

NATIONAL UNIVERSITY OF SINGAPORE  
DEPARTMENT OF STATISTICS & DATA SCIENCE  
**ST2334 PROBABILITY AND STATISTICS**  
SEMESTER I, AY 2025/2026

**Tutorial 03**

Please work on the questions before attending the tutorial.

**Exam-Like Questions**

**1. MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

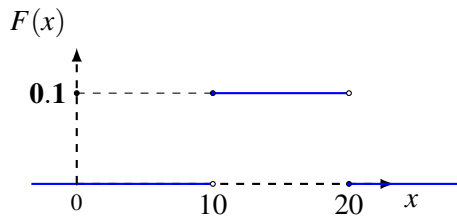
Which of the following can define probability distributions?

- (a)  $f(x) = x/14$  for  $x = 0, 1, 2, 3, 4$ .  
 (b)  $f(x) = \frac{3-x^2}{4}$  for  $x = 0, 1, 2$ .  
 (c)  $f(x) = 1/5$  for  $x = 5, 6, 7, 8, 9$ .  
 (d)  $f(x) = \frac{2x+1}{50}$  for  $x = 1, 2, 3, 4, 5$ .

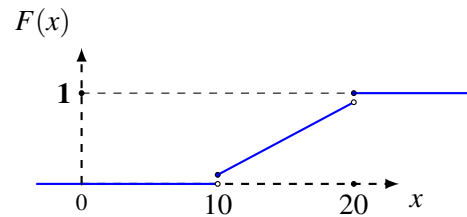
**2. MULTIPLE RESPONSE: CHOOSE ALL ANSWERS THAT APPLY**

Which of the graphs drawn (in blue) below can be the cumulative distribution function  $F(x)$  of some random variable  $X$ ?

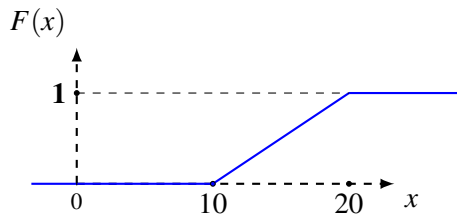
(a)



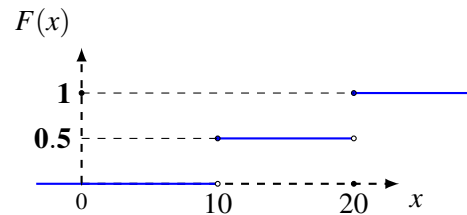
(c)



(b)



(d)



**3. MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let  $X$  be the number of months between successive payments. The cumulative distribution function of  $X$  is given as follows.

$$F_X(x) = \begin{cases} 0, & x < 1; \\ 0.3, & 1 \leq x < 3; \\ 0.4, & 3 \leq x < 4; \\ 0.45, & 4 \leq x < 6; \\ 0.6, & 6 \leq x < 12; \\ 1, & 12 \leq x. \end{cases}$$

Which of the following is **INCORRECT**?

(a)  $P(3 \leq X \leq 6) = 0.3$

(c)  $P(X = 3) = 0.15$

(b)  $P(X \geq 4) = 0.6$

(d)  $P(2 < X < 4) = 0.1$

4. **FILL IN THE BLANK**

The value of  $c$ , such that the following function can serve as the probability function of a random variable  $X$ , is \_\_\_\_\_.

$$f_X(x) = \begin{cases} c(x^2 + 4), & x = 0, 1, 2, 3; \\ 0, & \text{elsewhere.} \end{cases}$$

5. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let  $X$  be the number of months between successive payments. The cumulative distribution function of  $X$  is given as follows.

$$F_X(x) = \begin{cases} 0, & x < 1; \\ 0.3, & 1 \leq x < 3; \\ 0.4, & 3 \leq x < 4; \\ 0.45, & 4 \leq x < 6; \\ 0.6, & 6 \leq x < 12; \\ 1, & 12 \leq x. \end{cases}$$

Which of the following is the probability function of  $X$ ?

(a) 

$x$	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.05	0.15	0.4

(b) 

$x$	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.45	0.05	0.4

(c) 

$x$	1	3	4	6	12
$f_X(x)$	0	0.3	0.4	0.6	1

(d) None of the given options.

**Long Form Questions**

1. For customers purchasing a full set of tires at a particular tire store, consider the events

$A = \{\text{tires purchased were made in the United States}\},$

$B = \{\text{purchaser has tires balanced immediately}\},$

$C = \{\text{purchaser requests for front-end alignment}\}.$

Assume the following unconditional and conditional probabilities:

$$P(A) = 0.75, \quad P(B|A) = 0.9, \quad P(B|A') = 0.8, \quad P(C|A \cap B) = 0.8, \quad P(C|A' \cap B) = 0.7.$$

Compute the following probabilities:

(a)  $P(A \cap B \cap C)$

(d)  $P(B \cap C)$

(b)  $P(B)$

(e)  $P(A|B \cap C)$

(c)  $P(A|B)$

2. Total quality management (TQM) is a management philosophy and system of management techniques to improve product and service quality and worker productivity. TQM involves such techniques as teamwork, empowerment of workers, improved communication with customers, evaluation of work processes, and statistical analysis of processes and their output. One hundred Singapore companies were surveyed and it was found that 30 had implemented TQM. Among the 100 companies surveyed, 60 reported an increase in sales last year. Of those 60, 20 had implemented TQM. Suppose one of the 100 surveyed companies is to be selected randomly for additional analysis.
  - (a) What is the probability that a firm that implemented TQM is selected? That a firm whose sales increased is selected?
  - (b) Are the two events {TQM implemented} and {Sales increased} independent or dependent? Explain.
  - (c) Suppose that among the 60 firms reporting sales increases, there were only 18 TQM-implementers (instead of 20). Now are the events {TQM implemented} and {Sales increased} independent or dependent? Explain.
3. A company uses three different assembly lines,  $A_1, A_2$ , and  $A_3$ , to manufacture a particular component. Of those manufactured by line  $A_1$ , 5% need rework to remedy a defect, whereas 8% of  $A_2$ 's components need rework, and 10% of  $A_3$ 's components need rework. Suppose that 50% of all components are produced by line  $A_1$ , while 30% are produced by line  $A_2$ , and 20% come from line  $A_3$ . If a randomly selected component needs rework, what is the probability that it came from line  $A_2$ ?
4. Consider a group of five potential blood donors: A, B, C, D and E, of whom only A and B have type  $O^+$  blood. Five blood samples, one from each individual, will be typed in random order until an  $O^+$  individual is identified. Let  $Y$  be the number of typing needed to identify an  $O^+$  individual. Construct the probability function of  $Y$ .
5. Consider the probability function

$$f_X(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the value of the constant  $k$ .
  - (b) Find the cumulative distribution function  $F_X(x)$ , and use it to evaluate  $P(0.3 < X < 0.6)$ .
6. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution

$$F_X(x) = \begin{cases} 0, & x \leq 0; \\ 1 - e^{-8x}, & x > 0. \end{cases}$$

- (a) Find the probability of waiting 12 minutes or less between successive speeders.
- (b) Find the probability density function of  $X$ .

### Answers to some Long Form Questions

1. (a) 0.54; (b) 0.875; (c) 0.7714; (d) 0.68; (e) 0.7941.

2. (a) 0.3; 0.6. (b)  $A \not\perp B$ ; (c)  $A \perp B$ .

3. 0.3478;

$$5. \quad (a) 3/2; (b) F_X(x) = \begin{cases} 0, & x \leq 0 \\ x^{3/2}, & 0 < x < 1; \\ 1, & x \geq 1 \end{cases} \quad 0.3004$$

6. (a) 0.7981; (b)  $f_X(x) = 8e^{-8x}$  for  $x \geq 0$ .