EEEE3098

Integrated Circuits and Systems

Coursework 2: Solid State Devices

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1. The student ID is 20320941, so the 4th digit is 2, phosphorus concentrations "N" of the silicon resistor is $4.0 \times 10^{17}\text{cm}^{-3}$.

a) Carrier Type: Electrons. This is because for this bulk silicon resistor, the dopant is phosphorus (P), which is a pentavalent element that has one more valence electron than silicon. When a phosphorus atom is doped into the silicon lattice, that extra electron becomes relatively easy to move because it is not strongly bound to the atomic lattice points. Thus, this extra free electron increases the number of conducting electrons, turning silicon into an n-type semiconductor. In n-type semiconductors, the primary charge carriers are electrons, not holes.

Mobility: From the given curve it is observed that the mobility of electrons is about 425 $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ when the concentration is $4.0 \times 10^{17}\text{cm}^{-3}$. However, to get a more accurate value, the mobility "μ" is calculated using the Empirical equation as shown in Eqn 1.1, where μ_{min} is 92 $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$, μ_{max} is 1360 $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$, N_{ref} is $1.3 \times 10^{17}\text{cm}^{-3}$, α is 0.91 and N is $4.0 \times 10^{17}\text{cm}^{-3}$. These values can be available on the given graph.

$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + (\frac{N}{N_{ref}})^{\alpha}} \quad \textbf{Eqn 1.1}$$

$$\mu = 92\ \text{cm}^2\text{V}^{-1}\text{s}^{-1} + \frac{1360\ \text{cm}^2\text{V}^{-1}\text{s}^{-1} - 92\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}}{1 + (\frac{4.0\times10^{17}\text{cm}^{-3}}{1.3\times10^{17}\text{cm}^{-3}})^{0.91}} = 427.37\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$$

Therefore, the mobility "μ" is $427.37\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

b) According to Eqn 1.2, the function between the carrier velocity "v" and the electric field "E" can be found. Figure 1.1 shows the log-log plot based on this function.

$$v = \mu E \quad \textbf{Eqn 1.2}$$

$$v = 427.37\text{E}$$

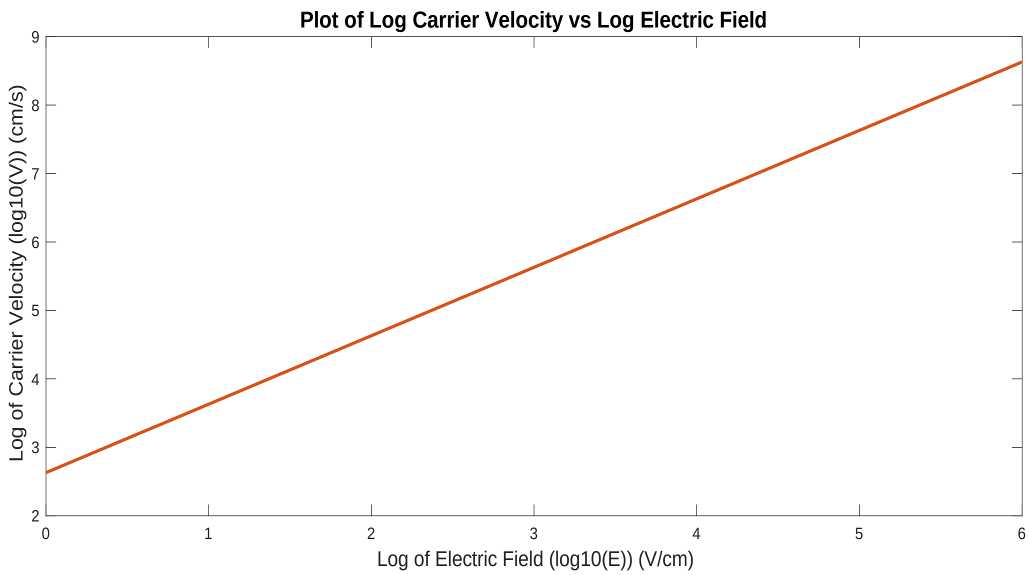

Figure 1.1: Log-log plot of the carrier velocity "v" as a function of electric field "E"

c) The range of electric field "E" can be calculated by Eqn 1.2 and $|\text{v}_{\text{sat}}| \cong 1.0 \times 10^{7}\text{c} \cdot \text{ms}^{-1}$.

$$E_{sat} = \frac{v_{sat}}{\mu} = \frac{1.0 \times 10^7 \text{cm} \cdot \text{s}^{-1}}{427.37 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}} = 23398.93 \text{ V} \cdot \text{cm}^{-1}$$

The electric field "E" at which the carrier velocity begins to saturate is 23398.93 $\text{V} \cdot \text{cm}^{-1}$. Figure 1.2 shows the labelled points for V = $1.0 \times 10^7 \text{cms}^{-1}$, and the range of electric fields. This means that when the electric field exceeds the blue line, the carrier velocity no longer increases with the electric field, but reaches a constant saturation velocity of $1.0 \times 10^7 \text{cms}^{-1}$.

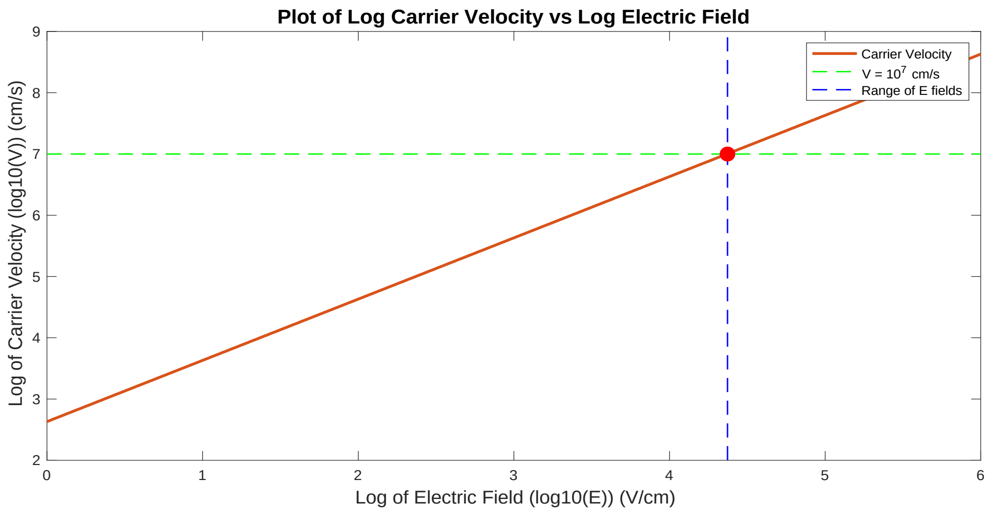

Figure 1.2: Log-log plot with carrier saturation velocity marker and the range of electric fields

d) Conductivity "σ" of the material: It can be calculated by Eqn 1.3, where q is 1.602×10^{-19}C, μ is $427.37 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, N is $4.0 \times 10^{17}\text{cm}^{-3}$.

$$\sigma = q\mu N \qquad \textbf{Eqn 1.3}$$

$$\sigma = 1.602 \times 10^{-19}\text{C} \times 427.37 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \times 4.0 \times 10^{17}\text{cm}^{-3} = 27.39 \text{ S} \cdot \text{cm}^{-1}$$

Justify for the units: $\text{C} \times \text{cm}^2\text{V}^{-1}\text{s}^{-1} \times \text{cm}^{-3} = \text{CV}^{-1}\text{s}^{-1}\text{cm}^{-1}$, the unit Coulomb per volt (CV^{-1}) is the unit of capacitance, farad (F), and since 1S =1Fs^{-1}, we can express the Coulomb per second as siemens (S). Therefore, the units simplify to $\text{S} \cdot \text{cm}^{-1}$, which is the correct unit for electrical conductivity.

Resistance "R": It can be calculated by Eqn 1.4. L is 1.0 μm and it is the length of the resistor. A is $5 \times 10^{-8}\text{cm}^2$ and it is the cross-sectional area. σ is $27.39 \text{ S} \cdot \text{cm}^{-1}$.

$$R = \frac{L}{\sigma A} = \frac{1.0 \text{ μm}}{27.39 \text{ S·cm}^{-1} \times 5 \times 10^{-8}\text{cm}^2} = 73.02 \ \Omega \qquad \textbf{Eqn 1.4}$$

Justify for the units: $\frac{\text{μm}}{\text{S·cm}^{-1} \times \text{cm}^2} = \frac{\text{cm}}{\text{S} \times \text{cm}} = \frac{1}{\text{S}}$, $\frac{1}{\text{S}}$ is the unit for resistance, ohms (Ω).

Therefore, the conductivity "σ" of the material is 27.39 $\text{S} \cdot \text{cm}^{-1}$ and its resistance is 73.02 Ω.

e) The electric field in the silicon when biased with a voltage of 1.0 V can be calculated by Eqn 1.5, where L is 1.0 μm and it is the length of the resistor.

$$E_{biasd} = \frac{V_{biasd}}{L} = \frac{1.0\ \mathrm{V}}{1.0\ \mu\mathrm{m}} = 10^4\ \mathrm{V \cdot cm^{-1}} \qquad \textbf{Eqn 1.5}$$

Therefore, the electric field "$\mathrm{E_{biasd}}$" is $10^4\ \mathrm{V \cdot cm^{-1}}$.

f) The carrier velocity for the electric field in (e) can be calculated by Eqn 1.2, where μ is $427.37\ \mathrm{cm^2V^{-1}s^{-1}}$:

$$v = \mu E_{biasd} = 427.37\ \mathrm{cm^2V^{-1}s^{-1}} \times 10^4\ \mathrm{V \cdot cm^{-1}} = 4.27 \times 10^6\ \mathrm{cm \cdot s^{-1}}$$

Therefore, the velocity is $4.27 \times 10^6\ \mathrm{cm \cdot s^{-1}}$. It is less than the saturation velocity $(1.0 \times 10^7 \mathrm{cm \cdot s^{-1}})$. This is because at low electric field strengths the carrier velocity is determined by the mobility, whereas at high electric field strengths the velocity reaches the saturation velocity limit. In this case, the calculated $4.27 \times 10^6\ \mathrm{cm \cdot s^{-1}}$is the carrier velocity before saturation is reached.

2. The 5th digit is 0 and the 6th digit is 9, so $\mathrm{I_{d0}}$ is 5×10^{-18}A and $\mathrm{I_{r0}}$ is 5×10^{-11}A.

a) The current, voltage behaviour of this diode is shown in Eqn 2.1, 2.2 and 2.3. q is $1.602 \times 10^{-19}\mathrm{C}$ and is the electron Charge. K is the Boltzmann's Constant and has a magnitude of $1.381 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$. T is the room temperature, which is 300 K. Based on these three equations, the semilogarithmic graph including curves for $\mathrm{I_d}$, $\mathrm{I_r}$ and $\mathrm{I_{total}}$ is shown in Figure 2.1. Here the values of the horizontal coordinate V range from -1 V to 1V. (There is a labelled point on this graph that are used for calculations in later questions.)

$$I_d = I_{d0}\{\exp\left(\frac{qV}{kT}\right) - 1\} \qquad \textbf{Eqn 2.1}$$

$$I_r = I_{r0}\{\exp\left(\frac{qV}{2kT}\right) - 1\} \qquad \textbf{Eqn 2.2}$$

$$I_{total} = I_d + I_r = I_{d0}\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} + I_{r0}\{\exp\left(\frac{qV}{2kT}\right) - 1\} \qquad \textbf{Eqn 2.3}$$

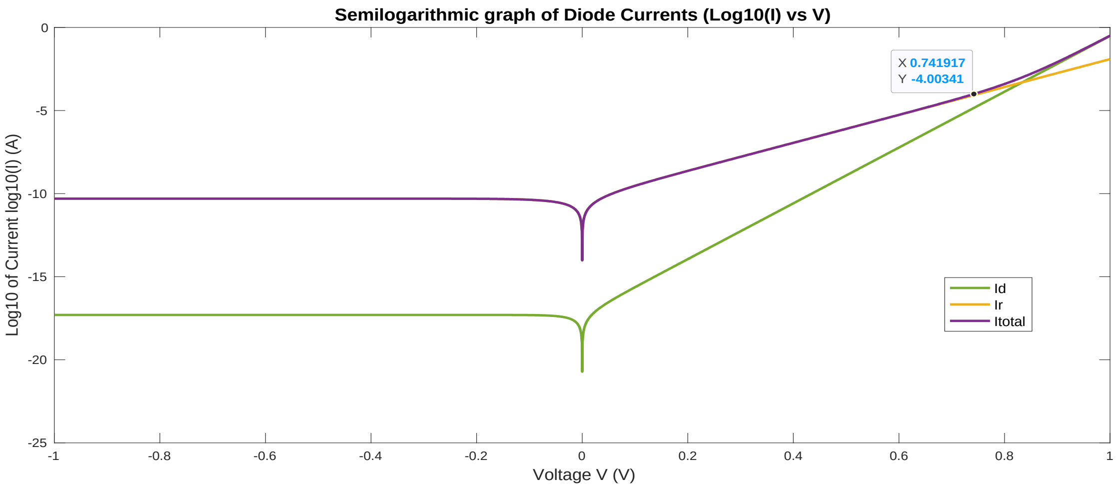

Figure 2.1: Semilogarithmic graph for $\mathrm{I_d}$, $\mathrm{I_r}$ and $\mathrm{I_{total}}$ vs V

b) Combining the three equations above gives Eqn 2.4 to calculate the voltage drop "V" across the diode when forward current " I_{total}" is 100 μA. For mathematical convenience, $\exp\left(\frac{qV}{2kT}\right)$ is set to x and $\exp\left(\frac{qV}{kT}\right)$ is set to x^2, as shown in Eqn 2.5 and this should be calculate first. Eqn 2.6 is the bring into Eqn 2.4 of Eqn 2.5. Details about calculation is as follows:

$$I_{total} = I_d + I_r = I_{d0}\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} + I_{r0}\left\{\exp\left(\frac{qV}{2kT}\right) - 1\right\} \quad \textbf{Eqn 2.4}$$

$$\exp\left(\frac{qV}{kT}\right) = x,\ \exp\left(\frac{qV}{2kT}\right) = x^2 \quad \textbf{Eqn 2.5}$$

$$I_{total} = I_{d0}(x^2 - 1) + I_{r0}(x - 1) \quad \textbf{Eqn 2.6}$$

$$I_{d0}x^2 + I_{r0}x - (I_{total} + I_{d0} + I_{r0}) = 0$$

Because I_{total} (100 μA) $\gg$ $I_{d0} + I_{r0}$ (5×10^{-11}A $+ 5 \times 10^{-18}$A), the equation can be simplified as:

$$I_{d0}x^2 + I_{r0}x - I_{total} = 0$$

$$x = \frac{-I_{r0} \pm \sqrt{{I_{r0}}^2 - 4I_{d0}(-I_{total})}}{2I_{d0}}$$

$$x = \frac{-5\times10^{-11}\ \text{A} + \sqrt{(5\times10^{-11}\text{A})^2 - 4\times5\times10^{-18}\text{A}(-100\ \mu\text{A})}}{2\times5\times10^{-18}\ \text{A}} = 1.71 \times 10^6$$

Based on Eqn 2.5, the function used to calulate V can be expressed as Eqn 2.7:

$$V = \frac{2\text{kT}}{q}\ln(x) = \frac{2\times1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times300\ \text{K}}{1.602\times10^{-19}\text{C}}\ln(1.71 \times 10^6) = 0.74\ \text{V} \quad \textbf{Eqn 2.7}$$

Therefore, the voltage drop "V" across the diode for a forward current of 100 μA is 0.74 V. This value can also be verified at the marked points in Figure 2.1 when $I_{total} = 10^{-4}$A, V = 0.74 V.

c) To draw this tangent line, the coordinates of the tangent point should be found from Eqn2.3, and when I_{total} = 100 mA, V is calculated similarly to the method used in b), and the process is not repeated here.

$$I_{total} = I_{d0}\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} + I_{r0}\left\{\exp\left(\frac{qV}{2kT}\right) - 1\right\} = 0.1\ \text{A} \qquad \text{V} = 0.969\ \text{V}$$

With the coordinates of the tangent point, the the slope "k" of the line can be known, which can be found from Eqn 2.8.

$$k_{(V=0.969\ \text{V})} = \frac{d(I_{total})}{d\,V} = \frac{d(I_d + I_r)}{d\,V} = \frac{d(I_{d0}\{\exp\left(\frac{qV}{kT}\right) - 1\} + I_{r0}\{\exp\left(\frac{qV}{2kT}\right) - 1\})}{d\,V} \quad \textbf{Eqn 2.8}$$

$$k_{(V=0.969\ \text{V})} = I_{d0}\frac{q}{kT}\exp\left(\frac{qV}{kT}\right) + I_{r0}\frac{q}{2kT}\exp\left(\frac{qV}{2kT}\right)$$

$$k_{(V=0.969\ \text{V})} = 5 \times 10^{-18}\ \text{A} \times \frac{1.602\times10^{-19}\text{C}}{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times 300\ \text{K}} \times \exp\left(\frac{1.602\times10^{-19}\text{C}\times0.969}{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times 300\ \text{K}}\right) +$$

$$5 \times 10^{-11}\ \text{A} \times \frac{1.602\times10^{-19}\text{C}}{2\times1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times 300\ \text{K}} \times \exp\left(\frac{1.602\times10^{-19}\text{C}\times0.969}{2\times1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times 300\ \text{K}}\right) = 3.75$$

Bringing in the coordinates of the tangent point (0.969 V, 0.1 A) gives the tangent expression as Eqn 2.9, Figure 2.2 shows this tangent of the line to I_{total} vs V at I_{total} = 100 mA. For the convenience of showing the tangent line, the horizontal coordinates of the I_{total} vs V curve here take the range of 0 to 1V.

$$y = 3.75k - 3.54 \qquad \textbf{Eqn 2.9}$$

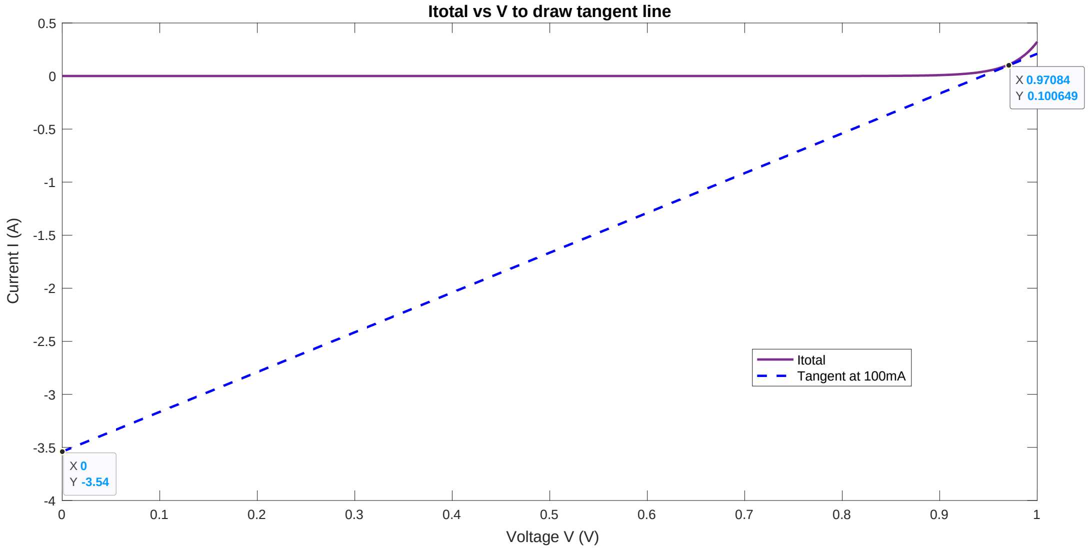

Figure 2.2: Line tangent to I_{total} vs V at I_{total} = 100 mA

Eqn 2.9 shows the non-ideal diode equation. Since the slope k of this tangent line has already been found, k is also equal to the point of derivation of Eqn 2.9 at V = 0.969 V, I_0 and n can also be worked out as follows:

$$I_{total} = I_0\{\exp\left(\frac{qV}{nkT}\right) - 1\} \qquad \textbf{Eqn 2.9}$$

$$I_{total(V=0.969\text{ V})} = I_0\left\{\exp\left(\frac{qV}{nkT}\right) - 1\right\} = 0.1\text{ A}$$

$$k_{(V=0.969\text{ V})} = \frac{d(I_{total})}{d\,V} = \frac{d(I_0\{\exp\left(\frac{qV}{nkT}\right)-1\}\ )}{d\,V} = I_0\frac{q}{nkT}\exp\left(\frac{qV}{nkT}\right) = 3.75$$

$$\frac{I_0\frac{q}{nkT}\exp\left(\frac{qV}{nkT}\right)}{I_0\left\{\exp\left(\frac{qV}{nkT}\right)-1\right\}} = \frac{\frac{q}{nkT}\exp\left(\frac{qV}{nkT}\right)}{\exp\left(\frac{qV}{nkT}\right)-1} = \frac{3.75}{0.1} = 37.5$$

$$\frac{q}{nkT}\exp\left(\frac{0.969q}{nkT}\right) = 37.5(\exp\left(\frac{0.969q}{nkT}\right) - 1)$$

For ease of calculation, set "$\frac{q}{nkT}$" as "x":

$$x\exp(0.969x) = 37.5\exp(0.969x) - 37.5)$$

$$\frac{d(x\exp(0.969x))}{d\,x} = \frac{d(37.5\exp(0.969x)-37.5))}{d\,x}$$

$$\exp(0.969x) + 0.969x\exp(0.969x) = 37.5 \times 0.969\exp(0.969x)$$

$$x = 36.47$$

$$n = \frac{q}{xkT} = \frac{1.602 \times 10^{-19}\mathrm{C}}{36.47 \times 1.381 \times 10^{-23}\ \mathrm{J \cdot K^{-1}} \times\ 300\ \mathrm{K}} = 1.06$$

$$I_0 = \frac{I_{total}}{\exp\left(\frac{qV}{nkT}\right) - 1} = \frac{0.1\ \mathrm{A}}{\exp\left(36.47 \times 0.969\ \mathrm{V}\right) - 1} = 4.49 \times 10^{-17}\ \mathrm{A}$$

Therefore, saturation current " I_0 " is 4.49×10^{-17} A and ideality factor "n" is 1.06.

3. The 7th digit is 4, so λ_0 is 725 nm. From the given graph it can be estimated that the absorption coefficient "α" is 1900 cm^{-1}.

a) The p-i-n photodiode consists of three layers, p^+ layer, intrinsic layer (i) and n^+ layer. Figure 3.1 shows the structure of the device, details of the design are explained below the figure.

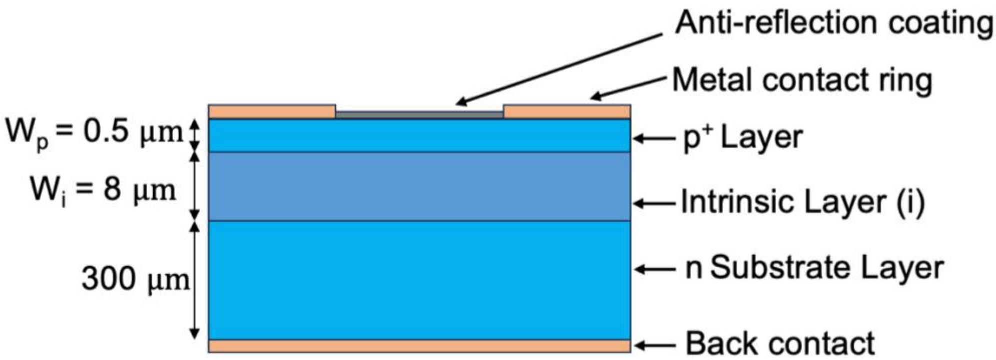

Figure 3.1: The structure of designed device

Size: This should not exceed the size of the optical signal, which usually means that the diameter of its active region should match or be slightly smaller than the diameter of the incoming fibre to ensure maximum capture of the optical signal. The diameter of the optical fibre in this question is 100 µm. therefore, the active region diameter of the p-i-n photodiode should be set to 100 µm. Based on this, the active region size of the p-i-n photodiode "A" can be calculated from Eqn 3.1:

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{100\ \mu\mathrm{m}}{2}\right)^2 = 7853.98\ \mu\mathrm{m}^2 \qquad \textbf{Eqn 3.1}$$

Therefore, the active region diameter and size of p-i-n photodiode is 100 µm and $7853.98\ \mu\mathrm{m}^2$.

Layer thickness: For the p^+ layer, this layer should be thin enough so that most of the incident light reaches the intrinsic layer. Eqn 3.2 is the function that needs to be satisfied for the layer thickness W_p. For the intrinsic layer (i), the layer should be thick enough to absorb the incident light, but not so thick that it increases the carrier transport time, thus affecting the frequency response. Eqn 3.3 is the function that needs to be satisfied for the layer thickness W_i.

$$\alpha W_p \ll 1 \qquad \textbf{Eqn 3.2}$$

$$W_p \ll \frac{1}{\alpha} = \frac{1}{1900\ \mathrm{cm}^{-1}} = 5.26\ \mu\mathrm{m}$$

$$\alpha W_i > 1 \qquad \textbf{Eqn 3.3}$$

$$W_i > \frac{1}{\alpha} = \frac{1}{1900\ \mathrm{cm}^{-1}} = 5.26\ \mu\mathrm{m}$$

After the calculation, it can be found that the thickness of p^+ layer W_p should be much less than 5.26 μm, while the thickness of intrinsic layer (i) W_i should be greater than 5.26 μm. Therefore, here W_p is chosen as 0.5 μm and W_i is chosen as 8 μm. For the n^+ layer, it is a silicon substrate with a thickness of 300 μm.

Doping type and levels: This design is chosen to use heavily doped n-type silicon as a substrate. ($N_d = 1 \times 10^{20}\ \mathrm{cm}^{-3}$) This is because the mobility of electrons (n-type silicon) is higher than that of holes (p-type silicon), and the n-type provides faster carrier transport, which helps to form a wide depletion region, thus contributing to fast response and efficient optical signal detection. In addition, considering that the design is for optical signals in the near-infrared to visible (NIR-VIS) wavelength range, the n-type silicon can provide a sufficient response range, as well as a low RC time constant.

b) Eqn 3.4 describes the information that light is absorbed in the diode, where $I_{(x)}$ is the intensity of light at position x. $I_{(0^+)}$ is the initial intensity of light incident on the material minus the reflection loss, and since the diode is anti-reflection coated (R ~ 0), $I_{(0^+)}$ is the initial intensity of light. α is the absorption coefficient of the material, which is 1900 cm^{-1}. x is the distance that the light travels in the material. Eqn 3.5 is used to calculate the absorption efficiency based on Eqn 3.4, which means absorbed light intensity/original light intensity.

$$I_{(x)} = I_{(0^+)} \exp(-\alpha x) \qquad \textbf{Eqn 3.4}$$

$$\eta = \frac{P_{absorbed}}{P_{incident}} = \frac{I_{absorbed}}{I_{(0^+)}} = \frac{I_{(0^+)} - I_{(x)}}{I_{(0^+)}} = \frac{I_{(0^+)} - I_{(0^+)} \exp(-\alpha x)}{I_{(0^+)}} = 1 - \exp(-\alpha x) \qquad \textbf{Eqn 3.5}$$

When talking about the absorption efficiency of a p-i-n photodiode, the usual focus is on the absorptive capacity of the intrinsic layer (i), since the absorption of light by this layer is directly converted into photocurrent. Therefore, as shown in Eqn 3.6, x can be directly carried as the thickness of the p^+ layer W_p and intrinsic layer W_i. The absorption efficiency is calculated as follows:

$$\eta = 1 - \exp[-\alpha(W_i + W_p)] \qquad \textbf{Eqn 3.6}$$

$$\eta = 1 - \exp(-1900\ \mathrm{cm}^{-1} \times (8\ \mu\mathrm{m} + 0.5\ \mu\mathrm{m})) = 80.11\%$$

Thus, this result shows that about 80.11% of the incident light is absorbed in the intrinsic layer, which is a high value for the efficiency of the photodiode.

c) The bandwidth limited by RC time constant can be calculated by Eqn 3.7. R is 50 Ω and Eqn 3.8 is used to calculate C, where relative permittivity "ε_s" is 11.7, dielectric constant of free space "ε_o" is $8.854 \times 10^{-12}\ \mathrm{Fm}^{-1}$ and A is 784 $\mu\mathrm{m}^2$.

$$f_{3dB} = \frac{1}{2\pi RC} \qquad \textbf{Eqn 3.7}$$

$$C = \frac{\varepsilon_s \varepsilon_o A}{W_{depl}} \cong \frac{\varepsilon_s \varepsilon_o A}{W_i} \qquad \textbf{Eqn 3.8}$$

$$C = \frac{\varepsilon_s \varepsilon_o A}{W_i} = \frac{11.7 \times 8.854 \times 10^{-12}\ \mathrm{Fm}^{-1} \times 7853.98\ \mu\mathrm{m}^2}{8\ \mu\mathrm{m}} = 10.17 \times 10^{-14}\ \mathrm{F}$$

$$f_{3dB} = \frac{1}{2\pi \times 50\ \Omega \times 10.17 \times 10^{-14}\ \mathrm{F}} = 31.3\ \mathrm{GHz}$$

The bandwidth limited by the carrier transit time can be calculated by Eqn 3.9, the transit time "τ_{tr}" can be calculated by Eqn 3.10, where $\mathrm{v_{sat}}$ is $10^7\ \mathrm{cm \cdot s^{-1}}$.

$$f_{3dB} = \frac{0.45}{\tau_{tr}} \qquad \textbf{Eqn 3.9}$$

$$\tau_{tr} = \frac{W_{depl}}{v} = \frac{W_i}{v_{sat}} = \frac{8\ \mu\mathrm{m}}{10^7\ \mathrm{cm \cdot s^{-1}}} = 8 \times 10^{-11}\ \mathrm{s} \qquad \textbf{Eqn 3.10}$$

$$f_{3dB} = \frac{0.45}{8 \times 10^{-11}\ \mathrm{s}} = 5.63\ \mathrm{GHz}$$

The two bandwidths calculated are higher than 5 GHz, so the operation is satisfied.

d) Measures to maximise the signal to noise ratio of the device:

1. Improvement of photoelectric conversion efficiency: Optimisation of the width of the i-layer to increase light absorption and photogenerated carrier generation to enhance the signal without significantly increasing the carrier transmission time.

2. Reducing carrier complexation: Reducing it by using high lifetime materials to extend the effective lifetime of carriers and reduce noise due to carrier complexation.

3. Use of anti-reflective coatings: The reflection of incident light can be reduced, thus increasing the effective received light intensity of the photodiode.

4. The 8th digit is 4, so operating current " I " is 100 mA.

a) The thicknesses of n- base " $\mathrm{W_n}$" and p- base "$\mathrm{W_p}$" selected here are 400 μm and 4 nm. Doping level of n- base " $\mathrm{N_a}$" is $1 \times 10^{18}\ \mathrm{cm^{-3}}$ and doping level of p- base " $\mathrm{N_d}$" is $1 \times 10^{16}\ \mathrm{cm^{-3}}$. Figure 4.1 shows this design.

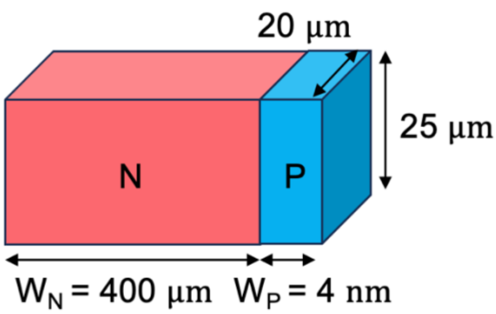

Figure 4.1: The layer thickness and doping levels of the n- and p- bases.

b) Eqn 4.1 and 4.2 are used to calculate the minority carrier diffusion lengths L_n and L_p. There are two unknown quantities in this equation, one is the lifetime of minority carrier τ_n (τ_p). τ_n refers to the lifetime of the minority of electrons that enter the p side and τ_p refers to the lifetime of the minority of holes that enter the n side. These values can be found from the given graphs, τ_n is 4×10^{-6} s when N_a is $1 \times 10^{18}\ cm^{-3}$, τ_p is 9×10^{-5} s when N_d is $1 \times 10^{16}\ cm^{-3}$.

Another value is the diffusion coefficients for electrons " D_n" (for holes " D_p"). These values can be calculated by Eqn 4.3 and 4.4, where q is $1.602 \times 10^{-19}C$ and is the electron Charge. K is the Boltzmann's Constant and has a magnitude of $1.381 \times 10^{-23}\ J \cdot K^{-1}$. T is 300 K. μ_n is the mobility of electrons that enter the p side (N = $N_a = 1 \times 10^{18}\ cm^{-3}$) and μ_p is the mobility of holes that enter the n side. (N = $N_d = 1 \times 10^{16}\ cm^{-3}$) These two values can be calculated by the equation (Eqn 1.1) in the given graph.

$$\mu_n = \mu_{min} + \frac{\mu_{max}-\mu_{min}}{1+(\frac{N}{N_{ref}})^{\alpha}} = 92 + \frac{1360-92}{1+(\frac{1\times10^{18}}{1.3\times10^{17}})^{0.91}} = 263.31\ cm^2V^{-1}s^{-1}$$

$$\mu_p = \mu_{min} + \frac{\mu_{max}-\mu_{min}}{1+(\frac{N}{N_{ref}})^{\alpha}} = 47.7 + \frac{495-47.7}{1+(\frac{1\times10^{16}}{6.3\times10^{16}})^{0.76}} = 406.43\ cm^2V^{-1}s^{-1}$$

$$L_n = \sqrt{D_n\tau_n} \qquad \textbf{Eqn 4.1}$$

$$L_p = \sqrt{D_p\tau_p} \qquad \textbf{Eqn 4.2}$$

$$D_n = \frac{kT}{q}\mu_n \qquad \textbf{Eqn 4.3}$$

$$D_p = \frac{kT}{q}\mu_p \qquad \textbf{Eqn 4.4}$$

$$D_n = \frac{kT}{q}\mu_n = \frac{1.381\times10^{-23}\ J{\cdot}K^{-1}\times 300\ K}{1.602\times10^{-19}C} \times 263.31\ cm^2V^{-1}s^{-1} = 6.81\ cm^2\ s^{-1}$$

$$D_p = \frac{kT}{q}\mu_p = \frac{1.381\times10^{-23}\ J{\cdot}K^{-1}\times 300\ K}{1.602\times10^{-19}C} \times 406.43\ cm^2V^{-1}s^{-1} = 10.51\ cm^2\ s^{-1}$$

$$L_n = \sqrt{\frac{kT}{q}\mu_n\tau_n} = \sqrt{6.81\ cm^2\ s^{-1}\ \times 4 \times 10^{-6}\ s} = 52.19\ \mu m$$

$$L_p = \sqrt{\frac{kT}{q}\mu_p\tau_p} = \sqrt{10.51\ cm^2\ s^{-1}\ \times 9 \times 10^{-5}\ s} = 307.55\ \mu m$$

Therefore, minority carrier diffusion lengths L_n and L_p are 52.19 μm and 307.55 μm. Because W_n(400 μm) $> L_p$, n- region is long base, W_p (4 nm) $< L_n$, p- region is short base.

c) The expression for the diode I-V characteristics is shown as Eqn 4.5.

$$I = I_0\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} \qquad \textbf{Eqn 4.5}$$

$$I = I_0\left\{\exp\left(\frac{1.602\times10^{-19}C\times V}{1.381\times10^{-23}\ J{\cdot}K^{-1}\times 300\ K}\right) - 1\right\} = I_0\{\exp(38.67\ V) - 1\}$$

Therefore, the expression is $I = I_0\{\exp(38.67\ V) - 1\}$.

d) The saturation current " I_0" can be calculated by Eqn 4.6, where junction area "A" is 20 μm × 25 μm, intrinsic carrier density " n_i " is 8.59×10^9 cm^{-3} and the other values can be found in b).

$$I_0 = qA{n_i}^2 \left[\frac{D_n}{N_a L_n \tanh\left(\frac{W_p}{L_n}\right)} + \frac{D_p}{N_d L_p \tanh\left(\frac{W_n}{L_p}\right)}\right] \qquad \textbf{Eqn 4.6}$$

$$I_0 = 1.602 \times 10^{-19}\text{C} \times 20\ \mu\text{m} \times 25\ \mu\text{m} \times (8.59 \times 10^9 \text{cm}^{-3})^2 \times$$
$$\left[\frac{6.81\ \text{cm}^2\,\text{s}^{-1}}{1\times10^{18}\ \text{cm}^{-3}\times52.19\ \mu\text{m}\times\tanh\left(\frac{4\ \text{nm}}{52.19\ \mu\text{m}}\right)} + \frac{10.51\ \text{cm}^2\,\text{s}^{-1}}{1\times10^{16}\ \text{cm}^{-3}\times307.55\ \mu\text{m}\times\tanh\left(\frac{400\ \mu\text{m}}{307.55\ \mu\text{m}}\right)}\right] = 1 \times 10^{-15}\ \text{A}$$

Therefore, I_0 is 1×10^{-15} A.

e) With Eqn 4.5 and the value calculated in d), the forward bias voltage for operating current " I " (100 mA) can be calculated as:

$$I = I_0\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} \rightarrow \frac{I}{I_0} + 1 = \exp\left(\frac{qV}{kT}\right) \rightarrow \ln\left(\frac{I}{I_0} + 1\right) = \frac{qV}{kT}$$

$$V = \frac{kT}{q}\ln\left(\frac{I}{I_0} + 1\right) = \frac{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times 300\ \text{K}}{1.602\times10^{-19}\text{C}}\ln\left(\frac{100\ \text{mA}}{1\times10^{-15}\ \text{A}} + 1\right) = 0.83\ \text{V}$$

Therefore, the forward bias voltage " V " is 0.83 V.

f) Because one of quasi-neutral base is much longer than the relevant minority carrier diffusion length ($W_n > L_p$), so this a long-base pn diode. It means that all excess minority carriers recombine before arriving at the ohmic contacts at the far edge of the depletion region. In this case, the corresponding minority electron and hole distributions are Eqn 4.7 and 4.8. As the excess minority carrier density moves away from the junction and recombines with the majority carriers, they fall exponentially to zero. Figure 4.2 shows the minority distribution based on these equations.

$$n_p'(x) = \left(\frac{n_i^2}{N_a}\right)\left[exp\left\{\frac{qV}{kT}\right\} - 1\right] exp\left\{\frac{(x+x_p)}{L_n}\right\} \qquad (-W_p \le x \le -x_p) \qquad \textbf{Eqn 4.7}$$

$$p_n'(x) = \left(\frac{n_i^2}{N_a}\right)\left[exp\left\{\frac{qV}{kT}\right\} - 1\right] exp\left\{\frac{-(x-x_n)}{L_p}\right\} \qquad (x_n \le x \le W_n) \qquad \textbf{Eqn 4.8}$$

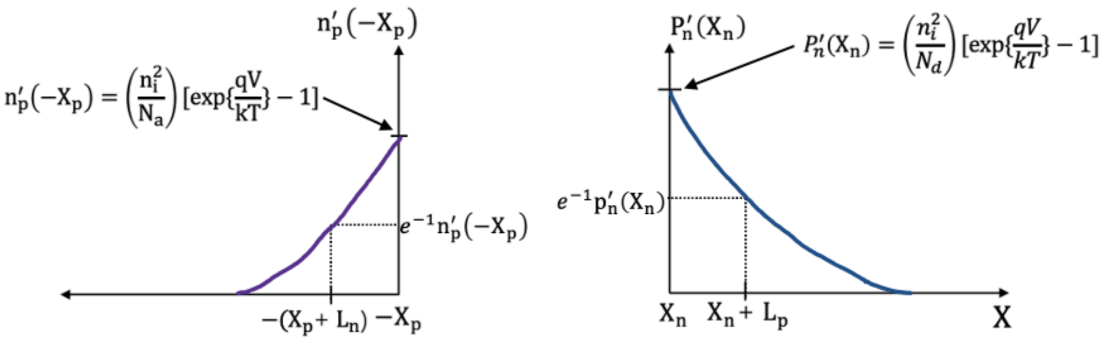

Figure 4.2: The minority distributions

g) Eqn 4.9 is used to calculate the fraction of the total current that is injected into the n-base $I_p(0)$. Eqn 4.10 is used to calculate the fraction of the total current that is injected into the p-base $I_n(0)$. For this design, there's a requirement that needs to be met. (Eqn 4.11)

$$I_p(0) = \frac{qAn_i^2D_p}{N_dL_p}(e^{\frac{qV}{kT}} - 1) \qquad \textbf{Eqn 4.9}$$

$$I_p(0) = \frac{1.602\times10^{-19}\text{C}\times20\ \mu\text{m}\times25\ \mu\text{m}\times(8.59\times10^{9}\text{cm}^{-3})^2\times10.51\ \text{cm}^2\ \text{s}^{-1}}{1\times10^{16}\ \text{cm}^{-3}\times307.55\ \mu\text{m}}(e^{\frac{1.602\times10^{-19}\text{C}\times0.83\ \text{V}}{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times\ 300\ \text{K}}} - 1)$$

$$I_p(0) = 1.75\times10^{-4}\ \text{A}$$

$$I_n(0) = \frac{qAn_i^2D_n}{N_aW_p}(e^{\frac{qV}{kT}} - 1) \qquad \textbf{Eqn 4.10}$$

$$I_n(0) = \frac{1.602\times10^{-19}\text{C}\times20\ \mu\text{m}\times25\ \mu\text{m}\times(8.59\times10^{9}\text{cm}^{-3})^2\times6.81\ \text{cm}^2\ \text{s}^{-1}}{1\times10^{18}\ \text{cm}^{-3}\times4\ \text{nm}}(e^{\frac{1.602\times10^{-19}\text{C}\times0.83\ \text{V}}{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times\ 300\ \text{K}}} - 1)$$

$$I_n(0) = 0.0875\ \text{A}$$

$$\frac{I_n(0)}{I_{toatl}} \geq 0.995 \rightarrow \frac{I_n(0)}{I_n(0)+I_p(0)} \geq 0.995 \rightarrow I_n(0) \geq 199\ I_p(0) \qquad \textbf{Eqn 4.11}$$

$$\frac{I_n(0)}{I_p(0)} = \frac{0.0875}{1.75\times10^{-4}} = 500\ > 199$$

Therefore, the fraction of the total current that is injected into the n-base $I_p(0)$ is 1.75×10^{-4} A, and this meets the requirements of the title.

h) Dynamic resistance: Based on Eqn 4.12, r_d can be calculated as follows, where ideality factor "n" is 1, because this diode is ideal. Operating Current " I " is 100 mA. Other values can be seen in above questions.

$$r_d = \frac{nkT}{qI} = \frac{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times\ 300\ \text{K}}{1.602\times10^{-19}\text{C}\times100\ \text{mA}} = 0.26\ \Omega \qquad \textbf{Eqn 4.12}$$

Depletion Capacitance: Eqn 4.13 is used to calculate C_{depl}, where potential barrier V_{bi} can be calculated by Eqn 4.14. Other values can be seen in above questions.

$$C_{depl} = \frac{\varepsilon_s\varepsilon_oA}{W} = A[\frac{q\varepsilon_s\varepsilon_o}{2(V_{bi}-V)}\left(\frac{N_aN_d}{N_a+N_d}\right)]^{\frac{1}{2}} \qquad \textbf{Eqn 4.13}$$

$$V_{bi} = \frac{kT}{q}ln\left(\frac{N_aN_d}{n_i^2}\right) = \frac{1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1}\times\ 300\ \text{K}}{1.602\times10^{-19}\text{C}}ln\left(\frac{1\times10^{18}\ \text{cm}^{-3}\times1\times10^{16}\ \text{cm}^{-3}}{(8.59\times10^{9}\text{cm}^{-3})^2}\right) = 0.84 \qquad \textbf{Eqn 4.14}$$

$$C_{depl} = 20\ \mu\text{m}\ \times25\ \mu\text{m}[\frac{1.602\times10^{-19}\text{C}\times11.7\times8.854\times10^{-12}\ \text{Fm}^{-1}}{2(0.84-0.83)}\left(\frac{1\times10^{18}\ \text{cm}^{-3}\times\ 1\times10^{16}\ \text{cm}^{-3}}{1\times10^{18}\ \text{cm}^{-3}+\ 1\times10^{16}\ \text{cm}^{-3}}\right)]^{\frac{1}{2}}\ ¥$$

$$C_{depl} = 1.43\times10^{-15}\ \text{F}$$

Diffusion Capacitance: Eqn 4.15 is used to calculate C_{diff}.

$$C'_{diff}(V) = \frac{d}{dV}\left\{\frac{qn_i^2L_n}{N_a} + \frac{qn_i^2W_n}{2N_d}\right\}[\exp\left(\frac{qV}{kT}\right) - 1] \qquad \textbf{Eqn 4.15}$$

i) Figure 4.3 shows the small signal equivalent circuit for diode. L_w is the wire inductance. R_s is the series resistance of the quasi-neutral bases and the contact resistances. r_d is the differential resistance. C_j (C_{depl}) is the depletion capacitance of junction E-field. C_{diff} is the diffusion capacitance of injected minority carriers.

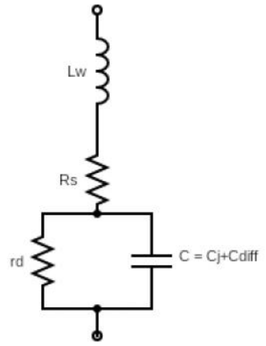

Figure 4.3: The small signal equivalent circuit of the diode