Filter Design Coursework

EEEE2046 （UNUK）

Practical Engineering Design Solutions and Project Development

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1. Introduction

In this section, the advantages, disadvantages, and applications of the Butterworth, Chebyshev, and Bessel filters are presented.

Butterworth filters have a very flat frequency response, which means that they pass through all frequencies at the same level of gain, up to the cut-off frequency. This makes it very suitable for applications where a linear phase response is required, for example in audio systems. As its Q (Quality factor of the filter) is a constant value, the ratio between the resistance and capacitance of the Butterworth filter is unique. Due to their ease of design and relatively low implementation costs, Butterworth filters are very widely used. The disadvantage of the Butterworth filter is that its frequency response is not as sharp as other types of filters. This means that it may not attenuate out-of-band frequencies as much as these other types of filters. In addition, the Butterworth filter has a slower roll-off rate, which means that the gain falls off more slowly at frequencies above the cut-off point, which can lead to higher levels of noise and distortion in the passband [1].

Due to the flat frequency response of the Butterworth filter, it can be used in medical imaging systems such as MRI or CT scanners to remove noise and artifacts from images to improve image accuracy and clarity [2]. It can also be used in automotive systems such as ECUs to smooth out noise signals from sensors such as temperature sensors or oxygen sensors, thereby improving the performance and reliability of the vehicle [3].

Chebyshev filters provide a steep roll-off in the frequency response, this is achieved by allowing ripple in the passband, which provides a clearer transition between the passband and the stopband compared to other types of filters. This means that the gain drops off more quickly at frequencies above the cut-off point. This allows them to better suppress high frequency noise and is well suited to applications that require a high degree of frequency selectivity, such as control systems. The disadvantage is that they have ripple in their passband, which can lead to higher distortion and noise compared to other types of filters. Furthermore, Chebyshev filters are more complex and expensive to implement [1].

Because the Chebyshev filter provides a steep roll-off in the frequency response to reject high frequency noise, it can be used in radar systems as a transmit and receive filter to reduce clutter and improve the detection range and resolution of the system [4]. It can also be used in audio and video equipment as an equaliser or crossover network to improve the clarity and separation of the audio or video signal [5].

One advantage of using a Bessel filter is its linear phase response, which means that it does not introduce any phase distortion into the signal. This is important in applications where the phase of the signal is critical, for example in communication systems where the signal passing through them will not have any delay, phase delays can lead to instability. The disadvantage of Bessel filters is that they have a relatively slow roll-off, making them less effective than other filter designs in rejecting high frequency noise, which can lead to higher levels of noise and distortion in the passband. In addition, Bessel filters are more complex and expensive to implement than other types of filters [6].

Bessel filters are used in communication systems such as transmit and receive filters for radio or cellular telephone systems to provide a linear phase response, reduce phase distortion and improve system performance [6].

2. Filter circuit design calculation and description

This section shows how a high pass Butterworth filter with a gain of 1, a pass band frequency of 141 Hz, a stop band frequency of 50 Hz, and a stop band attenuation of 30 dB was designed. To design the filter, two main calculations are made, the order of the filter and the magnitude of the components in the circuit.

Calculation of the high pass Butterworth filter order:

Equation 1 shows the transfer function for the N^{th} order high pass Butterworth filter. To calculate the order of the filter for this design, several variables about this function need to be known. 'ϵ' is the maximum gain in the pass band, which in this case is the unity gain filter, so it is 1. 'ω_p' is the pass band frequency (141 Hz). Because the stop band attenuation for this design is 30 dB, so the response of the filter is -33 dB when it is at the stop frequency (50 Hz). In addition, a relationship exists between 'ω' and frequency in Equation 2. This relationship calculation should be completed first when using frequency. Based on the values and relationships illustrated above, the calculation of the order N proceeds as follows:

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2(\frac{\omega p}{\omega})^{2N}}} \qquad \textbf{Eqn.1}$$

$$\omega = 2\pi f \qquad \textbf{Eqn.2}$$

$$33\ dB = 44.668$$

$$\frac{1}{44.668} = \frac{1}{\sqrt{1+(\frac{2\pi \times 141}{2\pi \times 50})^{2N}}}$$

$$N = 3.66$$

Since n must always be an integer, then the next highest value of 3.66 is n = 4, so a fourth order filter is required. To produce a fourth order Butterworth filter, two second order filters need to be cascaded together.

Calculate the magnitude of the components in the high pass Butterworth filter circuit:

Butterworth filter response can be implemented in the Sallen Key topology. Figure 2.1 shows a second order Sallen Key filter with the unity gain. For a high pass filter, 'Z_1' and 'Z_2' should be two capacitors, 'Z_3' and 'Z_4' should be two resistors.

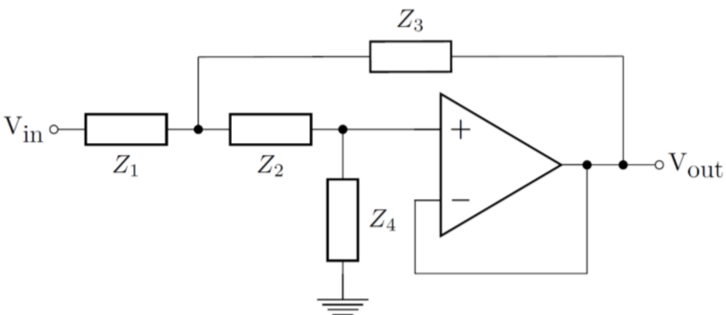

Figure 2.1: Second Order Sallen Key Filter Topology

To determine the individual capacitance and resistance values in the two second order circuits, Equation 3 is a simplified transfer function for the second order high pass Sallen Key filter expressed through the resistance and capacitance.

$$H(s) = \frac{s^2 R_1 R_2 C_1 C_2}{s^2 R_1 R_2 C_1 C_2 + s(R_2 C_2 + R_2 C_1) + 1} \qquad \textbf{Eqn.3}$$

Equation 4 is a more general form of the frequency response of a unity gain high pass filter, where ‘ ω_c’ is the corner frequency (cut-off frequency) between the pass band and stop band and ‘Q’ is the quality factor of the filter.

$$H(j\omega) = \frac{-(\frac{\omega}{\omega_c})^2}{-\left(\frac{\omega}{\omega_c}\right)^2 + \frac{j\omega}{Q\omega_c} + 1} \qquad \textbf{Eqn.4}$$

Combining Equation 3 and Equation 4 gives ‘ ω_c’ and ‘Q’ expressed in terms of capacitance and resistance, these expressions are Equations 5 and 6.

$$\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad \textbf{Eqn.5}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_2 C_2 + R_2 C_1} \qquad \textbf{Eqn.6}$$

The response of the filter is -3dB at the cut-off frequency, the following process can be listed according to Equation 1:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega_p}{\omega_c})^8}}$$

$$3\ dB = 1.414$$

$$\frac{1}{1.414} = \frac{1}{\sqrt{1 + (\frac{\omega_p}{\omega_c})^8}}$$

$$\omega_p = \omega_c$$

Therefore, the cut-off frequency is equal to the pass band frequency (141 Hz).

For the fourth order Butterworth filter, the polynomials are represented by Equation 7, which is the multiplication of two second order filters. From Equation 4 it is known that the quadratic term coefficient in Equation 7 is ‘ $\frac{1}{{\omega_c}^2}$ ’ and the primary term coefficient is ‘ $\frac{1}{Q\omega_c}$ ’. Since the quadratic term coefficient in Equation 7 is 1, then ‘ ω_c’ is 1 and its primary term coefficient is ‘ $\frac{1}{Q}$ ’. The two primary term coefficients are 0.765 and 1.848 respectively and the ‘Q’ values for the two filters are calculated as follows:

$$(1+0.765s+s^2)(1+1.848s+s^2) \qquad \textbf{Eqn.7}$$

$$Q_1=\frac{1}{0.765}=1.307$$

$$Q_2=\frac{1}{1.848}=0.541$$

With the values of ‘ ω_c’ and ‘Q’, the capacitance and resistance required for the filter can be found. For ease of calculation, the capacitances in the circuit are set to 100 nF. According to Equation 5, and 6, the resistance of the first second order filter is calculated as follows:

$$Q_1=\frac{\sqrt{R_1R_2C_1C_2}}{R_2C_2+R_2C_1}=\frac{1}{\omega_c(R_2C_2+R_2C_1)}=1.307$$

$$C=C_2=C_1=100\text{ nF}$$

$$R_2=\frac{1}{2\times 2\pi\times 141\times 100\times 10^{-9}\times 1.307}=4318.13\ \Omega$$

$$\omega_c=\frac{1}{\sqrt{R_1R_2C_1C_2}}=2\pi\times 141$$

$$R_1=(\frac{1}{2\pi\times 141})^2\div 4318.13\div(100\times 10^{-9})^2=29505.75\ \Omega$$

For another second order circuit, the resistance is calculated as follows:

$$Q_2=\frac{\sqrt{R_3R_4C_3C_4}}{R_4C_4+R_4C_3}=\frac{1}{\omega_c(R_4C_4+R_4C_3)}=0.541$$

$$R_4=\frac{1}{2\times 2\pi\times 141\times 100\times 10^{-9}\times 0.541}=10432.15\ \Omega$$

$$\omega_c=\frac{1}{\sqrt{R_3R_4C_3C_4}}=2\pi\times 141$$

$$R_3=(\frac{1}{2\pi\times 141})^2\div 10432.15\div(100\times 10^{-9})^2=12213.16\ \Omega$$

The component values for the two second-order filter circuits are shown below:

$$\begin{cases}R_1=29505.75\ \Omega\\ R_2=4318.13\ \Omega\\ C_2=C_1=100\text{ nF}\end{cases}\qquad\begin{cases}R_3=12213.16\ \Omega\\ R_4=10432.15\ \Omega\\ C_3=C_4=100\text{ nF}\end{cases}$$

Based on the two sets of resistor and capacitor values selected above, the fourth order filter is constructed as shown in Figure 2.2.

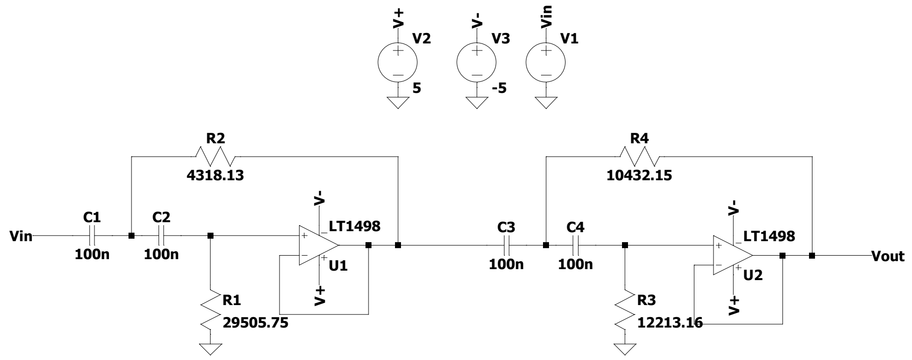

Figure 2.2: Designed fourth order Butterworth filter

Select the appropriate component value from the E24 range:

Considering that in practice the components cannot be selected so precisely, the resistors and capacitors of the circuit are reselected according to the component values of the E24 range. Table 1 shows how the components in the circuit have been reselected. Figure 2.3 shows the filter circuit built from the reselected components.

Table 1: Components reselected according to E24

	Second filter 1				**Second filter 2**			
	R_1 (Ω)	**R_2 (Ω)**	**C_1 (nF)**	**C_2 (nF)**	**R_3 (Ω)**	**R_4 (Ω)**	**C_3 (nF)**	**C_4 (nF)**
Precise values	29505.75	4318.13	100	100	12213.16	10432.15	100	100
E24 range	30000	4300	100	100	12000	10000	100	100

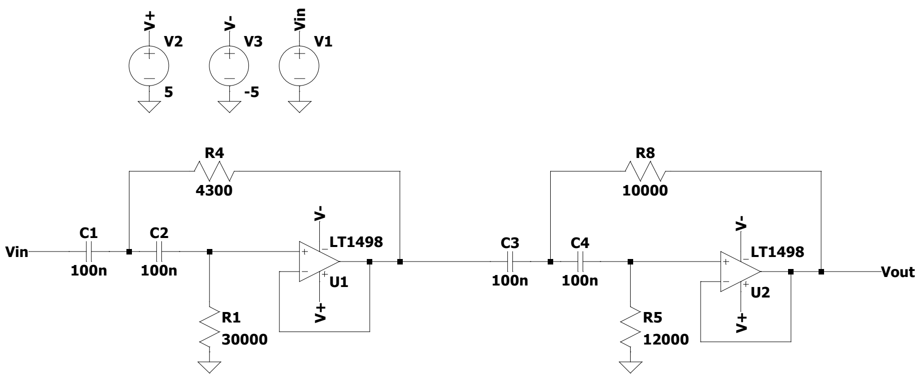

Figure 2.3: Designed fourth order Butterworth filter in E24 range

3. Results: simulation results

For AC simulation:

Figure 3.1 illustrates the output results of the design circuit with the AC input set to 1 V. Here 1000 points were used in the AC sweep to obtain a smooth response. The frequency range was set to be 1-1000 Hz.

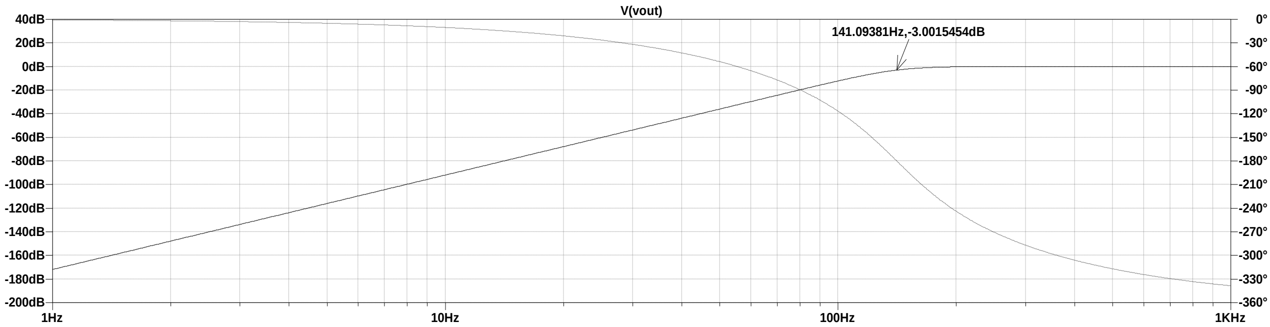

Figure 3.1: AC simulation result

Figure 3.1 marks the low cut off frequency of 141.09381 Hz when the response of the circuit is -3 dB. For this high pass filter, the expected low cut off frequency is 141 Hz, and the simulated results are negligibly different from the expected value. The simulation results show that the designed filter fully satisfies the requirements, and this circuit is in accordance with the design.

For transient response simulation.:

Figures 3.2, 3.3, and 3.4 show the results of the transient response simulation when the input voltage source has an amplitude of 1 V and frequencies of 10 Hz, 141 Hz, and 500 Hz respectively. To be able to see about 10 cycles of input, the simulation lengths are selected as 100 ms, 100 ms, and 20 ms respectively.

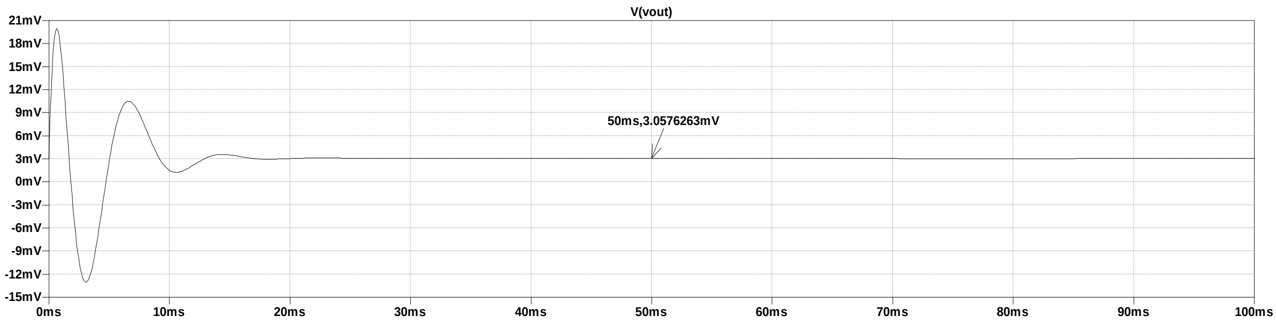

Figure 3.2: Transient response simulation result (Input frequency = 10 Hz)

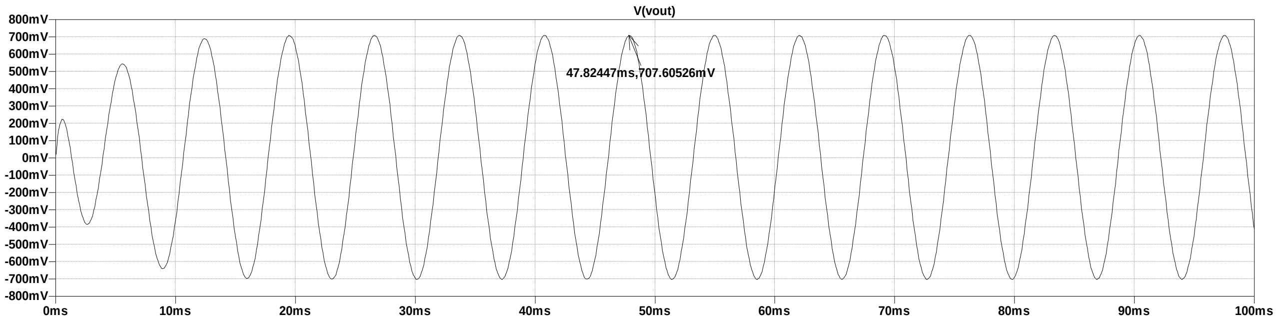

Figure 3.3: Transient response simulation result (Input frequency = 141 Hz)

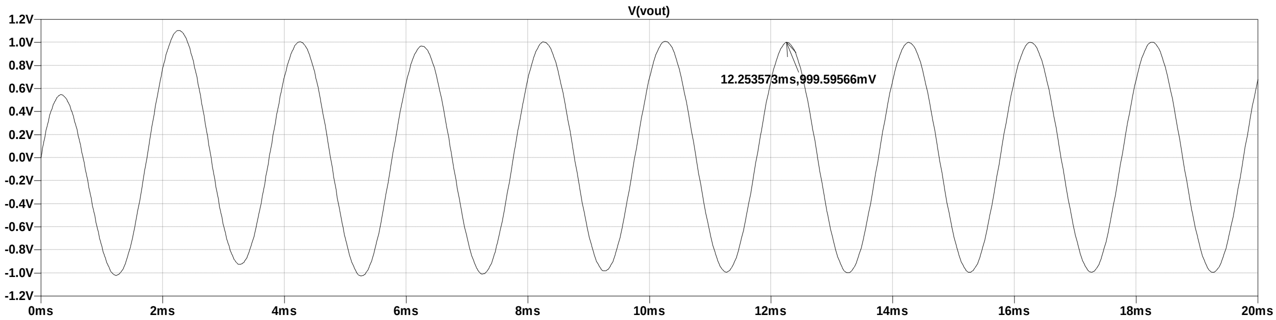

Figure 3.4: Transient response simulation result (Input frequency = 500 Hz)

As can be observed in the three figures, the magnitude of the output voltage at which the simulation results stabilise is marked. The output voltage results at different frequencies and their expected values are recorded in Table 2.

Table 2: Comparison of transient response simulation results with expected values

Input frequency (Hz)	Output amplitude (V)	Expected output amplitude (V)
10	0.031	0
141	0.707	0.707
500	0.999	1

In theory, for this high pass filter, the desired output voltage should be equal to 0 when the input frequency is less than the stopband (50 Hz) and equal to 1 when the input frequency is greater than the passband (141 Hz). This is because input frequencies below the passband are filtered out. When the input frequency is equal to the passband (141 Hz), the desired voltage should be $\frac{1}{\sqrt{2}}$ (0.707). In a realistic situation, the voltage does not change immediately, so there may be errors.

A comparison of the data obtained from Table 2 shows that the output voltages obtained at the three different input frequencies do not differ much from the expected values and are almost exactly as expected. This means that the circuit meets the expected design and has a good performance.

4. Discussion

High cut-off frequency:

Theoretically, this high pass filter should not have a high cut-off frequency, but the LT1498 element used in this circuit has a limited frequency of 10 MHz. The frequency range of the AC simulation is enlarged as shown in Figure 4.1. It can be observed that the response of the circuit is -3dB when the frequency is 10 MHz. Therefore, for this circuit, the high cut-off frequency is 10 MHz due to the frequency limitation of the chosen amplifier element LT1498.

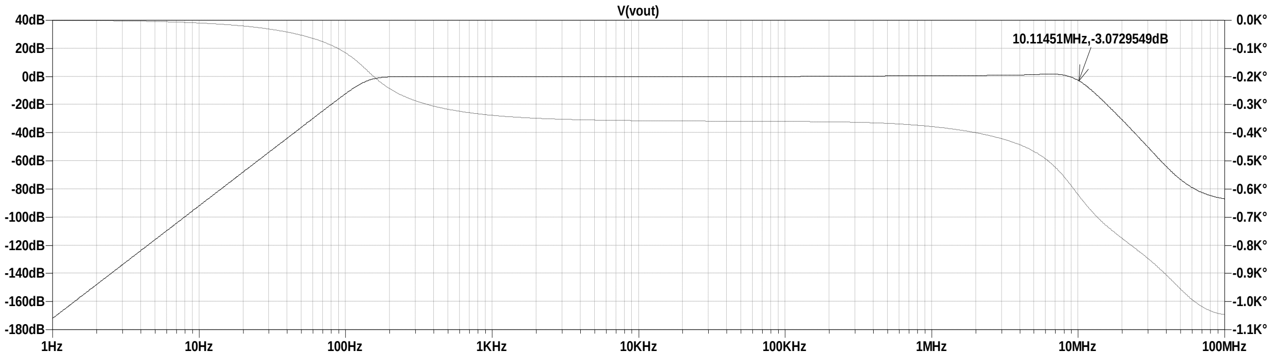

Figure 4.1: AC simulation result with high frequency range

Results of changing to E24:

Figure 4.2 shows the AC simulation results of the circuit when the component values in the design are replaced with E24. As can be observed from the graph, the new -3 dB low cut off frequency is 141.09381 Hz, which has not changed from the original. This indicates that the components chosen for the circuit are very suitable and sufficient to support the design in the E24 range.

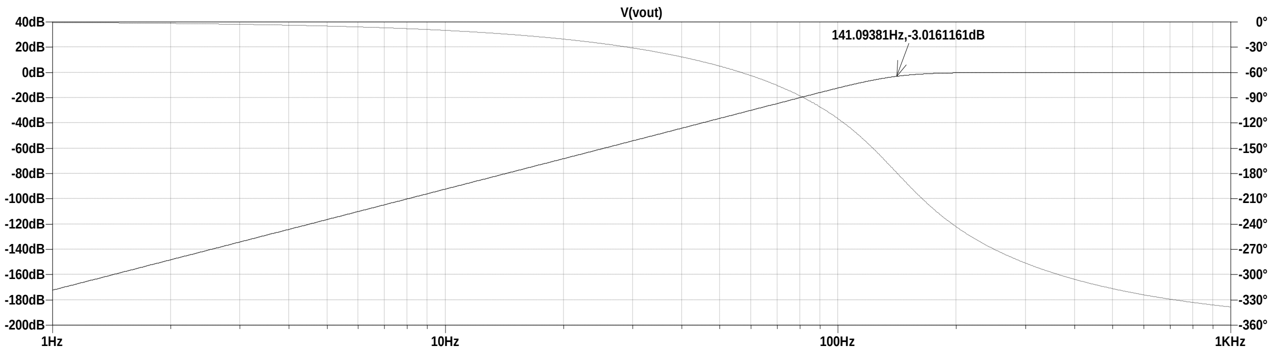

Figure 4.2: AC simulation result with E24 range

Tolerance:

In a realistic circuit design, resistors may differ by $\pm 5\%$ from their labeled size and capacitors may differ by $\pm 10\%$. This is called the tolerance of component. Here the effect of this tolerance on the result of the circuit design is investigated. Figure 4.3 shows the results of the AC simulation with all resistances increased by 5% and capacitances increased by 10% for a design filter with components selected in the E24 range. Figure 4.4 shows the results of the AC simulation with all resistors reduced by 5% and capacitance reduced by 10%.

The -3 dB low cut off frequency observed in these figures and their corresponding component selection values are recorded in Table 3. The first row of data in the table shows the selected values in the original E24 range, the second row shows the increasing component values, and the third row shows the decreasing values.

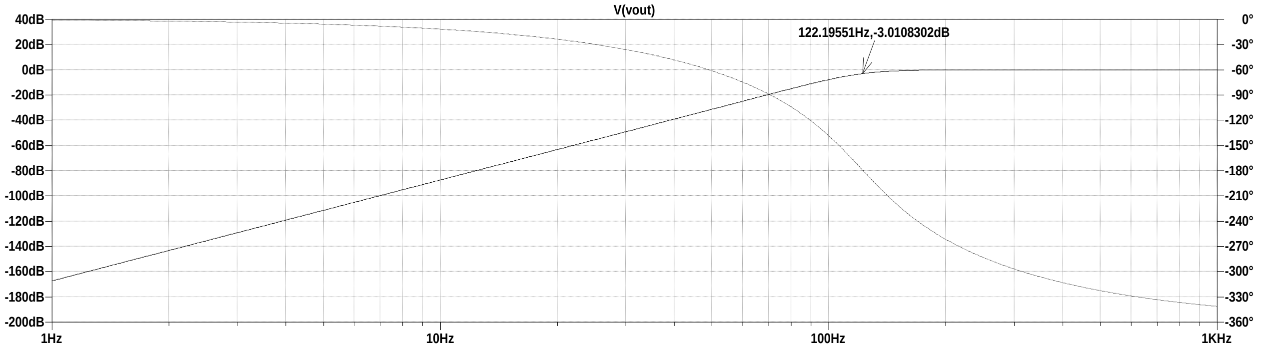

Figure 4.3: AC simulation result with +5% resistors and +10% capacitors

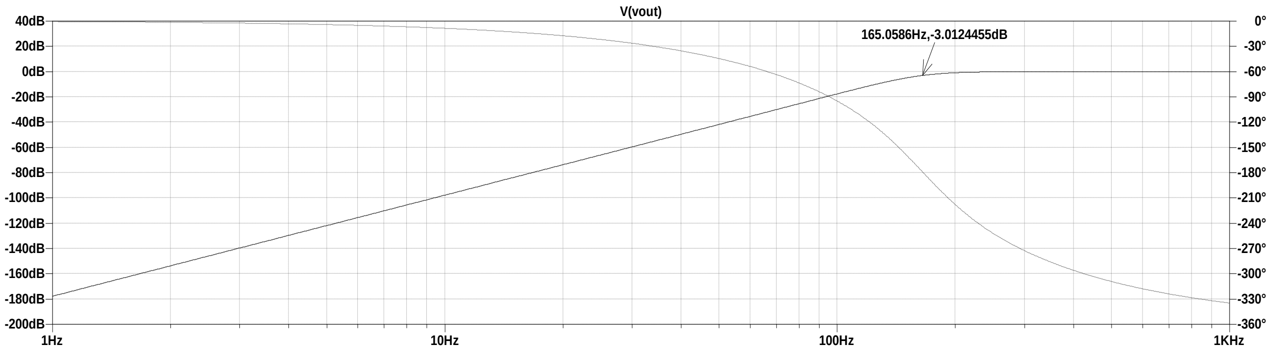

Figure 4.4: AC simulation result with -5% resistors and -10% capacitors

Table 3: Cut-off frequency for different component selection values

R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)	C_1, C_2, C_3, C_4 (nF)	Cut-off Frequency (Hz)
30000	4300	12000	10000	100	141.1
31500	4515	12600	10500	110	122.2
28500	4085	11400	9500	90	165.1

With the cut off frequencies for each case, here is the procedure for calculating the frequency error: (α when the components are both increasing and β when they are decreasing)

$$\alpha = \frac{F_{E24} - F_+}{F_{E24}} = \frac{141.1 - 122.2}{141.1} = \frac{18.9}{141.1} = 13.4\% \qquad \textbf{Eqn.8}$$

$$\beta = \frac{F_- - F_{E24}}{F_{E24}} = \frac{165.1 - 141.1}{141.1} = \frac{24}{141.1} = 17.0\% \qquad \textbf{Eqn.9}$$

From the results of the calculations, it can be concluded that the selection of smaller components causes more errors in the design of the circuit compared to larger components.

Amplification:

To change the amplification within the pass band, the circuit was modified as shown in Figure 4.5.

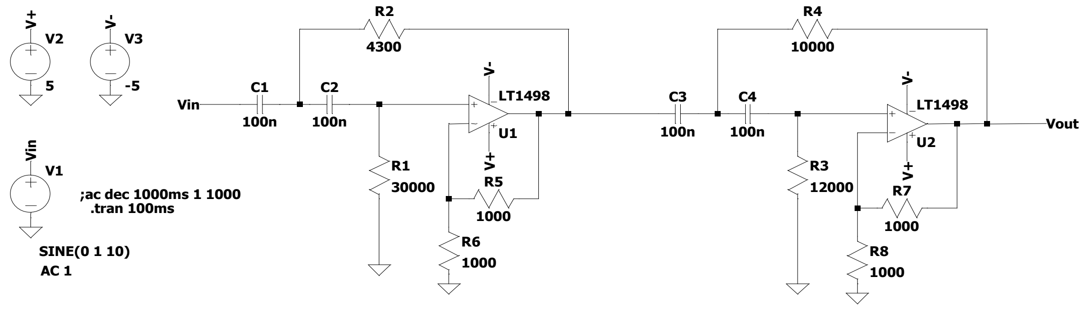

Figure 4.5: Designed fourth order Butterworth filter with changed amplification

The gain of the circuit is determined by the addition of four resistors (R_5, R_6, R_7, R_8) and can be represented by the following equation.

$$Gain = \left(1+\frac{R_5}{R_6}\right)\left(1+\frac{R_7}{R_8}\right) = \left(1+\frac{1000}{1000}\right)\left(1+\frac{1000}{1000}\right) = 4 \qquad \textbf{Eqn.10}$$

5. References

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