Chapter 4 - Probability:

Rare Event Rule for Inferential Statistics: If under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

Vocabulary -

1. **Event: An event is any collection of results or outcomes of a procedure.**
2. **Simple Event: A single outcome (A simple event is an outcome or an event that cannot be further broken down into simpler components.)**
3. **Sample Space: The sample space for a procedure consists of all possible simple events. That is, the sample space consists of all outcomes that cannot be broken down any further.**

Eg:

Procedure	Event	Sample space
Flipping a coin Flip one time	Head	Head/Tail {H, T}

Procedure	Event	Sample space
Flipping a coin Flip a coin 3 times	1 Head 2 Tails	{H H H / H H T / H T T / *H T H / T T T / *T T H / *T H T / T H H} * = 3 Ways we can achieve the event. All single outcomes are separately called a simple event. If we put everything in flower brackets it's called sample space.

Procedure	Event	Sample space
Flipping a coin Flip a coin 3 times	1 Tail 2 Heads 3 Heads 3 Tails	{**H H H / *H H T / H T T / *H T H / ++T T T / T T H / T H T / *T H H} * = First event can be achieved 3 times ** = Second event can be achieved 1 time ++ = Third event can be achieved one time. All single outcomes are separately called a simple event. If we put everything in flower brackets it's called sample space.

Probability - **The likelihood of an event occurring.**

Notation for Probabilities:

- **P denotes a probability.**
- **A, B, and C denote specific events.**
- **P(A) denotes the probability of event A occurring.**

3 Types of Probability:

1. **Observed Probability(Relative Frequency Approximation of Probability) - Probability that is estimated based on observation.**

$$P(A) = \frac{number\ of\ times\ A\ occurred}{number\ of\ times\ the\ procedure\ was\ repeated}$$

**** What did Happen**

2. **Classical Probability - Probability based on the chance of an event occurring. Each simple event must have an equal chance of occurring.**

$$P(A) = \frac{number\ of\ times\ A\ could\ occur}{number\ of\ simple\ events}$$

**** What should happen.**

3. **Subjective Probability - Educated Guess**

Eg:

1. **The probability of selecting a Heart from a standard deck of cards.**

    P(Heart) = 13/52 = 25%

    Classical Probability - Should Happen

2. **Flip a coin 100 times, you get 64 tails.**

    P(T) = 64/100 = 64%

    Observed Probability - What did Happen

3. **Peyton completed 385 out of his first 528 passes. Find the probability that Peyton will complete a pass.**

    P(Complete a pass) = 385/528 = 72.91%

    Observed Probability - What did Happen

4. **From a deck of cards find the probability of randomly selecting A2**

    P(2) = 4/52 = 7.69%

    Classical Probability - Should Happen

5. **Cloning people:**

    Poll: 91 people said cloning is good, 901 people said cloning is bad, and 20 people had no opinions. Find the probability of Randomly selecting a person who thinks cloning is a good idea.

    P(Cloning Good) = 91/1012 = 8.99%

    Observed Probability - What did Happen

6. **Find the probability of A bird will poop on your car today.**

    Subjective Probability - Educative Guess

7. **Find the probability that if a couple has 3 kids two will be boys. (Assuming equal chance of a Boy or a Girl)**

   Procedure - Having 3 Children
   Event - 2 Boys, 1 Girl
   Simple Events - { BBB / *BBG / *BGB / BGG / GGG / GGB / GBG / *GBB }
   Sample Space - All the events.

   P(2B, 1G) = 3/8 = 37.5%

   Classical Probability - Should Happen

- **The probability of an impossible event is 0.**
- **The probability of an event that is certain to occur is 1.**
- **For any event A, the probability of A is between 0 and 1 inclusive. That is, 0 >= P(A) <= 1.**
- **The more of a procedure is repeated, the closer the observed probability will get to classical probability.**

Complementary Events - Events that are mutually exclusive (Can't happen at the same time). The complement of event A, denoted by $\overline{A}$ consists of all outcomes in which event A does not occur.

Eg:

Event - When we roll a dice of Getting 5 as a result.

Complement - Not getting 5 complements in 1,2,3,4,6.

P(5) = 1/6 = 16.67%

P($\overline{A}$) = 5/6 = 83.33%

The probability of an event plus the probability of the complement must be equal to 1

P(A) + P($\overline{A}$) = 1

Addition Rule:

Compound event - A compound event is any event combining two or more simple events.

Eg:

1. **The Probability of Rolling dice for 1 OR 5.**

   P(A or B) = Probability of A occurring or B occurring or Both A and B occurring in a single trial.

2. **P(Blond or Female) Not mutually exclusive.**

	Didn't Do It	Did It
Guilty	11 False Positive	72 True Positive
Not Guilty	85 True Positive	9 False Negative

1. **How many people were Guilty OR Did it**

   = 11 + 72 + 72 + 9 = Not Mutually Exclusive.

   = 11 + 72 + 9 = 92.

   P(G or NG) = 92/177 = 52%

Addition Rule -

P(A or B) = Requires elimination of Any "Double Count".

P(A or B) = P(A) + P(B) - P(A and B)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

where P (A and B) denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.

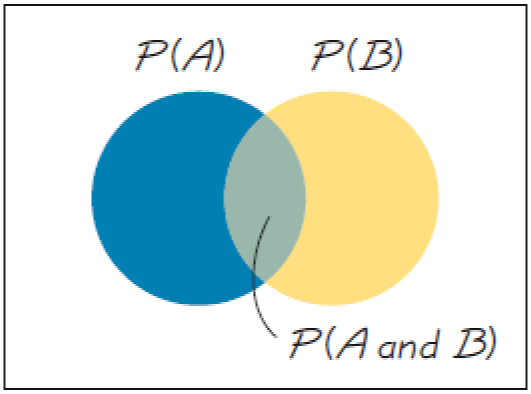

Figure 4-3 Venn Diagram for Events That Are Not Disjoint

Disjoint - Events which are mutually exclusive, meaning events can not happen simultaneously.

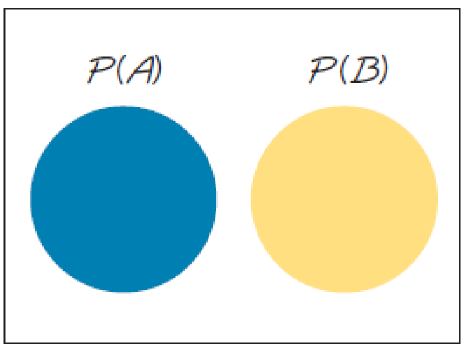

Figure 4-4 Venn Diagram for Disjoint Events

Eg:

1. **Probability of selecting a "Heart" or a "Spade" from a standard deck of cards.**

   P(Hearts or Spades) = P(Heart) + P(Spade) - P(Heart and Spade)

   = 13/52 + 13/52 - 0

   = 50%

2. **Probability of selecting a "Blond" or "Female"**

   Blond = 0.18
   Female = 0.60
   Blond Female = 0.12

   P(Blond or Female) = P(Blond) + P(Female) - P(Blond and Female)

   = 0.18 + 0.60 - 0.12

   = 66%

3. **P(Diamond or King) = P(D) + P(K) - P(D and K)**

   = 13/52 + 4/52 - 1/52

   = 0.25 + 0.076 - 0.019

   = 0.307 = 30.7%

Complementary events:

$$P(A \text{ or } \overline{A}) = P(A) + P(\overline{A}) - P(A \text{ and } \overline{A}) = 1$$

$$P(A) + P(\overline{A}) = 1$$

$$P(A) = 1 - P(\overline{A})$$

$$P(\overline{A}) = 1 - P(A)$$

Eg:

P(Girl) = 0.512

P(Boy) = $P(\overline{Girl)}$ **= 1 - 0.512 = 0.488**

Multiplication Rule:

P(A and B) = P(event A occurs in a first trial and event B occurs in a second trial)

Eg: Two Questions

1. **(T/F) - Pradeep drives an Audi**

2. **(MCQ) - Pradeep's favourite colour is**
    a. **Red**
    b. **Black**
    c. **Blue**
    d. **White**
    e. **Purple**

If you guess randomly, Find the Probability of Getting both Q correct.

(T, a) (T,b) (T,c) (T,d) (T,e)
(F,a) (F,b) (F,c) (F,d) (F,e)

Probability of getting both right = 1/10

Q: Find the probability of Selecting someone guilty and then Someone not guilty

	Didn't Do It	Did It
Guilty	11 False Positive	72 True Positive
Not Guilty	85 True Positive	9 False Negative

P(Guilty) = 83/177 = 46.89 %

And then

P(Not Guilty) = 94/176 = 53.40 %   (176 because we already found one guilty)

Conditional Probability: The Probability of an event occurring given that some other event has already occurred.

P(B|A) represents the probability of event B occurring after it is assumed that event A has already occurred. (We can read (B|A) as "B given A" or as "event B occurring after event A has already occurred.")

Independent Events: The occurrence of one event does not depend on the occurrence of another event or subsequent event. (Non-Independent events are Dependent).

If A & B are independent P(B|A) = P(B)

Dependent Events: The occurrence of one event depends on the occurrence of another event or subsequent event. (Dependent events are independent)

Eg:

1. If we roll a dice:

   P(2|3) = P(2) — — Independent

2. If we draw a card:

   With Replacement : P(Q|9) = 4/52 = 1/13 = 7.69 % – – – Independent

   Without Replacement : P(Q|9) = 4/51 = 7.84 % — — — dependent

   With Replacement: P(Q|Q) = 4/52 = 1/13 – – – Independent

   Without Replacement : P(Q|Q) = 3/51 = — — — dependent (One Q was already taken)

   With Replacement: P(H|J of D) = 13/52 = — — — Independent

   Without Replacement: P(H|J of D) = 13/51 = — — — dependent

Multiplication Rule:

P(A and(Then) B) = P(A) * P(B|A)

If A and B are independent events, P(B|A) is the same as P(B).

P(A and B) = P(A)*P(B)

Eg:

1. In a bag of marbles there were 3 Red, 2 Blue and 4 Green. Find,

   P(G and B) with Replacement = P(G) * P(B|G) = 4/9 * 2/9 = 9.87 %

   P(G and B) without Replacement = P(G) * P(B|G) = 4/9 * 2/8 = 11.11 %

   P(R and R) without Replacement = P(R) * P(R|R) = 3/9 * 2/8 = 8.33 %

   P(B and B and B) without Replacement = P(B) * P(B|B) * P(B|B) = 2/9 * 1/8 * 0/7 = 0 %

2. Roll a dice

   P(1 and 2 and 3 and 4) = P(1) * P(2) * P(3) * P(4) = ⅙ * ⅙ *⅙ * ⅙ = 1/1296

3. Cards without Replacement

   P(A and K and Q and J and 10) = 4/52 * 4/51 * 4/50 * 4/49 * 4/48 = 1024/31,18,75,200

Multiplication Rule: Complements and Conditional Probability:

Complements: The Probability of "At Least One":

Let's suppose that we want to find the probability that among 3 children, there is "at least one" girl. In such cases, the meaning of the language must be clearly understood:

1. "At least one" is equivalent to "one or more."

2. The complement of getting at least one item of a particular type is that you get no items of that type. For example, not getting at least 1 girl among 3 children is equivalent to getting no girls (or 3 boys).

3. **The complement of at least one is none.**

P(At least one) = 1 - P(None)

Eg:

1. **Flip a coin 3 times. What is the probability of getting at least one head.**

{ HHH, HTH, HHT, HTT, TTT, TTH, THT, THH }

P (At least one head) = 7 / 8

P (At least one head) = 1 - P (NONE)

= 1 - P (No Heads)

= 1 - P (All Tails)

= 1 - P(T and T and T)

= 1 - ½ * ½ * ½

= 1 - ⅛

= ⅞

2. **Flip a coin 20 times. What is the probability of getting at least one head.**

P (At least one head) = 1 - P (NONE)

= 1 - P (No Heads)

= 1 - P (All Tails)

= 1 - P(T and T and T……and T)

= 1 - ½ * ½ * ½ …….*½

= 1 - $1/2^{20}$
= 1 - 1/ 1048576

= 0.999

Counting:

Fundamental Counting Rule:

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of m * n ways.

1. **Security code - 1st digit cannot be zero -**

   ___9___ ___10_____ ___10___ ___10___ = 9 because 0 cannot be the choice

   9*10*10*10 = 9000 total combinations

   If you have only one chance = 1/9000

2. **Security code - 1st digit can't be 0 or 1 and the last digit cannot be zero -**

   ___8___ ___10_____ ___10___ ___10___ ___9___

   8 because can't be 0 or 1 & 9 because 0 cannot be the choice

   8*10*10*10*9 = 72000 total combinations

   If you have only one chance = 1/72000

3. **Apple, Orange, Banana, Cherry, Kiwi**

   ___5___ _____4_____ ___3___ ___2___ ___1___ = 5 * 4 * 3 * 2 * 1 = 120 Ways

   Can also write as 5! = 5*4*3*2*1 = 120

Factorial:

- **0! = 1**
- **For any set of 'n' different items there are n! Diff arrangements possible.**

Eg:

1. **There are 7 cool rides at Disneyland. How many diff ways could you ride all of them?**

   7! = 7*6*5*4*3*2*1 = 5040 ways.

Permutations Rule (When Items Are All Different):

Permutation - A different arrangement of items.

Requirements:

1. There are n different items available.

2. We select r of the n items (without replacement).

3. We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately)

If the preceding requirements are satisfied, the number of permutations (or sequences) of r items selected from n different available items (without replacement) is

$$_nP_r = \frac{n!}{(n-r)!}$$

Eg:

1. A bet on an exact in a race is won by correctly selecting the horses that finish first and second, and you must select those two horses in the correct order. The 132nd running of the Kentucky Derby had a field of 20 horses. If a bettor randomly selects two of those horses for an exact bet, what is the probability of winning?

We have n = 20 horses available, and we must select r = 2 of them without replacement. The number of different sequences of arrangements is found as shown:

$$_nP_r = \frac{n!}{(n-r)!} = \frac{20!}{(20-2)!} = 380$$

There are 380 different possible arrangements of 2 horses selected from the 20 that are available. If one of those arrangements is randomly selected, there is a probability of 1/380 that the winning arrangement is selected.

2. There are 10 people who want to be president, VP, Speaker, VS. We want to select 4 people to lead the country.

10P4 = 10! / 6! = 5040 ways.

Permutations Rule (When Some Items Are Identical to Others):

Requirements:

1. There are n items available, and some items are identical to others.

2. We select all of the n items (without replacement).

3. We consider rearrangements of distinct items to be different sequences. If the preceding requirements are satisfied, and if there are n1 alike, n2 alike,……nk alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

n! = Total items

N1!, n2!…… = The counts of non-distinct items.

Eg:

1. **3 Apples, 2 Oranges and 5 Bananas.**

   $\frac{10!}{3!\,2!\,5!} = \frac{10!}{3!\,2!\,5!} =$

2. **How many different ways could you arrange the word "Statistics"?**

   $\frac{10!}{3!\,3!\,2!}$ **= 50400**

3. **How many could you plan to have a family with 7 girls and 5 boys?**

   $\frac{12!}{7!\,5!} =$

Combinations Rule:

Requirements:

1. There are n different items available.

2. We select r of the n items (without replacement).

3. We consider rearrangements of the same items to be the same. (The combination ABC is the same as CBA.)

If the preceding requirements are satisfied, the number of combinations of r items selected from n different items is

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

1. **Go on 5 rides out of 25 rides at magic mountain you don't care about which order you go on the rides, how many combinations of 5 rides could we select?**

   25C5 = $\frac{25!}{(25-5)!\,5!}$ **= 53130 Combinations.**

Baye's Theorem: **Bayes' Theorem states that the conditional probability of an event, based on the occurrence of another event, is equal to the likelihood of the second event given the first event multiplied by the probability of the first event.**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad where \;\; P(B) \neq 0$$

Ch. 4: Probability

$P(A \text{ or } B) = P(A) + P(B)$ if A, B are mutually exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

if A, B are not mutually exclusive

$P(A \text{ and } B) = P(A) \cdot P(B)$ if A, B are independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if A, B are dependent

$P(\overline{A}) = 1 - P(A)$ Rule of complements

$${}_nP_r = \frac{n!}{(n-r)!}$$ Permutations (no elements alike)

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$ Permutations (n_1 alike, . . .)

$${}_nC_r = \frac{n!}{(n-r)!\, r!}$$ Combinations