NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & DATA SCIENCE

ST2334 PROBABILITY AND STATISTICS

SEMESTER I, AY 2025/2026

Tutorial 03: Suggested Solutions

Exam-Like Questions

1. The answer is (c).

   (a) No, because $\sum f(i) = 10/14 < 1$.
   (b) No, because $f(2) = -1/4 < 0$.
   (c) Yes. $0 \le f(i) \le 1$, and $\sum f(i) = 1$.
   (d) No, because $\sum f(i) = 35/50 < 1$.

2. The answers are (b), (c) and (d).

   They are right continuous everywhere, have a maximum value of 1, and non-decreasing.

3. The answer is (c).

   Note that

$$\begin{aligned} P(3 \le X \le 6) &= F(6) - F(3-) = 0.6 - 0.3 = 0.3; \\ P(X \ge 4) &= 1 - P(X < 4) = 1 - F_X(4-) = 1 - 0.4 = 0.6; \\ P(X = 3) &= F(3) - F(3-) = F(3) - F(2) = 0.4 - 0.3 = 0.1; \\ P(2 < X < 4) &= P(X = 3) = 0.1. \end{aligned}$$

4. In order for $f_X(x)$ to be a probability function, we need

$$1 = \sum_{x=0}^{3} f_X(x) = c[(0^2+4) + (1^2+4) + (2^2+4) + (3^2+4)] = 30c.$$

   This gives $c = 1/30$.

5. The answer is (a).

   The possible values of X are those values at which $F_X(x)$ jumps, and the probability of each of those values is the size of the jump at that value. Thus we have

x	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.05	0.15	0.4

Long Form Questions

1. (a) $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B) = 0.75(0.9)(0.8) = 0.54$.
   (b) $P(B) = P(A)P(B|A) + P(A')P(B|A') = (0.75)(0.9) + (0.25)(0.8) = 0.875$.
   (c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.75 \times 0.9}{0.875} = 0.7714$.

(d) Note that

$$P(B \cap C) = P(A \cap (B \cap C)) + P(A' \cap (B \cap C)).$$

Now,

$$P(A' \cap (B \cap C) = P(A')P(B|A')P(C|A' \cap B) = 0.25(0.8)(0.7) = 0.14.$$

Therefore $P(B \cap C) = 0.54 + 0.14 = 0.68$.

(e) $P(A|B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = \dfrac{0.54}{0.68} = 0.7941$.

2. Let $A = \{\text{TMQ implemented}\}$ and $B = \{\text{sales increased}\}$.

(a) $P(A) = 0.3$; $P(B) = 0.6$.

(b) Since $P(A|B) = 20/60$, therefore

$$P(A \cap B) = P(A|B)P(B) = (1/3)0.6 = 0.2.$$

As $P(A \cap B) \neq P(A)P(B) = 0.18$, A and B are not independent events.

(c) Since $P(A|B) = 18/60$, therefore

$$P(A \cap B) = P(A|B)P(B) = (0.3)0.6 = 0.18.$$

As $P(A \cap B) = P(A)P(B)$, A and B are independent events.

3. Let B be the event that a component needs rework. Then

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3) = 0.5(0.05) + 0.3(0.08) + 0.2(0.1) = 0.069.$$

We then have $P(A_2|B) = \frac{P(A_2)P(B|A_2)}{P(B)} = \frac{0.3(0.08)}{0.069} = 0.3478$.

4. Let O_i and O_i' be the events that an O^+ and a non-O^+ individual is typed on the ith typing.

$$\begin{aligned}
f(1) &= P(Y=1) = P(O_1) \\
&= 2/5 = 0.4, \\
f(2) &= P(Y=2) = P(O_1')P(O_2|O_1') \\
&= 3/5 \times 2/4 = 0.3, \\
f(3) &= P(Y=3) = P(O_1')P(O_2'|O_1')P(O_3|O_1' \cap O_2') \\
&= 3/5 \times 2/4 \times 2/3 = 0.2, \\
f(4) &= P(Y=4) \\
&= P(O_1')P(O_2'|O_1')P(O_3'|O_1' \cap O_2')P(O_4|O_1' \cap O_2' \cap O_3') \\
&= 3/5 \times 2/4 \times 1/3 \times 2/2 = 0.1, \\
f(y) &= 0, \quad \text{if } y \neq 1,2,3,4.
\end{aligned}$$

The probability function of Y is then

y	1	2	3	4
$f(y)$	0.4	0.3	0.2	0.1

5. (a) For $f_X(x)$ to be a valid probability density function, we need

$$1 = \int_{-\infty}^{\infty} f_X(x)\,\mathrm{d}x = \int_0^1 kx^{1/2}\,\mathrm{d}x = \frac{k}{1+1/2}\left[x^{1+1/2}\right]_0^1 = \tfrac{2}{3}k.$$

This gives $k = \frac{3}{2}$.

(b) Clearly, when $x \leq 0$, $F_X(x) = 0$, and when $x \geq 1$, $F_X(x) = 1$.
When $0 < x < 1$, we have

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,\mathrm{dt} = \int_0^x \tfrac{3}{2}t^{1/2}\,\mathrm{dt} = \left[t^{3/2}\right]_0^x = x^{3/2}.$$

In summary,

$$F_X(x) = \begin{cases} 0, & x \leq 0; \\ x^{3/2}, & 0 < x < 1; \\ 1, & x \geq 1. \end{cases}$$

As a consequence,

$$P(0.3 < X < 0.6) = F_X(0.6) - F_X(0.3) = 0.6^{3/2} - 0.3^{3/2} = 0.3004.$$

6. (a) 12 minutes is equivalent to $\frac{1}{5}$ hours, so the required answer is

$$P\left(X < \tfrac{1}{5}\right) = F_X\left(\tfrac{1}{5}\right) = 1 - e^{-\frac{8}{5}} = 0.7981.$$

(b) Note that $f_X(x) = 0$ for $x \leq 0$. When $x > 0$,

$$f_X(x) = \frac{\mathrm{d}}{\mathrm{dx}} F_X(x) = \frac{\mathrm{d}}{\mathrm{dx}}\left(1 - e^{-8x}\right) = 8e^{-8x}.$$