Three

Supplementary Materials

Checking Independence

We have a handy way to check independence.

X and Y are independent if and only if both of the following hold:

(a) $R_{X,Y}$, the range where the probability function is positive, is a product space.

(b) For any $(x,y) \in R_{X,Y}$, we have

$$f_{X,Y}(x,y) = C \times g_1(x) \times g_2(y).$$

That is, $f_{X,Y}(x,y)$ can be "factorized" as the product of two functions g_1 and g_2, where g_1 **depends on** x **only**, g_2 **depends on** y **only**, and C is a constant not depending on both x and y.

Note: $g_1(x)$ and $g_2(y)$ on their own NEED NOT be probability functions.

Example 3.1

Consider the joint probability function

$$f(x,y) = \frac{1}{36}xy, \quad \text{for } x = 1,2,3,\ y = 1,2,3.$$

Note that we can write

$$A_1 = \{1,2,3\}, \quad A_2 = \{1,2,3\}.$$

So $R_{X,Y} = A_1 \times A_2$ is a product space.

We can also let

$$C = 1/36, \quad g_1(x) = x, \quad g_2(y) = y.$$

Then $f_{X,Y}(x,y) = C \times g_1(x) \times g_2(y)$. Thus X and Y are independent.

The advantage of this method is that we do not need to find the marginal distributions $f_X(x)$ and $f_Y(y)$ and check if $f_{X,Y}(x,y) = f_X(x)f_Y(y)$.

EXAMPLE 3.2
The joint probability function of (X,Y) is given below.

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{3}x(1+y), & \text{for } 0 < x < 2, 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Are X and Y independent?

Solution:
We can set $A_1 = (0,2)$ and $A_2 = (0,1)$. Then

$$R_{X,Y} = A_1 \times A_2$$

is a product space.

Notice that $f_{X,Y}(x,y)$ in $R_{X,Y}$ can be factorized by letting

$$C = \tfrac{1}{3}, \quad g_1(x) = x, \quad g_2(y) = 1+y.$$

Therefore, we conclude that X and Y are independent.