

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS AND DATA SCIENCE
ST2334 PROBABILITY AND STATISTICS
SEMESTER I, AY 2025/2026

Midterm Test (Sample Paper B)

- This assessment contains 15 questions.
- The total marks is 30; each question is worth 2 marks.
- Please answer ALL questions.
- Calculators of any kind are allowed.
- Solutions to this paper will be published by the Friday of the Recess/Reading Week.
Try working on it till then.

1. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

$A \cup (B \cap C)$ is the same as

- (a) $(A \cup B) \cap (A \cup C)$ (c) $A \cup B' \cup C'$
 (b) $(A \cup B) \cap C$ (d) $(A \cap B) \cup (A \cap C)$

2. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

How many different committees of 5 can be formed from 6 men and 4 women on which exactly 3 men and 2 women serve?

Pick the option closest to the answer.

- (a) 6 (c) 60
 (b) 20 (d) 120

3. **FILL IN THE BLANK**

Consider the digits 0,1,2,3,4,5, and 6. If each digit can be used at most once, how many **odd** 3-digit numbers, which are **equal to or greater than** 301, can be formed?

ANSWER: _____. (Give your answer in numerical form.)

4. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

Consider the following statements about Peter whom you have not met before.

- (A) He is not married. (C) He is married.
 (B) He is not married and smokes. (D) He is married and does not smoke.

You are to assign probabilities to these statements. Which answer below is consistent with the Laws of Probability?

- (a) $P(A) = 0.45$, $P(B) = 0.50$, $P(C) = 0.55$, $P(D) = 0.40$
 (b) $P(A) = 0.45$, $P(B) = 0.10$, $P(C) = 0.60$, $P(D) = 0.30$
 (c) $P(A) = 0.45$, $P(B) = 0.20$, $P(C) = 0.55$, $P(D) = 0.50$
 (d) $P(A) = 0.45$, $P(B) = 0.40$, $P(C) = 0.55$, $P(D) = 0.60$

5. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

In an oral exam, a student needs to answer a question correctly to pass. The question will be randomly drawn from a box containing 6 hard and 4 easy questions. If an easy question is drawn, with 80% chance, the student can answer it correctly; otherwise, with 20% chance, the student can answer it correctly. If the student passed the exam, what is the probability that an easy question was drawn?

Pick the option closest to the answer.

- (a) $2/3$ (c) $8/11$
 (b) $4/5$ (d) $7/12$

6. TRUE/FALSE

Let A and B be mutually exclusive events. If $P(A) = 0.1$, $P(B) = 0.01$, then A and B are not independent.

- TRUE
- FALSE

7. TRUE/FALSE

Let $f(x)$ be the probability function of a discrete random variable X . Then, for any real numbers $x_1 < x_2 < \dots < x_{1000}$ we must have

$$f(x_1) + f(x_2) + \dots + f(x_{1000}) \leq 1.$$

- TRUE
- FALSE

8. MULTIPLE CHOICE: CHOOSE **ONE** ANSWER ONLY

Let X be a random variable with density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 \leq x \leq 1; \\ ke^{\frac{1-x}{2}}, & x > 1; \\ 0 & \text{otherwise.} \end{cases}$$

What is the value of the constant k ?

Pick the option closest to the answer.

- | | |
|-----------|-----------|
| (a) $1/2$ | (c) $3/8$ |
| (b) $2/5$ | (d) $4/9$ |

9. FILL IN THE BLANK

Let X be a random variable, whose cumulative distribution function is given by

$$F(x) = \begin{cases} 0, & x < 0; \\ 0.2, & 0 \leq x < 2; \\ 0.6, & 2 \leq x < 3; \\ 0.7, & 3 \leq x < 5; \\ 1 & x \geq 5. \end{cases}$$

Compute $E(X)$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

10. **FILL IN THE BLANK**

Suppose X has a probability mass function given by the following table.

x	0	1	4	9
$f(x)$	0.3	0.5	0.1	0.1

Compute $E(\sqrt{X})$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

11. **FILL IN THE BLANK**

Let X be a random variable. $E(X) = 4$; $E[X(X - 1)] = 20$. Compute $V(X)$.

ANSWER: _____. (Round your answer to 2 decimal places, if necessary.)

12. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**

Which of the following is **possibly** the cumulative distribution function of a random variable X ?

$$\begin{aligned}
 \text{(a) } F(x) &= \begin{cases} 0 & x < -2 \\ 0.3 & -2 \leq x < -1 \\ 0.5 & -1 \leq x < 3 \\ 0.6 & 3 \leq x < 4 \\ 1 & x \geq 4 \end{cases} & \text{(c) } F(x) &= \begin{cases} 0 & x \leq -2 \\ 0.3 & -2 < x \leq -1 \\ 0.4 & -1 < x \leq 3 \\ 0.6 & 3 < x \leq 4 \\ 1 & x > 4 \end{cases} \\
 \text{(b) } F(x) &= \begin{cases} 0 & x < -2 \\ 0.3 & -2 \leq x < -1 \\ 0.6 & -1 \leq x < 2 \\ 0.5 & 2 \leq x < 4 \\ 1 & x \geq 4 \end{cases} & \text{(d) } & \text{None of the given options}
 \end{aligned}$$

13. **FILL IN THE BLANK**

A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint probability mass function of X and Y is given in the table below.

x	y		
	0	1	2
0	0.10	0.04	0.02
1	0.08	0.20	0.06
2	0.06	0.14	0.30

Compute $E(X|Y = 1)$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

14. **TRUE/FALSE**

Let $f(x, y)$ be the joint probability function of a discrete random vector (X, Y) . If $f_X(1) = 0$, then $f(1, y) = 0$ for any real number y .

- TRUE
- FALSE

15. **FILL IN THE BLANK**

The joint probability function of (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{1}{8}(x+y), & 0 \leq x \leq 2; 0 \leq y \leq 2; \\ 0, & \text{elsewhere.} \end{cases}$$

Compute $P(Y \geq 1|X \geq 1)$.

ANSWER: _____. (Round your answer to 3 decimal points, if necessary.)

END OF PAPER