Abstract questions about systems consisting of an ensemble of components have preoccupied the minds

of humans for millennia. For example, the Greek philosopher Aristotle stated that “. . . the whole is

something over and above its parts, and not just the sum of them all.” In The Republic, fellow Greek

philosopher Plato introduced the concept of “level of knowledge,” ranging from total ignorance to total

knowledge. “True knowledge” exists only if a foundation of axioms or a priori knowledge exists [172],

and this cannot be the case for complex systems.

At first we can turn for guidance on how to measure system complexity to thermodynamics, a

branch of physics concerned with heat and its relation with energy and work. Thermodynamics defines

macroscopic properties such as temperature, pressure, and entropy to characterize large assemblies of

microscopic particles, e.g., gases, and establishes laws governing the behavior of such systems. The

analogy to large-scale systems is inescapable; indeed, we are interested in high-level properties such

as reliability; performance measured by throughput and response time, security, and elasticity; and the

ability to respond to a sudden increase of the load of very large collections of servers, each one powered

by many processors, each processor with millions of transistors. From the “microscopic” properties of

these elements we have to estimate the “macroscopic” properties of the system.

The concepts of thermodynamic entropy, von Neumann entropy, and Shannon entropy are related to

the number of states of a system; thus, they reflect to some extent the system’s complexity [92]. The

thermodynamic entropy of a microscopic system, e.g., N molecules of gas, is

S = kB ln  , (10.1)

with kB the Boltzmann’s constant and the number of microstates of the system.When the N molecules

are grouped together in m macrostates depending on their energy, then the number of bits required to label the individual microstates is

Q = H(p1, p2, . . . , pm), (10.2)

with H(p1, p2, . . . , pm) the Shannon entropy of a systemwithm states. If ni is the number of molecules

in state i , then pi = ni /N is the probability of the system being in state i .

In turn, the von Neumann entropy of a quantum system with the density matrix ρ

S(ρ) = −tr[ρ log ρ] (10.3)

is equal to the Shannon entropy if the system is prepared in a maximally mixed state, a state where all

pure states are equally likely.

A measure of complexity is the relative predictive efficiency, denoted by e and defined as

e = E/C (10.4)

with E the excess entropy and C the statistical complexity [95]. The excess entropy measures the

complexity of the stochastic process and can be regarded as the fraction of historical information about

the process that allows us to predict the future behavior of that process. The statistical complexity reflects

the size of the model of the system at a certain level of abstraction.

The scale of organization considered by an external observer plays a critical role in assessing the

relative predictive efficiency. For example, at the microscopic level the calculation of e for a volume of

gas requires very complex molecular dynamic computations in order to accurately predict the excess

entropy; both E and C are very high and the predictive efficiency is low. On the other hand, at the

macroscopic level the relationship among the pressure P, the volume V, and the temperature T is a

very simple PV = nRT, with n the number of moles of gas and R the universal gas constant. In this

case, E maintains a high value, but now C is low and the predictive efficiency E/C is large.

Physical systems in equilibrium display their most complex behavior at critical points. In thermodynamics

a critical point specifies the conditions of temperature and pressure at which a phase boundary,

e.g., between liquid and gas, ceases to exist. The time to reach equilibrium becomes very high at critical

points, a phenomenon called critical slowing. Wolpert and Macready [377] argue that self-similarity

can be used to quantify complexity; the patterns exhibited by complex systems at different scales are

very different, whereas the patterns exhibited by simple systems such as gases and crystals do not vary

significantly from one scale to another.

As discussed earlier, we could use the complexity of a program that simulates the system as a

measure of complexity of the system. This will reflect not only the number of states but also the pattern

of transitions among states. This idea has its own limitations because, generally, in our simulations we

use approximate models of a system rather than exact ones.

This measure of complexity is consistent with the concept of depth, defined as the number of computational

steps needed to simulate a system’s state. The author of [225] argues that the emergence

of complexity requires a long history, but we need a measure stricter than physical time to reflect this

history. The depth reflects not how long the system remains in equilibrium but how many steps are

necessary to reach equilibrium following some efficient process. The rate of change of the system state

and the communication time do not reflect the complexity of a system. Indeed, two rotating structures

involving very different physical processes, a hurricane and a spiral celestial galaxy, are at the limit of today’s realistic computer simulations, thus, are of similar depth and, consequently, of similar complexity.

Yet, galaxy formation occurs at a scale of millions of light years and is bounded by communication

at the speed of light, whereas the time for hurricane formation is measured in days, the atmospheric

disturbances propagate more slowly, and the scale is only hundreds of kilometers.

Complexity could be related to the description of a system and may consist of structural, functional,

and, possibly, other important properties of the system. The question of how to measure the descriptive

complexity of an object was addressed by Kolmogorov [198] and, independently, by Solomonoff [328]

and Chaitin [69]. An application of Kolmogorov complexity to the characterization of scheduling on a

computational grid is discussed in [229].

TheKolmogorov complexity KV(s) of the string s with respect to the universal computer V is defined

as the minimal length over all programs ProgV that print s and halt

KV(s) = min[Length(s)] over all Prog: V(ProgV) = s. (10.5)

The intuition behind Kolmogorov complexity is to provide the shortest possible description of any

object or phenomenon, and its roots can be traced back to wisdom formulated centuries ago. “Nunquam

ponenda est pluritas sine necesitate,” the famous principle formulated by William of Ockham (1290–

1349), states; it means that an explanation should not be extended beyond what is necessary [351].

Bertrand Russell translates this as “It is vain to do with more what can be done with fewer.”