Department of Electrical and Electronic Engineering

EEEE2045

– Control Coursework

Student id： 20320941

Student name： Kexin Yu

1. Aim of the Lab

The aim of this coursework is to give students an overall idea of what they have learned in the control section over the past month and to enhance their understanding of the content of the lectures. Become familiar with the terminology used in the control topics, the use of MATLAB software, and accomplish the following aims through MATLAB:

- Analyse the first or second order system and learn the calculation of its relevant variables and the connection to the response function, the effect of the root on the response curve.
- Analyse closed loop systems and study the effect of various types of controllers on the system, and be able to complete a closed-loop system design problem and verify it through MATLAB

2. Approach

The Control System Toolbox is installed on MATLAB to facilitate the analysis of the transfer function by some of its functions. The commands used in MATLAB fall into two main categories, one is to convert the functions into some type of figure and the other is how to manipulate the figure so that it can be viewed more clearly and intuitively. The commands used in this coursework are illustrated below:

- Use the "tf" command to get Transfer functions into MATLAB in a simple way.
- Use the "step" command to see how the system reacts to the input of a unit step.
- Use the "rlocus" command to see if any of the values can reach the design point.
- Use the "feedback" command to see how the closed loop system responds and calculate the CLTF.
- Use the 'bode' command to see the bode plot of the function.
- Use the commands 'figure', 'hold' and curve colour to view the image to be analysed and save the various curves in a single plot for comparison.
- Use the cursor in "figure" to find the values that need to be observed on the figure.

3. Results and Discussion

This section contains the results of all the exercises, the correlation graphs generated by MATLAB, and a discussion of these results.

3.1 Exercise 1

The exercise begins with a simple plant transfer function, which is shown in Equation 1. For the system represented by this function, Figure 1.1 is a plot of its step response through MATLAB.

$$G_p(s) = \frac{10}{s+20} \qquad \textbf{Eqn.1}$$

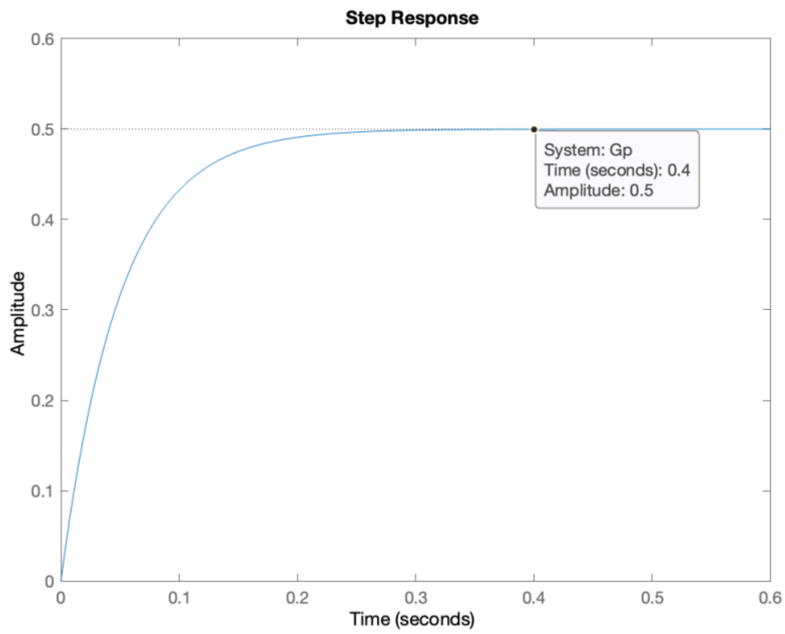

Figure 1.1: The step response of the simple transfer function

Equation 2 is used to determine the steady state gain of the transfer function and for Equation 1 the result of its gain is 0.5.

$$Gain = \lim_{s \to 0} \left(G_p(s) \right) \qquad \textbf{Eqn.2}$$

To explore the unit gain of the system, 'a' in Equation 3 is calculated by Equation 2.

$$G_p(s) = \frac{a}{s+20} \qquad \textbf{Eqn.3}$$

$$Gain = \lim_{s \to 0}\left(G_p(s)\right) = \lim_{s \to 0}\left(\frac{a}{s+20}\right) = \frac{a}{20} = 1$$

$$a = 20$$

The above calculation shows that the transfer function can have unity gain when the numerator is changed to 20 ('a' = 20). Figure 1.2 shows the simulation results after adjusting the value of 'a'.

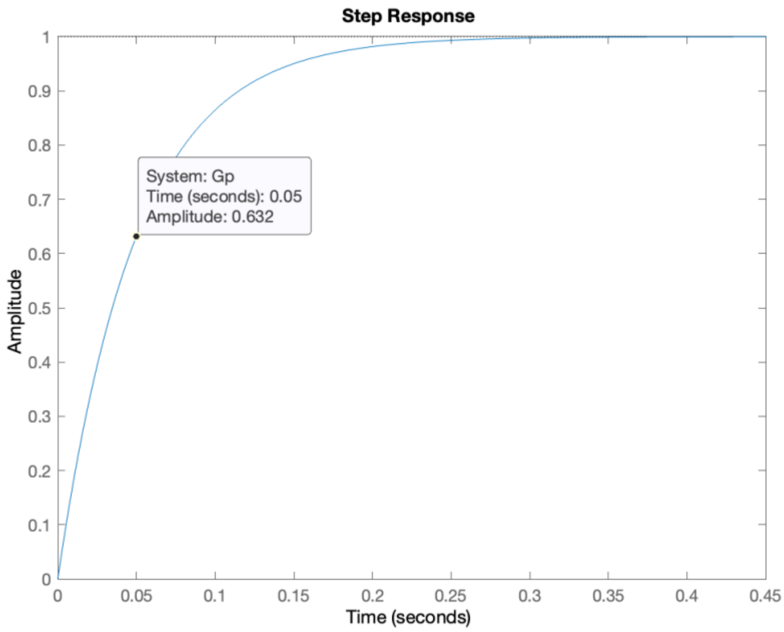

Figure 1.2: The step response of the modified transfer function with unit gain

As can be observed from Figure 1.2, the time constant for this first order system is 0.05.

From the above results, it can be proven that the words of the title, when the input to this system is changed (in this case 'a' goes from 10 to 20), its final value is scaled by the magnitude of the input. As shown in Figures 1.1 and 1.2, the final values are 0.5 and 1, respectively, just as their inputs differ by a factor of 2.

3.2 Exercise 2

i)

For Equation 4, Figure 2.1 shows the simulation of the step response of this transfer function when changing the value of ‘a’.

$$G_p(s) = \frac{a}{s+a} \qquad \textbf{Eqn.4}$$

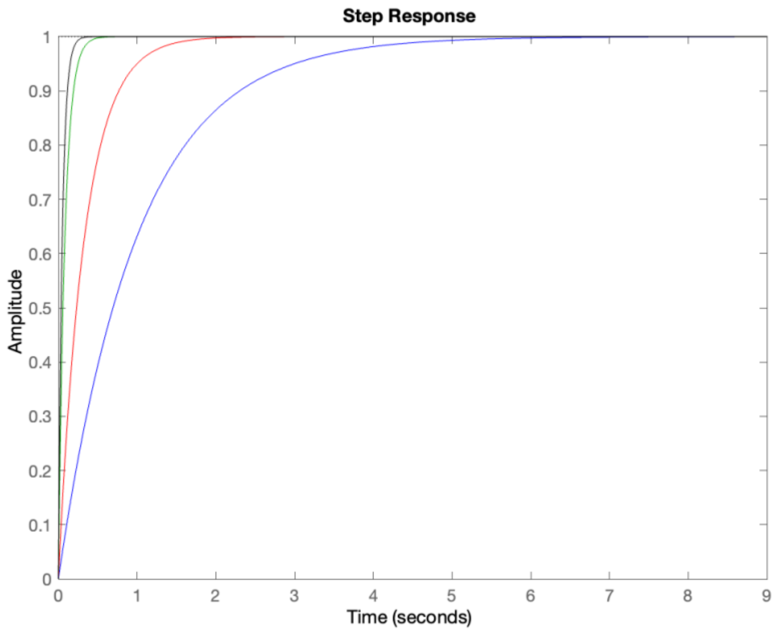

Figure 2.1: The step responses of the transfer function with different ‘a’

Table 1 records the time constants for each function when the value of ‘a’ varies and their corresponding colours in the Figure 2.1.

Table 1: Time constants of the transfer function for different values of ‘a’

Value of ‘a’	Curve Color	Time Constant τ (s)
1	Blue	1.0000
3	Red	0.3333
12	Green	0.0834
20	Black	0.0500

From the results in the table, as ‘a’ increases, the time constant is decreasing. It can be concluded that the time constant of the step response for a first order system has an inverse mathematical relationship with the 'a' in the transfer function, and the relationship between its values can be listed as (‘τ’ is the time constant):

$$\tau = \frac{1}{a} \qquad \textbf{Eqn.5}$$

ii)

For the transfer function shown in Equation 6, the simulation results are shown in Figure 2.2.

$$G_p(s) = \frac{20}{s-20} \qquad \textbf{Eqn.6}$$

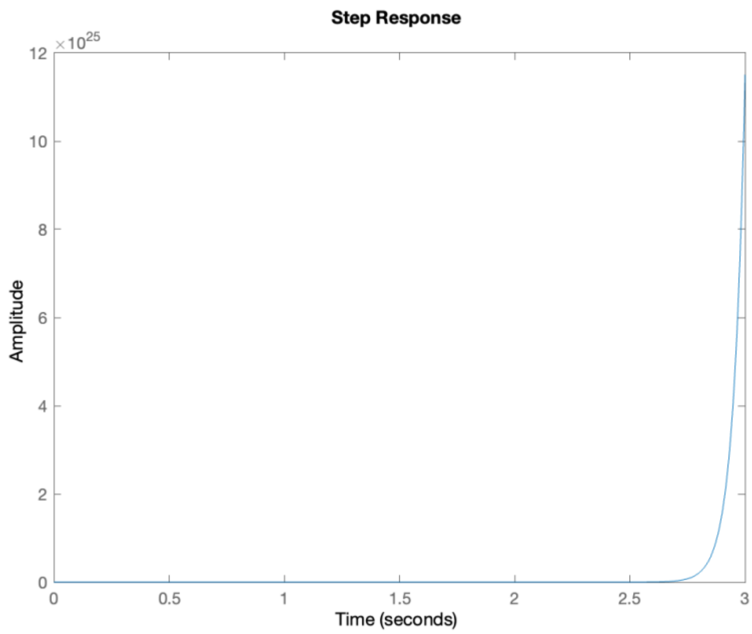

Figure 2.2: The step response of the extra transfer function

Comparing Figure 2.1 with Figure 2.2 it can be observed that when the ‘a’ in the denominator of the transfer function changes from a positive value (20) to a negative value (-20), the value of the step response curve does not have an upward trend, and then levels off to a constant value as in the case shown in Figure 2.1. It rises sharply to infinity after a relatively gentle rise. This indicates that the system is non-stationary when the a in the denominator of the expression becomes negative, which also means that the plant has a "right-hand" (positive) pole.

3.3 Exercise 3

Equation 7 and Equation 8 are used to investigate the properties of dominant roots. Figure 3.1 shows the unit step response of these functions for different values of 'a'

$$G_p(s) = \frac{10}{s+10} \quad \textbf{Eqn.7}$$

$$G_p(s) = \frac{10a}{(s+10)(s+a)} \quad \textbf{Eqn.8}$$

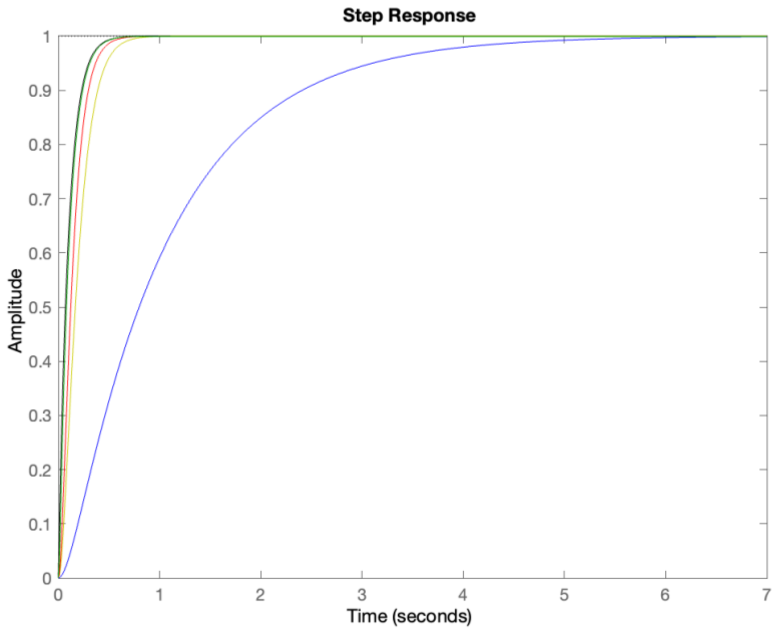

Figure 3.1: The step responses of the transfer function to investigate dominant roots

Table 2 records the time constants of this equation for different values of 'a' and their corresponding curve colours in Figure 3.1.

Table 2: The time constant of the transfer function with different 'a' and original one

Value of 'a'	Curve Color	Time Constant τ (s)
Original	Black	0.100
1	Blue	1.110
10	Yellow	0.215
20	Red	0.159
100	Green	0.111

The response for Equation 7 is referred to as the original response curve and is also a first order response system. It can be summarised for the second order response system corresponding to Equation 8, when 'a' is much less than 10, the response curve is similar to the original response curve and its time constant is also similar. When 'a' is much larger than 10, the response curve is more different from the original response curve and its time constant is relatively large compared to the original one.

This occurs because for the transfer function, Equation 8, there are two sets of poles, ‘s = -10’ and ‘s = -a’. When one set has approximately five times or more the real component than the other, the pole with the smaller real component is known as the dominant root.

For Equation 7, which has only one pole for ‘s = -10’. For Equation 8, when ‘a’ is much greater than 10, pole ‘s = -10’ is the dominant root, this means that the response curve for the second order at this point has similar characteristics to the original response curve, they have similar curve shapes and time constants. When ‘a’ is much less than 10, pole ‘s = -a’ is said to be the dominant root and the response curve differs significantly from the original curve, as does its time constant.

3.4 Exercise 4a

Figure 4.1 illustrates the step response when the value of ‘a’ in Equation 9 varies.

$$G_p(s) = \frac{10}{s^2+as+10} \qquad \textbf{Eqn.9}$$

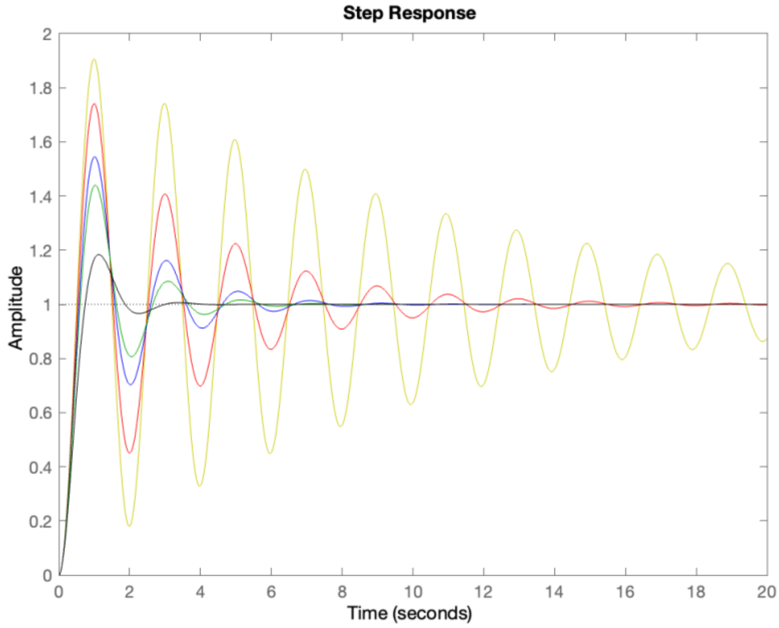

Figure 4.1: The step responses of the transfer function to investigate the damping factor

The damping factors for these second order responses can be calculated using Equation 10 and 11. When a = 0.2, the calculation procedure is as follows:

$$Gp = \frac{{\omega_0}^2}{s^2+2\omega_0\xi s+{\omega_0}^2} \qquad \textbf{Eqn.10}$$

$$Gp = \frac{{\omega_0}^2}{s^2+2\omega_0\xi s+{\omega_0}^2} = \frac{10}{s^2+as+10}$$

$$\omega_0 = \sqrt{10}$$

$$\xi = \frac{a}{2\omega_0} \qquad \textbf{Eqn.11}$$

$$\xi = \frac{a}{2\omega_0} = \frac{0.2}{2\sqrt{10}} = 0.032$$

When ‘a’ equals other values, they are also calculated by the above procedure. The damping factors obtained from the calculations, the max overshoot, and the peak time obtained from Figure 4.1 are listed in Table 3.

Table 3: Values used for discussion in Exercise 4a

Value of 'a'	Curve Color	Max overshoot M_p	Peak Time t_p (s)	Damping Factor ξ
0.2	Yellow	0.91	1.000	0.032
0.6	Red	0.74	1.000	0.095
1.2	Blue	0.54	1.000	0.192
1.6	Green	0.44	1.030	0.253
3.0	Black	0.18	1.130	0.474

Based on the results in Table 3, it can be analysed that the damping factor has a significant effect on the size of the max overshoot in the step response, which decreases as the damping factor increases. For all cases in the question, the damping coefficient is less than 0.707, so all are "Underdamped". In general, an increase in the damping factor stabilises the system.

Exercise 4b

Figure 4.2 shows the step response for Equation 12 with different values of 'a'

$$G_p(s) = \frac{100a}{(s^2+12s+100)(s+a)} \qquad \textbf{Eqn.12}$$

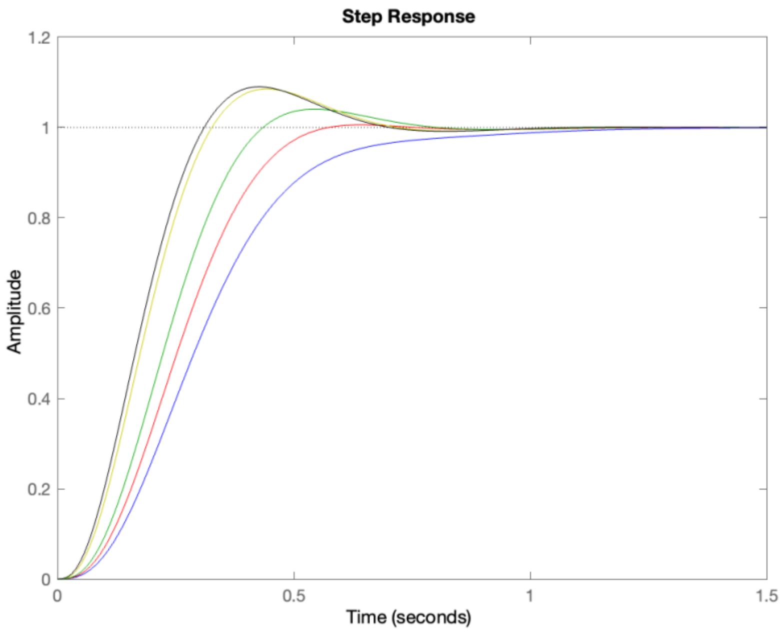

Figure 4.2: The step responses of the higher order transfer functions

Table 4 lists the poles of the functions and the colours of the curves they correspond to when ‘a’ equals different values.

Table 4: The corresponding poles of different values of ‘a’

Value of ‘a’	Curve Color	Poles
5	Blue	(-6, j8), (-6, -j8), (-5, 0)
7	Red	(-6, j8), (-6, -j8), (-7, 0)
10	Green	(-6, j8), (-6, -j8), (-10, 0)
25	Yellow	(-6, j8), (-6, -j8), (-25, 0)
35	Black	(-6, j8), (-6, -j8), (-35, 0)

Figure 4.3 shows these poles plotted in the complex plane for each value of ‘a’. The different poles are represented in the diagram by different colour symbols.

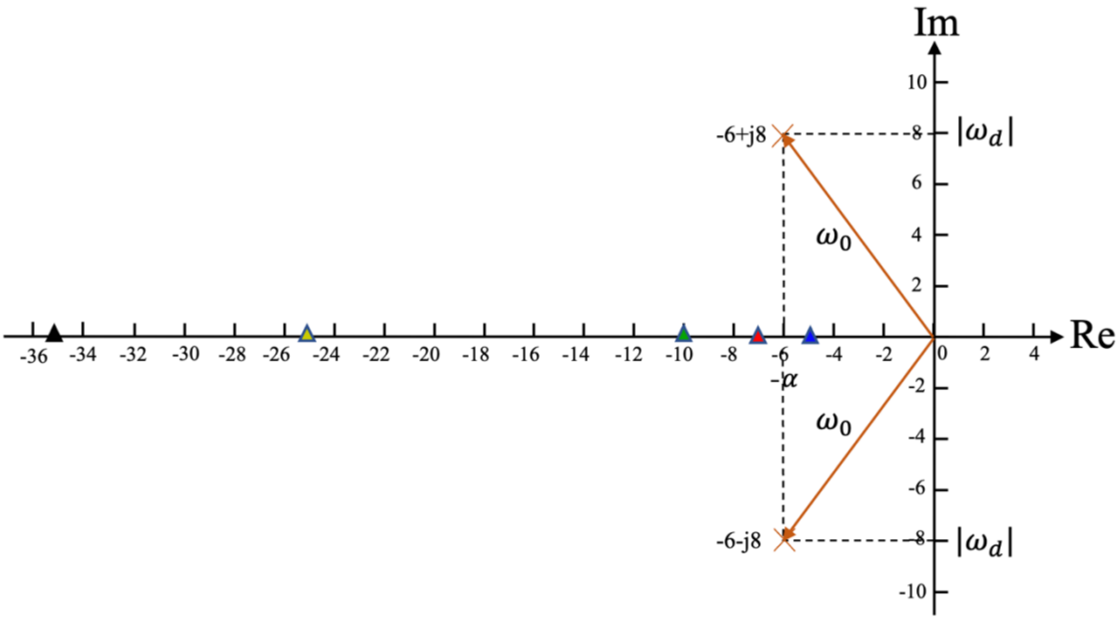

Figure 4.3: The poles in the complex plane

It can be concluded from Figure 4.3 that the dominant root remains influential in higher order systems. When two roots are related by a factor of five or more, the root with the lowest real part are said to be dominant. The root with the highest real part can be negligible.

As ‘a’ increases, especially after ‘a’ is greater than 6, the real part of the complex pole ‘-6±8j’ becomes increasingly dominant and the shape of the response curve is much closer to that of Equation 13. Equation 13 only contains the complex pole ‘-6±8j’ and Figure 4.4 illustrates the step response of this function.

$$G_p(s) = \frac{100}{(s^2+12s+100)} \quad \textbf{Eqn.13}$$

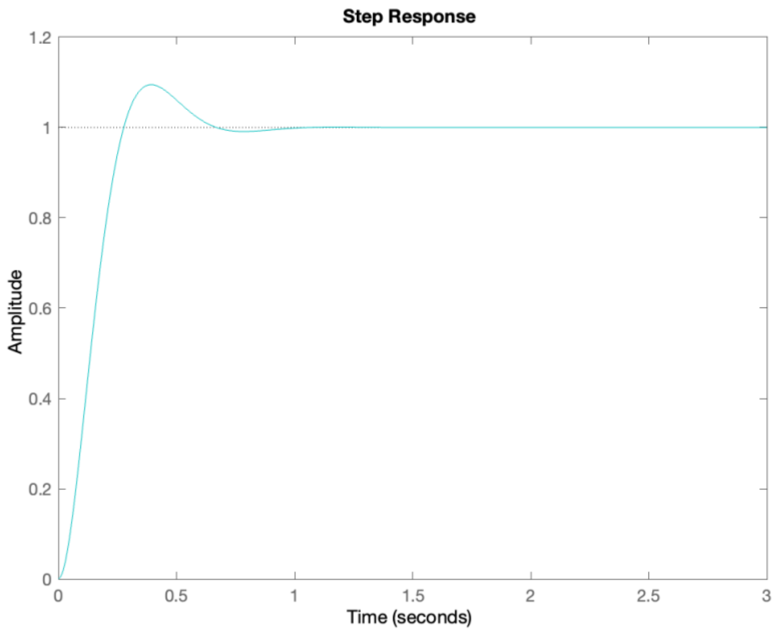

Figure 4.4: The step responses of the second-order transfer functions without ‘a’

3.5 Exercise 5

Figure 5.1 shows that only the proportional controller for $G_p(s)$. Two equations are shown below:

$$G_p(s) = \frac{10}{(s+10)} \qquad \textbf{Eqn.14}$$

$$G_C(s) = K \qquad \textbf{Eqn.15}$$

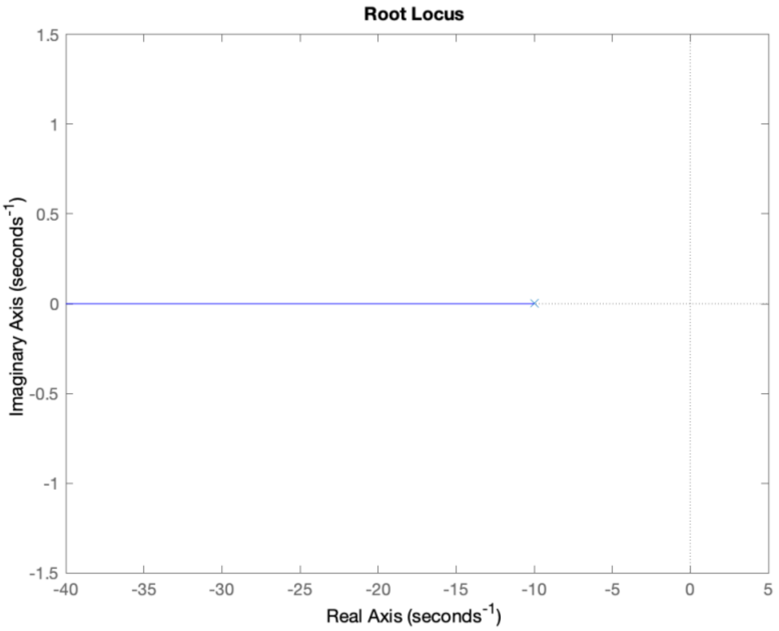

Figure 5.1: The root locus figure with P controller

As can be observed from Figure 5.1, the imaginary part of the root is zero for all values of 'K'. The root locus does not pass through the design point. This is because Equation 14 is a first order system and does not have an imaginary part.

Figure 5.2 shows the proportional and integral controllers for $G_p(s)$. The transfer function is shown in Equation 16.

$$G_C(s) = \frac{K(s+a)}{s} \qquad \textbf{Eqn.16}$$

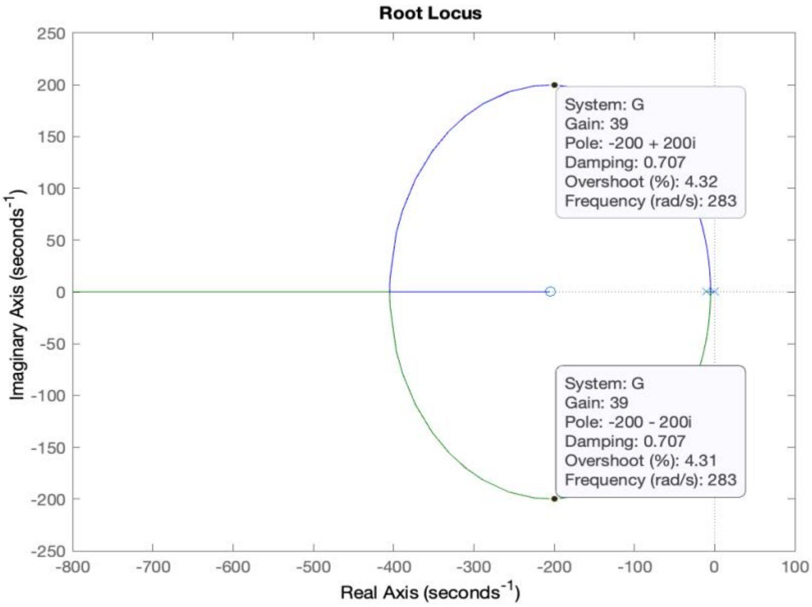

Figure 5.2: The root locus figure with PI controller

As can be seen from Figure 5.2, the gain value at the design point is 39 when a = 205, which indicates that 'K' is equal to 39. Now the forward transfer function is below:

$$G(s) = \frac{390(s+205)}{s(s+10)} \qquad \textbf{Eqn.17}$$

Equation 17 is checked by CLTF command, and the result is shown in Figure 5.3. As shown in Figure 5.3, the response amplitude at 20ms is 0.999. The values within 2% of the target range are 0.98-1.02, so this response is in line with expectations.

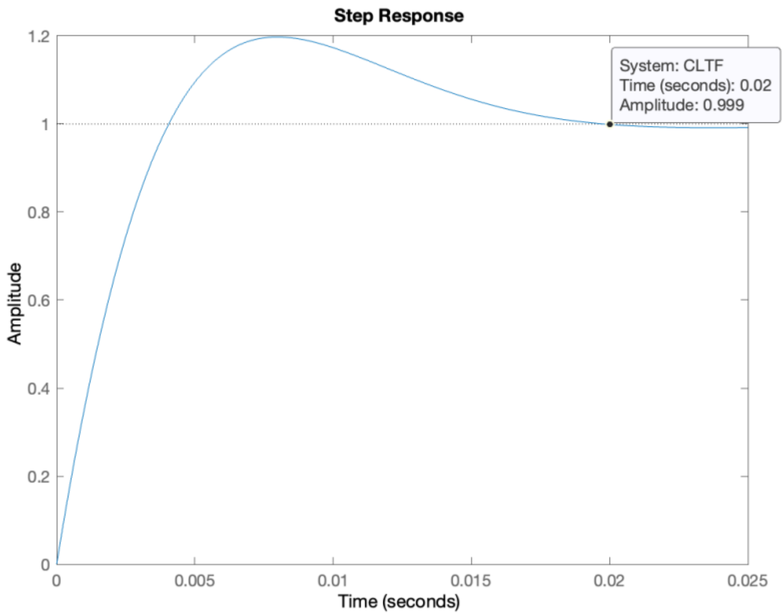

Figure 5.3: The step response of CLTF

4. Design Questions

Question 1:

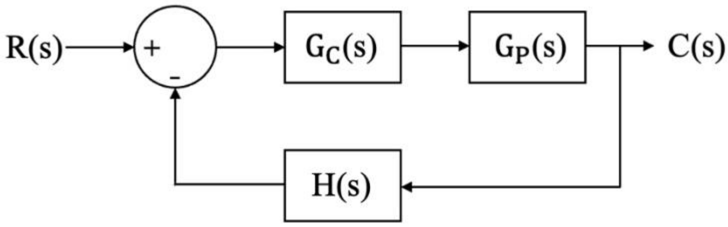

Figure 6.1: The simple block diagram for Design Question

Figure 6.1 shows a simple block diagram for this design, $G_C(s)$ for the transfer function of the PI controller, $G_P(s)$ for the plant transfer function, and H(s) for the unit feedback transfer function, and their expressions are shown below:

$$G_c(s) = K_p + \frac{K_i}{s} \quad \textbf{Eqn.18}$$

$$G_c(s) = K + \frac{Ka}{s} = \frac{K(s+a)}{s}$$

$$G_P(s) = \frac{6}{15s+1} \quad \textbf{Eqn.19}$$

$$H(s) = 1 \quad \textbf{Eqn.20}$$

Thus, the CLTF of this system can be expressed as:

$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+H(s)G(s)} = \frac{G_c(s)G_P(s)}{1+H(s)G_c(s)G_P(s)} \quad \textbf{Eqn.21}$$

To obtain the poles of the system, the denominator of the function is set to 0.

$$1 + H(s)G_c(s)G_P(s) = 0 \quad \textbf{Eqn.22}$$

$$\frac{6K(s+a)+s(15s+1)}{s(15s+1)}=0$$

To obtain the values of 'K' and 'a' in this equation, the above equation is rewritten in standard form with natural frequencies and damping factors, since the values of natural frequency ($\omega_0 = 15\ \text{rads}^{-1}$) and damping factor ($\xi = 0.9$) of the system are already known.

$$s^2+2\omega_0\xi s+{\omega_0}^2 = s^2+\frac{(6K+1)}{15}s+\frac{6Ka}{15} \qquad \textbf{Eqn.23}$$

$$\frac{(6K+1)}{15}=27 \qquad \frac{6Ka}{15}=225$$

$$K=67.333 \qquad a=8.354$$

Therefore, the transfer function of this system can be written as:

$$G(s)=G_c(s)G_P(s)=\frac{403.98(s+8.36)}{s(15s+1)} \qquad \textbf{Eqn.24}$$

Based on the above equation, the bode plot for this closed-loop system is shown in Figure 6.2. From this figure, it can be found that the bandwidth of this system is 34.8 rads^{-1}.

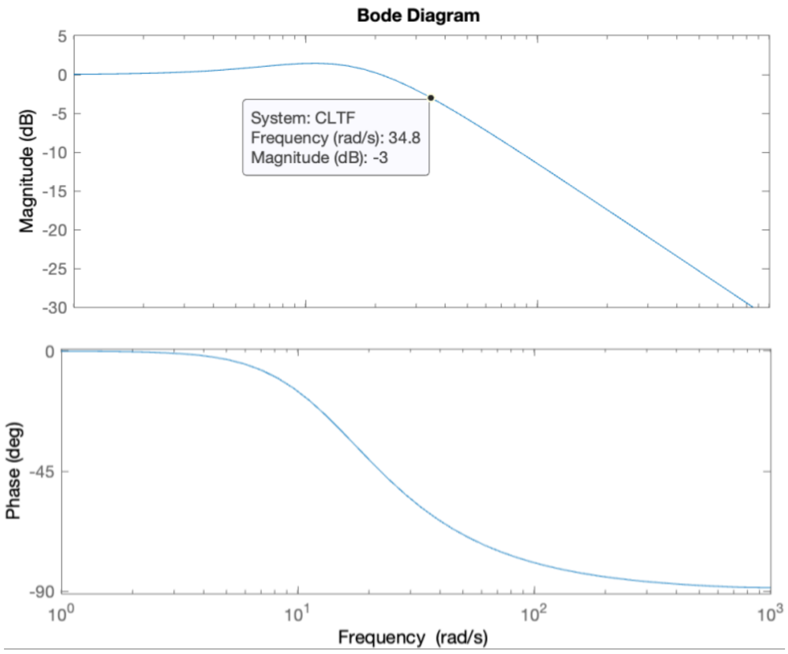

Figure 6.2: The bode plot of the designed closed loop system

Question 2:

The simple block diagram for this problem is also like Figure 6.1. The control transfer function and the feedback transfer function for this problem are the same as in Question 1, and the plant transfer function is represented as follows:

$$G_P(s) = \frac{15}{(s+150)(s+8)} \qquad \textbf{Eqn.25}$$

The plant transfer function can be simplified according to the properties of the dominant root mentioned earlier. In this equation, the real component of the pole 's = -150' is five times or more than ($\frac{150}{8}$ = 18.75 > 5) the pole 's = -8', so we can say that the pole 's = -8' with the lowest real component is dominant, the transfer function can be simplified as:

$$G_P(s) = \frac{\frac{15}{150}}{(s+8)} = \frac{1}{10(s+8)} \qquad \textbf{Eqn.26}$$

Therefore, the forward transfer function for this system is:

$$G(s) = G_c(s)G_p(s) = \frac{K(s+a)}{10(s+8)s} \qquad \textbf{Eqn.27}$$

The CLTF of this system is just like Equation 20 and its dominator can be expressed as:

$$1 + H(s)G_c(s)G_P(s) = 1 + G(s) = 1 + \frac{K(s+a)}{10(s+8)s} = 0 \qquad \textbf{Eqn.28}$$

$$\frac{s^2 + (0.1K + 8)s + 0.1Ka}{(s+8)s} = 0$$

Based on the title, the time constant T is 750 ms, and the damping factor ξ is 0.6, the value of ω_0 can be calculated as:

$$4T = \frac{4}{\xi\omega_0} = 750\ ms \qquad \textbf{Eqn.29}$$

$$\omega_0 = \frac{4}{0.75 \times 0.6} = 8.89\ rads^{-1}$$

Since ω_0 and ξ are known, the denominator of the CLTF can be written in the standard form containing these two values to find 'K' and 'a'.

$$s^2 + 2\omega_0\xi s + {\omega_0}^2 = s^2 + (0.1K + 8)s + 0.1Ka = 0 \qquad \textbf{Eqn.30}$$

$$s^2 + 10.668s + 79.032 = s^2 + (0.1K + 8)s + 0.1Ka = 0$$

$$\begin{cases} 10.668 = 0.1K + 8 \\ 79.032 = 0.1Ka \end{cases}$$

$$\begin{cases} K = 26.67 \\ a = 29.63 \end{cases}$$

The forward transfer function and CLTF can be expressed as:

$$G(s) = \frac{26.67(s+29.63)}{10(s+8)s} \qquad \textbf{Eqn.31}$$

$$CLTF = \frac{G(s)}{1+G(s)} = \frac{26.67s+790.2}{10s^2+106.7s+790.2} = \frac{2.667s+79.02}{s^2+10.67s+79.02} \qquad \textbf{Eqn.32}$$

Figure 6.3 shows the simulation results for Equation 32 CLTF. From this figure, it can be observed that the response amplitude at 750ms is 0.995. The target range of 2% is 0.98-1.02, so this value achieves the desired design.

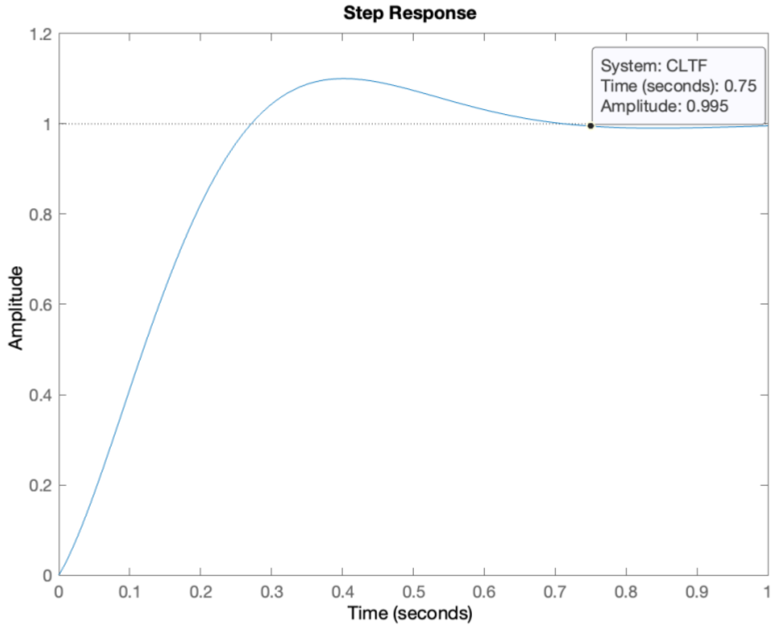

Figure 6.3: The step response of CLTF for Q2

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5. Conclusion

To conclude, this report has completed the simulation of seven exercises using various commands in MATLAB and has analysed the individual figures. For Exercise 1, it is analysed how varying the transfer equation gives different steady-state gains. For Exercise 2, the effect of a change in the root value on the time constant of the response is analysed, which has an inverse proportional relationship. The stability of the system is also affected by the positive or negative value of the roots, with a negative root value making the system stable and vice versa. For Exercise 3, the effect of the dominant root on the transfer equation is analysed, when the real part of one set of roots is five times or more than another set of roots then it is negligible. For exercise 4, the damping factor is calculated from the overshoot observed in the figure and it is found that the overshoot decreased as the damping coefficient increased. Dominant roots have the same effect on higher order systems. For Exercise 5, when only the proportional controller is added, there is no root locus through the design point because the response is first order and has no imaginary part. When both proportional and integral controllers are added, the values of 'K' and 'a' for the control transfer equation can be found mathematically and verified by the feedback command of MATLAB. With the correct control transfer function, the response is verified to be correct within 2% of the settling point in 20 ms. The two design problems for this coursework are a combination of the methods devised in the previous exercises. Both problems use the same method from Exercise 5 to find the control transfer function, and finally use different commands to verify that it is correct. And the second problem also needs to use the characteristics of dominant roots to simplify the higher order system.