NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS AND DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Midterm Test (Sample Paper B): Suggested Solutions

- This assessment contains 15 questions.
- The total marks is 30; each question is worth 2 marks.
- Please answer ALL questions.
- Calculators of any kind are allowed.

1. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
$A \cup (B \cap C)$ is the same as

(a) $(A \cup B) \cap (A \cup C)$
(b) $(A \cup B) \cap C$
(c) $A \cup B' \cup C'$
(d) $(A \cap B) \cup (A \cap C)$

SOLUTION
The answer is (a).

This is one of the identities given in the notes.

2. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
How many different committees of 5 can be formed from 6 men and 4 women on which exactly 3 men and 2 women serve?

Pick the option closest to the answer.

(a) 6
(b) 20
(c) 60
(d) 120

SOLUTION
The answer is (d).

We choose 3 men from 6, and 2 women from 4. There are $\binom{6}{3} \times \binom{4}{2} = 120$ ways to do it.

3. **FILL IN THE BLANK**
Consider the digits 0,1,2,3,4,5, and 6. If each digit can be used at most once, how many **odd** 3-digit numbers, which are **equal to or greater than** 301, can be formed?

ANSWER: ______________. (Give your answer in numerical form.)

SOLUTION
The answer is 50.

- If 3 is used at ones place, and 4, 5, or 6 is used at the hundreds place, we have $3 \times 5 \times 1 = 15$ ways.
- If 1 is used at ones place, we can arbitrarily choose a number from 3, 4, 5, and 6 to fill in the hundreds place, and choose arbitrarily a number from the leftover to fill in the tens place. Therefore, we have $4 \times 5 \times 1 = 20$ ways.
- If 5 is used at ones place, we can arbitrarily choose a number from 3, 4, and 6 to fill in the hundreds place, and choose arbitrarily a number from the leftover to fill in the tens place. Therefore, we have $3 \times 5 \times 1 = 15$ ways.

Altogether, we have $15 + 20 + 15 = 50$ ways.

4. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Consider the following statements about Peter whom you have not met before.

(A) He is not married.
(B) He is not married and smokes.
(C) He is married.
(D) He is married and does not smoke.

You are to assign probabilities to these statements. Which answer below is consistent with the Laws of Probability?

(a) $P(A) = 0.45$, $P(B) = 0.50$, $P(C) = 0.55$, $P(D) = 0.40$
(b) $P(A) = 0.45$, $P(B) = 0.10$, $P(C) = 0.60$, $P(D) = 0.30$
(c) $P(A) = 0.45$, $P(B) = 0.20$, $P(C) = 0.55$, $P(D) = 0.50$
(d) $P(A) = 0.45$, $P(B) = 0.40$, $P(C) = 0.55$, $P(D) = 0.60$

SOLUTION
The answer is (c).

We will need the given events to satisfy all of the following

(i) $P(A) + P(C) = 1$
(ii) $P(B) \leq P(A)$
(iii) $P(D) \leq P(C)$

Only (c) satisfies all of them.

5. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
In an oral exam, a student needs to answer a question correctly to pass. The question will be randomly drawn from a box containing 6 hard and 4 easy questions. If an easy question is drawn, with 80% chance, the student can answer it correctly; otherwise, with 20% chance, the student can answer it correctly. If the student passed the exam, what is the probability that an easy question was drawn?

Pick the option closest to the answer.

(a) 2/3
(b) 4/5
(c) 8/11
(d) 7/12

SOLUTION
The answer is (c).

Define events

$$E = \{\text{Easy question drawn}\}, \quad C = \{\text{Student answered correctly}\}$$

We have $P(E) = 2/5$; $P(C|E) = 4/5$; $P(C|E') = 1/5$. It follows that

$$P(E|C) = \frac{P(E \cap C)}{P(C)} = \frac{P(E)P(C|E)}{P(E)P(C|E) + P(E')P(C|E')} = \frac{(2/5)(4/5)}{(2/5)(4/5) + (3/5)(1/5)} = 8/11.$$

6. **TRUE/FALSE**
Let A and B be mutually exclusive events. If $P(A) = 0.1$, $P(B) = 0.01$, then A and B are not independent.

SOLUTION
The claim is TRUE.

Otherwise, $P(A \cap B) = P(A)P(B) > 0$, which contradicts the fact that A and B are mutually exclusive.

7. **TRUE/FALSE**
Let $f(x)$ be the probability function of a discrete random variable X. Then, for any real numbers $x_1 < x_2 < \ldots < x_{1000}$ we must have

$$f(x_1) + f(x_2) + \ldots + f(x_{1000}) \leq 1.$$

SOLUTION
The claim is TRUE.

This follows from the fact that $\sum_{x \in R_X} f(x) = 1$.

8. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Let X be a random variable with density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 \leq x \leq 1; \\ ke^{\frac{1-x}{2}}, & x > 1; \\ 0 & \text{otherwise.} \end{cases}$$

What is the value of the constant k?

Pick the option closest to the answer.

(a) 1/2
(b) 2/5
(c) 3/8
(d) 4/9

SOLUTION
The answer is (c).

Note that

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = \int_0^1 k\sqrt{x}\,\mathrm{d}x + \int_1^{\infty} ke^{\frac{1-x}{2}}\,\mathrm{d}x = \left[\tfrac{2}{3}kx^{\frac{3}{2}}\right]_0^1 + \left[-2ke^{\frac{1-x}{2}}\right]_1^{\infty} = \tfrac{2}{3}k + 2k = \tfrac{8}{3}k.$$

So $\frac{8}{3}k = 1$, which gives $k = \frac{3}{8}$.

9. **FILL IN THE BLANK**
Let X be a random variable, whose cumulative distribution function is given by

$$F(x) = \begin{cases} 0, & x < 0; \\ 0.2, & 0 \leq x < 2; \\ 0.6, & 2 \leq x < 3; \\ 0.7, & 3 \leq x < 5; \\ 1 & x \geq 5. \end{cases}$$

Compute $E(X)$.

ANSWER: ______________. (Round your answer to 3 decimal points, if necessary.)

SOLUTION
The answer is 2.6.

X is a discrete random variable, whose probability mass function is given by $f(x) = 0.2, 0.4, 0.1, 0.3$, for $x = 0, 2, 3, 5$ respectively. Therefore

$$E(X) = 0 \times 0.2 + 2 \times 0.4 + 3 \times 0.1 + 5 \times 0.3 = 2.6.$$

10. **FILL IN THE BLANK**
Suppose X has a probability mass function given by the following table.

x	0	1	4	9
$f(x)$	0.3	0.5	0.1	0.1

Compute $E(\sqrt{X})$.

ANSWER: ______________. (Round your answer to 3 decimal points, if necessary.)

SOLUTION
The answer is 1.

$E(\sqrt{X}) = \sqrt{0} \cdot 0.3 + \sqrt{1} \cdot 0.5 + \sqrt{4} \cdot 0.1 + \sqrt{9} \cdot 0.1 = 1.$

11. **FILL IN THE BLANK**
Let X be a random variable. $E(X) = 4$; $E[X(X-1)] = 20$. Compute $V(X)$.

ANSWER: ______________. (Round your answer to 2 decimal places, if necessary.)

SOLUTION
The answer is 8.

$E(X^2) - E(X) = 20$, so $E(X^2) = 20 + 4 = 24$. Thus

$$V(X) = E(X^2) - [E(X)]^2 = 24 - 4^2 = 8.$$

12. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Which of the following is **possibly** the cumulative distribution function of a random variable X?

(a) $F(x) = \begin{cases} 0 & x < -2 \\ 0.3 & -2 \le x < -1 \\ 0.5 & -1 \le x < 3 \\ 0.6 & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$

(b) $F(x) = \begin{cases} 0 & x < -2 \\ 0.3 & -2 \le x < -1 \\ 0.6 & -1 \le x < 2 \\ 0.5 & 2 \le x < 4 \\ 1 & x \ge 4 \end{cases}$

(c) $F(x) = \begin{cases} 0 & x \le -2 \\ 0.3 & -2 < x \le -1 \\ 0.4 & -1 < x \le 3 \\ 0.6 & 3 < x \le 4 \\ 1 & x > 4 \end{cases}$

(d) None of the given options

SOLUTION
The answer is (a).

(b) is not non-decreasing.

(c) is not right continuous at $x = -2$.

13. **FILL IN THE BLANK**
A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint probability mass function of X and Y is given in the table below.

x	y		
	0	1	2
0	0.10	0.04	0.02
1	0.08	0.20	0.06
2	0.06	0.14	0.30

Compute $E(X|Y = 1)$.

ANSWER: ________. (Round your answer to 3 decimal points, if necessary.)

SOLUTION
The answer is 1.263.

Consider the column $y = 1$, which sum gives $P(Y = 1) = 0.04 + 0.20 + 0.14 = 0.38$. The conditional distribution of $X|Y = 1$ is then

$$\begin{aligned} P(X = 0|Y = 1) &= 0.04/0.38; \\ P(X = 1|Y = 1) &= 0.20/0.38; \\ P(X = 2|Y = 2) &= 0.14/0.38. \end{aligned}$$

Therefore, we obtain

$$E(X|Y = 1) = 0(0.04/0.38) + 1(0.20/0.38) + 2(0.14/0.38) = 0.48/0.38 \approx 1.263.$$

14. **TRUE/FALSE**

Let $f(x,y)$ be the joint probability function of a discrete random vector (X,Y). If $f_X(1) = 0$, then $f(1,y) = 0$ for any real number y.

SOLUTION

The claim is TRUE.

$f_X(1) = 0$ implies that $\sum_{y \in R_Y} f(1,y) = 0$.

But $f(1,y) \geq 0$. So $f(1,y) = 0$ for all $y \in R_Y$, and thus $f(1,y) = 0$ for any real number y.

15. **FILL IN THE BLANK**

The joint probability function of (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{8}(x+y), & 0 \leq x \leq 2; 0 \leq y \leq 2; \\ 0, & \text{elsewhere.} \end{cases}$$

Compute $P(Y \geq 1 | X \geq 1)$.

ANSWER: ____________. (Round your answer to 3 decimal points, if necessary.)

SOLUTION

The answer is 0.6.

We need to compute

$$P(Y \geq 1 | X \geq 1) = \frac{P(Y \geq 1; X \geq 1)}{P(X \geq 1)}.$$

We shall evaluate the numerator and denominator separately.

The marginal density of X is given by

$$f_X(x) = \int_0^2 \tfrac{1}{8}(x+y)\,\mathrm{dx} = \tfrac{1}{8}(2x+2) = \tfrac{1}{4}(x+1).$$

Therefore

$$P(X \geq 1) = \int_1^2 \tfrac{1}{4}(x+1)\,\mathrm{dx} = 0.625.$$

For the numerator:

$$P(X \geq 1; Y \geq 1) = \int_1^2 \int_1^2 \tfrac{1}{8}(x+y)\,\mathrm{dx}\,\mathrm{dy} = 0.375.$$

As a consequence,

$$P(Y \geq 1 | X \geq 1) = \frac{0.375}{0.625} = 0.6.$$

END OF PAPER