

# Introduction to **Information Retrieval**

Lecture 7: Scoring and results assembly

# Recap: tf-idf weighting

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- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = (1 + \log \text{tf}_{t,d}) \times \log_{10}(N / \text{df}_t)$$

- Best known weighting scheme in information retrieval
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

# Recap: Queries as vectors

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- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors

# Recap: cosine(query,document)

$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \bullet \vec{d}}{\|\vec{q}\| \|\vec{d}\|} = \frac{\vec{q}}{\|\vec{q}\|} \bullet \frac{\vec{d}}{\|\vec{d}\|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

Dot product      Unit vectors



$\cos(\vec{q}, \vec{d})$  is the cosine similarity of  $\vec{q}$  and  $\vec{d}$  ... or,  
equivalently, the cosine of the angle between  $\vec{q}$  and  $\vec{d}$ .

# This lecture

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- Speeding up vector space ranking
- Putting together a complete search system
  - Will require learning about a number of miscellaneous topics and heuristics

**Question:** Why don't we just use the query processing methods for Boolean queries?

# Computing cosine scores

COSINESCORE( $q$ )

- 1 *float Scores[N] = 0*
- 2 *float Length[N]*
- 3 **for each** query term  $t$
- 4 **do** calculate  $w_{t,q}$  and fetch postings list for  $t$
- 5     **for each** pair( $d, tf_{t,d}$ ) in postings list
- 6         **do**  $Scores[d] += w_{t,d} \times w_{t,q}$
- 7     Read the array  $Length$
- 8     **for each**  $d$
- 9         **do**  $Scores[d] = Scores[d] / Length[d]$
- 10     **return** Top  $K$  components of  $Scores[]$

# Efficient cosine ranking

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- Find the  $K$  docs in the collection “nearest” to the query  $\Rightarrow K$  largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the  $K$  largest cosine values efficiently.
    - Can we do this without computing all  $N$  cosines?

# Efficient cosine ranking

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- What we're doing in effect: solving the  $K$ -nearest neighbor problem for a query vector
- In general, we do not know how to do this efficiently for high-dimensional spaces
- But it is solvable for short queries, and standard indexes support this well

# Special case – unweighted queries

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- No weighting on query terms
  - Assume each query term occurs only once
- Then for ranking, don't need to normalize query vector
  - Slight simplification of algorithm from Lecture 6

# Faster cosine: unweighted query

FASTCOSINESCORE( $q$ )

- 1 float  $Scores[N] = 0$
- 2 **for each**  $d$
- 3 **do** Initialize  $Length[d]$  to the length of doc  $d$
- 4 **for each** query term  $t$
- 5 **do** calculate  $w_{t,q}$  and fetch postings list for  $t$
- 6     **for each** pair( $d$ ,  $tf_{t,d}$ ) in postings list
- 7         **do** add  $\underline{wf}_{t,d}$  to  $Scores[d]$
- 8     Read the array  $Length[d]$
- 9 **for each**  $d$
- 10 **do** Divide  $Scores[d]$  by  $Length[d]$
- 11 **return** Top  $K$  components of  $Scores[]$

Figure 7.1 A faster algorithm for vector space scores.

# Computing the $K$ largest cosines: selection vs. sorting

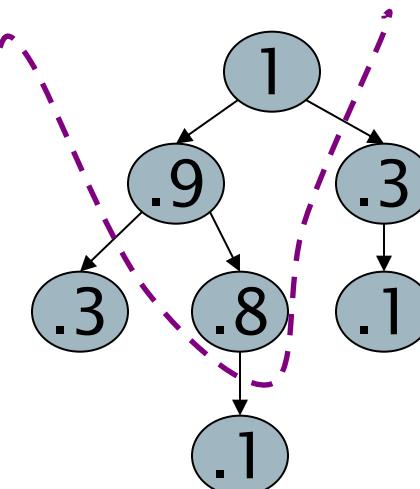
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- Typically we want to retrieve the top  $K$  docs (in the cosine ranking for the query)
  - not to totally order all docs in the collection
- Can we pick off docs with  $K$  highest cosines?
- Let  $n$  of docs with nonzero cosines
  - We seek the  $K$  best of these  $n$

[http://en.wikipedia.org/wiki/Binary\\_heap](http://en.wikipedia.org/wiki/Binary_heap)

# Use heap for selecting top $K / 1$

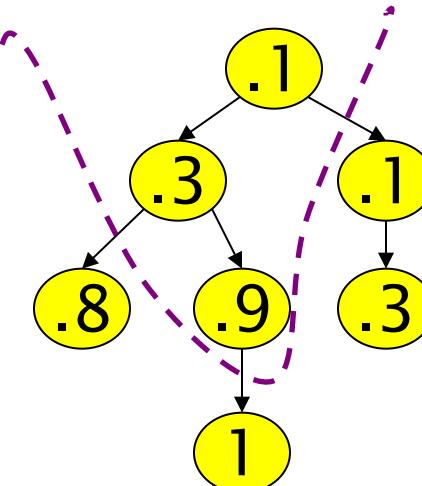
- Max-heap:
  - Binary tree in which each node's value > the values of children
- Takes  $2n$  operations to construct, then each of  $K$  “winners” read off in  $2\log n$  steps
- Total time is  $O(n + K * \log(n))$ ; space complexity is  $O(n)$
- For  $n=1M$ ,  $K=100$ , this is about 10% of the cost of sorting.



[http://en.wikipedia.org/wiki/Binary\\_heap](http://en.wikipedia.org/wiki/Binary_heap)

# Use heap for selecting top $K / 2$

- What about using a min-heap?
- Use the min-heap to maintain the top  $k$  scores so far.
- For each new score,  $s$ , scanned:
  - H.push ( $s$ )
  - H.pop()
- Total time is  $O(n * \log(k) + k * \log(k))$ ; space complexity is  $O(k)$



# Quick Select

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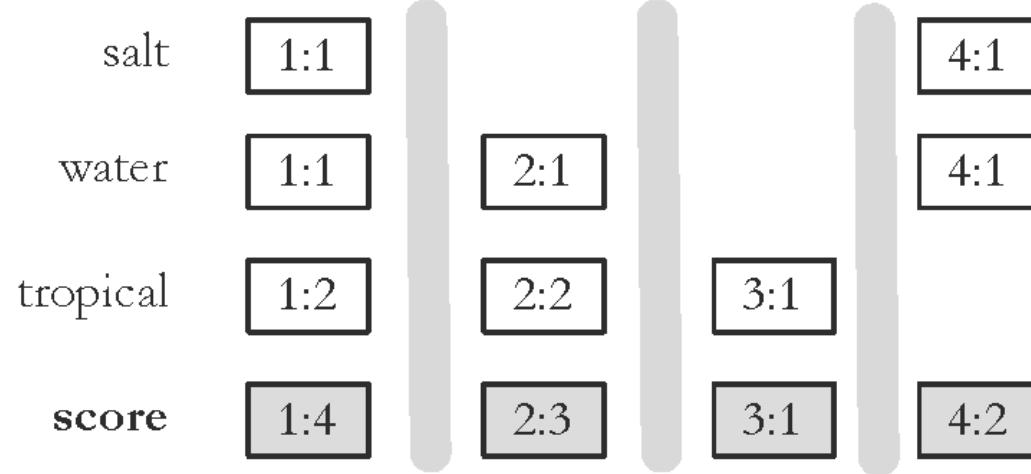
- QuickSelect is similar to QuickSort to find the top-K elements from an array
  - Takes  $O(n)$  time (in expectation)
- Sorting the top-K items takes  $O(K * \log(K))$  time
- Total time is  $O(n + K * \log(K))$

# Query Processing

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- Document-at-a-time
  - Calculates complete scores for documents by processing all term lists, one document at a time
- Term-at-a-time
  - Accumulates scores for documents by processing term lists one at a time
- Both approaches have optimization techniques that significantly reduce time required to generate scores
  - Distinguish between safe and heuristic optimizations

# Document-At-A-Time



# Document-At-A-Time

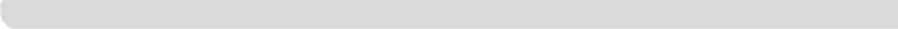
```
procedure DOCUMENTATATIMERETRIEVAL( $Q, I, f, g, k$ )
     $L \leftarrow \text{Array}()$ 
     $R \leftarrow \text{PriorityQueue}(k)$ 
    for all terms  $w_i$  in  $Q$  do
         $l_i \leftarrow \text{InvertedList}(w_i, I)$ 
         $L.\text{add}( l_i )$ 
    end for
    for all documents  $d \in I$  do
        for all inverted lists  $l_i$  in  $L$  do
            if  $l_i$  points to  $d$  then
                 $s_D \leftarrow s_D + g_i(Q)f_i(l_i)$            ▷ Update the document score
                 $l_i.\text{movePastDocument}( d )$ 
            end if
        end for
         $R.\text{add}( s_D, D )$ 
    end for
    return the top  $k$  results from  $R$ 
end procedure
```

# Term-At-A-Time

salt	1:1	4:1
<b>partial scores</b>	1:1	4:1



old partial scores	1:1	4:1	
water	1:1	2:1	4:1
<b>new partial scores</b>	1:2	2:1	4:2



old partial scores	1:2	2:1	4:2	
tropical	1:2	2:2	3:1	
<b>final scores</b>	1:4	2:3	3:1	4:2

# Term-At-A-Time

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```
procedure TERMATATIMERETRIEVAL( $Q, I, f, g, k$ )
```

$A \leftarrow \text{HashTable}()$

// accumulators

$L \leftarrow \text{Array}()$

$R \leftarrow \text{PriorityQueue}(k)$

**for all** terms  $w_i$  in  $Q$  **do**

$l_i \leftarrow \text{InvertedList}(w_i, I)$

$L.\text{add}( l_i )$

**end for**

**for all** lists  $l_i \in L$  **do**

**while**  $l_i$  is not finished **do**

$d \leftarrow l_i.\text{getCurrentDocument}()$

//  $A_d$  contains partial score

$A_d \leftarrow A_d + g_i(Q)f(l_i)$

$l_i.\text{moveToNextDocument}()$

**end while**

**end for**

**for all** accumulators  $A_d$  in  $A$  **do**

$s_D \leftarrow A_d$                                    ▷ Accumulator contains the document score

$R.\text{add}( s_D, D )$

**end for**

**return** the top  $k$  results from  $R$

**end procedure**

# Optimization Techniques

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- Term-at-a-time uses more memory for accumulators, but accesses disk more efficiently
- Two classes of optimization
  - Read less data from inverted lists
    - e.g., skip lists
    - better for simple feature functions
  - Calculate scores for fewer documents
    - e.g., conjunctive processing
    - better for complex feature functions

# Conjunctive Processing

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- Requires the result document containing all the query terms (i.e., conjunctive Boolean queries)
  - More efficient
  - Can also be more effective for **short queries**
  - Default for many search engines
- Can be combined with both DAAT and TAAT (see pseudocodes next)

# Conjunctive Term-at-a-Time

```

1: procedure TERMATATIMERETRIEVAL( $Q, I, f, g, k$ )
2:    $A \leftarrow \text{Map}()$ 
3:    $L \leftarrow \text{Array}()$ 
4:    $R \leftarrow \text{PriorityQueue}(k)$ 
5:   for all terms  $w_i$  in  $Q$  do
6:      $l_i \leftarrow \text{InvertedList}(w_i, I)$ 
7:      $L.\text{add}(l_i)$ 
8:   end for
9:   for all lists  $l_i \in L$  do
10:     $d_0 \leftarrow -1$ 
11:    while  $l_i$  is not finished do
12:      if  $i = 0$  then
13:         $d \leftarrow l_i.\text{getCurrentDocument}()$ 
14:         $A_d \leftarrow A_d + g_i(Q)f(l_i)$ 
15:         $l_i.\text{moveToNextDocument}()$ 
16:      else
17:         $d \leftarrow l_i.\text{getCurrentDocument}()$ 
18:         $d' \leftarrow A.\text{getNextAccumulator}(d)$ 
19:         $A.\text{removeAccumulatorsBetween}(d_0, d')$ 
20:        if  $d = d'$  then
21:           $A_d \leftarrow A_d + g_i(Q)f(l_i)$ 
22:           $l_i.\text{moveToNextDocument}()$ 
23:        else
24:           $l_i.\text{skipForwardToDocument}(d')$ 
25:        end if
26:         $d_0 \leftarrow d'$ 
27:      end if
28:    end while
29:  end for
30:  for all accumulators  $A_d$  in  $A$  do
31:     $s_d \leftarrow A_d$             $\triangleright$  Accumulator contains the document score
32:     $R.\text{add}(s_d, d)$ 
33:  end for
34:  return the top  $k$  results from  $R$ 
35: end procedure

```

Fig. 5.20. A term-at-a-time retrieval algorithm with conjunctive processing

# Conjunctive Document-at-a-Time

```
1: procedure DOCUMENTATATIMERETRIEVAL( $Q, I, f, g, k$ )
2:    $L \leftarrow \text{Array}()$ 
3:    $R \leftarrow \text{PriorityQueue}(k)$ 
4:   for all terms  $w_i$  in  $Q$  do
5:      $l_i \leftarrow \text{InvertedList}(w_i, I)$ 
6:      $L.\text{add}(l_i)$ 
7:   end for
8:    $d \leftarrow -1$ 
9:   while all lists in  $L$  are not finished do
10:     $s_d \leftarrow 0$ 
11:    for all inverted lists  $l_i$  in  $L$  do
12:      if  $l_i.\text{getCurrentDocument}() > d$  then
13:         $d \leftarrow l_i.\text{getCurrentDocument}()$ 
14:      end if
15:    end for
16:    for all inverted lists  $l_i$  in  $L$  do
17:       $l_i.\text{skipForwardToDocument}(d)$ 
18:      if  $l_i.\text{getCurrentDocument}() = d$  then
19:         $s_d \leftarrow s_d + g_i(Q)f_i(l_i)$             $\triangleright$  Update the document score
20:         $l_i.\text{movePastDocument}(d)$ 
21:      else
22:         $d \leftarrow -1$ 
23:        break
24:      end if
25:    end for
26:    if  $d > -1$  then  $R.\text{add}(s_d, d)$ 
27:    end if
28:  end while
29:  return the top  $k$  results from  $R$ 
30: end procedure
```

Fig. 5.21. A document-at-a-time retrieval algorithm with conjunctive processing

# Threshold Methods

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- Threshold methods use number of top-ranked documents needed ( $k$ ) to optimize query processing
  - for most applications,  $k$  is small
- For any query, there is a *minimum score* that each document needs to reach before it can be shown to the user
  - score of the  $k$ th-highest scoring document
  - gives *threshold*  $\tau$
  - optimization methods estimate  $\tau'$  to ignore documents

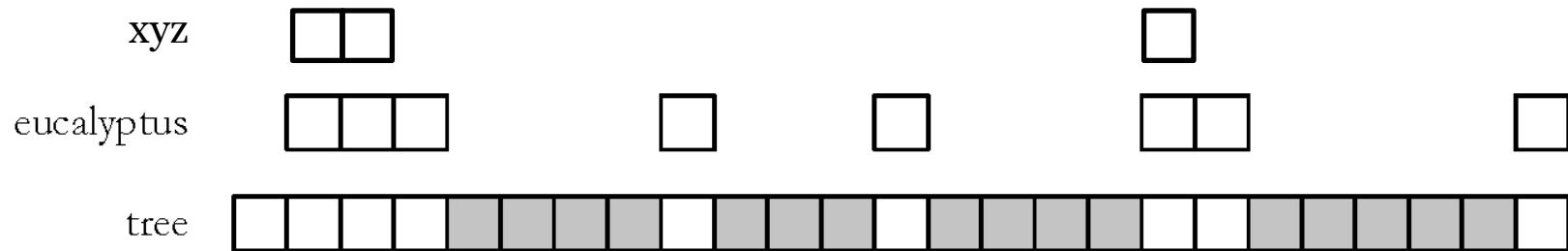
# Threshold Methods

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- For document-at-a-time processing, use score of lowest-ranked document so far for  $\tau'$ 
  - for term-at-a-time, have to use  $k_{th}$ -largest score in the accumulator table
- *MaxScore* method compares the maximum score that remaining documents could have to  $\tau'$ 
  - *safe* optimization in that ranking will be the same without optimization

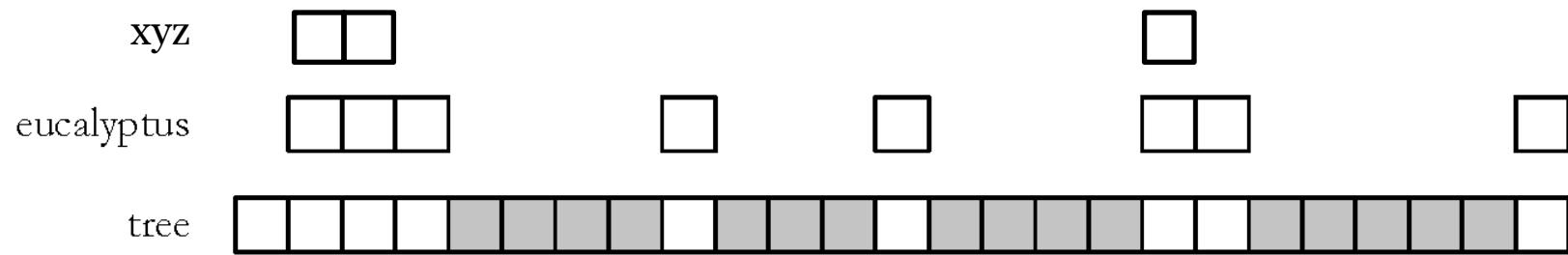
Better than the example in the textbook. See my Note 2 too.

# MaxScore Example



- Compute max term scores,  $\mu_t$ , of each list and sort them in decreasing order (fixed during query processing)
  - Assume  $k = 3$ ,  $\tau'$  is lowest score of the **current** top- $k$  documents
  - If  $\mu_{tree} < \tau' \rightarrow$  any doc that scores higher than  $\tau'$  must contains *at least one of* the first two keywords (aka *required term set*)
    - Use postings lists of required term set to “drive” the query processing
    - Will only check **some of** the white postings in the list of “tree” to compute score  $\rightarrow$  at least all gray postings are skipped.

# MaxScore



# Other Approaches

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- Early termination of query processing
  - ignore high-frequency word lists in term-at-a-time
  - ignore documents at end of lists in doc-at-a-time
  - *unsafe* optimization
- List ordering
  - order inverted lists by quality metric (e.g., PageRank) or by partial score
  - makes unsafe (and fast) optimizations more likely to produce good documents

# Bottlenecks

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- Primary computational bottleneck in scoring: cosine computation
- Can we avoid all this computation?
- Yes, but may sometimes get it wrong
  - a doc *not* in the top  $K$  may creep into the list of  $K$  output docs
  - Is this such a bad thing?

# Cosine similarity is only a proxy

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- Justifications
  - User has a task and a query formulation
  - Cosine matches docs to query
  - Thus cosine is anyway a proxy for user happiness
- Approximate query processing
  - If we get a list of  $K$  docs “close” to the top  $K$  by cosine measure, should be ok

# Generic approach

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- Find a set  $A$  of *contenders*, with  $K < |A| \ll N$ 
  - $A$  does not necessarily contain the top  $K$ , but has many docs from among the top  $K$
  - Return the top  $K$  docs in  $A$
- Think of  $A$  as pruning non-contenders
- The same approach is also used for other (non-cosine) scoring functions
- Will look at several schemes following this approach

# Index elimination

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- Basic algorithm FastCosineScore of Fig 7.1 only considers docs containing at least one query term
- Take this further:
  - Only consider high-idf query terms
  - Only consider docs containing many query terms

# High-idf query terms only

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- For a query such as *catcher in the rye*
- Only accumulate scores from *catcher* and *rye*
- Intuition: *in* and *the* contribute little to the scores and so don't alter rank-ordering much
- Benefit:
  - Postings of low-idf terms have many docs → these (many) docs get eliminated from set *A* of contenders

# Docs containing many query terms

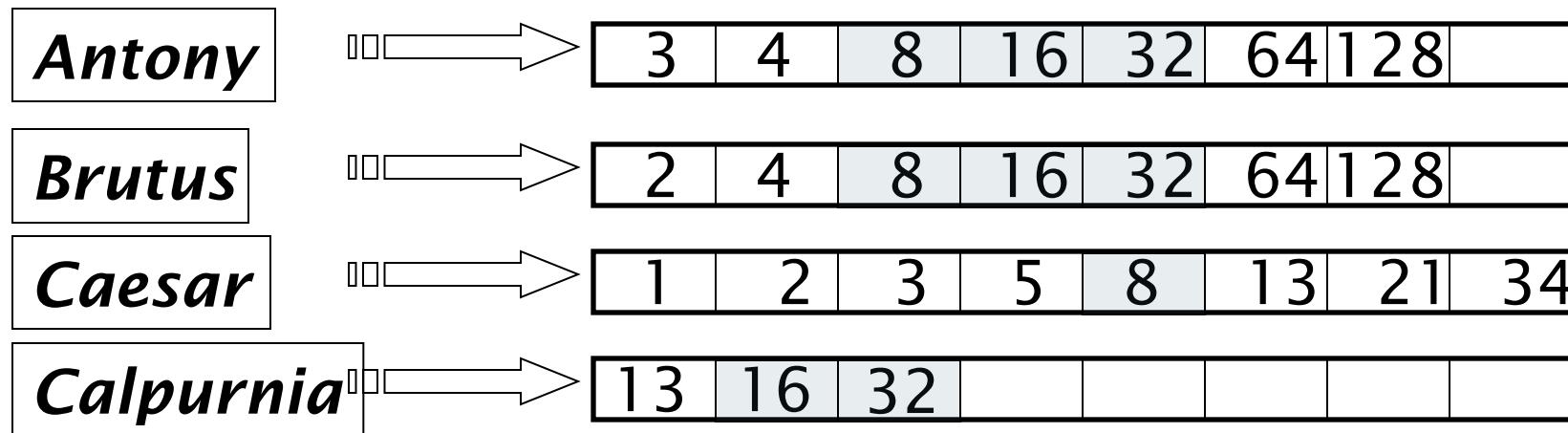
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- Any doc with at least one query term is a candidate for the top  $K$  output list
- For multi-term queries, only compute scores for docs containing several of the query terms
  - Say, at least 3 out of 4
  - Imposes a “soft conjunction” on queries seen on web search engines (early Google)
- Easy to implement in postings traversal

Can generalize to WAND method (safe)

## 3 of 4 query terms

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Scores only computed for docs 8, 16 and 32.

# Champion lists

- Precompute for each dictionary term  $t$ , the  $r$  docs of highest weight in  $t$ 's postings
  - Call this the champion list for  $t$
  - (aka fancy list or top docs for  $t$ )
- Note that  $r$  has to be chosen at index build time
  - Thus, it's possible that  $r < K$
- At query time, only compute scores for docs in  $A = \bigcup_{t \in Q} \text{ChampionList}(t)$ 
  - Pick the  $K$  top-scoring docs from amongst these

Inspired by “fancy lists” of Google:

<http://infolab.stanford.edu/~backrub/google.html>

# Exercises

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- How do Champion Lists relate to Index Elimination?  
Can they be used together?
- How can Champion Lists be implemented in an inverted index?
  - Note that the champion list has nothing to do with small docIDs

# Static quality scores

- We want top-ranking documents to be both *relevant* and *authoritative*
  - *Relevance* is being modeled by cosine scores
  - *Authority* is typically a query-independent property of a document
  - Examples of authority signals
    - Wikipedia among websites
    - Articles in certain newspapers
    - A paper with many citations
    - Many diggs, Y!buzzes or del.icio.us marks
    - (Pagerank)
- 
- The diagram consists of five red text items listed under the 'Examples of authority signals' heading. From each of these items, a thin black arrow points upwards and to the right towards a light gray rectangular box with a black border. Inside the box, the word 'Quantitative' is written in a black sans-serif font. The arrows originate from the bottom of each text item and converge towards the center of the box.

# Modeling authority

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- Assign to each document a *query-independent quality score* in  $[0,1]$  to each document  $d$ 
  - Denote this by  $g(d)$
- Thus, a quantity like the number of citations is scaled into  $[0,1]$ 
  - Exercise: suggest a formula for this.

# Net score

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- Consider a simple total score combining cosine relevance and authority
- $\text{net-score}(q,d) = g(d) + \cosine(q,d)$ 
  - Can use some other linear combination than an equal weighting
  - Indeed, any function of the two “signals” of user happiness
    - more later
- Now we seek the top  $K$  docs by net score

# Top $K$ by net score – fast methods

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- First idea: Order all postings by  $g(d)$
- Key: this is a common ordering for all postings
- Thus, can concurrently traverse query terms' postings for
  - Postings intersection
  - Cosine score computation
- Exercise: write pseudocode for cosine score computation if postings are ordered by  $g(d)$

# Why order postings by $g(d)$ ?

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- Under  $g(d)$ -ordering, top-scoring docs likely to appear early in postings traversal
- In time-bound applications (say, we have to return whatever search results we can in 50 ms), this allows us to stop postings traversal early
  - Short of computing scores for all docs in postings

# Champion lists in $g(d)$ -ordering

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- Can combine champion lists with  $g(d)$ -ordering
- Maintain for each term a champion list of the  $r$  docs with highest  $g(d) + \text{tf-idf}_{td}$
- Seek top- $K$  results from only the docs in these champion lists

# High and low lists

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- For each term, we maintain two postings lists called *high* and *low*
  - Think of *high* as the champion list
- When traversing postings on a query, only traverse all the *high* lists first
  - If we get more than  $K$  docs, select the top  $K$  and stop
    - Only union the high lists
  - Else proceed to get docs from the *low* lists
- Can be used even for simple cosine scores, without global quality  $g(d)$
- A means for segmenting index into two tiers

# Impact-ordered postings

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- We only want to compute scores for docs for which  $wf_{t,d}$  is high enough
- We sort each postings list by  $wf_{t,d}$
- Now: not all postings in a common order!
- How do we compute scores in order to pick off top  $K$ ?
  - Two ideas follow

# 1. Early termination

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- When traversing  $t$ 's postings, stop early after either
  - a fixed number of  $r$  docs
  - $wf_{t,d}$  drops below some threshold
- Take the union of the resulting sets of docs
  - One from the postings of each query term
- Compute only the scores for docs in this union

## 2. idf-ordered terms

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- When considering the postings of query terms
- Look at them in order of decreasing idf
  - High idf terms likely to contribute most to score
- As we update score contribution from each query term
  - Stop if doc scores relatively unchanged
- Can apply to cosine or some other net scores

Why  $N^{1/2}$  leaders?

# Cluster pruning: preprocessing

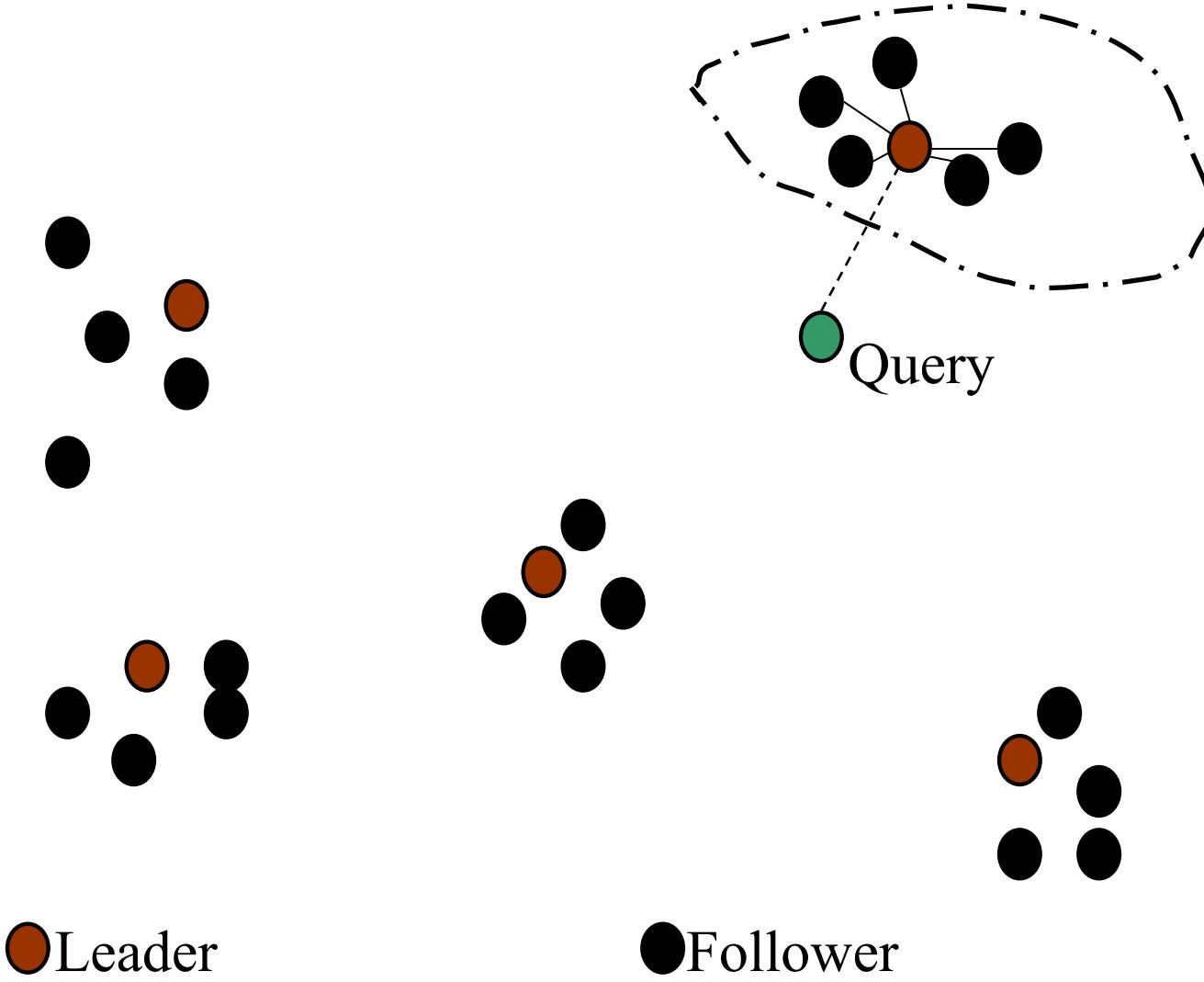
- Pick  $\sqrt{N}$  docs at random: call these *leaders*
- For every other doc, pre-compute nearest leader
  - Docs attached to a leader: its *followers*;
  - Likely: each leader has  $\sim \sqrt{N}$  followers.

# Cluster pruning: query processing

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- Process a query as follows:
  - Given query  $Q$ , find its nearest *leader*  $L$ .
  - Seek  $K$  nearest docs from among  $L$ 's followers.

# Visualization



# Why use random sampling

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- Fast
- Leaders reflect data distribution

# General variants

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- Have each follower attached to  $b1=3$  (say) nearest leaders.
- From query, find  $b2=4$  (say) nearest leaders and their followers.
- Can recur on leader/follower construction.

# Exercises

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- To find the nearest leader in step 1, how many cosine computations do we do?
  - Why did we have  $\sqrt{N}$  in the first place?
  - Hint: write down the algorithm, model its cost, and minimize the cost.
- What is the effect of the constants  $b_1, b_2$  on the previous slide?
- Devise an example where this is *likely to fail* – i.e., we miss one of the  $K$  nearest docs.
  - *Likely* under random sampling.

# Parametric and zone indexes

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- Thus far, a doc has been a sequence of terms
- In fact documents have multiple parts, some with special semantics:
  - Author
  - Title
  - Date of publication
  - Language
  - Format
  - etc.
- These constitute the metadata about a document

# Fields

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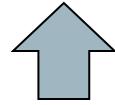
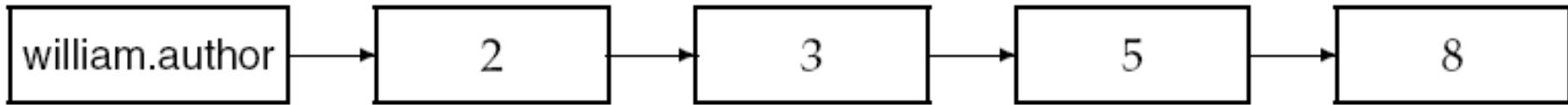
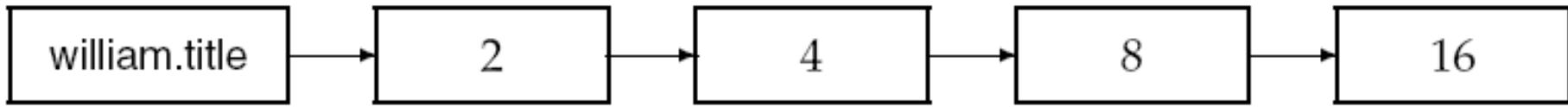
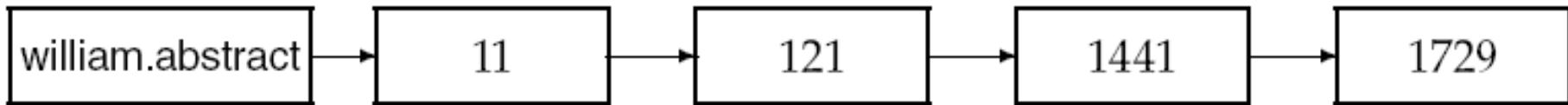
- We sometimes wish to search by these metadata
  - E.g., find docs authored by William Shakespeare in the year 1601, containing *alas poor Yorick*
- Year = 1601 is an example of a field
- Also, author last name = shakespeare, etc
- Field or parametric index: postings for each field value
  - Sometimes build range trees (e.g., for dates)
- Field query typically treated as conjunction
  - (doc *must* be authored by shakespeare)

# Zone

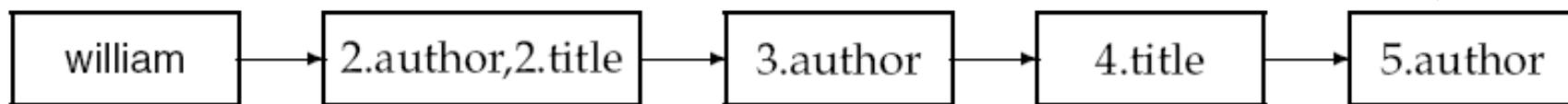
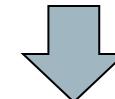
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- A zone is a region of the doc that can contain an arbitrary amount of text e.g.,
  - Title
  - Abstract
  - References ...
- Build inverted indexes on zones as well to permit querying
- E.g., “find docs with *merchant* in the title zone and matching the query *gentle rain*”

# Example zone indexes



Encode zones in dictionary vs. postings.

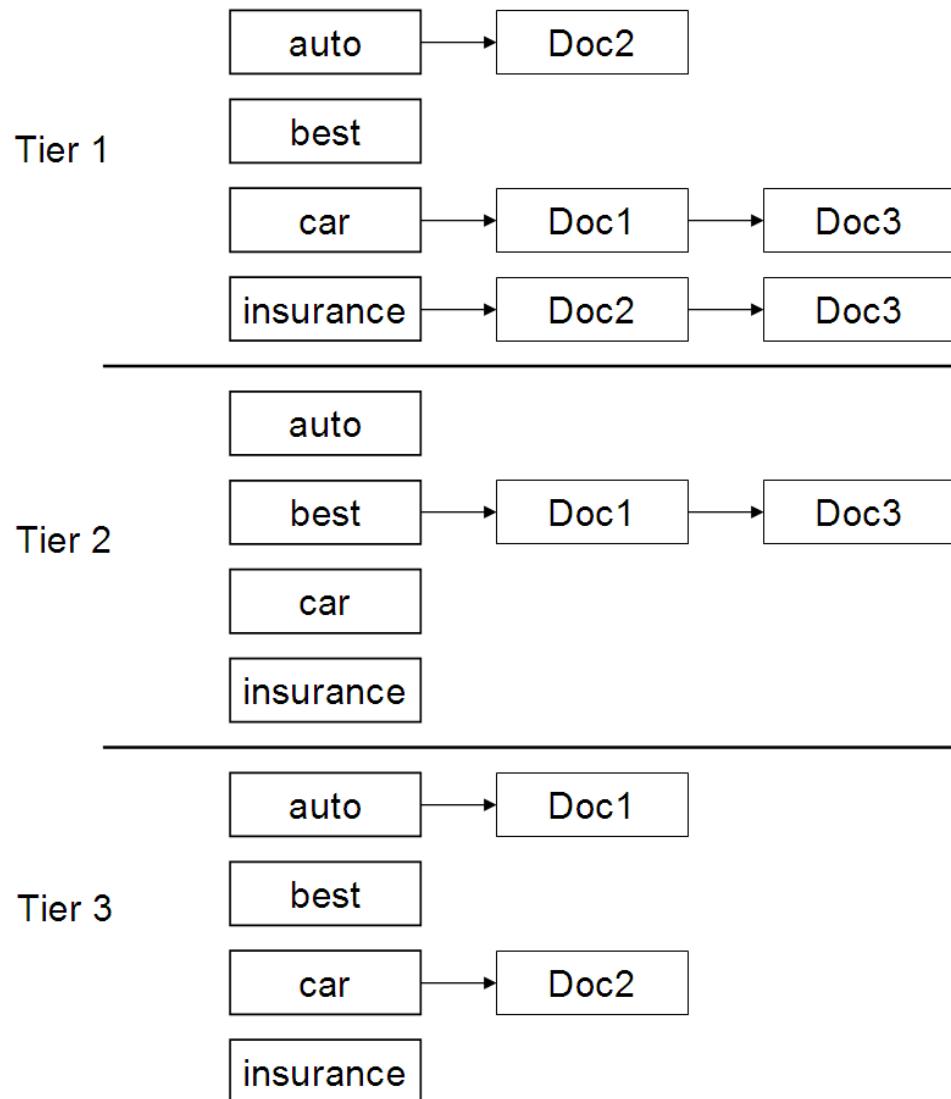


# Tiered indexes

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- Break postings up into a hierarchy of lists
  - Most important
  - ...
  - Least important
- Can be done by  $g(d)$  or another measure
- Inverted index thus broken up into tiers of decreasing importance
- At query time use top tier unless it fails to yield  $K$  docs
  - If so drop to lower tiers

# Example tiered index



# Query term proximity

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- Free text queries: just a set of terms typed into the query box – common on the web
- Users prefer docs in which query terms occur within close proximity of each other
- Let  $w$  be the smallest window in a doc containing all query terms, e.g.,
- For the query *strained mercy* the smallest window in the doc *The quality of mercy is not strained* is 4 (words)
- Would like scoring function to take this into account – how?

# Query parsers

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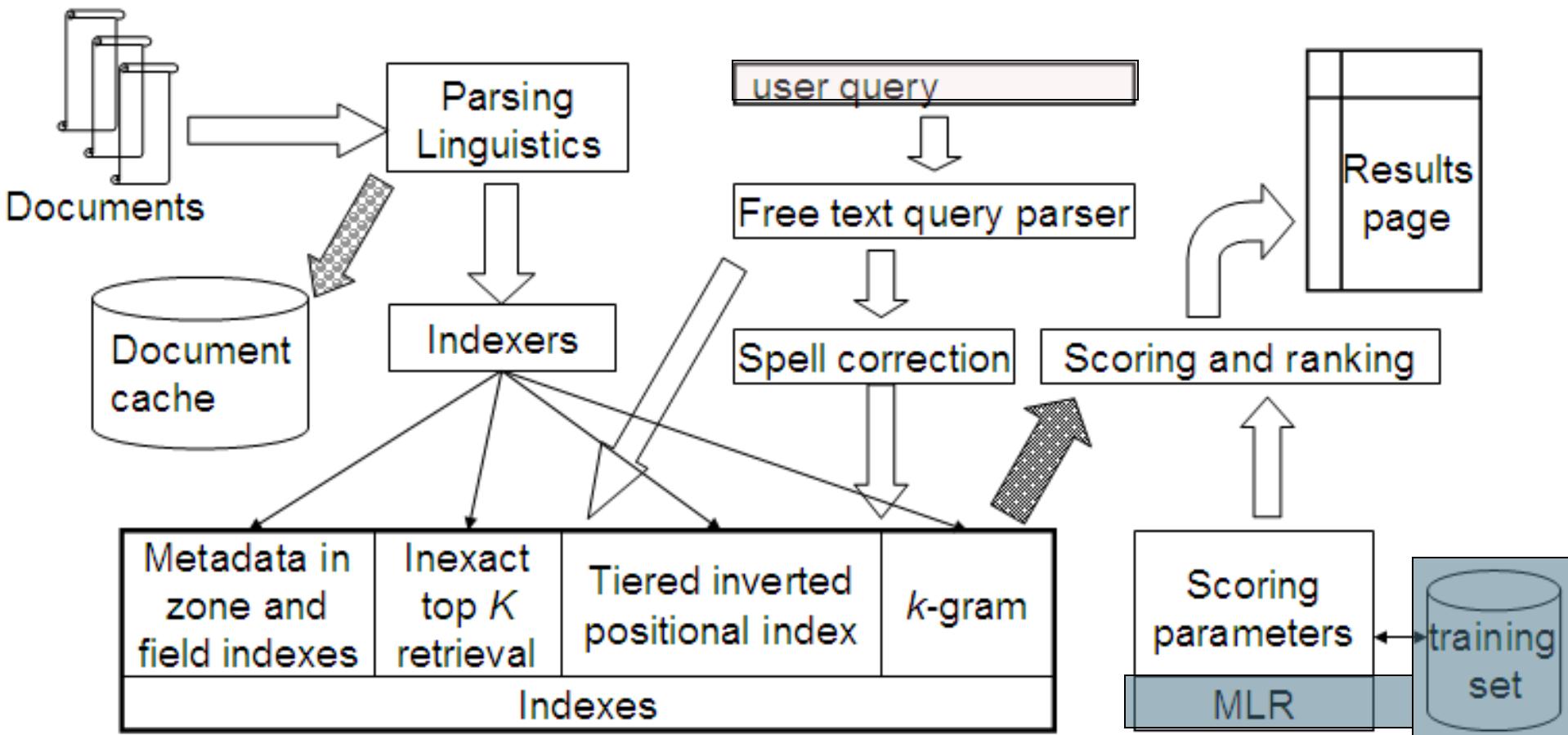
- Free text query from user may in fact spawn one or more queries to the indexes, e.g. query *rising interest rates*
  - Run the query as a phrase query
  - If  $<K$  docs contain the phrase *rising interest rates*, run the two phrase queries *rising interest* and *interest rates*
  - If we still have  $<K$  docs, run the vector space query *rising interest rates*
  - Rank matching docs by vector space scoring
- This sequence is issued by a query parser

# Aggregate scores

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- We've seen that score functions can combine cosine, static quality, proximity, etc.
- **How do we know the best combination?**
- Some applications – expert-tuned
- Increasingly common: machine-learned

# Putting it all together



# Resources

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- IIR 7, 6.1