1. Modeling of carbon cycle

In [16]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import scipy
```

In [457…
```
CO2=pd.read_csv("C:\\Users\\yuanwenting\\Documents\\ESE5023\\assignment\\global.1751_2008.csv",",")
CO2
```

```
C:\ProgramData\Anaconda3\lib\site-packages\IPython\core\interactiveshell.py:3369: FutureWarning: In a future version of pandas all arguments of read_csv except for the argument 'filepath_or_buffer' will be keyword-only.
  exec(code_obj, self.user_global_ns, self.user_ns)
```

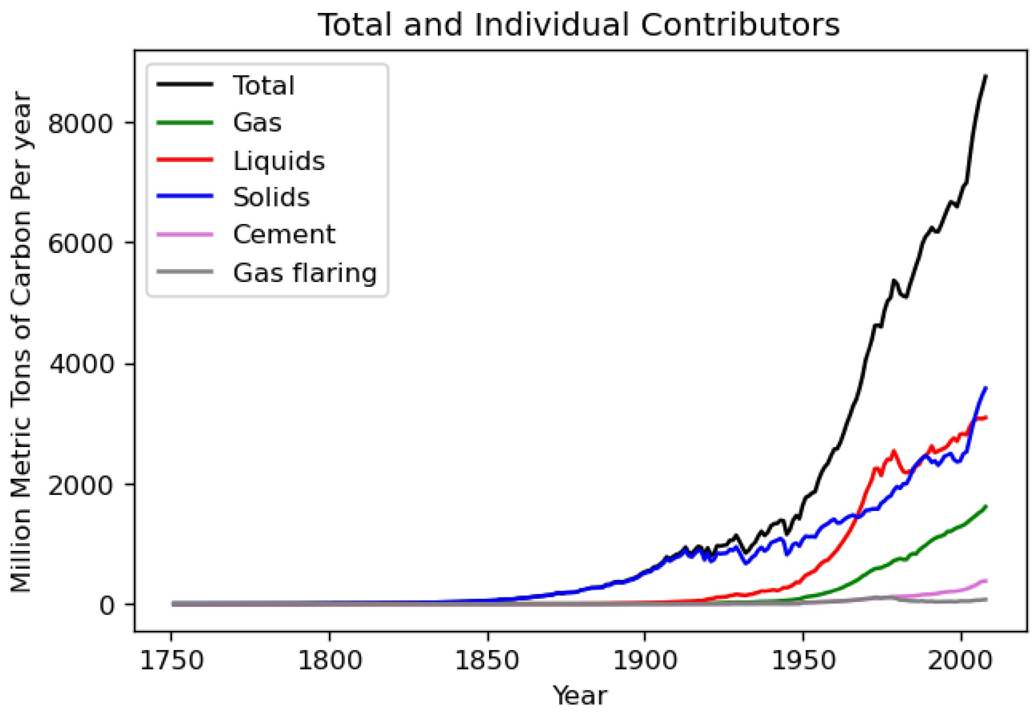
Out[457]:

	Year	Total	Gas	Liquids	Solids	cement production	gas flaring	per capita
0	1751	3	0	0	3	0	0	NaN
1	1752	3	0	0	3	0	0	NaN
2	1753	3	0	0	3	0	0	NaN
3	1754	3	0	0	3	0	0	NaN
4	1755	3	0	0	3	0	0	NaN
...	...	...	...	...	...	...	...	...
253	2004	7782	1431	3027	2971	298	55	1.21
254	2005	8086	1473	3071	3162	320	61	1.24
255	2006	8350	1519	3080	3333	355	62	1.27
256	2007	8543	1551	3074	3468	382	68	1.28
257	2008	8749	1616	3095	3578	386	73	1.30

258 rows × 8 columns

In [458…
```
plt.figure(figsize=(6,4),dpi=120)
plt.plot(CO2['Year'],CO2['Total'],c='k',linestyle='-')
plt.plot(CO2['Year'],CO2['Gas'],c='g',linestyle='-')
plt.plot(CO2['Year'],CO2['Liquids'],c='r',linestyle='-')
plt.plot(CO2['Year'],CO2['Solids'],c='b',linestyle='-')
plt.plot(CO2['Year'],CO2['cement production'],c='orchid',linestyle='-')
plt.plot(CO2['Year'],CO2['gas flaring'],c='grey',linestyle='-')
plt.legend(['Total','Gas','Liquids','Solids','Cement','Gas flaring'],loc=2)

plt.title('Total and Individual Contributors')
plt.xlabel('Year');plt.ylabel('Million Metric Tons of Carbon Per year')
```

Out[458]: Text(0, 0.5, 'Million Metric Tons of Carbon Per year')



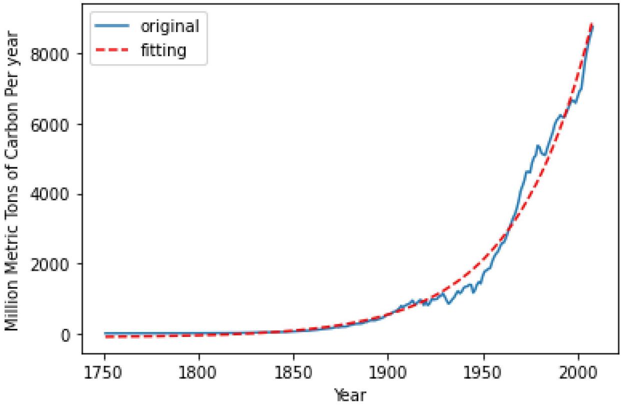
1.1 without the buffer effect

In [459…

```
#首先确定排放速率r, inspired by 焦小乔同学
#进行Curve fitting, 参考section 10: Optimization and fitting with scipy.optimize部分
from scipy.optimize import curve_fit

#从上面的图可以看出排放大致成指数上升, 因此这里我们定义一个指数函数
#指数拟合参考https://blog.csdn.net/Dontla/article/details/104081795
def r_func(x, a, b, c):
    return a * np.exp(b * x) + c

#p0是猜测,maxfev是最大迭代次数,参考https://codebug.vip/questions-307354.htm
popt,pcov = curve_fit(r_func, CO2['Year'], CO2['Total'], p0=[1,0,1], maxfev=2000)

#popt[0]是a, popt[1]是b, popt[2]是c
y_fit= [r_func(i, popt[0], popt[1], popt[2]) for i in CO2['Year']]

#输出原曲线和拟合曲线
plt.plot(CO2['Year'], CO2['Total'],label='original')
plt.plot(CO2['Year'], y_fit,'r--',label='fitting')
plt.legend(loc=2)
plt.xlabel('Year');plt.ylabel('Million Metric Tons of Carbon Per year')
```

```
#输出参数最佳值
print(popt)
```

```
[ 4.82316522e-18  2.43954472e-02 -1.12313831e+02]
```



In [460…

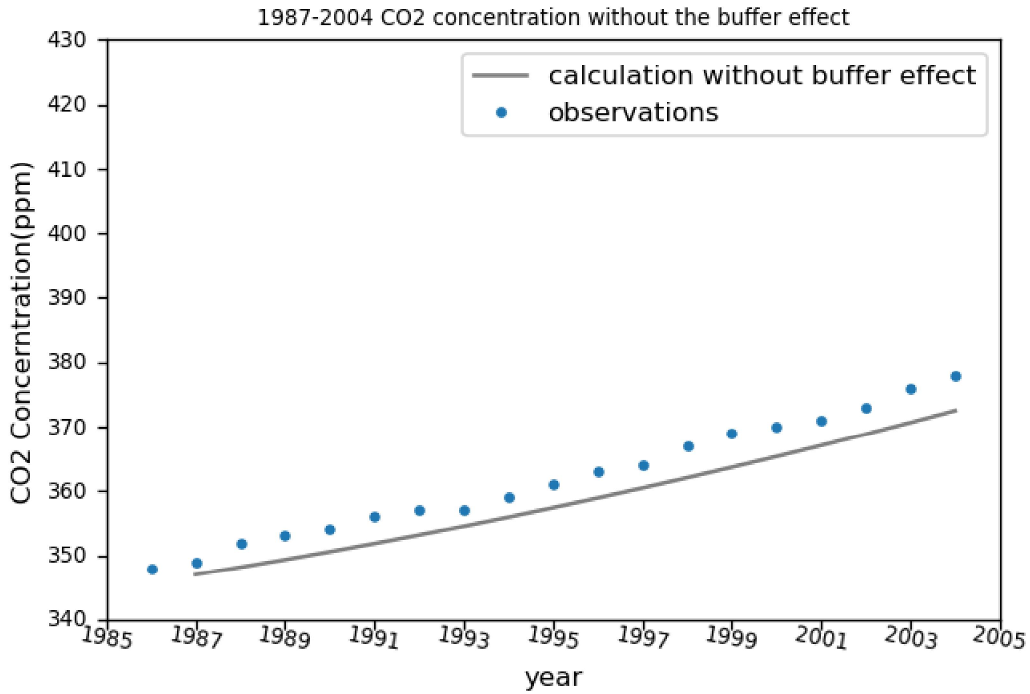
```
#排放速率r=4.82316522e-18*exp(2.43954472e-02 * x)-1.12313831e+02; r=a*e^(bx)+c,r'=a*b*e^(bx)
#scipy.integrate.odeint使用参照https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.odeint.html
from scipy.integrate import odeint
def pend(y,x1,k12,k21):
    N1, N2, r = y
    dydt = [-k12*N1+k21*N2+r,k12*N1-k21*N2,2.43954472e-02*(r+1.12313831e+02)]
    return dydt

#from 1987 to 2004
x1=np.arange(1987,2005)
r1=r_func(x1, 4.82316522e-18,2.43954472e-02,-1.12313831e+02)

#初始条件，来自paper,1pg=10^15g=1000百万公吨
k12=105/740;k21=102/900;N1=740*1000;N2=900*1000
y0=[N1,N2,r1[0]]
sol=odeint(pend, y0,x1, args=(k12,k21))/1000/740*347 #转化为ppm

#画根据公式1-2的积分曲线
plt.figure(figsize=(6,4),dpi=120)
plt.plot(x, sol[:,0], color='gray', label='calculation without buffer effect')

#根据链接Mauna Loa CO2 annual mean data中数据画实际观测图，选取1986-2004年段数据
CO2_ppm=pd.read_csv("C:\\Users\\yuanwenting\\Documents\\ESE5023\\assignment\\co2_annmean_mlo.csv",",")
CO2_ppm_1986_2004=CO2_ppb.iloc[27:46,:]
plt.plot(CO2_ppm_1986_2004['year'],CO2_ppm_1986_2004['mean'],".",label='observations')

#设置横纵坐标轴刻度，设置legend位置，设置横纵坐标轴标签，设置标题
plt.yticks(ticks=np.arange(340,440,10), fontsize=8, rotation=0, ha='right', va='center')
plt.xticks(ticks=np.arange(1985,2006,2), fontsize=8, rotation=-10, ha='center', va='center')
plt.legend(loc='best')
plt.xlabel('year');plt.ylabel('CO2 Concerntration(ppm)')
plt.title('1987-2004 CO2 concentration without the buffer effect',fontsize=8)
```

```
#相比paper中的fig.2，拟合值似乎偏低，因为我们的拟合初始值存在一定程度取小（与观测值原始数据对比发现）
#此外，排放速率从1751年开始拟合，相比直接从1987年拟合可能偏小，因此造成without buffer effect拟合曲线偏低
```

```
C:\ProgramData\Anaconda3\lib\site-packages\IPython\core\interactiveshell.py:3369: FutureWarning: In a future version of pandas all arguments of read_csv except for the argument 'filepath_or_buffer' will be keyword-only.
  exec(code_obj, self.user_global_ns, self.user_ns)
```

Out[460]:

```
Text(0.5, 1.0, '1987-2004 CO2 concentration without the buffer effect')
```



1.2 with the buffer effect

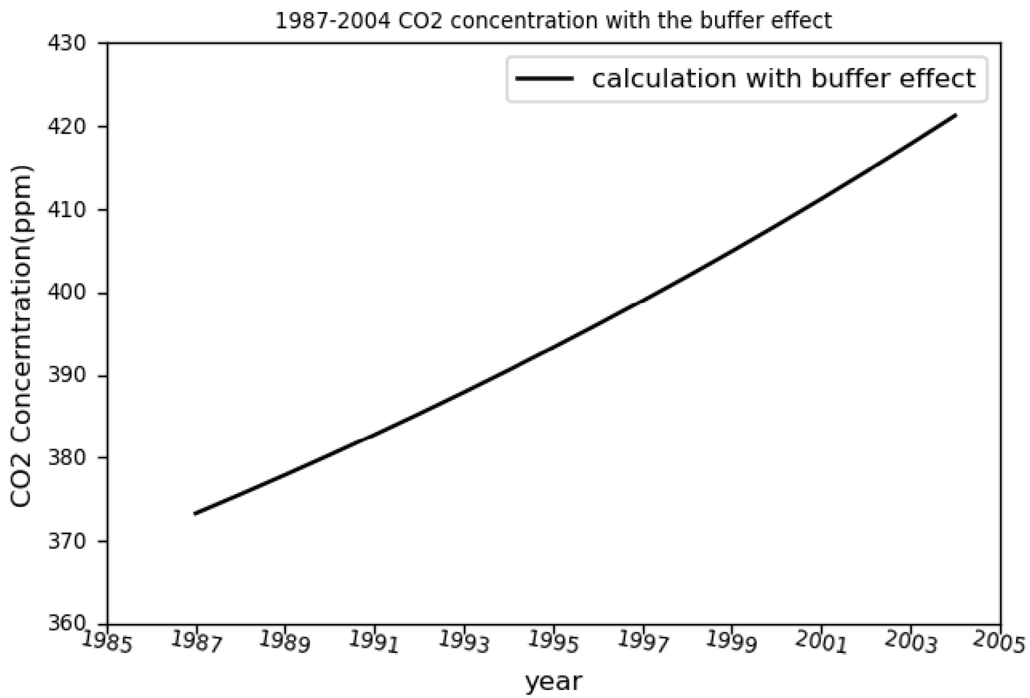
In [461…

```
def pend_2(y,t2,k12,k21):
    N1,N2,r2 = y
    z = N1/1000/740*347  #z is the atmospheric CO2 concentration of ppm unit.
    N2_0 = 842*1000      #In the preindustrial era the equilibrium value of carbon in the surface ocean(842pg/yr)
    ξ  = 3.69+1.86e-2*(z)-1.8e-6*(z**2)  #the buffer factor
    dydt = [-k12*N1+k21*(N2_0+ξ*(N2-N2_0))+r2, k12*N1-k21*(N2_0+ξ*(N2-N2_0)),2.43954472e-02*(r2+1.12313831e+02)]
    return dydt

#从前工业时代开始计算（1750年）
x2=np.arange(1750,2005)
r2=r_func(x2, 4.82316522e-18,2.43954472e-02,-1.12313831e+02)

#初始条件，来自paper fig.3,1pg=10^15g=1000百万公吨
k12=105/740;k21=102/900;N1=615*1000;N2=842*1000
```

```
y0=[N1,N2,r2[0]]
sol2 = odeint(pend_2, y0, x2, args=(k12,k21))/740/1000*347 #转化为ppm

#根据公式3-4画1987-2004年的CO2浓度积分曲线
plt.figure(figsize=(6,4),dpi=120)
plt.plot(x2[237:255], sol2[237:255,0], 'k', label='calculation with buffer effect')

#设置横纵坐标轴刻度，设置legend位置，设置横纵坐标轴标签，设置标题
plt.yticks(ticks=np.arange(360,440,10), fontsize=8, rotation=0, ha='right', va='center')
plt.xticks(ticks=np.arange(1985,2006,2), fontsize=8, rotation=-10, ha='center', va='center')
plt.legend(loc='best')
plt.xlabel('year');plt.ylabel('CO2 Concerntration(ppm)')
plt.title('1987-2004 CO2 concentration with the buffer effect',fontsize=8)
```

Out[461]:

```
Text(0.5, 1.0, '1987-2004 CO2 concentration with the buffer effect')
```



In [462…

```
#辅助理解
x2=np.arange(1750,2005)
x2[237:255]
```

Out[462]:

```
array([1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997,
       1998, 1999, 2000, 2001, 2002, 2003, 2004])
```

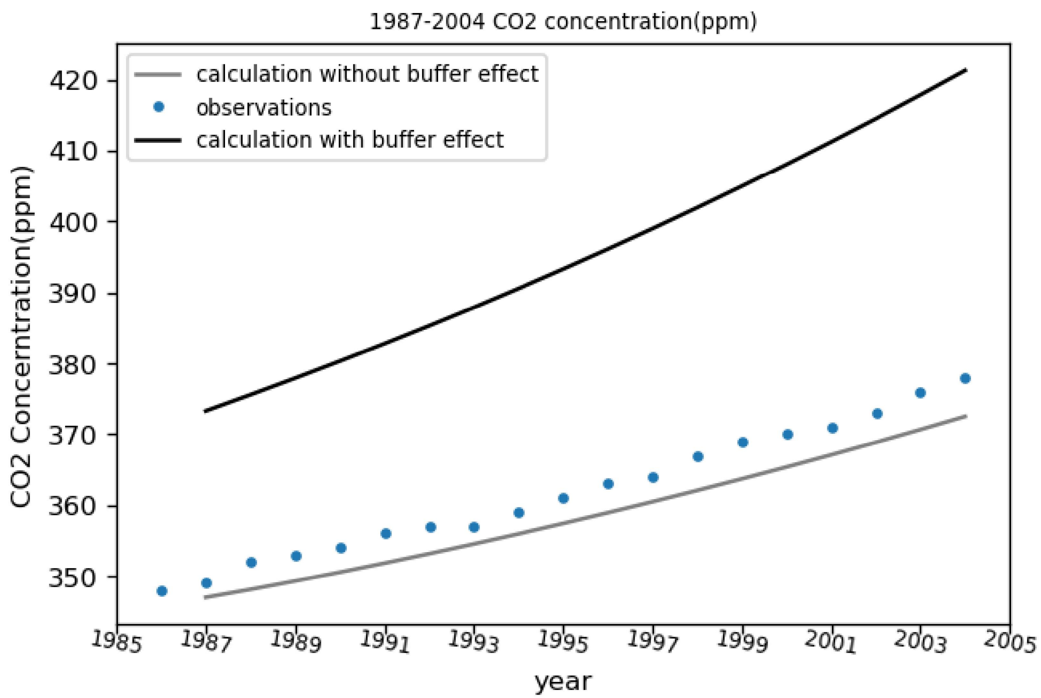
1.3 reproduce Figure 2 in Tomizuka (2009)

In [463…

```
plt.figure(figsize=(6,4),dpi=120)

plt.plot(x, sol[:,0], color='gray', label='calculation without buffer effect')
plt.plot(CO2_ppm_1986_2004['year'],CO2_ppm_1986_2004['mean'],".",label='observations')
plt.plot(x2[237:255], sol2[237:255,0], 'k', label='calculation with buffer effect')

#设置横纵坐标轴刻度，设置legend位置，设置横纵坐标轴标签，设置标题
plt.xticks(ticks=np.arange(1985,2006,2), fontsize=8, rotation=-10, ha='center', va='center')
plt.legend(loc='best',fontsize=8)
plt.xlabel('year');plt.ylabel('CO2 Concerntration(ppm)')
plt.title('1987-2004 CO2 concentration(ppm)',fontsize=8)
```

Out[463]:

```
Text(0.5, 1.0, '1987-2004 CO2 concentration(ppm)')
```



[Bonus] reproduce Figure 4 in Tomizuka (2009)

In [464…

```
#读入Carbon Flux to the Atmosphere from Land-Use Changes 1850-2005数据再把paper中提到1750年为0.2pgc/yr的数据也加入
δ_1850_2005=pd.read_csv("C:\\Users\\yuanwenting\\Documents\\ESE5023\\assignment\\Annual Net Flux of Carbon to the Atmosphere from Land-Use Change1850-2005.csv",",")
δ_1850_2005
```

```
C:\ProgramData\Anaconda3\lib\site-packages\IPython\core\interactiveshell.py:3369: FutureWarning: In a future version of pandas all arguments of read_csv except for the argument 'filepath_or_buffer' will be keyword-only.
  exec(code_obj, self.user_global_ns, self.user_ns)
```

Out[464]:

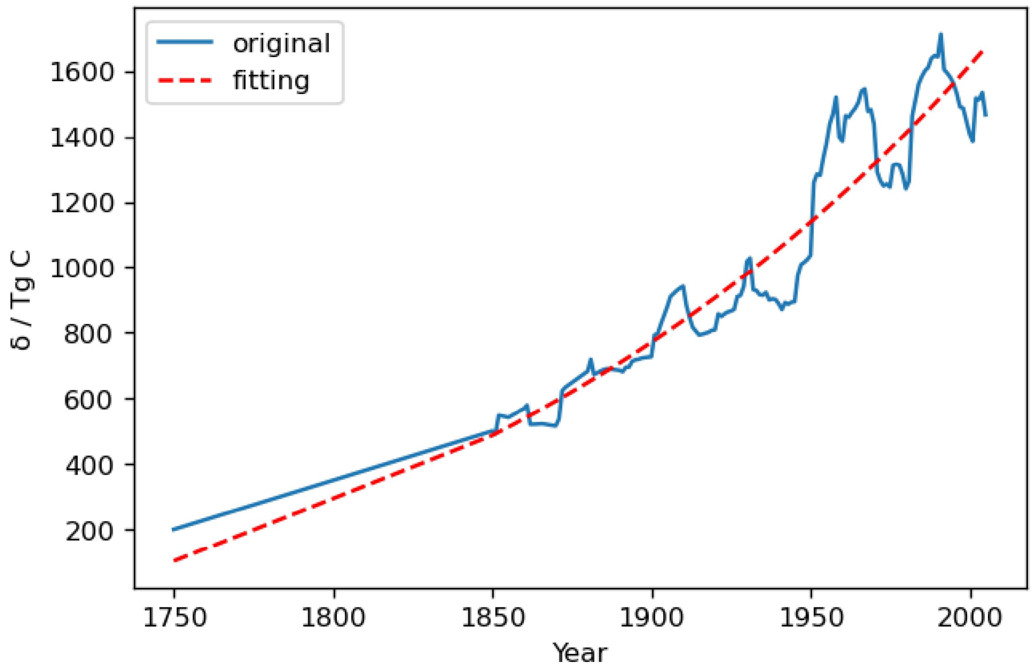
	year	Annual Net Flux/Tg
0	1750	200.0
1	1850	500.6
2	1851	492.7
3	1852	548.5
4	1853	546.8
...	...	...
152	2001	1385.4
153	2002	1517.7
154	2003	1513.2
155	2004	1534.9
156	2005	1467.3

157 rows × 2 columns

In [465…

```
#利用指数函数拟合the emission rate to the atmosphere by changes in land use: δ
def δ_func(x, a, b, c):
    return a * np.exp(b * x) + c

#p0是猜测,maxfev是最大迭代次数
popt,pcov = curve_fit(δ_func, δ_1850_2000['year'], δ_1850_2000['Annual Net Flux/Tg'], p0=[0.5,0,1], maxfev=2000)

#popt[0]是a, popt[1]是b, popt[2]是c
y_fit= [δ_func(i, popt[0], popt[1], popt[2]) for i in δ_1850_2000['year']]

#输出原曲线和拟合曲线
plt.figure(figsize=(6,4),dpi=120)
plt.plot(δ_1850_2000['year'], δ_1850_2000['Annual Net Flux/Tg'],label='original')
plt.plot(δ_1850_2000['year'], y_fit,'r--',label='fitting')
plt.legend(loc=2)
plt.xlabel('Year');plt.ylabel('δ / Tg C')

#输出参数最佳值
print(popt)
```

```
[ 5.85494050e-02  5.23778758e-03 -4.58368712e+02]
```



In [466…

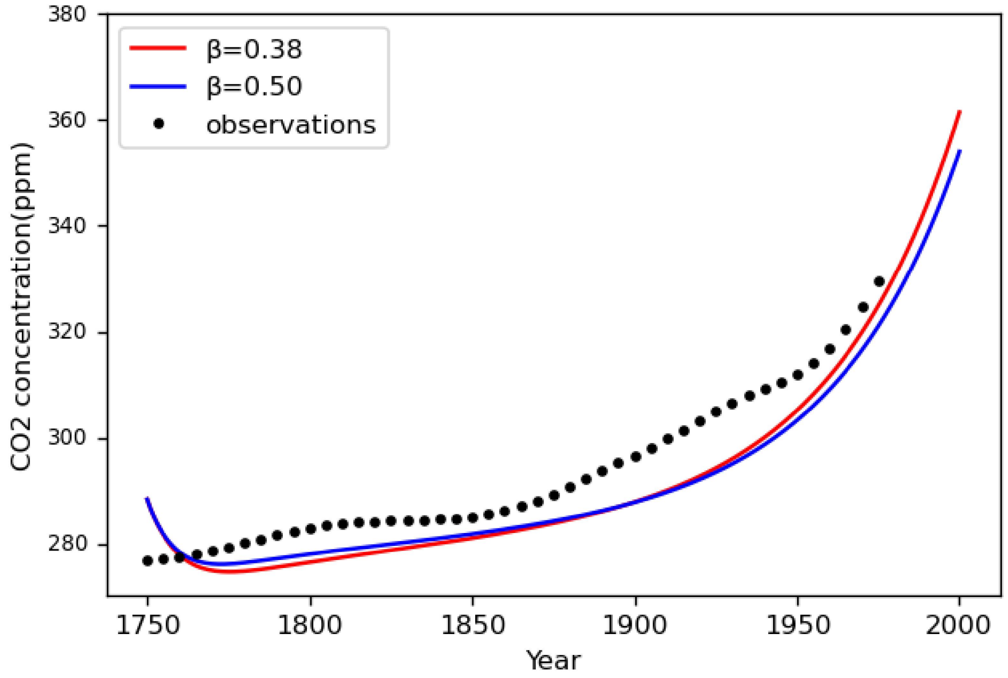
```
import math
#β=0.38
def pend_3(y,x3):
    N1,N2,N3,N4,N5,N6,N7,r3,δ = y
    z =N1/1000/740*347                    #z is the atmospheric CO2 concentration of ppm unit.
    N2_0 = 842*1000                       #In the preindustrial era the equilibrium value of carbon in the surface ocean(842pg/yr)
    p=N1; p0=615*1000; f0=62*1000         #P is the atmospheric CO2 concentration, and P0 is the preindustrial value of P
    ξ = 3.69+1.86e-2*(z)-1.8e-6*(z**2)    #the buffer factor
    f=f0*(1+0.38*math.log(p/p0,math.e))   #f is the net primary productivity
    dydt = [-60/615*N1+60/842*(N2_0+ξ*(N2-N2_0))+r3-f+δ+0.2/90000000*N5+62/1328*N7,
            60/615*N1-60/842*(N2_0+ξ*(N2-N2_0))-9/842*N2+52/9744*N3-43/842*N2,
            9/842*N2-52/9744*N3-162/9744*N3+205/26280*N4,
            162/9744*N3-205/26280*N4+43/842*N2-0.2/26280*N4,
            0.2/26280*N4-0.2/9000000*N5,
            f-62/731*N6-2*δ,
            62/731*N6-62/1328*N7+δ,
            2.43954472e-02*(r3+1.12313831e+02),
            5.23780754e-03*(r3+4.58363315e+02)]
    return dydt

#β=0.50
def pend_3_(y,x3):
    N1,N2,N3,N4,N5,N6,N7,r3,δ = y
    z =N1/1000/740*347                  #z is the atmospheric CO2 concentration of ppm unit.
    N2_0 = 842*1000                     #In the preindustrial era the equilibrium value of carbon in the surface ocean(842pg/yr)
    p=N1; p0=615*1000; f0=62*1000       #P is the atmospheric CO2 concentration, and P0 is the preindustrial value of P
    ξ = 3.69+1.86e-2*(z)-1.8e-6*(z**2)  #the buffer factor
```

```
    f=f0*(1+0.50*math.log(p/p0,math.e)) #f is the net primary productivity
    dydt = [-60/615*N1+60/842*(N2_0+ξ*(N2-N2_0))+r3-f+δ+0.2/90000000*N5+62/1328*N7,
            60/615*N1-60/842*(N2_0+ξ*(N2-N2_0))-9/842*N2+52/9744*N3-43/842*N2,
            9/842*N2-52/9744*N3-162/9744*N3+205/26280*N4,
            162/9744*N3-205/26280*N4+43/842*N2-0.2/26280*N4,
            0.2/26280*N4-0.2/9000000*N5,
            f-62/731*N6-2*δ,
            62/731*N6-62/1328*N7+δ,
            2.43954472e-02*(r3+1.12313831e+02),
            5.23780754e-03*(r3+4.58363315e+02)]
    return dydt

#from 1750 to 2000
x3=np.arange(1750,2001)
r3=r_func(x3,4.82316522e-18,2.43954472e-02,-1.12313831e+02)
δ=δ_func(x3,5.85469324e-02,5.23780754e-03,-4.58368712e+02)

#1750年初始值
N1=615*1000;N2=842*1000;N3=9744*1000;N4=26280*1000;N5=90000000*1000;N6=731*1000;N7=1238*1000
y0=[N1,N2,N3,N4,N5,N6,N7,r3[0],δ[0]]
sol3=odeint(pend_3, y0,x3)/1000/740*347 #转化为ppm
sol3_=odeint(pend_3_, y0,x3)/1000/740*347 #转化为ppm

#根据公式5-13画1987-2004年的CO2浓度积分曲线
plt.figure(figsize=(6,4),dpi=120)
plt.plot(x3, sol3[:,0], 'r', label='β=0.38')
plt.plot(x3, sol3_[:,0], 'b', label='β=0.50')

#根据链接Historical CO2 Records from the Law Dome DE08, DE08-2, and DSS Ice Cores中数据画实观测数据图，数据截至1975年
ice_CO2=pd.read_csv("C:\\Users\\yuanwenting\\Documents\\ESE5023\\assignment\\ice core records plus smoothed data.csv",",")
plt.plot(ice_CO2['year'],ice_CO2['CO2/ppm'],"k.",label='observations')

#调y轴刻度线，调legend位置，设置横纵坐标标签
plt.yticks(ticks=np.arange(280,400,20), fontsize=8, rotation=0, ha='right', va='center')
plt.legend(loc=2)
plt.xlabel('Year');plt.ylabel('CO2 concentration(ppm)')


#对比paper中fig.4 有较大差距，
#主要是由于δ(s the emission rate to the atmosphere by changes in land use)的1850年前的原始数据找不到，
#只能用1850年后的拟合数据来表示整个1750-2000年间的δ变化，
#此外，我们使用指数拟合结果并不是很好，造成δ的较大偏差，导致1750-1850之间有一凹陷峰，
#但是我们还是能观察到变化β值(fertilization factor),对CO2年浓度无明显影响
```

```
C:\ProgramData\Anaconda3\lib\site-packages\IPython\core\interactiveshell.py:3369: FutureWarning: In a future version of pandas all arguments of read_csv except for the argument 'filepath_or_buffer' will be keyword-only.
  exec(code_obj, self.user_global_ns, self.user_ns)
```

Out[466]:

```
Text(0, 0.5, 'CO2 concentration(ppm)')
```



380
360
340
320
300
280
CO2 concentration(ppm)
1750
1800
1850
1900
1950
2000
Year
β=0.38
β=0.50
observations