

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & DATA SCIENCE
ST2334 PROBABILITY AND STATISTICS
SEMESTER I, AY 2025/2026

Tutorial 05

Please work on the questions before attending the tutorial.

Exam-Like Questions

1. MULTIPLE RESPONSE: CHOOSE ALL ANSWERS THAT APPLY

Suppose $V(X) = 5$, $V(Y) = 3$ and $Z = -2X + 4Y - 3$. Which of the following is/are **TRUE**?

- (a) If X and Y are independent, $V(Z) = 68$. (c) If $\text{cov}(X, Y) < 0$, $V(Z) > 68$.
(b) If $\text{cov}(X, Y) > 0$, $V(Z) > 68$.

2. MULTIPLE RESPONSE: CHOOSE ALL ANSWERS THAT APPLY

If X and Y are independent with cumulative distribution functions $F_X(x)$ and $F_Y(y)$, which of the following is **TRUE**?

- (a) $P(X^2 > 2; Y^4 > 0) = [1 - F_X(\sqrt{2}) + F_X(-\sqrt{2}-)][1 - F_Y(0) + F_Y(0-)]$.
(b) $P(X \geq x; Y \leq y) = (1 - F_X(x-))F_Y(y)$.
(c) $E((X^2 - E(X^2))g(Y)) = 0$ for any function g .

3. MULTIPLE CHOICE: CHOOSE ONE ANSWER ONLY

A box contains 2 red marbles and 98 blue ones. Draws are made at random with replacement. In n draws from the box, there is a better than 50% chance for a red marble to appear at least once. What is the smallest possible value for n ?

- (a) 35 (c) 55
(b) 45 (d) None of the other options

Long Form Questions

1. Two random variables have the joint density

$$f(x_1, x_2) = \begin{cases} x_1 x_2, & \text{for } 0 < x_1 < 2, 0 < x_2 < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the probability that both random variables will take on values less than 1.
(b) Find the marginal densities of the two random variables, and check whether the two random variables are independent.
(c) Find the expected value of the random variable whose values are given by $g(x_1, x_2) = x_1 + x_2$.

2. Suppose that X and Y are random variables having the joint probability function below.

$f(x,y)$		x	
		2	4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

- (a) Determine if X and Y are independent. (d) Find $E(2X - 3Y)$.
 (b) Find $E(Y|X = 2)$. (e) Find $E(XY)$.
 (c) Find $E(X|Y = 3)$. (f) Find $V(X)$ and $V(Y)$.
3. A service facility operates with two service lines. On a randomly selected day, let X be the proportion of time that the first line is in use; whereas Y is the proportion of time that the second line is in use. Suppose that the joint probability density function for (X, Y) is given below.

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1; \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Determine if X and Y are independent. (c) Find the covariance of X and Y .
 (b) Find the mean and variance of X and Y . (d) Find the mean and variance of $X + Y$.
4. According to Chemical Engineering Progress (Nov, 1990), approximately 30% of all pipework failures in chemical plants are caused by operator error. We assume that pipework failures occur independently of one another. What is the probability that out of the next 20 pipework failures,
- (a) at least 10 are due to operator error?
 (b) no more than 4 are due to operator error?
 (c) exactly 5 are due to operator error?
5. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. 10,000 forms are selected at random and examined.
- (a) Use a suitable approximation to find the probability that 6, 7, or 8 forms contain an error.
 (b) Find the mean and variance of the number of persons among 10,000 who make an error in preparing their tax returns.
6. A couple decides they will continue to have children until they have two sons. Assume that $P(\text{son}) = 0.5$, and only one child is born each time.
- (a) What is the probability that their second son is their seventh child?
 (b) What is the expected number of children for the couple?

Answers to some Long Form Questions

1. (a) $1/4$; (b) $f_1(x_1) = x_1/2, 0 < x_1 < 2$; $f_2(x_2) = 2x_2, 0 < x_2 < 1$; (c) 2. 4. (a) 0.0480; (b) 0.2375; (c) 0.1789.
2. (a) Independent; (b) 3; (c) 3.2; (d) -2.6 ; (e) 9.6; (f) 0.96; 2. 5. (a) 0.2657; (b) 10; 9.99.
3. (a) Dependent; (b) $E(X) = E(Y) = 5/8$; $V(X) = V(Y) = 73/960$. (c) $-1/64$; (d) $5/4$; 29/240. 6. (a) 0.0469; (b) 4.