NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Tutorial 02: Suggested Solutions

Exam-Like Questions

1. The answer is (b).

   $P(A\cup B)=P(A)+P(B)$ holds if and only if $P(A\cap B)=0$.

   (a) is correct, since $P(A)P(B)=P(A\cap B)=0$.

   (b) is incorrect. For example, consider the experiment of selecting a random point from the interval $(0,1)$. Let A be the event of picking a point in $(0,0.5]$, and let B be the event of picking a point in $[0.5,1)$. Then it is easy to see that $A\neq B$, $P(A\cap B)=0$ but A and B are not mutually exclusive.

   (c) is correct. If not, $0<P(A)P(B)=P(A\cap B)=0$.

   (d) is correct, because $A\cap B=B$, and therefore $P(B)=P(A\cap B)=0$.

2. The answer is (d).

   The number of ways to draw 4 balls from 10 balls is $\binom{10}{4}=210$.

   The number of ways to get 2 blue balls, 1 green ball, and 1 red ball is $\binom{4}{2}\cdot\binom{4}{1}\cdot\binom{2}{1}=48$.

   So the required probability is $48/210=8/35$.

3. The answer is (c).

   Let

   $$A=\{\text{The factory will be set up in City A}\}\quad\text{and}\quad B=\{\text{The factory will be set up in City B}\}.$$

   It is given that $P(A)=0.7$, $P(B)=0.4$ and $P(A\cup B)=0.8$.

   (a) is correct, since $P(A\cap B)=P(A)+P(B)-P(A\cup B)=0.7+0.4-0.8=0.3$.

   (b) is correct, since $P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)=1-0.8=0.2$.

   (c) is incorrect, since $P(A\cap B)\neq P(A)P(B)$.

   (d) is therefore correct.

4. The answer is (a).

   Let

   $$A_i=\{\text{speeding at camera } i\},\quad B_i=\{\text{camera } i \text{ is on}\},\quad C_i=\{\text{speeding recorded at camera } i\}.$$

   Then $C_i=A_iB_i$, and $C_1'C_2'$ is the event that he will not receive a speeding ticket.

   We shall make the (reasonable) assumption that C_1 and C_2 are independent. This gives

   $$\begin{aligned}P(C_1'C_2')&=P(C_1')P(C_2')=[1-P(A_1B_1)][1-P(A_2B_2)]\\&=[1-P(A_1)P(B_1)][1-P(A_2)P(B_2)]\\&=(1-0.5\times0.4)(1-0.75\times0.4)=0.56.\end{aligned}$$

5. Number of possible hands of 5 cards is

$$\binom{52}{5} = \frac{52(51)(50)(49)(48)}{5!} = 2598960.$$

   (a) Number of spade flush hands is $\binom{13}{5} = 1287$.
   Similarly, the number of heart flush hands is also $\binom{13}{5} = 1287$, and so on.

$$P(A) = P(\text{a flush hand}) = \frac{4(1287)}{2598960} = 0.001981.$$

   (b) Number of straight hands with 1 as the smallest card is $\binom{4}{1}^5 - 4 = 1020$. Similarly, the number of straight hands with 2 as the smallest card is $\binom{4}{1}^5 - 4 = 1020$ and so on. The smallest card can be any one from 1 to 10.

$$P(\text{a straight hand}) = \frac{10(1020)}{2598960} = 0.003925.$$

Long Form Questions

1. (a) The number of ways to choose 5 out of 30 qualified applicants is $\binom{30}{5} = 142506$.
   (b) The number of ways to choose 5 out of 30 qualified applicants such that none of the minority is hired is $\binom{23}{5} = 33649$. Therefore the desired probability is $33649/142506 = 0.2361$.
   (c) The number of ways to choose 5 out of 30 qualified applicants such that one minority is hired is $\binom{7}{1} \times \binom{23}{4} = 61985$.
   Let A_0 and A_1 denote the events that no minority and one minority is hired respectively. Hence $P(A_1) = 61985/142506 = 0.4350$.
   From Part (b), $P(A_0) = 0.2361$.
   Therefore $P(\text{at most one minority is hired}) = P(A_0) + P(A_1) = 0.6711$.

2. Let $A_i, i = 1, 2$ denote the event that the motorist stops at light i. We have

$$P(A_1) = 0.4, \quad P(A_2) = 0.5, \quad P(A_1 \cup A_2) = 0.6.$$

   (a) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.4 + 0.5 - 0.6 = 0.3$.
   (b) The event {stop at exactly one light} can be represented as $(A_1 \cap A_2') \cup (A_1' \cap A_2)$.
   Note that

$$P(A_1 \cap A_2') = P(A_1) - P(A_1 \cap A_2) = 0.4 - 0.3 = 0.1$$

   and

$$P(A_1' \cap A_2) = P(A_2) - P(A_1 \cap A_2) = 0.5 - 0.3 = 0.2.$$

   Hence $P(\text{stop at exactly one light}) = 0.1 + 0.2 = 0.3$.
   (c) $P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - 0.6 = 0.4$.
   (d) $P(A_2|A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{0.3}{0.4} = 0.75$.

   Thus $P(A_2|A_1) = 0.75 \neq 0.5 = P(A_2)$. So A_1 and A_2 are not independent.

3. Let

$$\begin{aligned} M_1 &= \{\text{the selected bottle was filled on machine I}\}, \\ M_2 &= \{\text{the selected bottle was filled on machine II}\}, \\ N &= \{\text{a nonconforming bottle was selected}\}. \end{aligned}$$

   It is given that

$$P(M_1 \cap N) = 0.01, \quad P(M_2 \cap N) = 0.025, \quad P(M_1) = P(M_2) = 0.5.$$

(a) $P(N) = P((M_1 \cap N) \cup (M_2 \cap N)) = 0.01 + 0.025 = 0.035$.

(b) $P(M_2) = 0.5$;

(c) $P(M_2 \cap N') = P(M_2) - P(M_2 \cap N) = 0.5 - 0.025 = 0.475$.

(d) $P(M_1 \cup N') = P(M_1) + P(N') - P(M_1 \cap N')$.
On the other hand $P(N') = 1 - P(N) = 1 - 0.035 = 0.965$; $P(M_1 \cap N') = P(M_1) - P(M_1 \cap N) = 0.5 - 0.01 = 0.49$.
Therefore
$$P(M_1 \cup N') = 0.5 + 0.965 - 0.49 = 0.975.$$

(e) $P(N|M_1) = P(M_1 \cap N)/P(M_1) = 0.01/0.5 = 0.02$.

(f) $P(M_1|N) = P(M_1 \cap N)/P(N) = 0.01/0.035 = 0.2857$.

The events are different and the conditions are different.

The answer in (3e) is the probability of having a nonconforming item given that it was from machine I, that is, $P(N|M_1)$.

The answer in (3f) is the probability of having an item from machine I given that it was a nonconforming item, that is, $P(M_1|N)$.