# Spontaneous Parametric Down Conversion

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## I. INTRODUCTION

Up until this point we have only studied photons as discrete, quantized packets, unchangeable in their make-up. However, nonlinear crystals, specifically engineered for this purpose, are able to destroy one photon of a certain wavelength, creating two photons of greater wavelength. Within the nonlinear crystal, the electric field of the light is warped by the dialectric polarization of the medium. This is a process called Spontaneous Parametric Down Conversion. By studying conservation of momentum and energy of the in and out coming photons, we are able to understand how these out coming photons will behave and pair. Once we have created paired photons, we can imagine that these photons are entangled. Although we do not study entanglement particularly in our experiment, the process by which entangled photons are created is identical to our experimental setup. We can imagine that the ability to study a single photon and its interaction with matter, different samples, or even the human eye becomes dramatically more straightforward if we have a pair coincidence photon that allows us to know the behavior of the other.

The theoretical background of this experiment is easy to grasp and can serve as a basis for experimental quantum mechanics. Additionally, the optical setup used is easy to conceptualize and does not require advanced parts. In fact, the optical setup is built from scratch! The majority of the excitement of this experiment arrives from properly aligning and investigating the paths of the alignment and experimental laser beams.

## II. THEORY

### A. A Brief History

Entanglement of particle was introduced into Quantum Mechanics by the Einstein-Podolsky-Rosen Gedankenexperiment. This proposed that in quantum mechanics, if particles are 'entangled' they can not be factored into single-particle states, and are inseparable; like twins who always wear inverted matching outfits. We can observe from this that entangled photons are correlated with regard to spin, and polarization even if independently remain neutral. We are reminded here of the nonlocality of the measuring process; observing one entangled photon will effect and collapse the state function of the other even if they are far away from each other. This quantum entanglement was met with suspicion across the board and many mathematical papers were published, such as one by EPR, rebuking the idea. Although we will not study entanglement here, and will only approach the studying of entanglement, experimental set-ups, able to be conducted by undergraduates nonetheless, can confirm this property with ease, acting as one of the first confirmations of basic quantum mechanics.

### B. Spontaneous Parametric Down Conversion

Spontaneous parametric down conversion one variation of the simple nonlinear process by by which light of one frequency is converted into light of a different frequency. Of course, using the word simple here is immensely hyperbolic. The process of nonlinear conversion, which is unlike linear processes like refraction, polarization, absorption, or reflection, is based on wave theory that is wholly complicated and will be in our case refereed to as inherently a part of the crystal; a black box of sorts. The main thing we need to know is that in a crystal capable of spontaneous parametric down conversion there is some probability that a single photon of one frequency will be converted into two photons of lower frequency. This crystal is inefficient however it is one of the most efficient methods to down convert photons.

We are able to use basic properties of conservation of momentum and energy to derive the theoretical background for this experiment. We may refer to the input beam of photons at the pump, with angular frequency $\omega_p$, while the two output beams of photons referred to as the signal and idler beams, with angular momentum of respectively $\omega_s$ and $\omega_i$. This process is spontaneous because the crystal is spontaneous in its efficiency and parametric because it depends on electric field attributes and not just field intensities. Therefore there is a definite relationship between input and output fields. Down conversion simply refers to the output photons frequency being lower.

Obviously, the input and output fields need to have the same momentum and energy. Recall that for photons $E = \hbar\omega$ Therefore energy conservation requires that

$$E_p = E_s + E_i$$

$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$

$$\omega_p = \omega_s + \omega_i$$

Recall also that because the process is parametric we can use the classical condition of "phase-matching" which requires that wave vectors coming in and our are equal or that

$$\vec{k_p} = \vec{k_s} + \vec{k_i}$$

we also recall the dispersion relationship

$$k_p = \frac{n_p(\omega_p)\omega_p}{c}$$

where $n_p(\omega_p)$ is the index of refraction of the crystal at the pump frequency, and the idler and signal beams. We see that the following conditions must be met

$$n_p\omega_p = n_s\omega_s cos(\theta)_s = +n_i\omega_i cos(\theta)_i$$

and

$$0 = n_s\omega_s sin(\theta)_s + n_i\omega_i sin(\theta)_i$$

where the angles are the formed by the momentum of signal and idler photon pairs created with respect to the incident light. Since down converted photons come out of the crystal at a range of wavelengths and therefore angles, we must place our photon detectors correctly, at specifically aligned identical angles at both sides of the incident beam, to actually detect photon pairs and not random beams.

In this experiment we use type 1 down conversion. This means that the signal and idler beams are polarized parallel to each other and perpendicular to that of the pump. We note that the polarizations are linear. If the orientation of the pump beam wavevector $\vec{k_p}$ and optical axis of the crystal is is possible to satisfy all equations noted above.

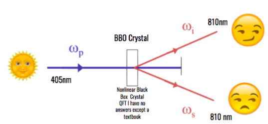


Fig. 1. Signal and Idler Beam Emission

Proper equipment is cleverly chosen so that the signal and idler beam come out of the crystal on opposite sides a few degrees from the pump, according to our understanding of angles formed by photons pairs. Here, only the relative angles between the pump, signal, and idler beams are relevant. We can from this understand that the beams are emitted spontaneously forming a cone surrounding the pump beam. Obviously, we can understand here that this cone is formed by pairs of coincidence photons matched via the equations given above. While emitted photons can come out any which way, they always come in signal-idler pairs satisfying conservation and coming out at the same time.

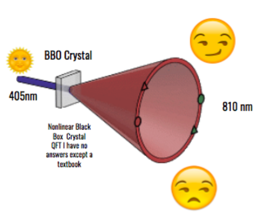


Fig. 2. Signal and Idler Beam Emission

The crux of our experiment is to find ways in which to monitor these specific pairs by using the technique of ”coincidence counting”. We, using our optical setup, look for two photons, detected within an extremely precise time interval, and assume they are coincident, and therefore confirm they constitute a signal-idler pair.

## III. Apparatus

A simple schematic of the experimental setup is show below

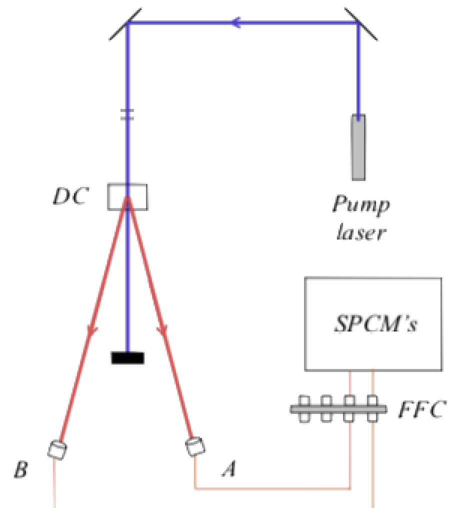


Fig. 3. Experimental Schematic

This experiment, at its core, consists of a source of light at a wavelength of 405 nm which is sent through a nonlinear optical crystal that down converts the photons entering the crystal into pairs of photons with wavelength around 810 nm, half the frequency of the incoming beam. These two photons are emitted from the crystal at the same time so therefore, at the far end of the optical table, are channeled into optical fibers by passing through a lens. The far end of the fiber is attached to the input of a single-photon coincidence detector. This detector has a relatively high efficiency of detecting single photons and therefore when both detectors are set up properly, coincidence pairs can be detected.

### A. The Lasers

The laser we use to create signal and idler beams has a wavelength of 405nm which creates beams of 810nm when split. These created beams however can be recognized as outside our range of visible light and therefore an alignment laser is needed. We choose a laser of about 650nm. We use this laser to simulate the beams created my parametric down conversion.

### B. Optical Setup

The optical setup primarily consists of a mirror that reflects the aligned 405nm beam into the crystal, which will send split beams into the photon detectors. A majority of the time spent aligning the optics is spent adjusting the alignment beam to find the location at which the photon detectors should be placed. We use a series of collimators, mirrors, and a plumb bob to do so.

*1) ThorLabs:* Aside from cutting environmental waste but substituting packing peanuts with lab snacks like real peanuts, ThorLabs is a great resource for optical parts that are customizable and easy to use. Most of our optical pieces are provided by ThorLabs.

### C. The Crystal

The crystal we use is beta-Barium Borate (BBO). Beta Barium Borate is a nonlinear optical crystal with a number of unique features such as a wide transparency region, broad phase-matching range, large nonlinear coefficient, high damage threshold. The actual physics of this crystal is withheld because quantum optics is a lot.

### D. Single Photon Detectors

Single photon detectors are located at the end of our optical setup. These detectors produce an electrical output pulse whenever the detector detects a photon. Although an ideal detector would produce a pulse whenever a photon entered the detector, our detectors do sometimes miss some photons that enter the detector and wont be recorded. In addition, the detector will produce output pulses at particular rate, even when no photons enter the detector, the dark count rate. However, in this experiment, our detector has a fairly high quantum efficiency, it picks up most photons, and a fairly low dark count rate. We are careful not to let too much light enter the detectors and damage them.

### E. Single Photon Counting Modules

The single-photon counting modules are what feed signals from the detector to the coincidence unit. Optical fibers from the photon detectors are converted into logic pulses connected to the coincidence counting module.

*1) Coincidence Unit:* Our ability to detect coincidence photons entering the Single Photon detectors is crucial to our experiment. We do this by using a coincidence unit to detect when a photon simultaneously arrives at different detectors. For example, if two photons hit detectors equally spaced from the pump beam, we can concur the photons are coincidence pair-photons.

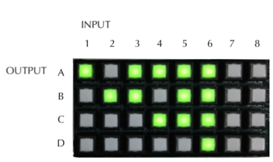


Fig. 4. Coincidence Unit

Each column in the figure (labeled by Outputs 1 through 8) corresponds to a particular coincidence configuration. For example, a count will be registered in an Output if a signal enters all lit up inputs. No signal will occur on an Output for any combination of inputs not listed. These coincidences are fed directly to out lab-view unit.

## IV. Methods

We begin this experiment by using a mirror to align the 405nm beam so the beam is collimated and at a steady height along the optical rail. We record, on the table, the path of the beam without the crystal in its path.

Our next pursuit is to find the specific locations at which to place the photon detectors to maximize the number of coincidence counts obtained while making sure proper pairs are being detected. We begin this step by using two mirror to make sure that the height of the alignment beam is the same as that of the 405nm beam. This is carefully checked at all stages of the process using several customized collimators. The second mirror is placed so that the path of the alignment laser is the same as that of our 405nm beam. From this point, we use the plumb bob to find the exact spot on the table the laser is propagating over at exactly 1 meter from the second mirror or our eventual placement of the crystal. We decide we want the split beams to be approximately $3^{\circ}$ from the incident beam. Since the incident beam is approximately 1 meter from the crystal, using basic trig we can determine that the beams $3^{\circ}$ from the incident beam will be 5.2cm from the point at which we marked using the plumb bob. Our task now is to use the alignment laser to hit these points, and then place the photon detectors appropriately.

We move the orientation of the second mirror until the alignment laser successfully hits the plumb bob at both 5.2cm distance spots remembering to check laser height at all times. Once this is set, we place the photon detectors appropriately. We check that the photon detectors are placed at the right height and spot by seeing if we can spot a red light through the optical fiber attached to each photon detector. Once this is checked we turn off the alignment laser. Additionally, we place an additional photon detector of to the side where no beams are pointed as a control.

We then set up the coincidence counters appropriately. We use coincidence counters A+B, A+C, C+B, A+B+C. We anticipate that if our alignment is set up properly, Coincidence counts for AB should be high where AC and BC should only consist of dark counts. The data is observed quite easily in the Lab View setup.

Finally, to actually take data, we turn all of the lights of in the room and attach the optical fibers to the data taking software. We then start the lab view software and carefully exit the room. We acquire data once for 40 minutes and once for an hour. For consistency, and to exclude noise from the start and end of the run when the door is opened, we cut the data to be approximately 32.5 minutes in run length. The data is analyzed below.

## V. Analysis

We expect that if our experimental set-up has been aligned properly, there will be some large number of coincidence counts for photon detectors angled $3^{\circ}$ from the incident beam, which are signal-idler pair photon. We also assume that the control photon detector will have very few coincidence photons with either photon detector positioned on the optical axis of the split beams.

Data is recorded from the photon detectors approximately every 128th of a second. This data is summed over the run time and coincidences are calculated below.

TABLE I
COINCIDENCE COUNTS

| Photon Detectors | Run 1 | Run 2 |
|---|---|---|
| A+B | 153461 | 190463 |
| A+C | 4403 | 5165 |
| B+C | 4701 | 5554 |
| A+B+C | 9 | 5 |

We notice here that our hypothesis was correct and theoretical approach is confirmed. There seems to be a dramatic increase in counts of photon pairs detected via the photon detectors placed in the signal-idler beams as compared to the control coincidence pairs. We can consider these coincidence counts counted via the control noise and use this as general error for our coincidence counts of actual photon pairs.

Additionally, we can visualize the count rate simply by plotting coincidence counts per 0.01 second which we choose in lab view, the approximate time per count collection. We label A+B as coincidence counts and B+c and A+C as background.

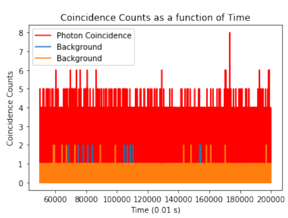


Fig. 5.

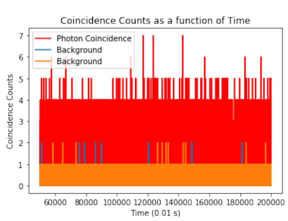


Fig. 6.

We can also bin every 1000 count collection times together to see a more distinct increase in detected photon pairs in the A+B Photon detector coincidence counts. Once we do this, it begins to make more sense to reasonably use our coincidence control counts set up in A+C and B+C to count noise. For roughly 4500 control coincidence counts in 140 bins of 10 seconds we can see that every 10 seconds roughly 3-5 noise counts were detected. Since this is comparably very small, we realize these counts are noise and in the figures use this noise as error that should contain all contributions to deviation from norm rates.

Using this binned noise as error for our coincidence counts of photon pairs we can visualize the counts further noticing the error makes very little difference.

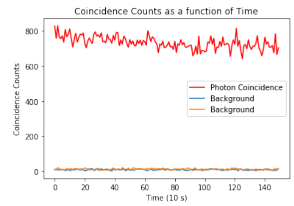


Fig. 7.

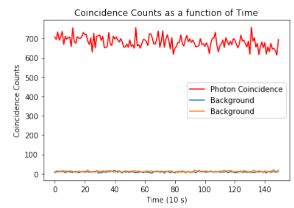


Fig. 8.

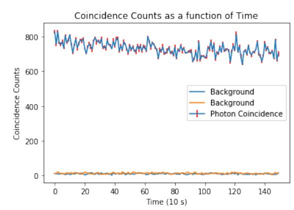


Fig. 9.

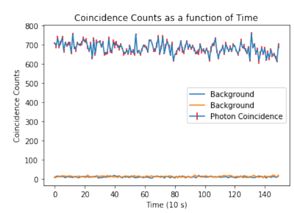


Fig. 10.

Zooming in on one part of the graph we can study the error bars further to realize that the noise really does not have a large influence on our ability to observe a large influx of signal-idler photon pairs.

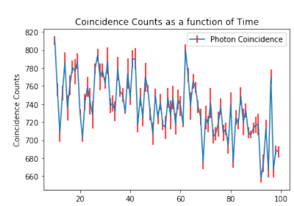


Fig. 11.

Additionally, we can check the validity of our coincidence counts and compare them to expected values. We can insist that there should be more than just a probabilistic relationship of coincidence between counters AB because it relates to photon pairs and less or equal coincidence counts to probable values for coincidence CB and CA.

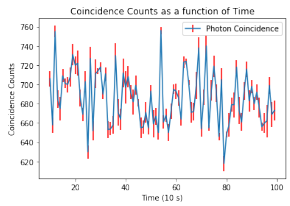


Fig. 12.

We first calculate the number of expected coincidences the detectors would count if the beams were completely independent of each other. The coincidence unit obtained from labVIEW was 1 reading for every 10 ms and the coincidence window hard-coded into the coincidence unit was about 10 ns. From this we obtain $10^6$ coincidence windows per reading. Using data from both experiments, the average photon count for a single detector per reading was for A:675 photons per reading, B:925 photons per reading, and for C:20 photons per reading.Therefore, the number of photons, $\gamma$, per coincidence window, CW, is given by

$$n = (\frac{\gamma}{reading})(10^{-6}\frac{CW}{reading})$$

From this we obtain for A: $675 \cdot 10^{-6}\frac{\gamma}{CW}$, for B: $925 \cdot 10^{-6}\frac{\gamma}{CW}$, and for C: $20 \cdot 10^{-6}\frac{\gamma}{CW}$.

In order to determine the expected number of photons that would share a coincidence window for two independent beams for AB, BC, and AC, we can use a Poisson distribution. We find the probability of 2 photons sharing the same coincidence window, given that the average number of photons per coincidence window is n. Therefore,

$$P(k=2) = \frac{n^k e^{-n}}{k!}$$

We obtain from this that for AB: $6.4 \cdot 10^{-7}\frac{CW}{reading}$, for AC: $1.2 \cdot 10^{-7}\frac{CW}{reading}$ and for BC: $2.0 \cdot 10^{-7}\frac{CW}{reading}$ or by multiplying this by the number of coincidence windows per reading for AB: $0.64\frac{coin}{reading}$, for AC: $0.12\frac{coin}{reading}$ and for BC: $0.20\frac{coin}{reading}$.

Lets compare these expected counts of coincidence per reading to out actual coincidence counts per reading which we have calculated based on data shown above.

TABLE II
MEASURED AND EXPECTED COINCIDENCE

| Detectors | Expected Coincidence | Measured Coincidence |
|---|---|---|
| A+B | $0.64\frac{coin}{reading}$ | $0.70\frac{coin}{reading}$ |
| A+C | $0.12\frac{coin}{reading}$ | $0.02\frac{coin}{reading}$ |
| B+C | $0.20\frac{coin}{reading}$ | $0.02\frac{coin}{reading}$ |

These comparisons are to be expected in the setup of our experiment. We expect that there is a higher coincidence between AB measured and expected because we are aligning an apparatus to have coincidence pairs, although it is concerning that the measured value is only slightly higher than the expected value. We also can realize that our control is truly functioning as a control since the coincidences measured for AC and BC are so much less than the expected values. Overall, these comparisons lead us to grant out data and coincidence detectors a bit more validity. It seems we accurately could have been detecting down converted photon pairs.

We have successfully observed coincidence signal-idler photon pairs created by the process of spontaneous parametric down conversion.

## VI. CONCLUSIONS

Overall the experiment we conducted was, once the optics were aligned, very straightforward and easy to conduct. This lab provided a good hands on experience and was able to be built from the ground up once supplies were provided. This lab is easily accessible to students first learning quantum mechanics and might be a very cool addition to some of the earlier lab classes as well, or at least, conceptually. Additionally, this lab, once set up, can be built on dramatically to do things like prove the existence of photons, monitor polarization, and learn more about quantum entanglement.

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