rate $\omega_y$is zero.
- The yaw rate ω_z is zero.

We can then express the angular velocity

$$\boldsymbol{\omega} = [\omega_x, 0, 0]^T. \tag{10}$$

Using the quaternion dynamics equations:

For the vector part $\boldsymbol{e}$:

$$\frac{d\mathbf{e}}{dt} = \frac{\mathbf{1}}{2}\delta\boldsymbol{\omega} - \frac{\mathbf{1}}{2}\boldsymbol{\omega} \times \mathbf{e} \tag{11}$$

Simplifies to:

$$\frac{d\mathbf{e}}{dt} = \frac{1}{2}\delta\omega_x\mathbf{i} \tag{12}$$

For the scalar part:

$$\frac{d\delta}{dt} = \frac{1}{2}\omega_x e_x \tag{13}$$

```

% Constants
roll_rate = 1; % positive constant roll rate (rad/s)
total_time = 2 * 2 * pi / roll_rate; % time for two complete revolutions
time_steps = linspace(0, total_time, 1000); % time vector for simulation

% Initial conditions: e = 0 and delta = 1
initial_conditions = [0, 0, 0, 1];

% Solve the differential equations
[t, solutions] = ode45(@(t, q) quaternion_dynamics(t, q, roll_rate), time_steps,
    initial_conditions);

% Extracting the solutions
ex_values = solutions(:, 1);
ey_values = solutions(:, 2);
ez_values = solutions(:, 3);
delta_values = solutions(:, 4);

% Calculate roll attitude (phi)
phi_values = roll_rate * time_steps;

% Plot the results

% Plot phi
figure;
```

```
subplot(3, 1, 1);
plot(time_steps, rad2deg(phi_values), 'DisplayName', 'Roll Attitude (phi)',LineWidth=2);
title('Roll Attitude (phi) vs Time');
xlabel('Time (s)');
ylabel('Phi (degrees)');
grid on;
legend;

% Plot e vector components
subplot(3, 1, 2);
plot(time_steps, ex_values, 'DisplayName', 'e_x',LineWidth=2);
hold on;
plot(time_steps, ey_values, 'DisplayName', 'e_y',LineWidth=2);
plot(time_steps, ez_values, 'DisplayName', 'e_z',LineWidth=2);
hold off;
title('Quaternion Vector Components (e) vs Time');
xlabel('Time (s)');
ylabel('e components');
grid on;
legend;

% Plot delta
subplot(3, 1, 3);
plot(time_steps, delta_values, 'DisplayName', 'delta',LineWidth=2);
title('Quaternion Scalar Component (delta) vs Time');
xlabel('Time (s)');
ylabel('delta');
grid on;
legend;

% Adjust layout
sgtitle('Quaternion Dynamics Simulation');


% Quaternion dynamics for roll only motion

function dqdt = quaternion_dynamics(t, q, omega_x)
    ex = q(1);
    ey = q(2);
    ez = q(3);
    delta = q(4);
    % Derivatives of the quaternion components
    dexdt = 0.5 * delta * omega_x;
    deydt = 0;
    dezdt = 0;
    ddeltadt = -0.5 * omega_x * ex;
    dqdt = [dexdt; deydt; dezdt; ddeltadt];
end
```

3.2 (b)

When an aircraft rolls upside down by $\phi = 180$, the rotation matrix is:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{14}$$

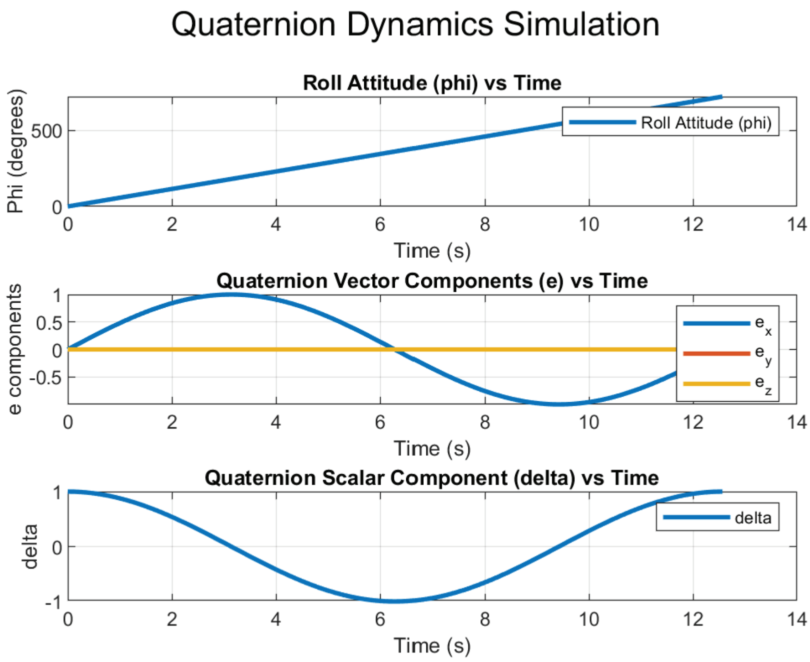


Figure 4: Results of HW9-3-a

$$R_x(180°) = \begin{bmatrix} \mathbf{1} & 0 & 0 \\ 0 & -\mathbf{1} & 0 \\ 0 & 0 & -\mathbf{1} \end{bmatrix} \tag{15}$$

To define a quaternion for a rotation matrix where R represents the transformation and using the notation where the quaternion q is defined as:

$$q = \begin{bmatrix} e_x \\ e_y \\ e_z \\ \delta \end{bmatrix} \tag{16}$$

$$\vec{e} = [e_x, e_y, e_z]^T = \sin\left(\frac{\theta}{2}\right)\vec{v_1} \tag{17}$$

$$\delta = \cos\left(\frac{\theta}{2}\right) \tag{18}$$

$$\theta = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right) \tag{19}$$

$$\theta = \cos^{-1}\left(\frac{-1-1}{2}\right) = \cos^{-1}(-1) = 180° \tag{20}$$

and $\vec{v_1}$ is the unit eigenvector of R associated with the eigenvalue 1.

$$V = [1, 0, 0]^T \tag{21}$$

$$\begin{aligned} e_x &= 1 \times 1 = 1 \\ e_y &= 0 \times 1 = 0 \\ e_z &= 0 \times 1 = 0 \\ \delta &= 0 \end{aligned} \tag{22}$$

Therefore, the quaternion representing a 180 rotation about the x-axis is:

$$q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

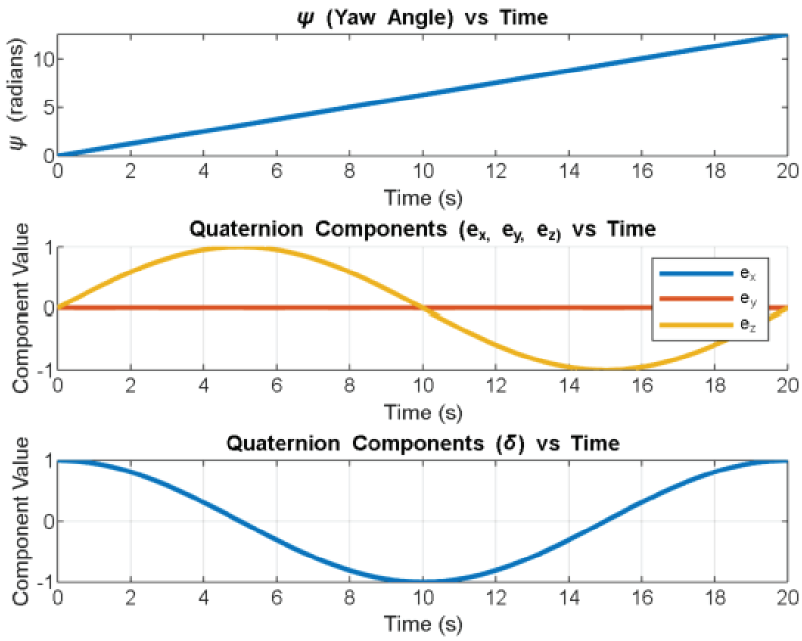


Figure 5: HW9-3-b-result

Once θ and $\vec{v_1}$ are known, the quaternion representing the rotation can be constructed as:

$$q = [\sin(\frac{\theta}{2})v_{1x}, \sin(\frac{\theta}{2})v_{1y}, \sin(\frac{\theta}{2})v_{1z}, \cos(\frac{\theta}{2})] \tag{24}$$

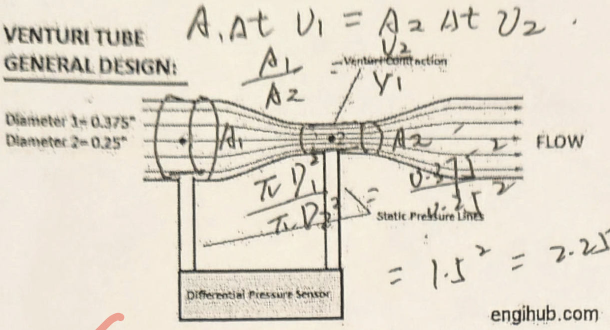
References

[1] Reconstructing the motion of a tossed iPhone | Rotations. (n.d.). Rotations.berkeley.edu. Retrieved April 8, 2024, from .`https://rotations.berkeley.edu/reconstructing-the-motion-of-a-tossed-iphone/`

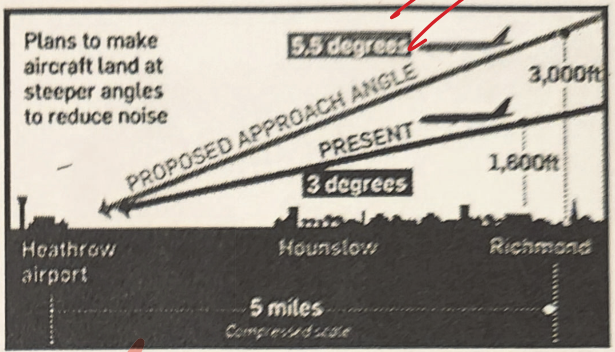
Name: Zhanqian Wu. 52463471 5×3=15

2024-01-29 – MEAM 543 – Quiz01

$A_1 \Delta t\, v_1 = A_2 \Delta t\, v_2$. $\frac{A_1}{A_2} = \frac{v_2}{v_1}$



$\frac{\pi D_1^2}{\pi D_2^2} = \frac{0.375^2}{0.25^2} = 1.5^2 = 2.25$



C 1. Water flows through a venturi tube as shown above. Which of the following is MOST ACCURATE about the flow speeds?

mass is constant

$A_1 v_1 \Delta t = A_2 v_2 \Delta t$

$\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2}$

A. $\frac{V_2}{V_1} \approx 0.67$

B. $\frac{V_2}{V_1} \approx 0.44$

C. $\frac{V_2}{V_1} \approx 2.25$

D. $\frac{V_2}{V_1} \approx 1.5$

E. I Know I Don't Know

B 4. The figure above (from airportwatch.org.uk) is a graphical presentation of a plan to reduce noise for aircraft landing at Heathrow airport in London. Which of the following is the best description of the proposed change?

A. New approach pitch attitude: $\theta = -5.5°$

B. New approach flight-path angle: $\gamma = -5.5°$

C. New approach angle of attack: $\alpha = -5.5°$

D. None of the other answers.

E. I Know I Don't Know

θ pitch altitude

α angle of attack

γ flight path angle

ΔP equal)

D 2. Continuing with the venturi tube, which of the following is MOST ACCURATE about the pressure difference $\Delta p = p_2 - p_1$?

Bernoulli equation

$p + \frac{1}{2}\rho v^2 = \text{constant}$

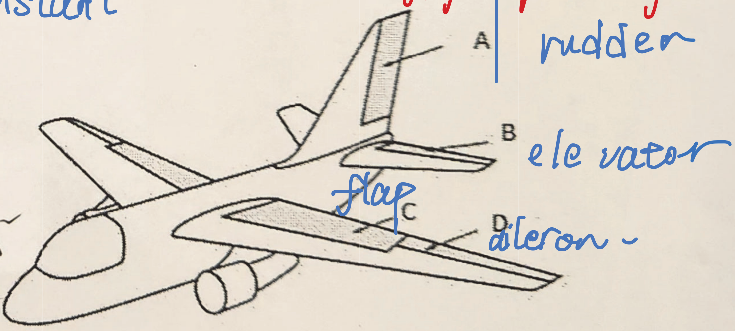
A. None of the other answers.

B. Δp will be proportional to $\sqrt{V_1}$

C. Δp will be proportional to V_1

D. Δp will be proportional to V_1^2

E. I Know I Don't Know

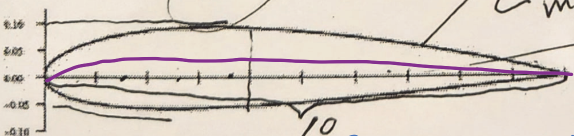


rudder

elevator

flap

aileron



0.15 t_{max} $C_{max} = 15$ 10

2: max camber

B 3. Which NACA airfoil might be shown in the figure above?

4: max camber position 40%

A. NACA 2406

B. NACA 2415 → 40%

C. NACA 0006

D. NACA 6715

E. I Know I Don't Know

15: thickness / chord = 0.15

B 5. Which of the following is NOT a correct label for an aircraft element shown in the figure above?

A. A = rudder

B. B = propeller

C. C = flap

D. D = aileron

E. I Know I Don't Know

Name: Zhangian Wu 12403471 $3\times3+2\times1+1=12$ / 15

2024-02-05 – MEAM 543 – Quiz02

C 1. Which of the following is most accurate about the procedure for submitting a homework solution after the assignment on canvas has closed, making the homework "rogue"?

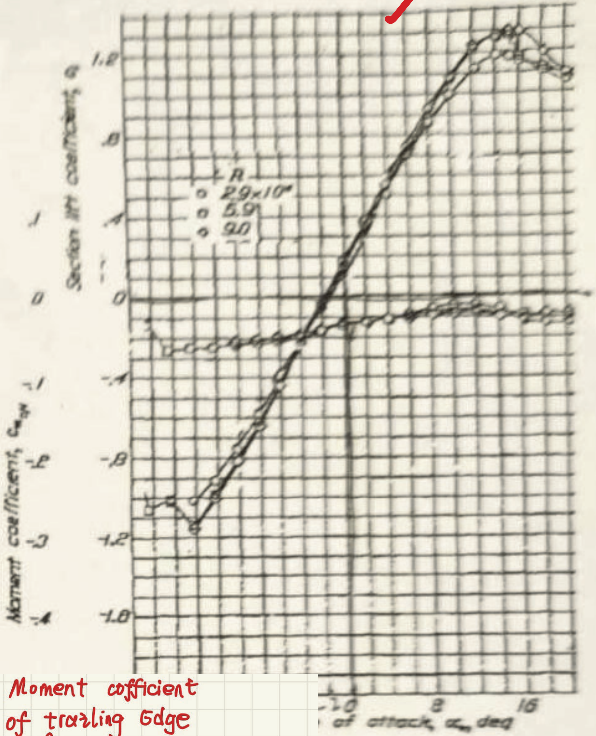
A. Email the solution to the professor.

B. On Canvas, click on the assignment, then click on the "go rogue" button in the lower left corner.

C. On Canvas, click on "Grades" then click the assignment, then click on "upload comment" and attach the file.

D. This isn't relevant for me, because if I accept the very loose due dates, I just know I will fall behind and then I won't learn much. So, I'm going to force myself to do the work in a timely manner by asking the professor to give me a zero on any assignment that would be rogue.

E. I Know I D…



D 2. Which of the follow… "stall" on an airplan…

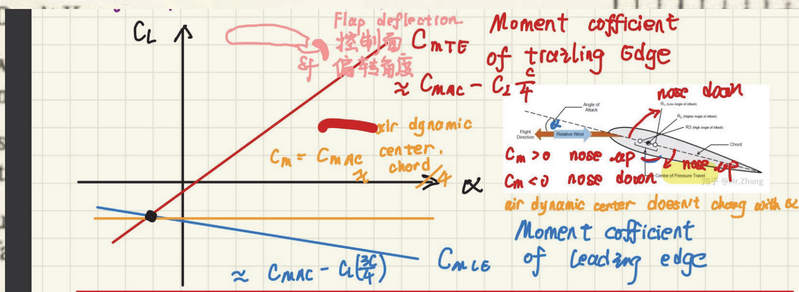
A. Stall limits… coefficient…

B. Stall is cau… upper surfa…

C. Stall angle depends on Reynolds number.

D. All of the other answers.

E. I Know I Don't Know.



gliding $T=0$ $L = W\cos\gamma$

$T = D + W\sin\gamma$

3. Which of the following is the equation for the flight-path angle of a gliding aircraft (no thrust)?

C ✗ A

A. $\tan\gamma = -\frac{D}{L}$

B. $\sec\gamma \approx 0$

C. $\sin\gamma = -\frac{W}{L}$

D. None of the other answers.

E. I Know I Don't Know

$-\frac{D}{L} = \tan\gamma$

$$\frac{v}{V} = \frac{[\sin(\alpha_0+\phi)+\sin(\alpha_0+\epsilon_{TE})]\,e^{\psi_0}}{\sqrt{(\sinh^2\psi+\sin^2\theta)\left[\left(1-\frac{d\epsilon}{d\phi}\right)^2+\left(\frac{d\psi}{d\phi}\right)^2\right]}}$$

No question about the formula for velocity distribution on an airfoil from conformal mapping.

4. …easurements[1] for lift and …git airfoil. Which of the … ASONABLE inference … these data?

A. …e is about -2°, so the … positive camber.

B. Stall is gradual, so the airfoil is relatively thick.

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know

D 5. Continuing with the figure from the previous problem, which of the following is MOST ACCURATE about the moment data? ✗ B

A. The moment at zero lift is negative, so the airfoil has negative camber.

B. The moment at the quarter chord (shown in the figure) has positive slope, so the aerodynamic center is forward of the quarter chord. backward

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know

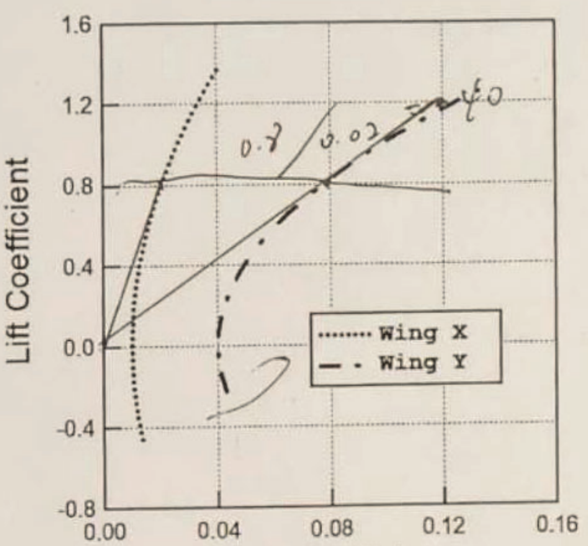
---

[1] NACA Report 824, *Summary of Airfoil Data*

Name: Zhanqian Wu.

12/15

2024-02-12 – MEAM 543 – Quiz03



$$C_D = C_{D_0} + rC_L^2 + \frac{C_L^2}{\pi e AR}$$

$AR \uparrow \quad \frac{dC_L}{dC_D} \uparrow$

E C

1. Which of the following is the MOST REASONABLE inference from the *wing* drag polars shown above?
   - A. Wing X is thicker than wing Y.
   - B. Wing X has more camber than wing Y.
   - C. Wing X has higher aspect ratio than wing Y.
   - D. None of the other answers is reasonable.
   - E. I Know I Don't Know

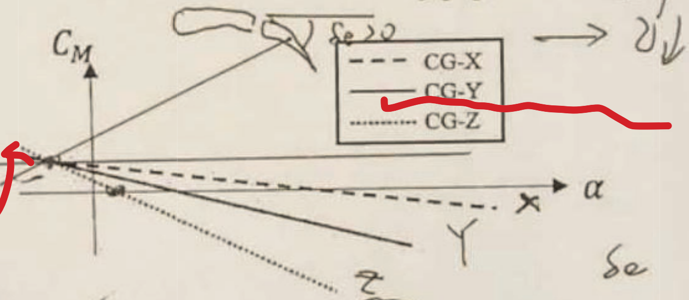
D

2. Continuing with the drag polar of the previous problem which of the followings is MOST ACCURATE?
   - A. Wing X has a maximum L/D of about 5.
   - B. Wing X has a maximum L/D of about 10.
   - C. Wing Y has a maximum L/D of about 5.
   - D. Wing Y has a maximum L/D of about 10.
   - E. I Know I Don't Know

A

3. Which of the following is MOST ACCURATE about the effect of ~~elevator deflection~~ on ~~steady-state speed~~ of a stable airplane operating in the linear portion of the lift curve? $\delta_e > 0$
   - A. Positive (downward) elevator deflection causes the airplane to fly faster. ✓
   - B. Positive (downward) ~~elevator deflection~~ causes the airplane to fly slower.
   - C. Elevator deflections affect steady climb rate, not steady speed.
   - D. It depends on whether the airplane is flying faster or slower than the speed for minimum drag.
   - E. I Know I Don't Know.

$\delta_e \downarrow \longrightarrow C_L \uparrow \longrightarrow v \downarrow$



D

4. The figure above shows the moment coefficient variation with α for a fixed airplane geometry with 3 different CG locations. Which of the following is the MOST REASONABLE inference from the figure?
   - A. CG-Z is forward of CG-Y which is forward of CG-X.
   - B. The zero-lift angle of the airplane is negative.
   - C. The airplane is stable for all of the CG locations shown in the figure.
   - D. All of the other answers.
   - E. I Know I Don't Know

~~B~~ D

5. Continuing with the previous problem, if no elevator deflection is used, which CG location will result in the highest steady speed?
   Careless, misread
   - A. Steady speed depends on thrust, not on CG location.
   - B. CG-X will be fastest.
   - C. CG-Y will be fastest. $\alpha \downarrow$
   - D. CG-Z will be fastest.
   - E. I Know I Don't Know

a

$2\times 3 + 3\times 2 = 12$

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Name: Zhangian Wu

2024-02-19 – MEAM 543 – Quiz04

$D = (C_{D0} + kC_L^2)\frac{1}{2}\rho v_\infty^2 S$

$L = W = \frac{1}{2}\rho v_\infty^2 S C_L \Rightarrow C_L = \frac{2W}{\rho S v_\infty^2}$

D 1. The landing gear on an airplane fails to retract, causing the parasite drag (C_{D0}) to increase. The pilot believes they have plenty of fuel to safely continue the short flight. With the gear retracted, they would have flown at speed, V_o, and altitude, h_o, which together would minimize drag in steady level flight. Which of the following changes would allow them to minimize drag with the stuck landing gear?

A. Fly with $V > V_o$ and $h = h_o$.

B. Fly with $V = V_o$ and $h < h_o$.

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know.

$= \frac{1}{2}\rho v_\infty^2 S C_{D0} + w$

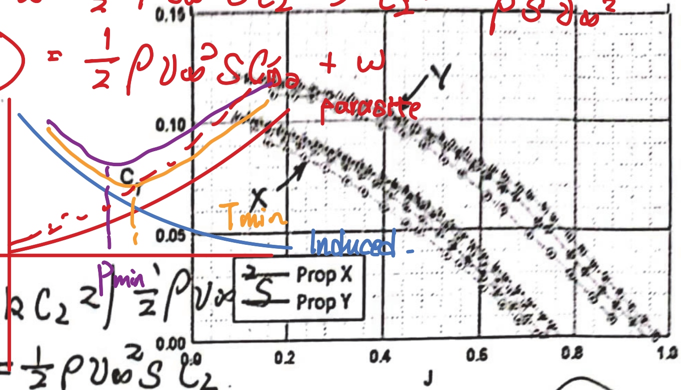
$v \nearrow$ $h > h_0$

$D = (C_{D0} + kC_L^2)\frac{1}{2}\rho v_\infty^2 S$

$L = W = \frac{1}{2}\rho v_\infty^2 S C_L$

$C_L = \frac{2W}{\rho S v_\infty^2}$

$D = \frac{1}{2}\rho v_\infty^2 C_{D0} S +$



C 2. The weight of an airplane on a long flight slowly decreases. Which of the following changes in flight condition would allow the airplane to maintain flight at minimum drag?

A. Gradually decrease speed.

B. Gradually increase altitude.

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know.

$k\frac{2W}{\rho v_\infty^2 S}$

$D = \frac{1}{2}\rho v_\infty^2 C_{D0} S + k\left(\frac{2W}{\frac{1}{2}\rho v_\infty^2 S}\right)^2 \frac{1}{2}\rho v_\infty^2 S$

$k\frac{2W^2}{\rho v_\infty^2 S}$

E 4. The figure above shows data from the UIUC propeller database for two different propellers, each at several different propeller speeds. Which of the following is the MOST REASONABLE inference from this figure?

A. Prop X has smaller chord than Prop Y.

B. Prop X has smaller pitch angle than Prop Y.

C. Prop X has smaller diameter than Prop Y.

D. All of the other answers.

E. I Know I Don't Know.

B

Next page

C 3. We defined the power required for level flight as P_R. Which of the following equations best describes the resulting steady flight condition for $P \neq P_R$?

A. $\tan\alpha = \frac{P-P_R}{WV}$

B. $V = \frac{P-P_R}{D}$

C. $\frac{dh}{dt} = \frac{P-P_R}{W}$

D. None of the other answers.

E. I Know I Don't Know

C

5. Continuing with the data of the previous problem, suppose that the propellers have the same diameter. Which of the following statements is the MOST REASONABLE inference about the propellers operating at the same RPM and zero speed?

A. Prop X has higher efficiency than Prop Y.

B. Prop X generates more thrust than Prop Y.

C. Both A and B.

D. Neither A nor B.

E. I Know I Don't Know.

D

T γ L D

$\frac{dh}{dt}$ v γ

$W = L\cos\gamma \approx L$

$T = D + W\sin\gamma$

$TV = DV + WV\sin\gamma$

P P_R $WV\sin\gamma$ dh/dt

$\frac{P - P_R}{W} = dh/dt$

Next page

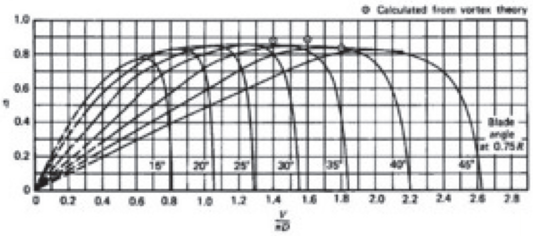


Figure 11.28: Typical propeller efficiency curves as a function of advance ratio ($J = u_0/nD$) and blade angle

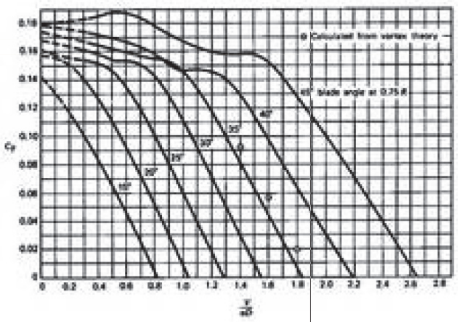


Figure 11.29: Typical propeller thrust curves as a function of advance ratio ($J = u_0/nD$) and blade angle (McCormick, 1979).

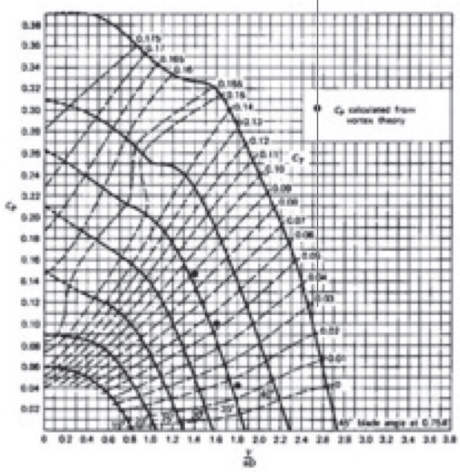


Figure 11.30: Typical propeller power curves as a function of advance ratio ($J = u_0/nD$) and blade angle (McCormick,

Name: Zhanqian Wu. 11/15

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E +1

C

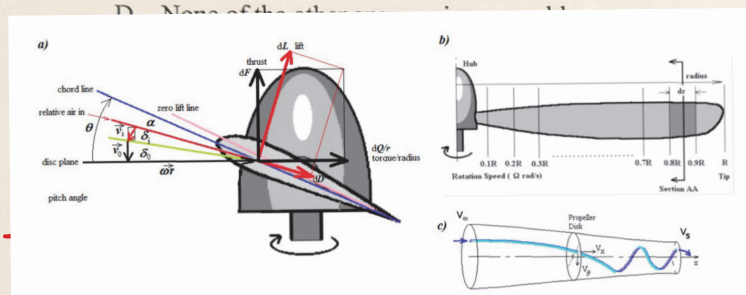
1. An RC airplane is sold with a LiPo battery having the following characteristics: 3S, ~~11.1V, 2200 mAh,~~ 180g. Which of the following would be the MOST REASONABLE inputs to our analysis based on this information?

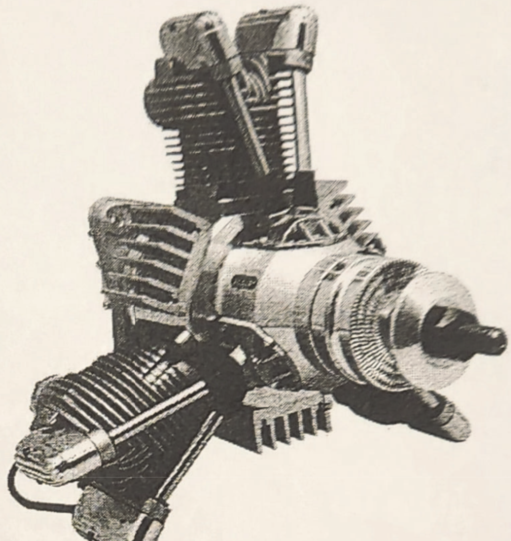
   A. $\beta_{TIP} \approx 5.5°$

   B. $i_o \approx 1.0$ amp

   C. $H_F \approx 50$ km

   D. None of the other answers is reasonable.





E

D

shown above. Which of the following would be the MOST REASONABLE inputs to our analysis based on this information?

$\beta_{tip} \neq$ Pitch

   A. $\beta_{TIP} \approx 5.5°$

   B. $i_o \approx 1.0$ amp

   C. $H_F \approx 50$ km

   D. None of the other answers is reasonable.

   E. I Know I Don't Know.

A

4. An RC airplane is sold with the 90cc IC engine shown above. Which of the following is the MOST REASONABLE statement about this engine?

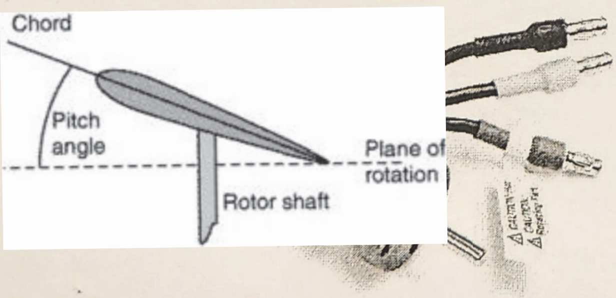
   A. All of the other answers.

   B. "90cc" refers to the total volume of the cylinders. At a given RPM, we expect power output of the engine to be roughly proportional to total cylinder volume.

   C. It has 3 cylinders.

   D. It is a radial engine.

   E. I Know I Don't Know.



+1

5. A small (1kg gross weight) airplane that is powered by a brushed DC motor and 11-inch propeller exhibits a maximum speed in level flight that is lower than the engineers expected. Which of the following is the MOST REASONABLE explanation for the discrepancy? B D↓ → v↑

D A

   A. None of the other answers.

   B. The wing induced drag is lower than expected. D↓ v↑

   C. The motor coil resistance is lower than expected. R↓ P↑ ↓ v↑ v↓

   D. The propeller C_P curve is lower than expected. P↓ v↓

   E. I Know I Don't Know.

B

3. An RC airplane is sold with the DC ~~motor shown~~ above. Which of the following would be the MOST REASONABLE inference about this motor?

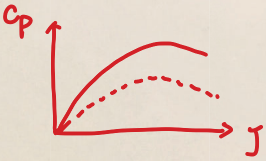
   A. Neither C nor D.

   B. Both C and D.

   C. It is a brushless motor.

   D. It requires an ESC to interface with the radio receiver.

   E. I Know I Don't Know



$P = C_P \rho n^3 D^5$

for equal Thrust

$P_{required} \downarrow$ ⇒ Can generate more thrust

⇒ fly faster

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Name: Zhangian Wu

2024-03-13 – MEAM 543 – Quiz06

~~E~~ B

1. A young engineer is using a software program to estimate the size of a small UAV. The company has decided to invest in a new structural design which they estimate will reduce the aircraft empty weight fraction, $\frac{W_E}{W_0}$, from 0.6 to 0.5. When the engineer puts this change into the software, but keeps the payload weight the same, the estimated gross weight of the aircraft drops by a factor of 2! Which of the following is the MOST REASONABLE inference about this result?

   A. The fuel weight fraction, $\frac{W_F}{W_0}$, is about 0.3.

   B. The engineer has obviously made a mistake.

   C. The engineer must have accidentally changed the propulsion system from hydrocarbon to electric by mistake.

   D. None of the other answers is reasonable.

   E. I Know I Don't Know.

C

2. We have generally talked about airplanes cruising at the C_L that maximizes $\frac{L}{D}$. Suppose we have an airplane with a propulsion system which burns fuel at a rate that is proportional to thrust. If we want to maximize the *range* of the aircraft, how does this change the optimal C_L for cruise?

   A. We should still fly at the C_L for maximum $\frac{L}{D}$.

   B. We should fly at a higher C_L than the C_L for maximum $\frac{L}{D}$.

   C. We should fly at a lower C_L than the C_L for maximum $\frac{L}{D}$.

   D. It depends on altitude.

   E. I Know I Don't Know.

$\frac{L}{D} = \frac{C_L}{C_{D0} + kC_L^2}$

~~A~~ D

3. Which of the following performance metrics generally increase with increased wing area, for a given drag polar, altitude, and aircraft weight?

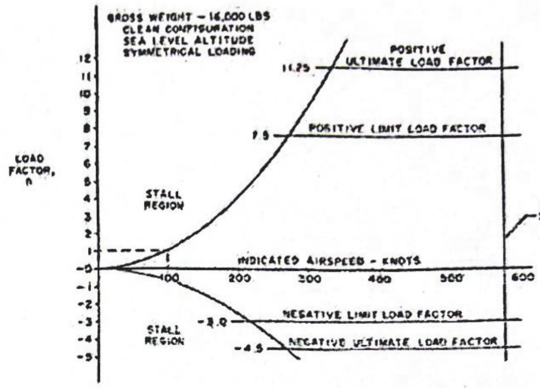
   A. Turn radius

   B. Stall speed

   C. Takeoff distance

   D. None of the other answers

   E. I Know I Don't Know



~~E~~ D

4. The figure above shows an example V-n diagram from the Air Training Command Manual (1963). The figure includes aircraft gross weight (16000 pounds) in the top left corner. Which of the following is the MOST REASONABLE conjecture about how the figure would change if the gross weight were increased without changing the altitude or the aerodynamic characteristics of the aircraft?

   A. All of the other answers.

   B. The "positive limit load factor" would decrease.

   C. The stall speed (currently about 100 knots) would decrease.

   D. The corner speed (currently about 300 knots) would decrease.

   E. I Know I Don't Know.

D

5. Continuing with the figure of the previous question, what does the annotation "clean configuration" mean?

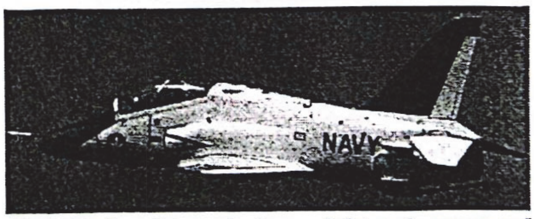
   A. Gear retracted.

   B. Flaps retracted.

   C. Neither A nor B.

   D. Both A and B.

   E. I Know I Don't Know.



T-45 Goshawk has similar weight and max speed

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2024-03-25 – MEAM 543 – Quiz07

$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$
$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma$$

$m\frac{dv}{dt} = T - D - mg\sin 90°$

$m\frac{dv}{dt} = T - D - mg$

$mv\frac{d\gamma}{dt}\gamma = L - mg\cos 90°$

1. According to the simple flight dynamics equations we wrote in lecture (above), which of the following ~~MUST be true about~~ an airplane that is flying vertically upward for 5 seconds?

   A. The airplane has zero drag. $v \neq 0$ Drag $\neq 0$

   B. The airplane has zero lift.

   C. The airplane has thrust greater than weight. depends on T-D-mg

   D. The airplane is decelerating.

   E. I Know I Don't Know.

E A

2. Continuing with the simple ~~flight dynamics~~ equations…While initially descending, an airplane executes a symmetric pull-up maneuver that is a circular arc of radius R. The maneuver is performed at constant thrust. At the end of the maneuver, the airplane is in steady level flight. Which of the following MUST be true during the maneuver?

   $L = W$ $L<W$ $L>W$ $L = W$

   A. $\frac{d\gamma}{dt}$ = constant throughout the maneuver.

   B. $L > W$ throughout the maneuver.

   C. $\frac{dV}{dt} = 0$ throughout the maneuver.

   D. None of the other answers.

   E. I Know I Don't Know.

B D -2

3. Which of the following is an assumption that we used when deriving the flight dynamics equations above?

   A. None of the other answers.

   B. The airplane is statically stable (moment coefficient versus angle of attack has negative slope).

   C. The airplane is flying above the speed for minimum thrust.

   D. Thrust is aligned with velocity.

   E. I Know I Don't' Know.

4. Given that $0.7^2 + 2.4^2 = 2.5^2$, which of the following is a solution of

$$\frac{d^2\gamma}{dt^2} + 1.4\frac{d\gamma}{dt} + 6.25\gamma = 0\,?$$

   A. $\gamma = 0.1e^{-0.7t}\cos(2.4t)$

   B. $\gamma = 0.7e^{-2.4t}\cos(0.1t)$

   C. $\gamma = 2.4e^{-0.1t}\cos(0.7t)$

   D. None of the other answers

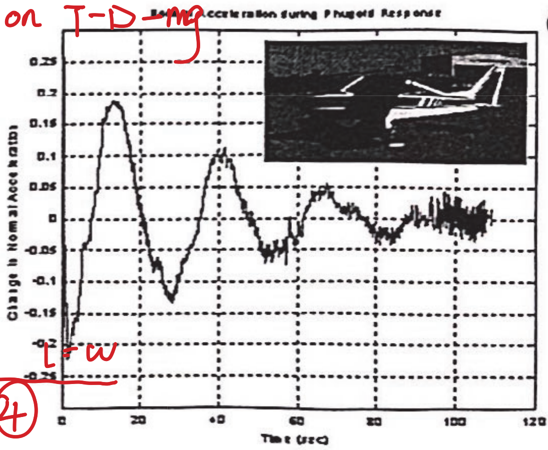
   E. I Know I Don't Know

A -1

1.4×-0.7 6.25 1.4

$a e^{-bt}\cos(ct)$

$-ab\, e^{-bt}\cos(ct)$ $-ab$ -0.7 $-ac$



5. The figure above shows the change in normal loadfactor (from the nominal value of 1g) during a phugoid evaluation flight test[2]. Which of the following is LEAST ACCURATE?

   A. Accelerometer data is often noisy due to a variety of sources of vibration in the aircraft.

   B. The variation in loadfactor implies that the pilot is moving the stick fore and aft to generate more or less lift. Pilot induced oscillation.

   C. A linear analysis of this motion would produce a complex eigenvalue with a real part that is less than zero. stable mode require real part <0

   D. A linear analysis of this motion would produce a complex eigenvalue with imaginary part of approximately 0.25.

   E. I Know I Don't Know.

C B Next page

$r^2 + pr + q = 0$ $\quad y'' + py' + qy = 0$

$r_1 \neq r_2$ $\quad y = C_1e^{r_1x} + C_2e^{r_2x}$

$r_1 = r_2$ $\quad y = (C_1 + C_2x)e^{r_1x}$

$r_{1,2} = \alpha \pm i\beta$ $\quad y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$

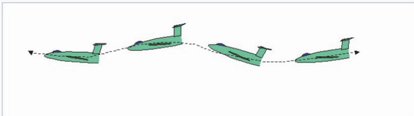
$$r = \frac{-1.4 \pm \sqrt{1.4^2 - 4\times 6.25}}{2} = -0.7 \pm 2.4i$$

[2] https://old.bd-4.org/kenkopp/kenkopp_flighttest_4.html

There can be two factors causing the landing mishaps mentioned:

Phugoid: the natural phenomenon of any aircraft.
Pilot Induced Oscillations: the pilot-in-the-loop oscillation of the aircraft while in control of a pilot.
To test the phugoid, trim the aircraft to a calm condition and do not touch the controls for several seconds.

In aviation, a **phugoid** or **fugoid** (/ˈfjuːɡɔɪd/) is an aircraft motion in which the vehicle pitches up and climbs, and then pitches down and descends, accompanied by speeding up and slowing down as it goes "downhill" and "uphill". This is one of the basic flight dynamics modes of an aircraft (others include short period, roll subsidence, dutch roll, and spiral divergence).



A diagrammatic representation of a fixed-wing airplane in phugoid

Detailed description [edit]

The phugoid has a nearly constant angle of attack but varying pitch, caused by a repeated exchange of airspeed and altitude. It can be excited by an elevator singlet (a short, sharp deflection followed by a return to the centered position) resulting in a pitch increase with no change in trim from the cruise condition. As speed decays, the nose drops below the horizon. Speed increases, and the nose climbs above the horizon. Periods can vary from under 30 seconds for light aircraft to minutes for larger aircraft. Microlight aircraft typically show a phugoid period of 15–25 seconds, and it has been suggested[by whom?] that birds and model airplanes show convergence between the phugoid and short period modes. A classical model for the phugoid period can be simplified to about (0.85 × speed in knots) seconds, but this only really works for larger aircraft.[further explanation needed]

Phugoids are often demonstrated to student pilots as an example of the speed stability of the aircraft and the importance of proper trimming. When it occurs, it is considered a nuisance, and in lighter airplanes (typically showing a shorter period) it can be a cause of pilot-induced oscillation.

The phugoid, for moderate amplitude,[1] occurs at an effectively constant angle of attack, although in practice the angle of attack actually varies by a few tenths of a degree. This means that the stalling angle of attack is never exceeded, and it is possible (in the <1g section of the cycle) to fly at speeds below the known stalling speed. Free flight models with badly unstable phugoid typically stall or loop, depending on thrust.[2]

An unstable or divergent phugoid is caused, mainly, by a large difference between the incidence angles of the wing and tail. A stable, decreasing phugoid can be attained by building a smaller stabilizer on a longer tail, or, at the expense of pitch and yaw "static" stability, by shifting the center of gravity to the rear.[why?][citation needed]

Aerodynamically efficient aircraft typically have low phugoid damping.[3]:464

The term "phugoid" was coined by Frederick W. Lanchester, the British aerodynamicist who first characterized the phenomenon. He derived the word from the Greek words φυγή and εἶδος to mean "flight-like" but recognized the diminished appropriateness of the derivation given that φυγή meant flight in the sense of "escape" (as in the word "fugitive") rather than vehicle flight.[4]

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2024A – MEAM 543 – Quiz 8 – 3-April

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1. ~~Beginning at $\phi = \theta = \psi = 0$, an airplane rotates for one second with constant angular velocity:~~ $p = \frac{\pi}{2}, q = r = 0.$
around body's x axis
It subsequently rotates one second with constant angular velocity: $p = r = 0, q = \frac{\pi}{2}$. The resulting Euler angles will be…
 A. $\phi = \frac{\pi}{2}, \theta = \frac{\pi}{2}, \psi = 0$
 B. $\phi = \frac{\pi}{2}, \theta = 0, \psi = \frac{\pi}{2}$
 C. $\phi = 0, \theta = \frac{\pi}{2}, \psi = \frac{\pi}{2}$
 D. None of the other answers.
 E. I Know I Don't Know.

C +1

$R_y(90°)R_x(90°)$

$\theta = \frac{\pi}{2}$ $\psi = \frac{\pi}{2}$

2. If $\phi = -\frac{\pi}{4}$ and $\theta = \psi = 0$, which of the following is correct when $\dot\phi = \dot\theta = 0$ and $\dot\psi > 0$?
 A. $p > 0$
 B. $q > 0$
 C. $r > 0$
 D. All of the other answers.
 E. I Know I Don't Know.

C +1

$\psi \Rightarrow$ body's z axis
$\theta \Rightarrow$ body's y axis
$\phi \Rightarrow$ body's x axis
Euler is angle $\Rightarrow$ world frame
p.q.r vehicle frame

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\cos\phi\sin\theta \\ 0 & 1 & \sin\phi \\ \sin\theta & 0 & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix}$$

3. Which of the following is NOT a 3D rotation matrix?

 A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

 B. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

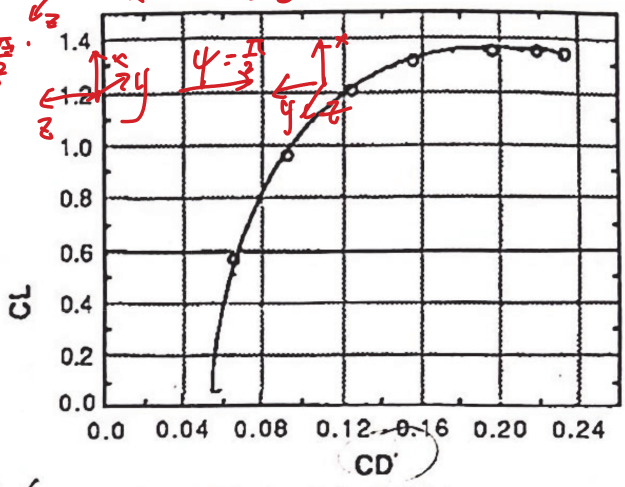
 C. $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

 D. All of the other answers. (In other words, none of the other answers is a 3D rotation matrix.)
 E. I Know I Don't Know.

D +1 B

- orthogonal. it's inverse is equal to transpose
- determinant must be +1
- columns & rows have a length of 1

B Not orthogonal & determinant $\neq +1$



4. Which of the following is the MOST REASONABLE statement about the trimmed aircraft drag polar shown above?
 A. Because $C_D \approx 0.95$ when $C_L \approx 1.0$, a reasonable value of K in our drag polar equation is 0.95
 B. The wing is stalled for $C_L \gtrsim 1.3$.
 C. Both A and B.
 D. Neither A nor B.
 E. I Know I Don't Know.

5. Continuing with the aircraft drag polar from the previous question…which of the following is the MOST REASONABLE expected consequence of reducing the span of the main wing, while keeping the main wing area constant?
 A. The drag coefficient would decrease at all values of C_L.
 B. The drag coefficient would increase mostly at high values of C_L.
 C. The drag coefficient would increase mostly at low values of C_L.
 D. None of the other answers. (Wingspan does not affect drag coefficient in our simple aero model.)
 E. I Know I Don't Know.

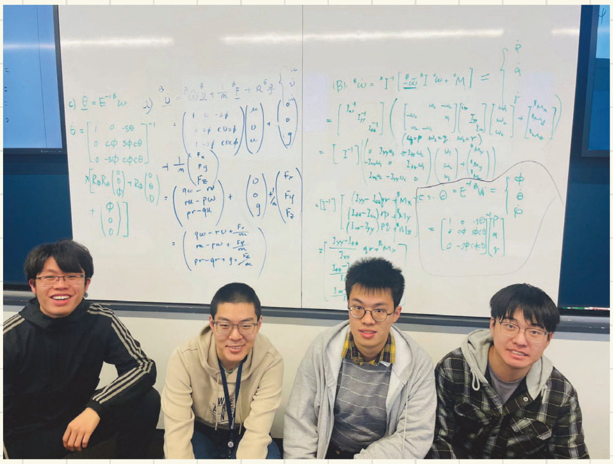
B +1

$C_D = C_{D_0} + kC_L^2$

$k = \frac{1}{\pi AR}$ $\frac{b^2}{S}$

$$C_D = C_{D_0} + kC_L^2$$

$$k = \frac{1}{ARe}$$ $\frac{b^2}{S}$



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Name: Zhanqian Wu.

2024A – MEAM 543 – Quiz 9 – 17-April

general rotation z-y-x order

1. Which rotation matrix corresponds to $\phi = 0, \theta = 0, \psi = \pi/3$?

$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -s\phi \\ 0 & s\phi & \cos\phi \end{pmatrix}$

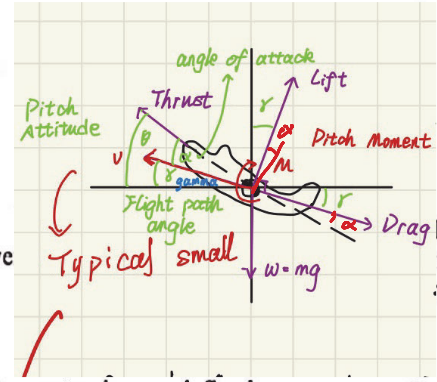
A. $\begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$

C. $\begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & -1/2 \end{bmatrix}$

D. None of the other answers.

E. I Know I Don't Know.



2. Which of the following is the quaternion, as defined in class, that corresponds to $\phi = 0, \theta = 0, \psi = \pi/3$?

$\mu = (1,0,0)$ $\theta = \psi = \frac{\pi}{3}$

$(e_x, e_y, e_z) = (\mu_x, \mu_y, \mu_z)\sin(\frac{\theta}{2})$

A. None of the other answers.

B. $\underline{q} = \begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}^T$ f

C. $\underline{q} = (\pi/3 \quad 0 \quad 0 \quad 1)^T$ f $\quad S = \cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{2}$

out of order

D. $\underline{q} = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}^T$ $(\frac{1}{2}, 0, 0)$ $S = \frac{\sqrt{3}}{2}$

E. I Know I Don't Know.

3. For flight at zero sideslip, which of the following is the correct relationship between the aerodynamic body-axis X force ~~and the lift~~ and drag forces?

C

A. $X = -D + L\tan\alpha$

B. $X = -D\cot\alpha$

C. $X = -D\cos\alpha + L\sin\alpha$ ✓

D. None of the other answers.

E. I Know I Don't Know.

4. Which of the following is a state in the 12-state dynamic model of aircraft dynamics that we developed in class?

position $(x, y, z)_E$
velocity (u, v, w)
attitude (φ, θ, ψ) roll pitch yaw
rate (p, q, r)

A. All of the other answers.

B. Pitch attitude, θ

C. Angle of attack, α

D. Flight-path angle, γ

E. I Know I Don't Know.

5. If we write the dynamic model that we developed in class as $\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u})$, which of the following is true about the derivatives of $\underline{f}$?

D

A. $\frac{\partial \underline{f}}{\partial w} = \underline{0}$

B. $\frac{\partial \underline{f}}{\partial \delta_e} = \underline{0}$

C. $\frac{\partial \underline{f}}{\partial q} = \underline{0}$

D. $\frac{\partial \underline{f}}{\partial \psi} = \underline{0}$

E. I Know I Don't Know.

In this section, we develop the small-disturbance equations for longitudinal motions in standard state-variable form. Recall that the linearized equations describing small longitudinal perturbations from a longitudinal equilibrium state can be written

$$\left[\frac{\mathrm{d}}{\mathrm{d}t} - X_u\right] u + g_0 \cos\Theta_0 \theta - X_w w = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T$$
$$-Z_u u + \left[(1 - Z_{\dot{w}})\frac{\mathrm{d}}{\mathrm{d}t} - Z_w\right] w - [u_0 + Z_q] q + g_0 \sin\Theta_0 \theta = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T \tag{5.37}$$
$$-M_u u - \left[M_{\dot{w}}\frac{\mathrm{d}}{\mathrm{d}t} + M_w\right] w + \left[\frac{\mathrm{d}}{\mathrm{d}t} - M_q\right] q = M_{\delta_e}\delta_e + M_{\delta_T}\delta_T$$

If we introduce the longitudinal state variable vector

$$\mathbf{x} = [u \quad w \quad q \quad \theta]^T \tag{5.38}$$

and the longitudinal control vector

$$\eta = [\delta_e \quad \delta_T]^T \tag{5.39}$$

these equations are equivalent to the system of first-order equations

$$\mathbf{I}_n \dot{\mathbf{x}} = \mathbf{A}_n \mathbf{x} + \mathbf{B}_n \eta \tag{5.40}$$

where $\dot{\mathbf{x}}$ represents the time derivative of the state vector $\mathbf{x}$, and the matrices appearing in this equation are

$$\mathbf{A}_n = \begin{pmatrix} X_u & X_w & 0 & -g_0\cos\Theta_0 \\ Z_u & Z_w & u_0 + Z_q & -g_0 \sin\Theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\mathbf{I}_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - Z_{\dot{w}} & 0 & 0 \\ 0 & -M_{\dot{w}} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{B}_n = \begin{pmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} & M_{\delta_T} \\ 0 & 0 \end{pmatrix} \tag{5.41}$$