NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS AND DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Midterm Test (Sample Paper A)

- This assessment contains 15 questions.
- The total marks is 30; each question is worth 2 marks.
- Please answer ALL questions.
- Calculators of any kind are allowed.
- Solutions to this paper will be published by the Friday of the Recess/Reading Week. Try working on it till then.

1. **TRUE/FALSE**
Consider an experiment of **rolling two six-faced regular FAIR dice**. Suppose that the problem of interest is "the numbers that show on the top faces".

   If the dice are **not distinguishable**, then the sample space S contains 36 elements.

   - TRUE
   - FALSE

2. **FILL IN THE BLANK**
Suppose we will choose THREE numbers from 1, 3, 5, 76, 125, 876, 987, and 1235, with each number being chosen at most once, to fill in the three boxes below such that the numbers are arranged in increasing order from left to right. For example, if we choose the numbers 987, 125, and 1, we shall put "1, 125, and 987" in the boxes from left to right. How many different ways do we have to fill in the boxes?

   ANSWER: _____. (Leave your answer in numerical form.)

3. **FILL IN THE BLANK**
A group of 8 friends A,B,C,D,E,F,G,H go to a restaurant. Due to safe-distancing measures, the group needs to split up into two groups of 4. How many ways are there to split the group such that A and B are together but away from C?

   ANSWER: _____. (Leave your answer in numerical form.)

4. **TRUE/FALSE**
$P(A \cup B \cup C) = 1 - P(A'|B' \cap C')P(B'|C')P(C')$.

   - TRUE
   - FALSE

5. **FILL IN THE BLANK**
Find $P(B \cap A)$, if

$$P(A') = 1/2, \quad P(B) = 3/8 \quad \text{and} \quad P(B'|A) = 3/4.$$

   ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

6. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Suppose A and B are two events where $P(A) = 0.4$ and $P(A \cap B) = 0.2$. What is $P(A|B)$?

   (a) 0.4
   (b) 0.5
   (c) Insufficient information to determine
   (d) None of the other options

7. **TRUE/FALSE**
Let A and B be two events in the sample space S. If $P(A) > P(B) = 0.8$, then A and B **ARE NOT** independent.

- TRUE
- FALSE

8. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
A new Covid test kit detects the virus 90% of the time if a patient is infected. However, it also detects the virus 5% of the time if a patient is not infected. Given that the overall Covid infection rate is 1%, what is the probability of being infected if your test kit detects the virus?

Pick the option closest to the answer.

(a) 0.114
(b) 0.215
(c) 0.154
(d) 0.322

9. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Which of the following is a legitimate probability function?

(a) $f(x) = \begin{cases} x^2/3 & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$

(b) $f(x) = \begin{cases} 2|x| & \text{for } -1 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$

(c) $f(x) = \begin{cases} x^2/30 & \text{for } x = -3,-2,1,2,3 \\ 0 & \text{elsewhere} \end{cases}$

(d) $f(x) = \begin{cases} |x|/11 & \text{for } x = -3,-2,-1,2,3 \\ 0 & \text{elsewhere} \end{cases}$

10. **FILL IN THE BLANK**
Assume that $F(x)$ below is the cumulative distribution function of a random variable X:

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.3, & 0 \le x < 1 \\ 0.7, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}.$$

Compute $E(X^2)$.

ANSWER: _____. (Round your answer 2 decimal places, if necessary.)

11. **FILL IN THE BLANK**
The cumulative distribution for a random variable X is given by

$$F(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x^2}{4}, & 0 < x < 2 \\ 1, & x \ge 2 \end{cases}.$$

Compute $V(X)$.

ANSWER: _____. (Round your answer 2 decimal places, if necessary.)

12. **TRUE/FALSE**
Let X be a continuous random variable; and let Y be a discrete random variable. It is possible for us to find a real number a, such that $P(X = a) > P(Y = a)$.

- TRUE
- FALSE

13. **FILL IN THE BLANK**
A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint probability mass function of X and Y is given in the table below.

x	y		
	0	1	2
0	0.10	0.04	0.02
1	0.08	0.20	0.06
2	0.06	0.14	0.30

Compute $P(X+Y \geq 2)$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

14. **TRUE/FALSE**
Let $f(x,y)$ be the joint probability function of a random vector (X,Y) (discrete or continuous). If $f_X(1) > 0$, then there exists a y such that $f(1,y) > 0$.

- TRUE
- FALSE

15. **FILL IN THE BLANK**
The joint probability function of (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{8}(x+y) & 0 \leq x \leq 2; 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Compute $P(Y \geq 1 | X = 1)$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

END OF PAPER