NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Tutorial 01: Suggested Solutions

Exam-Like Questions

1. The answer is (c).

   Note that

$$((A \cup B) \cap C)' = (A \cup B)' \cup C' = (A' \cap B') \cup C' = (A' \cup C') \cap (B' \cup C').$$

2. The total number of samples

   - with no restrictions is $\binom{26}{3} = 2600$.
   - without any vowels is $\binom{21}{3} = 1330$.

   Thus the number of samples with at least 1 vowel is $2600 - 1330 = 1270$.

3. Seat the men first.

$$\wedge\, M_1\ \wedge\ M_2\ \wedge\ M_3\ \wedge\ M_4\ \wedge$$

   The women can choose 3 out of 5 spaces between the men.

   Finally, arrange the women in those 3 chosen spaces.

   Number of ways required is $\binom{5}{3} \times 4! \times 3! = 1440$.

4. This is equivalent to lining up 9 houses. Once this is done, we can imagine that the first 6 would be automatically built on the South side, and the remaining 3 will be built on the North side.

   Thus the total number of ways to do so (i.e., permute 9 houses) is given by $P_9^9 = 9! = 362880$.

Long Form Questions

1. (i) $S = \{123, 124, 125, 13, 14, 15, 213, 214, 215, 23, 24, 25, 3, 4, 5\}$;

   (ii) $A = \{3, 4, 5\}$;

   (iii) $B = \{5, 15, 25, 125, 215\}$;

   (iv) $C = \{23, 24, 25, 3, 4, 5\}$;

   (v) $A \cap B = \{5\}$, $A \cup B = \{3, 4, 5, 15, 25, 125, 215\}$, $A \cap B \cap C = \{5\}$; A and B are not mutually exclusive.

2. (i) The number of choices for the hundreds, tens and ones positions are 5, 5 and 4 respectively. Hence the number of 3-digit numbers that can be formed is $5 \times 5 \times 4 = 100$.

   (ii) To ensure it is odd, we place 9 in the ones position. It follows that the number of choices for the ones, hundreds and tens positions are 1, 4 and 4 respectively. Hence the number of odd 3-digit numbers that can be formed is $4 \times 4 \times 1 = 16$.

   (iii) The number of odd 3-digit numbers greater than 620 with hundreds position greater than 6 is $1 \times 4 \times 1 = 4$; the number of odd 3-digit numbers greater than 620 with hundreds position being 6 is $1 \times 3 \times 1 = 3$. Hence the number of 3-digit numbers greater than 620 is $4 + 3 = 7$.

3. (i) The number of ways to choose 5 questions out of 7 is $\binom{7}{5} = 21$.

(ii) The number of ways to choose 2 questions from the first 2 is $\binom{2}{2} = 1$.

The number of ways to choose 3 questions from the remaining 5 is $\binom{5}{3} = 10$.

Using the Multiplication Principle, the number of ways required is $1 \times 10 = 10$.

(iii) The number of ways, to select exactly 1 question from the first 2 and 4 from the remaining 5, is $\binom{2}{1} \cdot \binom{5}{4} = 2 \times 5 = 10$.

From Part (b), the number of ways to select 2 questions from the first 2, and 3 from the remaining 5 is 10.

Using the Additional Principle, the number of ways required is $10 + 10 = 20$.

(iv) The number of ways, to select exactly 2 questions from the first 3 and 3 from the remaining 4, is $\binom{3}{2} \cdot \binom{4}{3} = 12$.

4. (i) Each path from A to B is composed of 21 steps, with 8 steps to the North (N) and 13 steps to the East (E). For example, `ENENNNEENEEEENEEEENNEE` is one such a path.

Therefore, "the number of ways from A to B" is equivalent to "the number of ways we can choose 8 North-ward steps out of 21 steps".
That is given as

$$\binom{21}{8} = 203490.$$

(ii) Using the argument in (i),

- The number of ways from A to Y is $\binom{16}{6} = 8008$; and
- the number of ways from Y to B is $\binom{5}{2} = 10$;
- So the number of ways from A to B with a stop at Y is $8008 \times 10 = 80080$.
- Consequently, the number of ways from A to B without stopping by Y is

$$203490 - 80080 = 123410.$$

(iii) The number of ways from A to B, stopping by X but not at Y, is

$$\binom{4}{2} \times \left[\binom{17}{6} - \binom{12}{4} \times \binom{5}{2}\right] = \frac{4!}{2!2!} \times \left(\frac{17}{6!11!} - \frac{12!}{4!8!} \cdot \frac{5!}{2!3!}\right) = 44556.$$