The Dissemination of Public Opinion

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Abstract

In this research, we present an extended version of the SIR model to examine the propagation of opinions by incorporating a conformity function that depends on how many people are in the infective compartment. To explain this model thoroughly, we examined the equilibrium of the systems of ODE we introduced and derived the width of dissemination by adapting from the idea of Information Bomb by Paul Virilio. The point of popularity subsided is also found. We also made the sensitivity analysis on a set of parameters to study the effect of their variation on the equilibrium. This research also tested the model's robustness by running simulations on different network structures and found that small-world networks possess a lower transmission threshold and faster dissemination than other structures. Suggestions for future investigations include combining our model with the IDRSI model and fitting real-world data into our model.

1 Background

In this age of advanced communication and interaction capabilities, the importance and influence of information dissemination may exceed our previous understanding. During the pandemic, we have witnessed a wide public divergence on topics like the validity of vaccinations which has deeply affected the policy enacted to combat the virus. Under this circumstance, the importance of understanding the formation and propagation of opinions has emerged.

Opinion propagation is the process by which people form and change their opinions based on the information they receive from various sources, such as media, friends, or experts. This complex phenomenon has been shaped by various factors, including the rise of social media, political polarization, confirmation bias, and the emergence of social media influencers. In particular, social media has played a significant role in recent years. According to a study by the Pew Research Center, 80% [5]of U.S. adults who use Twitter said that they retweeted others' posts. Another study by the University of Pennsylvania showed that the median time taken for a tweet to be retweeted is less than 18 minutes. The data from Twitter have indicated that public opinions are easily spread widely and quickly.

Despite the power of the retweet, the dissemination of public opinions shaped the influencer marketing. Influencers are those who have built a reputation on social media by creating content focused on a specific topic that resonates with their followers. By sharing their opinions and recommendations with their followers, influencers are able to shape the opinions and behaviors of others, which further can be seen in the sales of the influencers' recommended products and the growth of the whole industry. A survey by Morning Consult has found that 45% of U.S. adults made purchases based on influencers' recommendations and the influencer marketing industry was worth more than 13.8 billion in 2021. This growth has been driven in part by the ability of influencers to disseminate public opinions[6].

Another famous case is GameStop. In January 2021, a group of amateur investors on the social media platform Reddit coordinated to drive up the stock price of GameStop, a struggling video game retailer. The investors, who called themselves the "WallStreetBets" subreddit, used various tactics to push up GameStop's stock price, including buying large amounts of shares and encouraging others to do the same. The coordinated effort attracted widespread attention in the media and on social media platforms like Twitter, where the hashtag #Gamestop trended for several days. As a result of the investor activity, GameStop's stock price rose from around \$20 per share to a high of \$347 per share in just a few weeks, before ultimately crashing back down. The case indicated the power of social media and online networks to influence the stock market and how the dissemination of opinions can have real-world consequences in areas of finance.[7]

The dissemination of opinions can have serious negative consequences as well, particularly when those opinions are not real and lack scientific evidence. One has been seen in the COVID-19 pandemic. A study published in the Journal of Medical Internet Research found that false information about COVID-19 was widespread on social media, with rumors and conspiracy theories spreading rapidly. For example, in March 2020, the claim that drinking could cure COVID went viral on social media. Within a week, the US Centers for Disease Control and Prevention received more than 20 calls related to bleach ingestion. The spread of false information about vaccines has also led to a decline in vaccination rates, with some communities experiencing outbreaks if preventable diseases like measles.

2 Modelling Question

Under this background, we are curious about how influential public opinion could be and how we can deal with it.
In the way to examine the dissemination of opinions, we proposed the following questions:

- How wide an opinion could spread in society?
- Is there a peak for the popularity of an opinion? If there is, when is it?
- How convinced people could be when they receive information from others?

3 Modelling Approach

Based on a SIR model, we adapted the idea from Xue Zhou 2014 who uses a probability constant p to indicate the chances that a Susceptible transforms to Recovered. Under the intuition of how people may perceive a piece of information is significantly influenced by how many are currently spreading, we made a function for this probability p and make it dependent on the proportion of the public currently spreading the information.

We also created an idea of **Width of Dissemination** that describes the influenced population at equilibrium.

4 Formulation of Model

4.1 Modelling Assumptions

- People's reactions (the embodiment of opinions) towards a topic can be generally categorized into 3 states.
- People's opinions can be more or less influenced by what percentage of people who believe or are interested in a topic.
- Opinions can be spread homogeneously
- Once people have shown no interest in spreading an opinion, there is no flip.
- The rate of injection of an opinion to others is constant

4.2 Defining Compartments

Our first innovative alternation is taking credibility of the information into consideration

- Susceptible: Population that is unaware of the information and is possible to absorb
- Infective: Population that is aware of the information and is spreading it to Susceptible
- Recovered: Population that is either not convinced from the beginning or not interested in spreading anymore.

- Conformity: How likely a susceptible would be convinced and start spreading after absorbing the information

4.3 Variables and Parameters

Variables	Physical Interpretation	Unit
t	Time of Day	days
S	Susceptible Compartment	-
I	Infective Compartment	-
R	Recovered Compartment	-

Table 1: Variable Table

Parameters	Physical Interpretation
α	Infectious rate of Susceptible
λ	Recover rate of Infective
c	credibility of information

Table 2: Parameter Tables

A base compartment model can be described as a system of differential equations.

$$\frac{dS}{dt} = -\alpha SI$$
$$\frac{dI}{dt} = c\alpha SI - \lambda I$$
$$\frac{dR}{dt} = (1-c)\alpha SI + \lambda I$$

4.4 Conformity

Our second innovative alternative is to take credibility dependent on the proportion of people spreading the information, which means we try to fit a function of $C(I)$. We observed data from Tweets Dataset and interpreted as follows:

- Proportion of people spreading the information is interpreted as

$$\frac{\text{number of retweets}}{\text{Total Active Users}}$$

- Conformity is interpreted as

$$\frac{\text{number of retweets}}{\text{number of views}}$$

We observe the following characteristics:

- Generally, the more people retweeted, the more likely a susceptible choose to retweet
- People could be convinced even if no one has chosen to spread before, implies $C(0) > 0$

- There exist some points of I such that there is a radical increase in Conformity.

We adopted the sigmoid function to fit our observations

$$\frac{1}{1+e^{-k(\frac{I}{N}-\mu_0)}}$$

Our new Parameter table is given as:

Parameters	**Physical Interpretation**
α	Infectious rate of Susceptible
λ	Recover rate of Infective
k	rate of convincement at threshold
μ_0	threshold of convincement
N	Total Population

Table 3: Parameter Tables

Our final model is in the following form

$$\frac{dS}{dt} = -\alpha SI$$
$$\frac{dI}{dt} = c\alpha SI - \lambda I$$
$$\frac{dR}{dt} = (1-c)\alpha SI + \lambda I$$
$$c = \frac{1}{1+e^{-k(\frac{I}{N}-\mu_0)}}$$

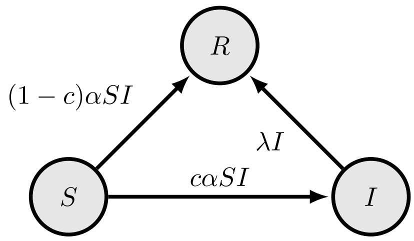


Figure 1: The three states of the SIR model: S - susceptibles, I - infectives, and R - recovered, as well as the fluxes between the states: infection at rate $p\alpha SI$ and recovery at rate λI.

5 Analysis

First, we guarantee that we have a fixed total population. Since $N = S + I + R$, we have $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$

5.1 Equilibrium and Stability

For simplicity of calculation, we define

$$p(I) = \frac{I}{1 + e^{-k(\frac{I}{N} - \mu_0)}}$$

and we evaluate

$$p'(I) = \frac{(\frac{Ik}{N} + 1)(e^{-k(\frac{I}{N} - \mu_0)}) + 1}{(1 + e^{-k(\frac{I}{N} - \mu_0)})^2}$$

the Jacobian for our system at equilibrium is evaluated to be

$$\begin{pmatrix} -\alpha I & -\alpha S & 0 \\ (p(I)\alpha - 1)I & \alpha S p'(I) - \lambda & 0 \\ (1-p)\alpha I & \alpha S + \lambda - \alpha S p'(I) & 0 \end{pmatrix}, \tag{1}$$

From observation, we find that an isolated equilibrium will be $I = 0$ with S and R free, at this equilibrium, $p'(0) = \frac{1}{1+e^{k\mu_0}}$ and the Jacobian we evaluate becomes

$$\begin{pmatrix} 0 & -\alpha S & 0 \\ 0 & \frac{\alpha S}{1+e^{k\mu_0}} - \lambda & 0 \\ 0 & \alpha S + \lambda - \frac{\alpha S}{1+e^{k\mu_0}} & 0 \end{pmatrix}, \tag{2}$$

Jacobian has eigenvalues $0, 0, \frac{\alpha S}{1+e^{k\mu_0}} - \lambda$, thus if $S \leq \frac{\lambda(1+e^{k\mu_0})}{\alpha}$, equilibria are stable. And since we have eigenvectors $([1,0,0]^T, [0,0,1]^T)$, this means as long as we have $I = 0$, we have an equilibrium

5.2 Power of Dissemination

Adapted from the idea of *Information Bomb* by Paul Virilio, we discuss the power of dissemination in the following criteria. Assuming at t_τ, our model reaches equilibrium $I = 0$

5.2.1 Basic Reproduction Number

Applying the method from Driessche 2002, we obtained an $R_0 = \frac{\alpha S}{\lambda(1+e^{k\mu_0})}$, which is the spectral radius of the next-generation matrix. Therefore if $R_0 < 1$, our model has an infectious-free equilibrium, which corresponds to the discussion in the previous section

5.2.2 Width of Dissemination

We interpret the Width of the Dissemination as the proportion of the population that has absorbed the information, which is the proportion of the population that has been Infective during the whole process. Viewing our model as a Markov chain, an Infectious only recovers or stays infected, leading to

$$W = \frac{I(t_\tau) + R(t_\tau)}{N} = \frac{R(t_\tau)}{N}$$

Taking either $\frac{dS}{dI}$ and $\frac{dR}{dI}$ results in an unsolvable differential equation, to make predictions on R_τ, we use

$$\begin{aligned}\frac{dR}{dS} &= \frac{(1-c)\alpha SI + \lambda I}{-\alpha SI} \\ &= c - 1 - \frac{\lambda}{\alpha S} \\ R(S) &= (c-1)S - \frac{\lambda}{\alpha} log(S) + K \quad \text{for some constant K}\end{aligned}$$

Plugging an initial condition of $S = S(0)$ and $R = 0$,

$$K = \frac{\lambda}{\alpha} log(S(0)) + (1-c)S(0)$$

Since $N = S + I + R, I = 0$ our Equilibrium condition is given as $R^{eq} = N - S^{eq}$, solving for equilibrium R gives us

$$R^{eq} = \frac{\lambda logN - c\alpha N}{\lambda logN - c\alpha} + \frac{\alpha(N-K)}{\lambda logN - \alpha c} = \frac{\lambda logN - c\alpha N + \alpha(N-K)}{\lambda logN - c\alpha}$$

We could also derive the following derivatives

$$\begin{aligned}\frac{dR^{eq}}{d\lambda} &= \frac{-c\alpha log(N) + c\alpha N log(N) - \alpha log(N)(N-K)}{(\lambda log(N) - c\alpha)^2} \\ \frac{dR^{eq}}{d\alpha} &= \frac{-\lambda c N \ln(N) + \lambda c \ln(N) + \lambda \ln(N)(N-K)}{(\lambda \ln(N) - c\alpha)^2}\end{aligned}$$

For any party that decides to make a social influence by modifying either the contact rate or recovery rate. he could plug in current data into derivative formulas and based on the costs find the most efficient way either widen or narrow the width of dissemination.

5.2.3 Popularity Subsided

We interpret the peak information popularity at the point when I reaches maximum.

- If $\frac{\alpha S}{1+e^{-k(\frac{I(0)}{N}-\mu_0)}} - \lambda < 0$, we conclude that $I_{max} = I_0$, an outbreak would not happen
- If $\frac{\alpha S}{1+e^{-k(\frac{I(0)}{N}-\mu_0)}} - \lambda > 0$, an outbreak brought by information bomb arrives. By solving $c\alpha S = \lambda$, the outbreak will bring Infectious to a peak of

  $$I = (\frac{-ln(\frac{\alpha S}{\lambda} - 1)}{k} + \mu_0)N$$

  which from that point, the popularity of that information starts to subside.

5.2.4 Sensitivity Analysis of Parameters

In order to conduct sensitivity analysis, we gave parameters the following values:

Parameters	Values
α	0.025
λ	1
k	100
μ_0	0.04
N	100

Table 4: Parameter Tables

Recall $I^{eq} = 0$, so $c^{eq} = \frac{1}{1+e^{k\mu_0}} = \frac{1}{1+e^4}$.
Under this setting, we can examine how sensitive the equilibrium R^{eq} is affected by λ and α:

$$
\begin{aligned}
S(R^{eq}, \lambda) &= \frac{dR^{eq}}{d\lambda}\frac{\lambda}{R^{eq}}\Big|_{\lambda=1} \\
&= \frac{-c\alpha log(N) + c\alpha Nlog(N) - \alpha log(N)(N-K)}{(\lambda log(N) - c\alpha)^2}\left[\frac{\lambda logN - c\alpha}{\lambda logN - c\alpha N + \alpha(N-K)}\right]\lambda\Big|_{\lambda=1} \\
&= \lambda\frac{-c\alpha log(N) + c\alpha Nlog(N) - \alpha log(N)(N-K)}{(\lambda log(N) - c\alpha)(\lambda logN - c\alpha N + \alpha(N-K))}\Big|_{\lambda=1} \\
&= 0.0098
\end{aligned}
$$

$$
\begin{aligned}
S(R^{eq}, \alpha) &= \frac{dR^{eq}}{d\alpha}\frac{\alpha}{R^{eq}}\Big|_{\alpha=0.025} \\
&= \frac{-\lambda cNlog(N) + \lambda clog(N) + \lambda log(N)(N-K)}{(\lambda log(N) - c\alpha)^2}\left[\frac{\lambda logN - c\alpha}{\lambda logN - c\alpha N + \alpha(N-K)}\alpha\right]\Big|_{\alpha=0.025} \\
&= \alpha\frac{-\lambda cNlog(N) + \lambda clog(N) + \lambda log(N)(N-K)}{(\lambda log(N) - c\alpha)(\lambda logN - c\alpha N + \alpha(N-K))}\Big|_{\alpha=0.025} \\
&= -0.0098
\end{aligned}
$$

This means the equilibrium of Recovery R^{eq} is affected by λ and α to the same very small extent. However, it changes positively with λ the Recover rate of the Infectives while it changes negatively with α the Infectious rate of Susceptible, which complies with our common sense.

6 Simulation

6.1 Stability of R_0

Using original numerical simulation, this parts simulate the stability of R_0. There are two cases: when $R_0 > 0$ and $R_0 < 0$.

- When $R_0 > 1$, the resulting population has an outbreak at around t = 0.5, as showing the graph below. The curves of S, I, R change within time during transmitting.S declines rapidly with I and R increasing and tends to be stable at last. I increases rapidly at its initial stage, after reaching its maximum(the outbreak), it gradually reduced to zero. The three variables tend to be stable when t tends to infinity.

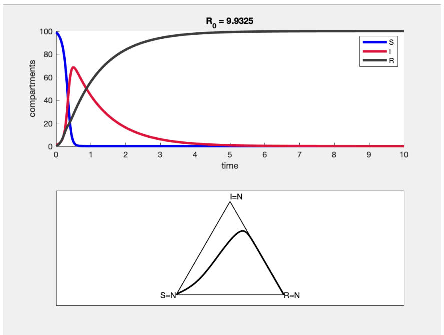


Figure 2: $R_0 > 1$.

- When $R_0 < 1$, the resulting population has no outbreaks, as shown in the graph below. The curves of S, I, R change slowly within time during transmitting. As t tends to infinity, S is approaching a constant, I and R are approaching 0.

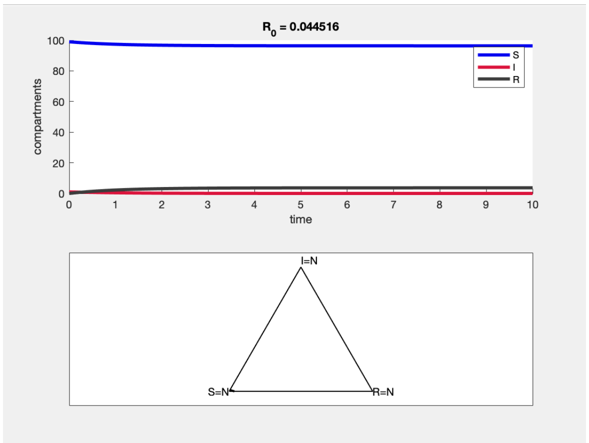
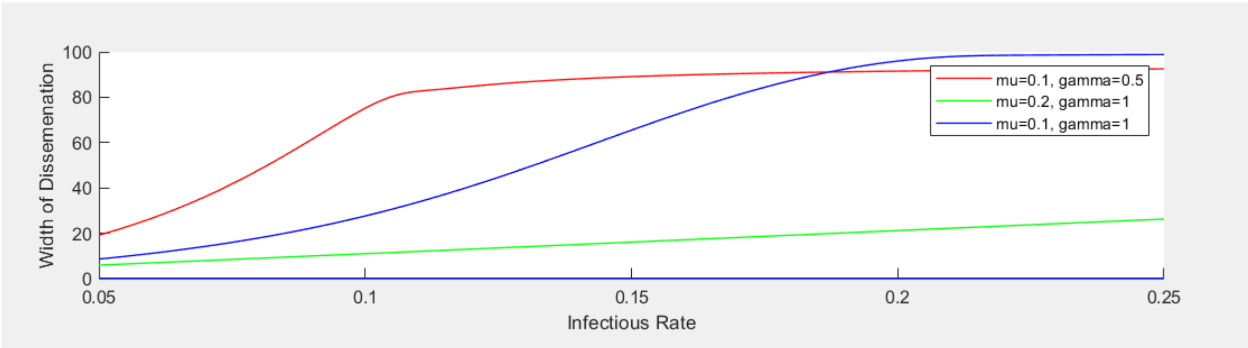


Figure 3: $R_0 < 1$.

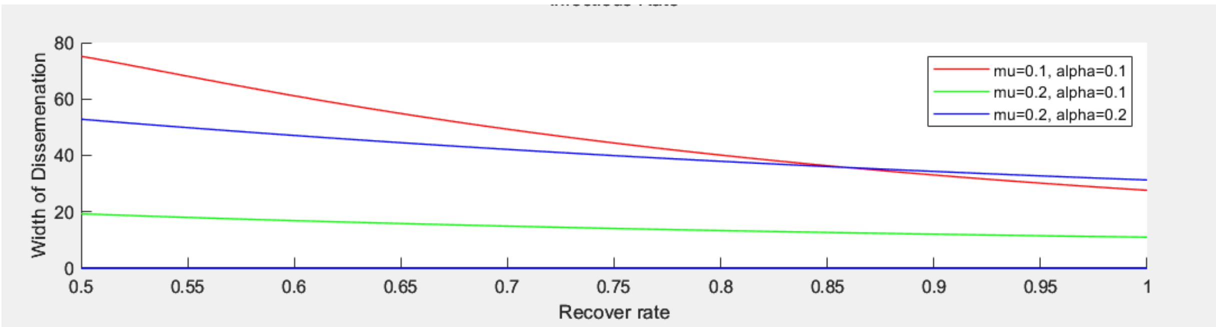
6.2 Width of dissemination & parameters

By choosing different other parameters we are able to model their relationship with width of dissemination.

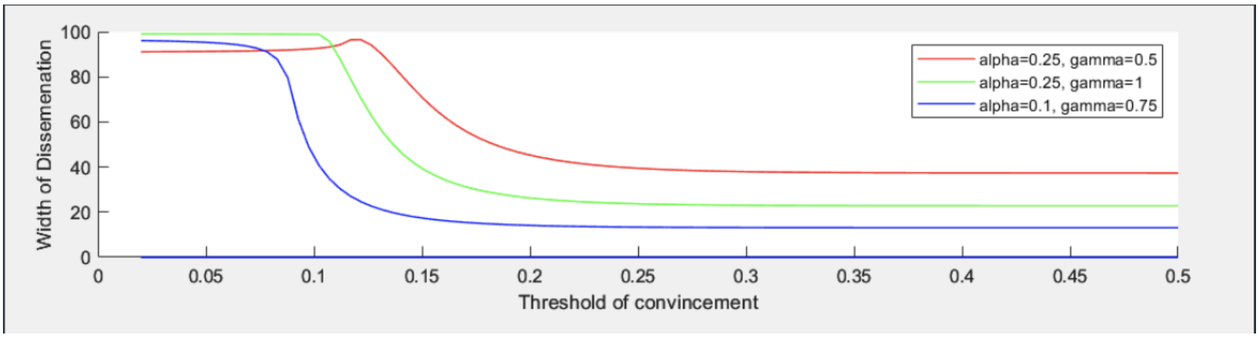
- By fixing μ and γ, varying α, we obtain the graph below. We can see that generally, when the infectious rate increases, the width of dissemination increases as well, which means that the infected were more widely spread. And there will be outbreaks occur under some parameters.(e.g.$\mu = 0.1$, $\gamma = 0.5$)



- By fixing μ and α, varying γ, we obtain the graph below. We can see that generally, when the recovery rate increases, the width of dissemination decreases instead, which means that the recovered were less widely spread. And there will be outbreaks occur under some parameters.



- By fixing α and γ, varying μ, we are able to model the width of dissemination with respect to threshold of convincement, which the percentage of people spreading that make the public start to be convinced. From the graph below, We could observe that although under different parameters conditions, at lower threshold, there is always outbreak but as it increases, width of dissemination drops dramatically.



6.3 Robustness of Assumptions

In reality, platforms are not completely neutral as they would promise and they have their own expertise. The model we have introduced may only work in a single platform as it naively assumed homogeneity in spreading and therefore did not take into account the potential information barriers across platforms. Zanette[9] studied rumor propagation in small-world networks and pointed out that the network topologies have a great influence on the rumor propagation. To address this problem, we can extend our model to the following:

$$\frac{dS_i}{dt} = -\alpha_i S_i I_i + \sum_j T^s_{i,j} S_j - \sum_j T^S_{j,i} S_i \tag{3}$$

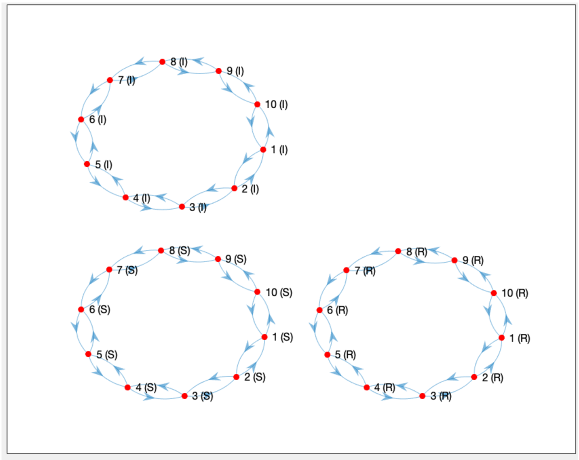
$$\frac{dI_i}{dt} = c_i \alpha_i S_i I_i - \lambda_i I_i + \sum_j T^I_{i,j} I_j - \sum_j T^I_{j,i} I_i \tag{4}$$

$$\frac{dR_i}{dt} = (1 - c_i)\alpha_i S_i I_i + \lambda_i I_i + \sum_j T^R_{i,j} R_j - \sum_j T^R_{j,i} R_i \tag{5}$$

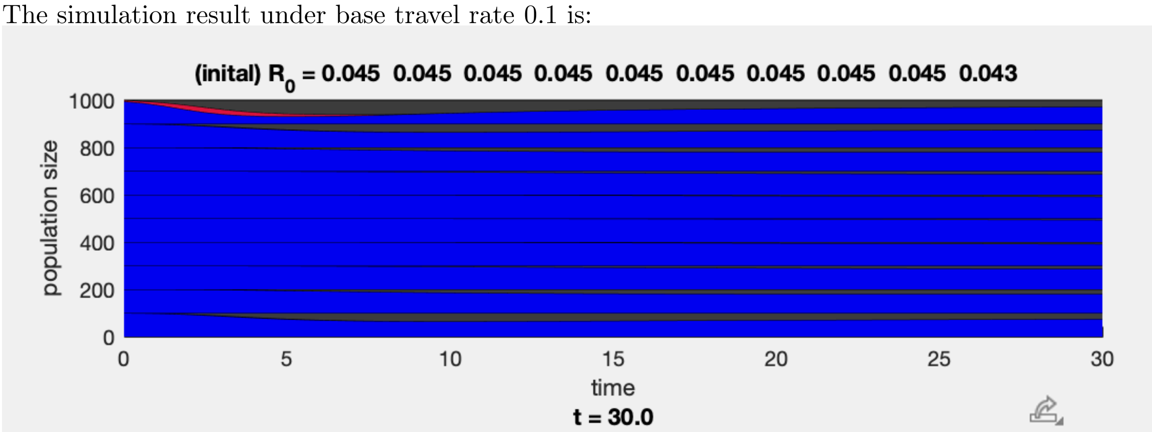
in which

$$c_i = c(I_i) = \frac{1}{1 + e^{-k(I_i/N_i - p)}}$$

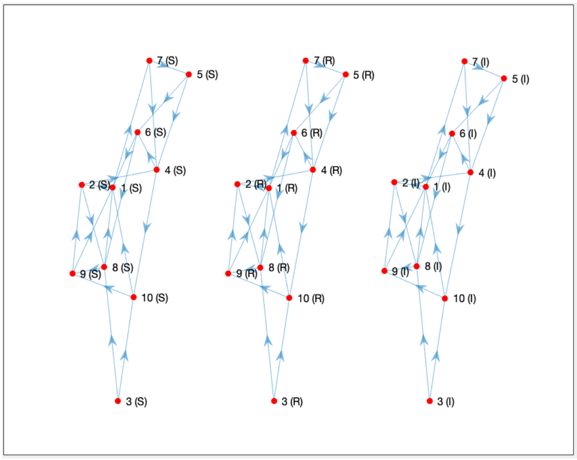
- Then we simulate it on Cartesian Networks which consist of nodes arranged in a regular grid pattern, with each node connected to its two nearest neighbors. In our context, this means users of each platform only spread information to the two closest platforms in terms of ideology or fields.
  The shape of the network is:



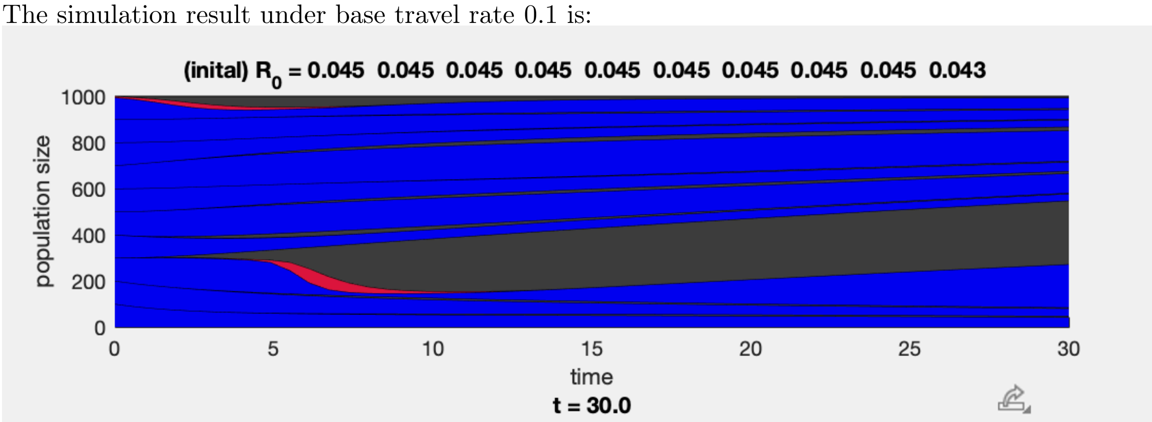
The simulation result under base travel rate 0.1 is:



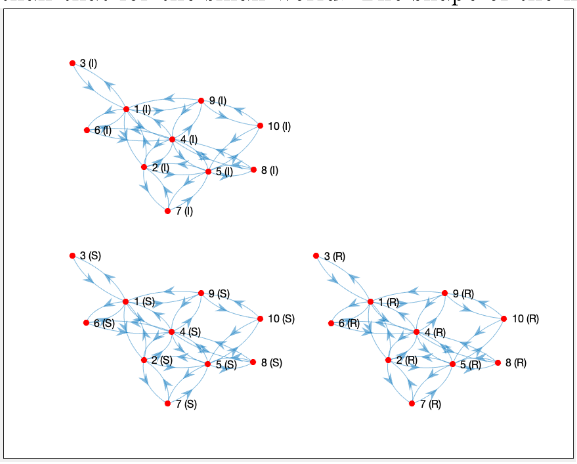
- Next, we simulate it on Small World Networks which are characterized by a high degree of local clustering and a few long-range connections. In our context, this network is a representation of the existence of niche/extremist platforms and also the widely popular platforms. There is a significant distinction in spreading scale between each platform which may be due to local clustering. And the overall speed is quicker in this structure compared with Cartesian Network. The shape of the network is:



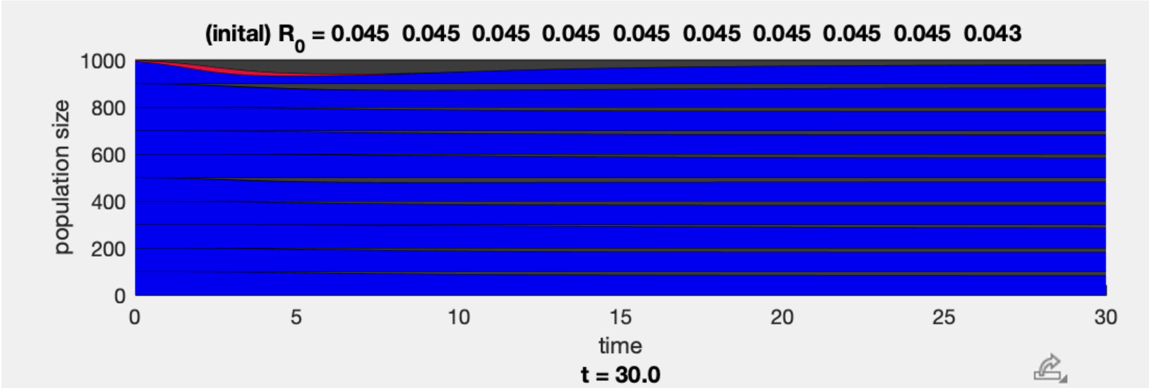
The simulation result under base travel rate 0.1 is:



- Finally, we simulate our model on Scale-Free Networks which are characterized by a few highly connected nodes and many poorly connected nodes. This simulates the situation when we have a monopoly or oligarchy in the channels of communication. It took this structure a bit longer than that for the small world. The shape of the network is:



The simulation result under base travel rate 0.1 is:



For these three topological structures of networks, we have found that under the same parameters, a small world network has a smaller transmission threshold and faster dissemination. The result coincides with the previous study of Zanette [9] even though we used different models. This shows in an environment with diversified platforms, rumors can be soon absorbed.

7 Response to Questions From Presentation

The main comments we had about our presentation were

- *Questions*: Is # of retweets = I or $\frac{dI}{dt}$?
  Response: The number of retweets is equal to **I**. Since I represents the infective compartment that is **aware** of the information and **is** spreading it to Susceptible(unaware of the information and is possible to absorb).

- *Question*: What is the meaning of R_0?
  Response: R_0 is the basic reproduction number, referring to the transmissibility of information during dissemination.

- *Question*: How could this model be used by regulators to improve social media environments?
  Response: By using this model, the regulators are able to gain insights in how public opinions are formed and disseminated across social media platforms. For example, they can identify the key factors that drive the dissemination of opinions and design methods to target these factors. They can also use the model to evaluate the impact of their policies and help them anticipate future risks or emergence of new controversial topic.

- *Question*: Have you compared different social media sites?(e.g. Twitter vs. Truth Social). What are the commonalities/differences?
  Response: We did not compare different social media sites. The dataset we used was mainly from Twitter since it's very hard to find such large dataset that includes all the information we wanted. After evaluating the dataset we found, we believe it's the best we can find that includes the following: total views, number of likes, number of tweets and number of retweets.

- *Question*: Did you compare your model against data?
  Response: The Tweets Dataset we found was categorized tweets by authors, then documented

the corresponding tweet content, following country, date time, id, language, latitude, longtitude, number of likes, and number of shares. We interpret the number of shares = population retweets = Infected, and the number of likes + number of shares = total views = N. We used this data to fit our assumption of conformity function and it fits well.

- *Question*:What does "stable equilibrium" mean in your context?
  Response: The stable equilibrium means that the population of Susceptible, Infected, and Recovered do not change at a specific point.

- *Question*: What parameters have you considered while choosing a conformity function?
  Response: Since conformity is interpreted as

  $$\frac{\text{number of retweets}}{\text{number of views}}$$

  , and after observing the characteristics, we adopted the sigmoid function. Then our parameters are: infectious rate of susceptible, recover rate of infective, rate of convincement at threshold, and the total population.

8 Conclusion

8.1 Summary of Results

In this project, we developed an extended version of the SIR model and examined the dissemination of public opinion. Our principal results are as follows:

- Overall our model was able to have an equilibrium as long as Infected = 0. If $R_0 < 1$, we will have an infectious-free equilibrium, and we do not expect an outbreak. In general, the width of dissemination is positively influenced by the infectious rate and negatively influenced by the recovery rate, however, depending on different choices of parameters and other conditions, the efficiency of changing those parameters may vary. Another parameter of significance we like to mention is the threshold of convincement, they can be seen as a general characteristic of the public but could be affected by education level. So the best to eradicate a rumor is to let people less likely to conform.

- We tested the robustness of our model assumptions by running simulations in small-world networks, Cartesian Networks, and Scale-Free Networks. We found that with equivalent parameters, a small world network possesses a lower transmission threshold and faster dissemination when compared to the other three topological structures. This outcome aligns with the findings of Zanette's[9] prior investigation, despite our use of distinct models. The implications are that in a setting featuring varied platforms, rumors can be assimilated rapidly.

- We included a brief sensitivity analysis using $\frac{dR^{eq}}{d\lambda}$ and $\frac{dR^{eq}}{d\alpha}$ and showed that the impact of λ and α on the equilibrium of Recovery R^{eq} is minimal. Nonetheless, R^{eq} increases with an increase in λ, which represents the recovery rate of the infectives, and decreases with an increase in α, which represents the infectious rate of susceptible individuals. This outcome is consistent with our intuition.

8.2 Influence on Dissemination

- For *Width of Dissemination*, according to R^{eq} formula we have derived and the according to derivatives we derive, either company that would like to enlarge the influence of a spread or a government trying to eradicate a rumor can find the most efficient way of affected the width of dissemination and act accordingly.
- For *Popularity Subsided*, we could expect the peak of opinion dissemination and predict when the popularity would fall. Therefore, in advertising a product, we could predict this time and make precautions that either continue to popularize the publicity or alternate to propagate another product.
- Another point of significance we would like to mention is the *threshold of convincement.* From the simulation we do, we indeed expect a drastic drop in the width of dissemination as the threshold of convincement increases. Although this can be viewed as an unchanged characteristic of the public, in rumor dissemination, this can indeed be affected by public awareness and education level...This might alert to the best way of stopping a rumor is to raise public awareness of authentication before propagating and make the public less convinced to unauthenticated information.

8.3 Next Steps

Here are several suggestions that we have for future investigations:

- A future investigation of our model is to extend and combine it with the IDRSI model proposed by Zhenhua Yu, 2021 and incorporate our idea of *conformity* into the IDRSI model
- Another future investigation is to extend the model to multi-platform dissemination and fit real-world data to it

9 Contributions of Each Group Member

9.1 Yifei Sun

- Found related literature papers and formulate the model, including necessary parameters and variables, generation of the conformity function, etc...
- Examined the equilibrium and stability of the model by using linear algebra and related course materials.
- Generated the output/idea of width of dissemination and popularity subsided analyzed their impact.
- Developed suggestions for future improvements of the model.
- Contributed the section 3, 4, 5, 7, 8 of the report.

9.2 Ruihan Guo

- Provided the necessary background that introduces the information needed to understand the problem.
- Found specific dataset to fit the assumption of conformity function.
- Implemented the code for simulation of stability of R_0 and the relationship of width of dissemination and parameters.
- Concluded the general results for the model.
- Investigated questions from presentations and responded.
- Contributed to section 1, 4, 6, 7, 8 of the report.

9.3 Siyuan Zhao

- Read related articles and provide ideas on how we should proceed with our project.
- Write the abstract and form our research questions and assumptions.
- Conduct the sensitivity analysis.
- Discuss the robustness of our model assumptions and propose how to deal with the assumption failure. Then I developed a new multi-platform model and simulated it on different networks of structures of platforms connection.

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