UNIVERSITY OF PENNSYLVANIA

ESE 650: LEARNING IN ROBOTICS

SPRING 2023

MIDTERM EXAM

03/22 WED 10.15 AM – 11.45 AM ET

DURATION: 75 MINUTES

MAXIMUM POINTS: 75

Read the following instructions carefully before you begin.

- This is a closed book exam. You are allowed to use one A4/Letter paper cheat sheet (front and back).
- You may not use laptops/phones/tablets or the Internet to look up a textbook/notes/any other material during this exam. You may not discuss with your peers.
- You will have 75 minutes from the time you start the exam. After that you will get 15 minutes to upload your solutions to Gradescope.
- You should use a pen and paper (do not use a pencil because it might not be readable when scanned, there will be plenty of empty space for you to use for scratch work) to write your solutions and click pictures and upload your solutions as a PDF (Dropbox or Apple Notes apps on your phone are great for scanning all pages as a single PDF). Make sure to write in legible handwriting; we cannot grade what we cannot read.
- You should bring a phone/tablet to take pictures of your exam. You will then click pictures of each page and upload these pages to Gradescope from the phone, or after transferring this to your laptop/tablet if you wish. Having all the exams on Gradescope makes it easy for us to grade, attend to regrade requests etc.
- Each question mentions how short/long we would like your responses to be. DO NOT write excessively verbose answers.
- None of the questions require long derivations. If you find yourself going through lots of equations reconsider your approach or consider moving on to the next question.
- If you are stuck, explain your answers and what you are trying to do clearly. We will give partial credit if you are on the right track.
- **This exam has 6 problems.** The questions are NOT arranged in order of difficulty. Try to attempt every question.

Problem 1 (27 points). Answer True or False with a 1-2 sentence explanation.

(1) **(3 points)** Any quaternion can be expressed as a product of three other quaternions.

Answer:

(2) **(3 points)** We know that any rotation matrix R in three dimensions can be represented as a product of three successive rotations around the co-ordinate axes $R_x(\alpha), R_y(\beta)$ and $R_z(\gamma)$ where (α, β, γ) are the Euler angles. The order in which these matrices are multiplied changes the resultant rotation matrix R.

Answer:

(3) **(3 points)** The Kalman filter outputs the *most likely state* given all past observations for linear dynamics, linear observations and Gaussian noise in both the dynamics and the observations.

Answer:

(4) **(6 points)** Describe briefly how an extended Kalman Filter works. You can use the nonlinear dynamical system

$$x_{k+1} = f(x_k, u_k) + \epsilon_k$$
$$y_k = g(x_k) + \nu_k$$

where ϵ_k, ν_k refer to zero-mean Gaussian noise with covariance R, Q respectively.

Answer:

(5) **(3 points)** Explain in 1-2 sentences why we resample particles in a particle filter. What will happen if we skip the resampling step, which one will be inaccurate, out of the mean and the covariance of the estimate?

Answer:

(6) **(3 points)** Answer True or False with a 1-2 sentence explanation. Dijkstra's algorithm returns the correct shortest path between two two nodes for an acyclic graph even if the dist variable (that maintains the best cost to go from that node) for the nodes is assigned randomly.

Answer:

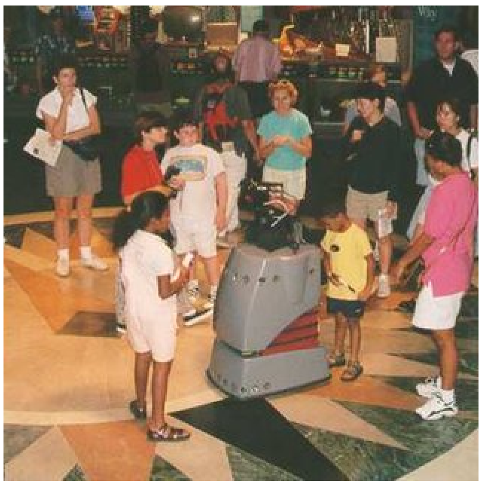
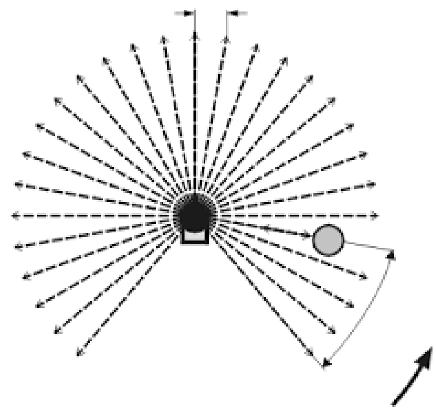
(7) **(3 points)** Answer True or False with a 1-2 sentence explanation. If policy iteration is initialized with the optimal policy, then we do not need to do the policy evaluation step before the algorithm terminates.

Answer:

(8) **(3 points)** For value iteration for a Markov Decision Process with n states, m control actions and a time-horizon of length T how much work does **each iteration** of the the algorithm do? Explain in 1-2 sentences.

Answer:

Problem 2 (8 points). Answer in 4-5 sentences.

Minerva was a robot deployed in the Smithsonian museum to educate and interact with museum-goers. This is a highly dynamic environment with many people constantly moving around the robot. Let us assume that Minerva has a 2D LiDAR (a planar LiDAR that shoots off rays in one single plane, see the right hand side picture) and would like to use an occupancy grid to keep track of people around it so that it does not collide with them when it moves. Explain how you will (a) mount the sensor, and (b) build an occupancy grid.

Remember that there are a lot of children in museums. So your answer should focus on making sure that the robot immediately calls a grid cell as occupied if it gets a LiDAR reading but at the same time clears out the grid cell immediately if the person moves to some other place; this helps ensure that there is open space in the occupancy grid and the robot can plan a trajectory to go to some new location without all cells around it being marked as obstacles.

Answer:

Problem 3 (8 points). How you would build a system with 10 drones that **together** count the number of apples in an orchard with many trees. Focus on what you think are the most important parts of this system for the purposes of localization and moving around; do not worry about details. An apple tree looks like this.

Answer:

Problem 4 (8 points). In the Unscented Kalman Filter (UKF) problem in Homework 2, we estimated the bias of the gyroscope manually using the training datasets. Gyroscope biases typically have a strong correlation with temperature variations. As the electronics heats up during the operation of the robot (say a drone), the bias of a gyroscope will change significantly, often in a way that is an unknown and a complex function of the temperature. Explain how you will tackle this variation in the bias if the goal is to estimate the orientation of the drone using observations from the gyroscope and an accelerometer.

Answer:

Problem 5 (14 points). Given a dynamical system

$$\begin{aligned} x_{k+1} &= x_k + \epsilon_k \\ y_k &= x_k + \nu_k. \end{aligned}$$

where $x_k, y_k \in \mathbb{R}$ are scalars, the initial distribution is $x_0 \sim N(\mu_0, \sigma_0^2)$ and noise $\epsilon_k \sim N(0,1)$, $\nu_k \sim N(0,2)$ are independent of each other and uncorrelated across time k. If we run a Kalman filter for this problem for a very long time k, the variance $\sigma_{k|k}^2$ reaches a steady-state, i.e., it does not change with respect to time.

(a) **(8 points)** Compute this steady-state variance.

(b) Answer with a 1-2 sentence explanation for each.

  (i) **(3 points)** If the initial variance of the state σ_0^2 is equal to the above steady-state variance, do you think that the variance of the state estimate of the KF will change over the next few time-steps before it settles again to the steady-state?

  (ii) **(3 points)** Answer True or False. The covariance of the Kalman filter for any system (not just the one provided above) reaches a steady-state if the time-horizon k is large enough.

You might find the following update equations for the KF useful.

$$\begin{aligned} \mu_{k+1|k} &= A\mu_{k|k} + Bu_k \\ \Sigma_{k+1|k} &= A\Sigma_{k|k}A^\top + R \\ \mu_{k+1|k+1} &= \mu_{k+1|k} + K_{k+1}(y_{k+1} - C\mu_{k+1|k}) \\ \Sigma_{k+1|k+1} &= \left(\Sigma_{k+1|k}^{-1} + C^\top Q^{-1} C\right)^{-1}. \end{aligned}$$

Answer:

Problem 6 (10 points). Consider a scalar dynamical system

$$x_{k+1} = x_k + u_k; \quad x_k, u_k \in \mathbb{R}$$

with initial state x_0. The cost function to be minimized is

$$J(x_0; u_0, u_1) = x_2^2 + u_0^2 + u_1^2.$$

Apply dynamic programming to find the optimal policy $u_k^*(x_k)$ for $k = 0, 1$ when the control input $u_k \in \{-1, 1\}$, i.e., it can take only two distinct values -1 and 1. You should write down an explicit expression for the optimal policy.

Hint: Notice that

$$-\text{sign}(x) = \underset{u \in \{-1,1\}}{\text{argmin}} \; xu$$

for any $x \in \mathbb{R}$.

Answer:

END OF EXAM