Two

Supplementary Materials

PROPERTIES OF THE CUMULATIVE DISTRIBUTION FUNCTION
Any cumulative distribution function has the following properties.

Right-continuous:
The cumulative distribution function is continuous except possibly for having some jumps. Wherever there is a jump, the cumulative distribution function is continuous from the right. That is, for any a, we have

$$F(a) = \lim_{x \to a^+} F(x).$$

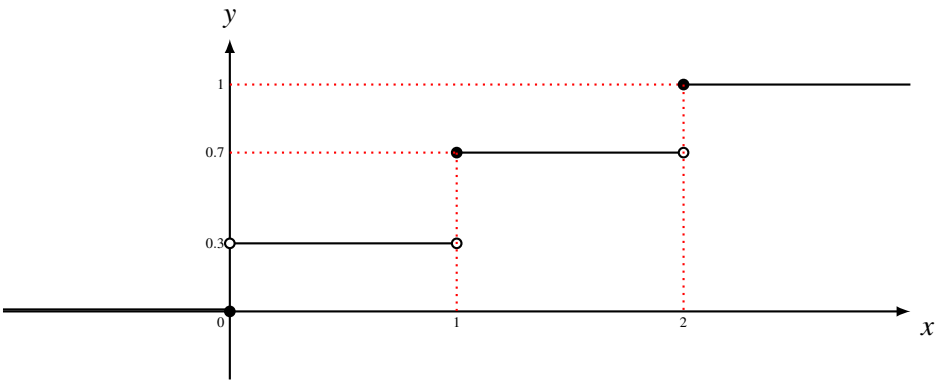
Convergence to 0 and 1 in the limits:

$$\lim_{x \to -\infty} F(x) = 0 \quad \text{and} \quad \lim_{x \to \infty} F(x) = 1.$$

EXAMPLE 2.1
Can the following be a valid cumulative distribution function?

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 0.3, & 0 < x < 1 \\ 0.7, & 1 \leq x < 2 \\ 1, & \text{elsewhere} \end{cases}$$

Solution:
$F(x)$ can be drawn as follows:



We see that $F(x)$ is not right-continuous at $x = 0$. So it cannot be a cumulative distribution function.