EEEE2055:
Modelling Methods and Tools
Coursework1
(Fourier Transforms) 22-23

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1.1

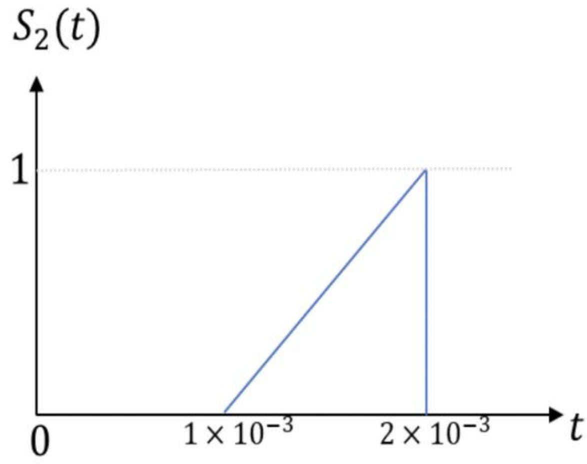

Figure 1.1.1: Time domain plot of S2

According to Figure 1.1.1, the expression of $S_2(t)$ can be identified as follow:

$$S_2(t) = \begin{cases} 0, & 0 < t \le 1\times10^{-3}s \\ 1000t - 1, & 1\times10^{-3}s < t \le 2\times10^{-3}s \\ 0, & t > 2\times10^{-3}s \end{cases} \qquad \textbf{Eqn.1}$$

The Fourier transform equation for $S_2(t)$ can be expressed as follows:

$$\widetilde{S_2}(\omega) = \int_{-\infty}^{\infty} S_2(t)e^{-j\omega t}\mathrm{dt} \qquad \textbf{Eqn.2}$$

According to Equation 1 and Equation 2, the Fourier transform of $S_2(t)$ can be derived below:

$$\widetilde{S_2}(\omega) = \int_{-\infty}^{0.001} 0 * e^{-j\omega t}\,\mathrm{dt} + \int_{0.001}^{0.002} (1000t - 1)e^{-j\omega t}\mathrm{dt} + \int_{0.002}^{\infty} 0 * e^{-j\omega t}\,\mathrm{dt}$$

$$\widetilde{S_2}(\omega) = \int_{0.001}^{0.002} (1000t - 1)e^{-j\omega t}\mathrm{dt}$$

$$\widetilde{S_2}(\omega) = \left[\frac{(1000t-1)e^{-j\omega t}}{-j\omega}\right]_{0.001}^{0.002} - \int_{0.001}^{0.002} \frac{1000e^{-j\omega t}}{-j\omega}\mathrm{dt}$$

$$\widetilde{S_2}(\omega) = \left[\frac{(1000t-1)e^{-j\omega t}}{-j\omega}\right]_{0.001}^{0.002} + \left[\frac{1000e^{-j\omega t}}{\omega^2}\right]_{0.001}^{0.002}$$

$$\widetilde{S_2}(\omega) = \frac{e^{-j0.002\omega}}{-j\omega} + \frac{1000e^{-j0.002\omega}}{\omega^2} - \frac{1000e^{-j0.001\omega}}{\omega^2}$$

Applying Euler's Theorem, the Fourier transform of $S_2(t)$ can be expressed as:

$$\widetilde{S_2}(\omega) = \frac{\cos(0.002\omega) - \mathrm{j}\sin(0.002\omega)}{-j\omega} + \frac{1000(\cos(0.002\omega) - \mathrm{j}\sin(0.002\omega))}{\omega^2} - \frac{1000(\cos(0.001\omega) - \mathrm{j}\sin(0.001\omega))}{\omega^2}$$

$$\widetilde{S_2}(\omega) = \frac{\sin(0.002\omega)}{\omega} + \frac{\mathrm{j}\cos(0.002\omega)}{\omega} + \frac{1000(\cos(0.002\omega) - \cos(0.001\omega))}{\omega^2} + \frac{\mathrm{j}1000(\sin(0.001\omega) - \sin(0.002\omega))}{\omega^2}$$

$$\widetilde{S_2}(\omega) = \frac{\omega\sin(0.002\omega) + 1000(\cos(0.002\omega) - \cos(0.001\omega))}{\omega^2} + j\frac{\omega\cos(0.002\omega) + 1000(\sin(0.001\omega) - \sin(0.002\omega))}{\omega^2}$$

1.2

Figure 1.2.1 illustrates the circuit schematic simulated to explore the FFT.

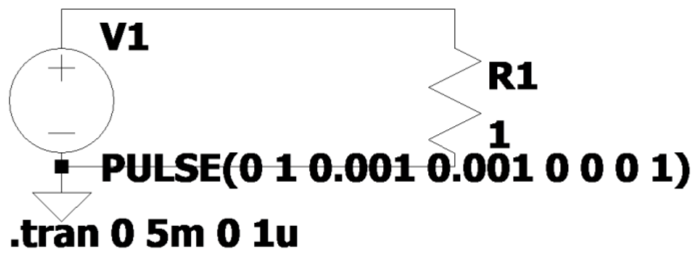

Figure 1.2.1: LTSpice simulation

Figure 1.2.2 explores the effect on LTSpice results when simulating different time-step. (Stop time = 5ms, Sample points = 262144) Figure 1.2.3 explores the effect on LTSpice results when simulating different stop time. (Time-step = 1μs, Sample points = 262144) Figure 1.2.4 explores the effect on LTSpice results when simulating different FFT sample settings. (Stop time = 5ms, Time-step = 1μs) Other variables are held constant when exploring the corresponding variable.

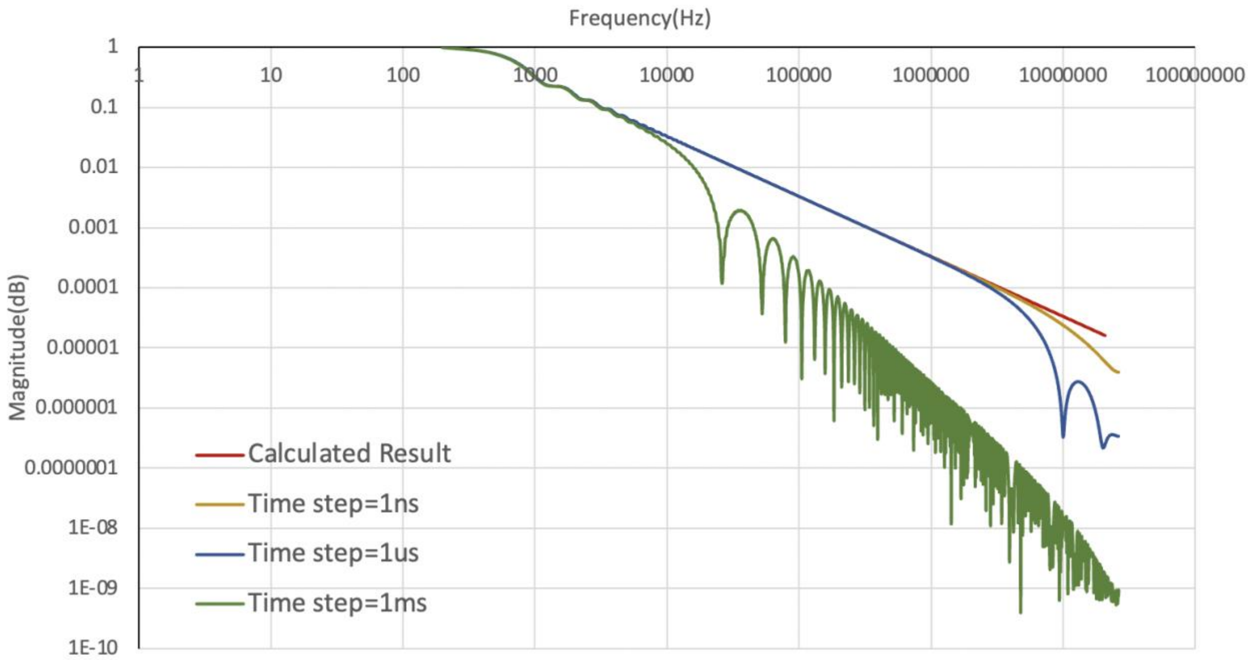

Figure 1.2.2: LTSpice results with different time-step

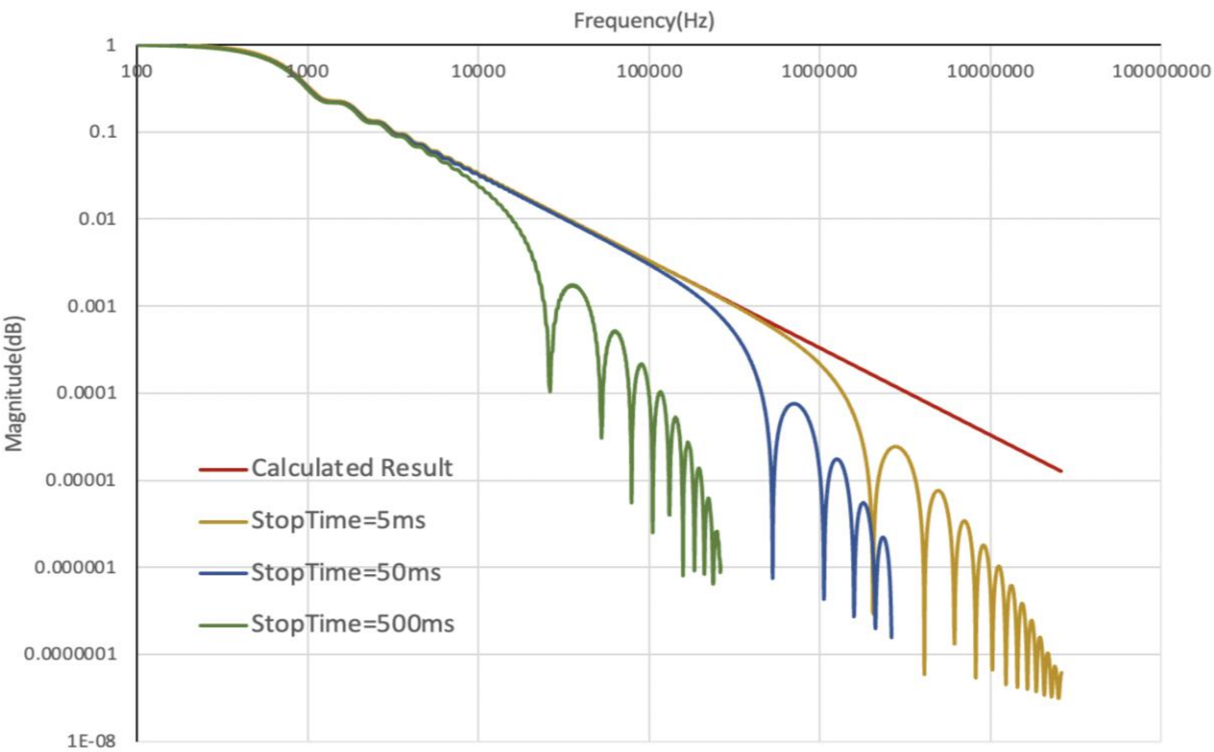

Figure 1.2.3: LTSpice results with different stop time

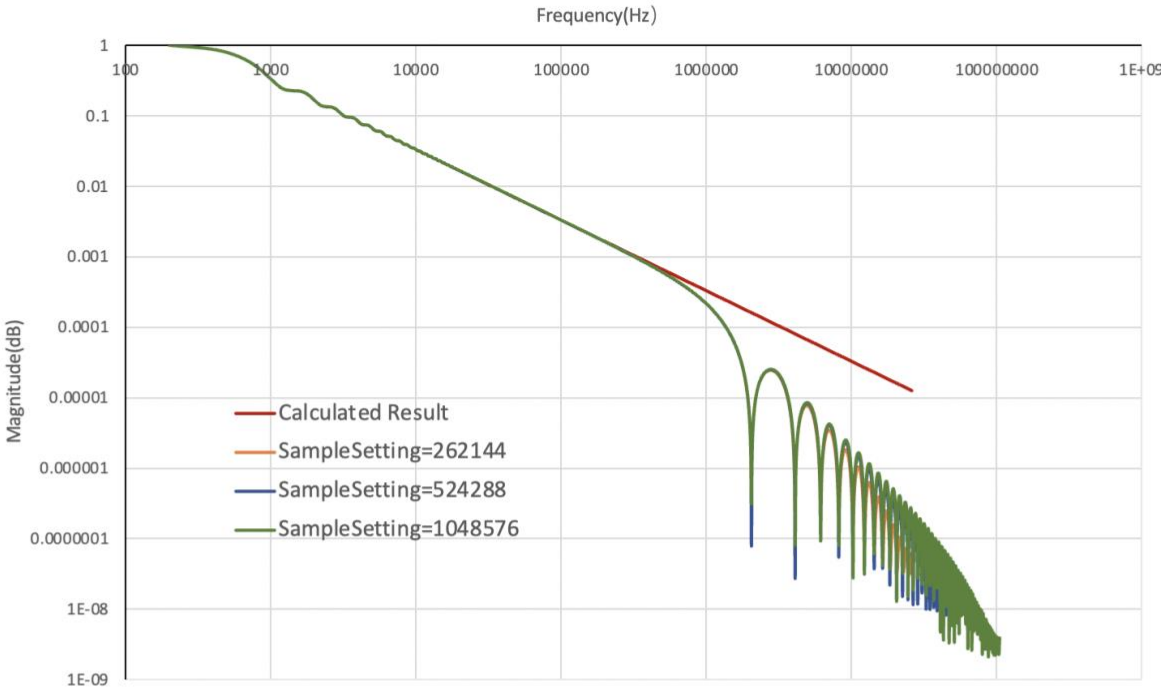

Figure 1.2.4: LTSpice results with different FFT sample settings

Figure 1.2.2 shows that the smaller the time step, the closer the simulation results are to those of the theoretical Fourier Transform. This is because the smaller the time step means that the more points appear in the simulation, the more accurate the FFT results will be and the closer they will be to their theoretical values.

As can be seen in Figure 1.2.3 the simulation results are closer to the theoretical values when the stop time is larger. However, the FFT at the same frequency point is the same value for different sampling times. The reason for this is that the resolution in the frequency domain changes for the different number of sampling points. So, the longer the sampling time and the more points sampled, the higher the accuracy.

Figure 1.2.4 shows that changing the sample points has no effect on the simulation results, but only on the visible frequency range.

In summary, the simulation results are almost identical to the theoretical values at low frequencies, and at high frequencies, the accuracy of the FFT results is affected by the different parameters set in the LTSpice simulation.

1.3

Figure 1.3.1 shows the effect of different falling edge times on the Fourier results in the LTSpice simulation. (Stop time = 5ms, Time-step = 1μs, Sample points = 262144)

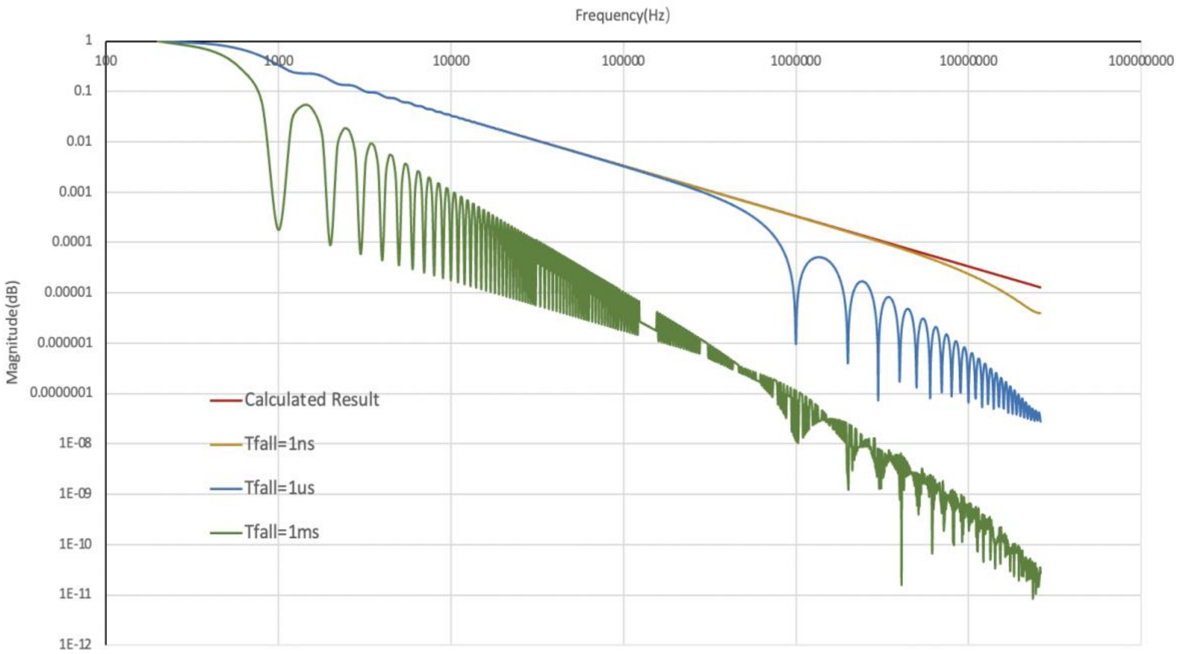

Figure 1.3.1: LTSpice results with different falling edge time

From this figure, the smaller the falling edge time the closer the simulation results are to the theoretical value. This is because in the theoretical case, the falling edge time should be zero, but in the LTSpice simulation it is not exactly zero, so there are some small differences with the theoretical results. It can be deduced that the simulation results should be closer to the theoretical value when the falling edge time is equal to 1 ps.

2.1

According to Figure 2, $V_i(t)$ is equal to the sum of the voltage values of the resistor and the inductor. $V_0(t)$ is equal to the voltage across the resistor. Therefore, $V_i(t)$ can be expressed as follows:

$$V_i(t) = L\frac{di(t)}{dt} + V_0(t) \qquad \textbf{Eqn.3}$$

According to the above equation, $i(t)$ can be expressed as:

$$i(t) = \int \frac{V_i(t)-V_0(t)}{L}dt \qquad \textbf{Eqn.4}$$

$V_0(t)$ can be expressed as:

$$V_0(t) = i(t)R \qquad \textbf{Eqn.5}$$

Substituting Equation 4 into Equation 5, $V_0(t)$ can be expressed as:

$$V_0(t) = R\int \frac{V_i(t)-V_0(t)}{L}dt \qquad \textbf{Eqn.6}$$

$$V_0(t) = \frac{R}{L}\int V_i(t) - V_0(t)\,dt$$

Applying the Fourier transform to equation 6, the corresponding $\widetilde{V_0}(\omega)$ and $\widetilde{V_\imath}(\omega)$ can be expressed as follows:

$$\widetilde{V_0}(\omega) = \frac{R}{j\omega L}(\widetilde{V_\iota}(\omega) - \widetilde{V_0}(\omega)) \qquad \textbf{Eqn.7}$$

$$\left(1 + \frac{R}{j\omega L}\right)\widetilde{V_0}(\omega)c\frac{R}{j\omega L}\widetilde{V_\iota}(\omega)$$

Equation 8 is the expression for the frequency response $\widetilde{\mathrm{F}}(\omega)$ of this circuit, and by substituting Equation 7, its simplified equation is obtained as:

$$\tilde{F}(\omega) = \frac{\widetilde{V_0}(\omega)}{\widetilde{V_\iota}(\omega)} \qquad \textbf{Eqn.8}$$

$$\tilde{F}(\omega) = \frac{\widetilde{V_0}(\omega)}{\widetilde{V_\iota}(\omega)} = \frac{\frac{R}{j\omega L}}{1+\frac{R}{j\omega L}} = \frac{R}{j\omega L+R} \qquad \textbf{Eqn.9}$$

Substituting the inductance (L = 100 mH) and resistance (R = 1 KΩ) values, this frequency response expression is:

$$\tilde{F}(\omega) = \frac{R}{j\omega L+R} = \frac{1000}{j\omega 0.1+1000} = \frac{10000}{j\omega+10000} \qquad \textbf{Eqn.10}$$

2.2

Since in this circuit, the Fourier transform of its impulse response F(t) is the frequency response $\tilde{\mathrm{F}}(\omega)$. The impulse response F(t) can be derived from the inverse Fourier transform of $\tilde{\mathrm{F}}(\omega)$.

$$\mathrm{F(t)} = \mathcal{F}^{-1}\{\tilde{\mathrm{F}}(\omega)\} = \mathcal{F}^{-1}\left\{\frac{10000}{j\omega+10000}\right\} \qquad \textbf{Eqn.11}$$

Equation 12 is in the Fourier transform Table. By substituting this equation into Equation 11, the impulse response expression for this circuit can be expressed as in Equation 13:

$$\mathrm{S(t)} = \mathcal{F}^{-1}\{\tilde{\mathrm{S}}(\omega)\} = \mathcal{F}^{-1}\left\{\frac{1}{j\omega+a}\right\} = e^{-at} \qquad \textbf{Eqn.12}$$

$$\mathrm{F(t)} = \mathcal{F}^{-1}\left\{\frac{10000}{j\omega+10000}\right\} = 10000\mathcal{F}^{-1}\left\{\frac{1}{j\omega+10000}\right\} = 10000e^{-10000t} \qquad \textbf{Eqn.13}$$

2.3

To simulate the Dirac delta function, the integration size of the power input in the circuit should be 1. The following five figures perform different delta approximations by varying the size of V_{on} and T_{on}. A summary of whether the results are correct is recorded in Table 1.

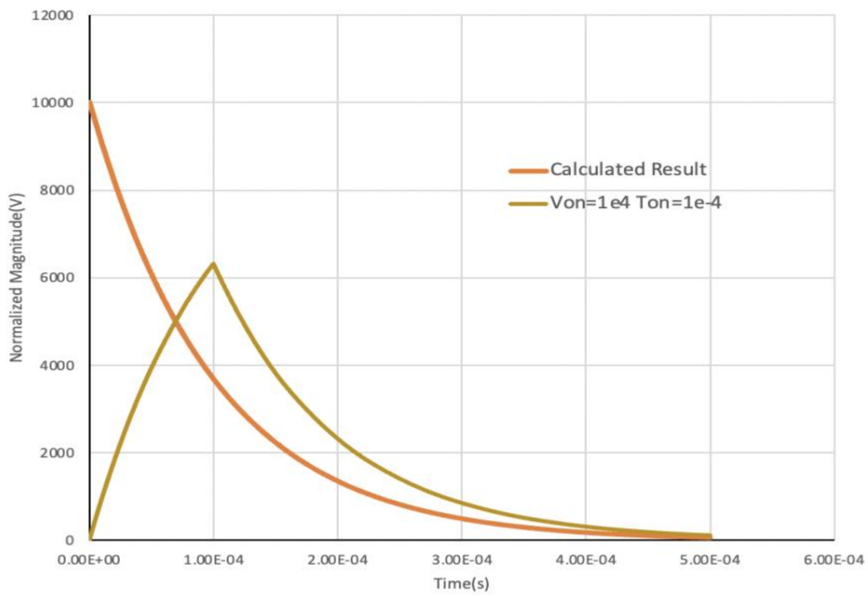

Figure 2.3.1: Time domain simulation with different delta approximation ($V_{on} = 10^4$, $T_{on} = 10^{-4}$)

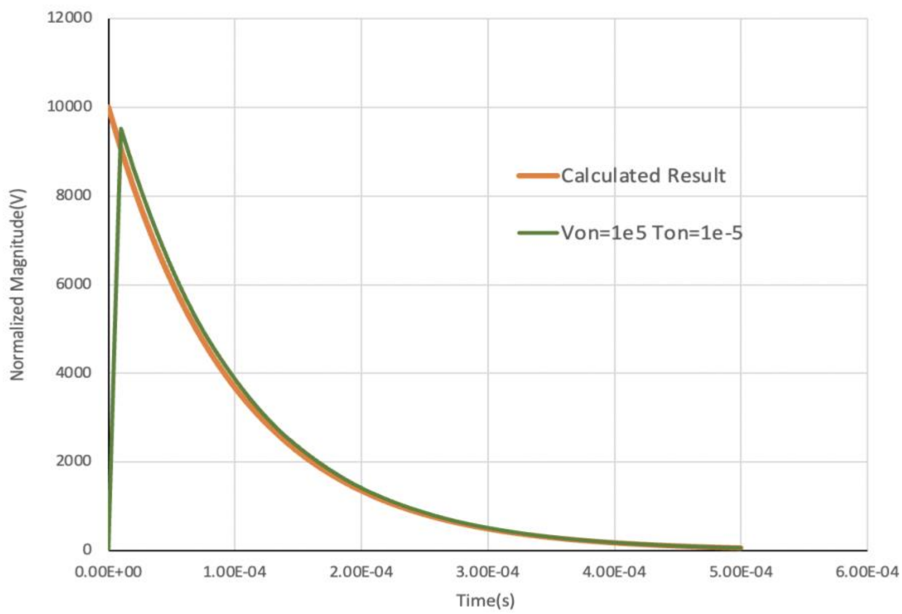

Figure 2.3.2: Time domain simulation with different delta approximation ($V_{on} = 10^5$, $T_{on} = 10^{-5}$)

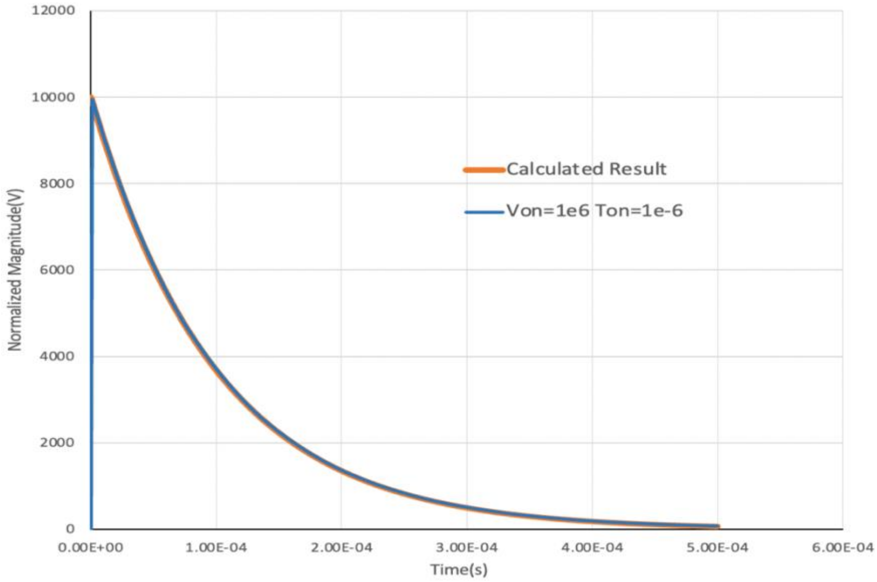

Figure 2.3.3: Time domain simulation with different delta approximation ($V_{on} = 10^6$, $T_{on} = 10^{-6}$)

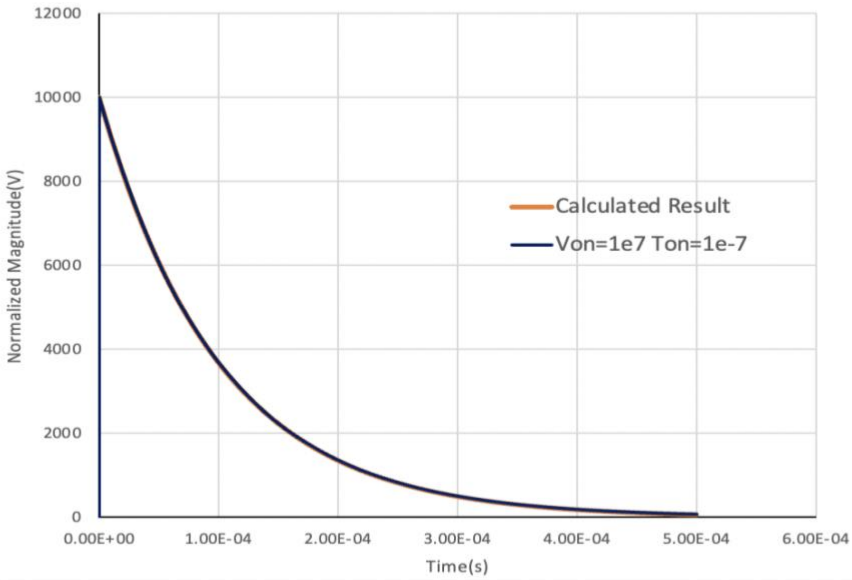

Figure 2.3.4: Time domain simulation with different delta approximation ($V_{on} = 10^7$, $T_{on} = 10^{-7}$)

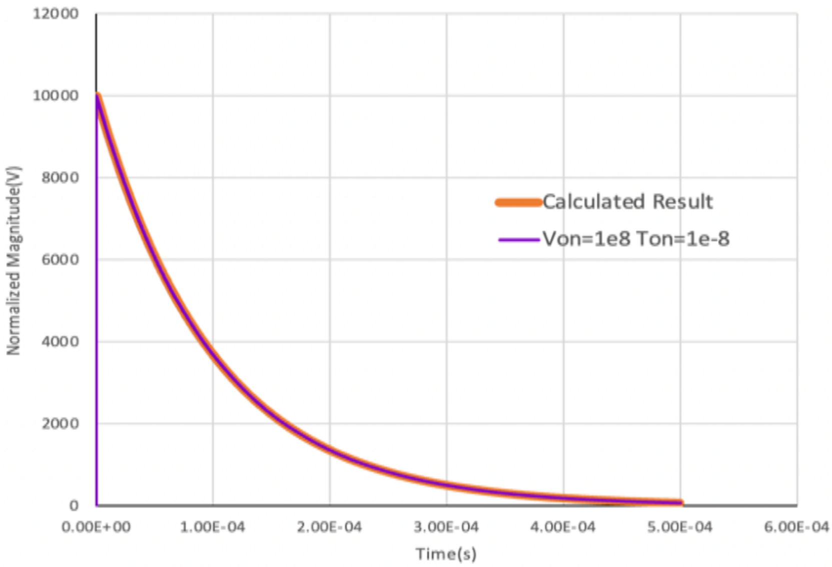

Figure 2.3.5: Time domain simulation with different delta approximation ($V_{on} = 10^8$, $T_{on} = 10^{-8}$)

Table1: Time domain simulation result with different delta approximation

V_{on} (V)	T_{on} (s)	Answer
10^4	10^{-4}	Incorrect
10^5	10^{-5}	Incorrect
10^6	10^{-6}	Correct
10^7	10^{-7}	Correct
10^8	10^{-8}	Correct

The delta function gives a correct simulation result when V_{on} is greater than 10^6 and T_{on} is less than 10^{-6}. In this case, the simulated values of LTSpice are highly coincident with the calculated theoretical values. This is because in the mathematical definition, the Dirac function is equal to zero at all points except zero, and its integral over the entire domain of the definition is equal to one. The smaller the T_{on}, the closer the equation is to zero at all points except zero, and the more accurate the mathematical model will be. Therefore, it is necessary to model the Dirac function with a shorter pulse time to get closer to the correct simulation result.

2.4

Figure 2.4.1 compares the LTSpice simulations for different Impulse Approximation cases and the calculated frequency response results. (Stop time = 1s, Time-step = 1μs, Sample points = 262144)

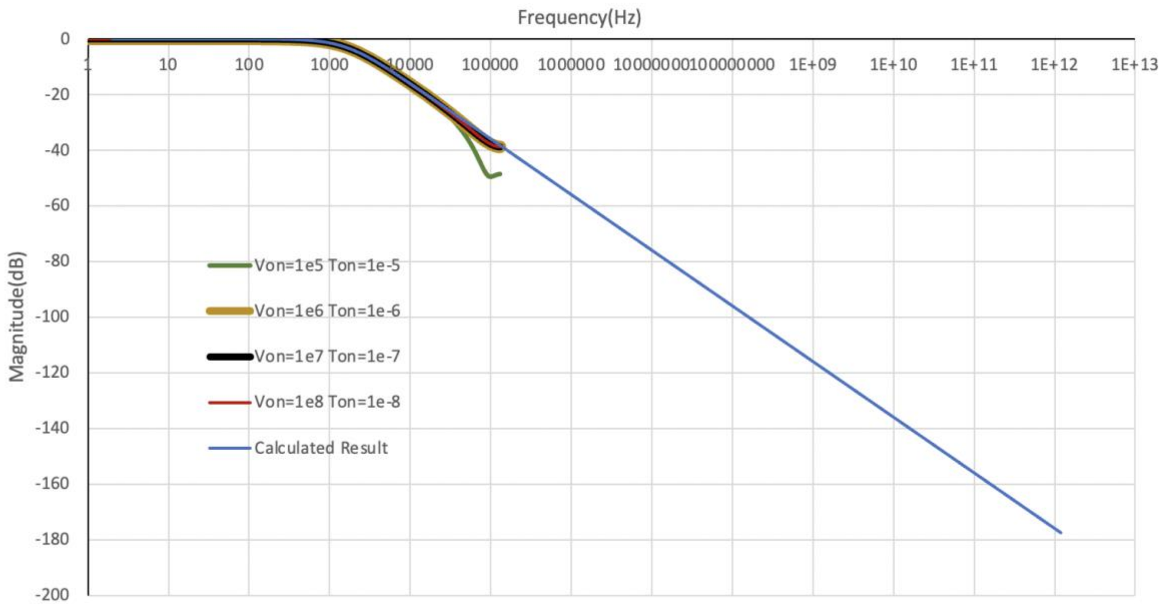

Figure 2.4.1: Comparison of the FR generated by the FFT of the simulated IR with the calculated FR

From this figure, it can be observed that for the values of IR approximation, the larger the V_{on} and the smaller the T_{on}, the closer the simulation results are to the calculated theoretical values. This is again because the smaller T_{on} is, the closer it is to the mathematical model of the delta function. When T_{on} is less than 10^{-6}, the simulation results are almost identical to the expected values.

FR is usually obtained in LTSpice by selecting the "AC Simulation" method directly from the "Simulation Command". This method selects the frequency response of a circuit by giving it different frequencies of AC signals.

3.1

By way of convolution, $V_0(t)$ can be written as:

$$V_o(t) = F(t) * V_i(t) = F(t) * S_2(t) = \int_{-\infty}^{\infty} F(\tau) * S_2(t-\tau)\mathrm{dt} \qquad \textbf{Eqn.14}$$

To facilitate the calculation of the convolution under each limit, this equation can be reduced to $(b < t < a)$:

$$V_o(t) = \int_{\mathrm{b}}^{\mathrm{a}} F(\tau) * S_2(t-\tau)\mathrm{dt} \qquad \textbf{Eqn.14}$$

$$V_o(t) = \int_{\mathrm{b}}^{\mathrm{a}} (10^4 e^{-10^4\tau})(10^3(t-\tau)-1)\mathrm{d}\tau$$

$$V_o(t) = [(10^3(\tau-t)+1)e^{-10^4\tau}]_b^a - \int_b^a 10^3 e^{-10^4\tau} d\tau$$

$$V_o(t) = [(10^3(\tau-t)+1)e^{-10^4\tau}]_b^a + [\frac{e^{-10^4\tau}}{10}]_b^a \qquad \textbf{Eqn.15}$$

According to the properties of convolution, since $S_2(t)$ has different expressions depending on t, the expressions of the convolution are also different at different t

When $0\mathrm{s} < \mathrm{t} \leq 0.001\mathrm{s}$, the product of $F(\tau)$ and $\mathrm{S}_2(t-\tau)$ is 0. So, it is not involved in the discussion of convolutional integration.

When $1\times10^{-3}\mathrm{s} < \mathrm{t} \leq 2\times10^{-3}\mathrm{s}$, according to Equation 15, the following equation can be listed:

$$V_o(t) = [(10^3(\tau-t)+1)e^{-10^4\tau}]_0^{t-0.001} + [\frac{e^{-10^4\tau}}{10}]_0^{t-0.001}$$

$$V_o(t) = ((10^3(t-0.001-t)+1)e^{-10^4(t-0.001)} - (-10^3t+1)) + (\frac{e^{-10^4(t-0.001)}-1}{10})$$

$$V_o(t) = 10^3 t + \frac{e^{-10^4(t-0.001)}}{10} - 1.1$$

When $2 \times 10^{-3}s < t$, according to Equation 15, the following equation can be listed:

$$V_o(t) = [(10^3(\tau - t) + 1)e^{-10^4\tau}]_{t-0.002}^{t-0.001} + [\frac{e^{-10^4\tau}}{10}]_{t-0.002}^{t-0.001}$$

$$V_o(t) = ((10^3(t - 0.001 - t) + 1)e^{-10^4(t-0.001)} - (10^3(t - 0.002 - t) + 1)e^{-10^4(t-0.002)}) + (\frac{e^{-10^4(t-0.001)}}{10} - \frac{e^{-10^4(t-0.002)}}{10})$$

$$V_o(t) = e^{-10^4(t-0.002)} + (\frac{e^{-10^4(t-0.001)}}{10} - \frac{e^{-10^4(t-0.002)}}{10})$$

$$V_o(t) = \frac{9e^{-10^4(t-0.002)}}{10} + \frac{e^{-10^4(t-0.001)}}{10}$$

Based on the above calculations, the expression for V_{out} in two different sets of limits for different ranges of t is listed below:

$$V_o(t) = \begin{cases} 10^3 t + \frac{e^{-10^4(t-0.001)}}{10} - 1.1, & 1 \times 10^{-3}s < t \leq 2 \times 10^{-3}s \\ \frac{9e^{-10^4(t-0.002)}}{10} + \frac{e^{-10^4(t-0.001)}}{10}, & t > 2 \times 10^{-3}s \end{cases}$$

3.2

Figure 3.2.1 shows the result of LTSpice simulation. Figure 3.2.2 compares the result of the data derived from the simulation with the result obtained using convolution.

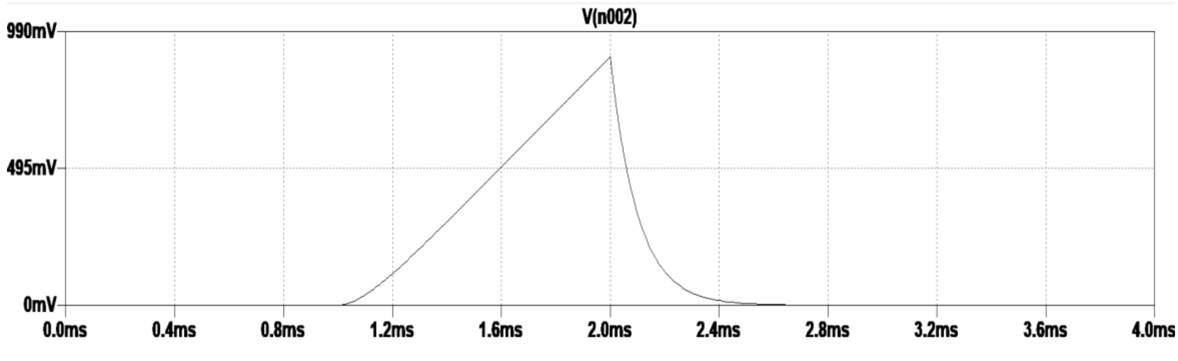

Figure 3.2.1: LTSpice simulation Result

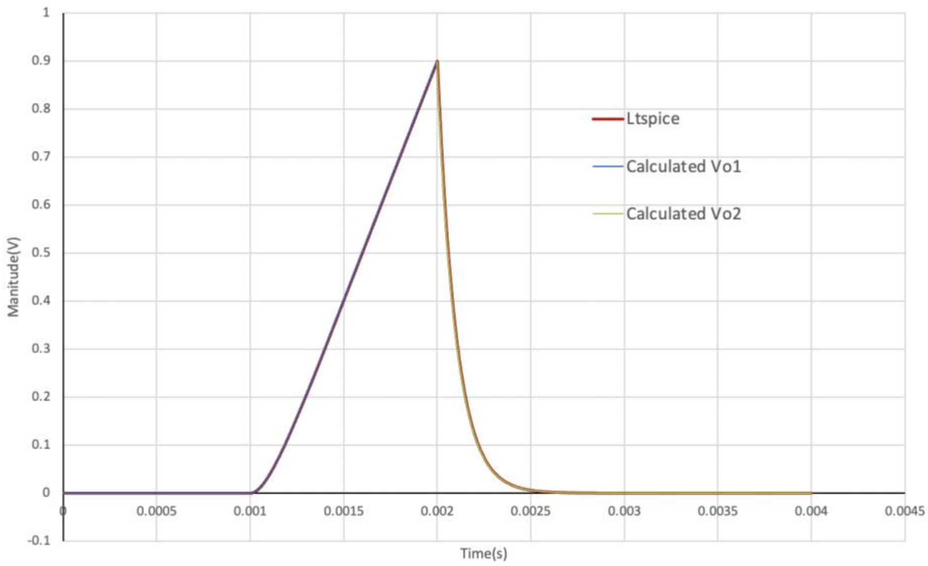

Figure 3.2.2: Comparison of results with convolution calculations and LTSpice simulation

Figure 3.2.2 shows that the simulation results are in high agreement with the calculated results. This proves that the convolution calculation in 3.1 is correct and that the convolution in the time domain is the product of the Fourier transform in the frequency domain.