NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Tutorial 04

Please work on the questions before attending the tutorial.

Exam-Like Questions

1. **FILL IN THE BLANK**
   Let X be the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day; and Y be the number of times a technician is called on an emergency call. Their joint probability distribution is given below. What is $P(Y = 3|X = 2)$?

$f_{X,Y}(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.10
	2	0.05	0.10	0.35
	3	0.00	0.20	0.10

2. **FILL IN THE BLANK**
   Let X be the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day; and Y be the number of times a technician is called on an emergency call. Their joint probability distribution is given below. What is $E(X|Y = 2)$?

$f_{X,Y}(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.10
	2	0.05	0.10	0.35
	3	0.00	0.20	0.10

3. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
   The joint probability function of (X,Y) is given below.

y	x	
	0	1
0	0.25	0.35
1	0.23	0.17

   Then $E(3X + 2Y)$ is

   (a) 1.87
   (b) 2.36
   (c) 2.45
   (d) 3.11
   (e) None of the given options.

4. **FILL IN THE BLANK**
The random variables X and Y have the joint probability density function given by

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1, 0 \le y \le 1; \\ 0, & \text{elsewhere.} \end{cases}$$

Compute $E(Y|X = 0.2)$.

5. **MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY**
Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let X be the actual air pressure (in 10 pound per square inch) for the right tire; and Y be the actual air pressure (in 10 pound per square inch) for the left tire. Suppose that X and Y are random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} k(x^2+y^2), & 3 \le x \le 5, 3 \le y \le 5; \\ 0, & \text{elsewhere.} \end{cases}$$

Which of the following is INCORRECT?

(a) We must have $k = 3/392$.

(b) $P(3 \le X \le 4, 4 \le Y \le 5) = 1/4$.

(c) The marginal probability function for X is given by

$$f_X(x) = \begin{cases} \frac{1}{196}(3x^2+49), & 3 \le x \le 5; \\ 0, & \text{elsewhere.} \end{cases}$$

(d) $P(3.5 < X < 4) = 0.1925$.

Long Form Questions

1. The random variable X, representing the number of errors per 100 lines of software code, has the following probability function:

x	2	3	4	5	6
$f_X(x)$	0.01	0.25	0.40	0.30	0.04

(a) Find $E(X)$, $E(X^2)$ and $V(X)$.

(b) Find the mean and variance of the discrete variable $Z = 3X - 2$.

(c) Find the probability function of the random variable Z, and use it to find $E(Z)$ and $V(Z)$.

2. The probability function of a random variable X is given by

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1; \\ 2-x, & \text{for } 1 \le x < 2; \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the probability that the random variable will take on a value between 0.6 and 1.2.

(b) Find $E(X)$ and $V(X)$.

3. From a sack of fruits containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

   (a) the joint probability distribution of X and Y;

   (b) $P(X = 1, Y = 1)$;

   (c) $P(X + Y \leq 2)$;

   (d) $f_X(x)$;

   (e) $f_{Y|X}(y|2)$ and hence $P(Y = 0|X = 2)$.

4. The joint probability function for (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{12}{13}x(x+y), & 0 \leq x \leq 1; 1 \leq y \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

   Compute $P(Y \leq 1.5|X = 0.5)$ and $E(Y|X = 0.5)$.

Answers to some Long Form Questions

1. (a) 4.11, 17.63, 0.7379; (b) 10.33, 6.6411; (c) 10.33, 6.6411;
2. (a) 0.5; (b) 1; (c) 1/6.
3. (a) $f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$, $x = 0,1,2,3$, $y = 0,1,2$, $1 \leq x+y \leq 4$; (b) 0.2571; (c) 0.5; (d) $f_X(x) = \frac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}}$, $x = 0,1,2,3$. (e) $f_{Y|X}(y|2) = \frac{1}{10}\binom{2}{y}\binom{3}{2-y}$, $y = 0,1,2$; 0.3.