

# Intermediate report part a - step 3

Julian Main(10541578), Yorick de Boer(10786015), Amor Frans(10897844)

February 16, 2016

## Introduction

For this assignment we constructed a bigram language model based on the trainings corpus. The user can choose whether it wants to use smoothing on the trainingsdata or not. There are two smoothings added to the model: (1) Add-one smoothing which will add one to the counter of a sentence/word if the sentence/word is not counted in the trainings corpus and (2) Good-turing smoothing which uses a standard Good-Turing formula to smooth the trainings-data.

## Method

### Question 1 :

For question 1 we used the program of step 2. The python script could be executed with the command:

```
python3.4 a1-step3.py -train-corpus [path] -test-corpus [path] -n [value] -smoothing [no|add1|gt]
```

### Question 2 :

The python script could be executed with the command:

```
python3.4 a1-step3.py -train-corpus [path] -test-corpus [path] -n [value] -smoothing [no|add1|gt]
```

To make a add-one smoothing program we used the following steps:

- *Get paragraph*

Use a insert start-stop method separate each paragraph in corpus

- *Count ngrams in paragraph*

Use the collections module to count the ngrams of each paragraph.

- *Add one if count is zero*

If the count returns a None then we add one to the counter, according to the

rules of the add-one smoothing.

- *Compute probability*

To compute the probability we use the following formula:  $\frac{\text{thecountofthengram}}{\text{thecountofthen} - 1 \text{gramaddedwiththevocabularysize}}$

If the count of the n-1 gram is null then the divisor is only the vocabulary size.

To retract the count of the n-1 gram we also used the collections module.

### Question 3 :

The python script could be executed with the command:

```
python3.4 a1-step3.py -train-corpus [path] -test-corpus [path] -n [value] -smoothing [no|add1|gt]
```

To make a good-turing smoothing program we used the following steps:

- *Get paragraph*

Use an insert start-stop method separate each paragraph in corpus

- *Make a count dictionary*

To make a good-turing smoothing we first retracted how many times each count occurred. We did by making a dictionary how many times each count occurred. Then using a get statement we can retract the value of the counts. -

*Count the ngram*

Use the collections module to count the ngrams of each paragraph

- *Use the good-turing algorithm*

As given in the assignment we only used the smoothing on counts  $k$  if  $k \leq 5$ .

The formula given in the assignment:

$$r^* = \frac{(r+1) \frac{n_r+1}{n_r} - r(k+1) \frac{(n_k+1)}{n_k}}{1 - (k+1) \frac{n_k+1}{n_k}}$$
 Using the method from the first action where we

made a dictionary for each count we can solve this equation.

- *Compute probability*

To compute the probability we use the following formula:  $\frac{\text{thecountofthengram}}{\text{thecountofthen} - 1 \text{gramaddedwiththevocabularysize}}$

If the count of the n-1 gram is null then the divisor is only the vocabulary size.

To retract the count of the n-1 gram we also used the collections module.

## Results

### Question 2 :

Execute the program with the command:

```
python a1_step3.py -n 2 -smoothing add1 austen.txt ja-pers-clean.txt
```

to see the results for this question. **Question 3 :**

(a)

percentage of sentences assigned zero probability using no smoothing: 0.31037827352085356

percentage of sentences assigned zero probability using add-1 smoothing: 0.31037827352085356

percentage of sentences assigned zero probability good turing smoothing: 0.028128031037827354

Output probabilities:

e-07, 3.974166041474985e-12, 2.196058548889562e-12, 2.4435323577344104e-  
 173, 1.2687186017179182e-22, 1.0662900982618058e-99, 1.0371362295021483e-  
 113, 1, 1.5853014117784715e-115, 7.217800331643056e-260, 3.4137784302160565e-  
 63, 0.0, 1.9223738767830318e-41, 2.983251105593473e-178, 2.4569665442709954e-  
 160, 0.0, 1.4307848649189902e-24, 9.085879778422319e-126, 4.5899989535008626e-  
 151, 2.653623550711697e-116, 4.344584217460401e-259, 4.804959217972987e-119,  
 0.0, 7.451596841408804e-82, 1.1647697150623053e-73, 2.196058548889562e-12,  
 2.334326012208205e-96, 3.40305618099831e-300, 2.7609842516965746e-185, 1.0203789688571675e-  
 247, 1.4588815261770657e-185, 8.316436975929572e-87, 8.770864564885226e-180,  
 1.6444548962618815e-47, 1.303963912352297e-124, 7.921168437721782e-150, 3.8782954060938226e-  
 165, 1.139487524403174e-108, 2.110609647614592e-93, 3.0107745585708797e-111,  
 2.0655790685126675e-187, 8.998515596028385e-256, 7.419210937020614e-75, 2.196058548889562e-  
 12, 3.137853860977097e-50, 6.004079629672395e-53, 3.600214107782885e-13, 5.081537637399784e-  
 308, 9.494946878368126e-37, 1.0481180929780173e-35, 2.579968243651909e-177,  
 1.2921468566552986e-159, 3.6599887453125364e-06, 4.107636306205382e-82, 1,  
 3.6025549028921416e-59, 4.5316759064164855e-60, 3.4098410675216864e-24, 9.471217448291052e-  
 12, 0.0, 0.0, 1.5553639780684972e-231, 1.4661959351498418e-17, 5.682899454909508e-  
 18, 3.0437247438423472e-33, 2.6373283905093593e-24, 1.2282643831085458e-160,  
 1.9834118863938197e-172, 5.882607412147326e-127, 2.40367876037254e-18, 8.653758183671296e-  
 31, 2.3485069557519434e-92, 2.0015024710217814e-06, 6.000178420499581e-07,  
 1.2009371935201481e-12, 2.320173091847477e-47, 6.137014587407681e-52, 7.518485678136113e-  
 159, 2.1773954924674468e-122, 1.921180318317482e-141, 8.377631915981182e-  
 41, 1.0404620483451127e-81, 2.196058548889562e-12, 6.2323372604718825e-174,  
 9.412890925867083e-97, 3.923969715533859e-167, 1.4422246435492666e-201, 3.659559282549938e-  
 305, 4.132585794251641e-75, 0.0, 0.0, 3.5005383176022813e-99, 0.0, 1.3627765697900316e-  
 75, 4.959751243781957e-41, 2.196058548889562e-12, 2.0271130468700575e-69,  
 2.914706015393513e-121, 1.9166103862463005e-35, 2.7502966308306705e-112, 5.905593691878134e-  
 35, 1.6767103745030589e-164, 5.125950544162512e-86, 6.292528662879751e-63,  
 2.097810953018139e-41, 3.03292877787825e-65, 1.8886175961133593e-146, 1.731021203719227e-  
 192, 7.724599882894984e-40, 3.5517353064876795e-250, 1.4422501427800499e-  
 24, 5.192409310988616e-37, 8.195568865994801e-17, 2.540807784130806e-46, 1.6123492739735646e-  
 192, 1.2596675066228015e-177, 4.6378292813561806e-269, 4.9174875453460224e-  
 23, 1.954291201209034e-34, 7.414934953069061e-57, 7.4781214292807e-50, 9.471217448291052e-  
 12, 2.8015602879062624e-65, 2.2448435843028387e-36, 6.377966094949761e-94,  
 2.244843584302838e-36, 4.010637906299089e-51, 1.3200535551169726e-103, 3.6599887453125364e-  
 06, 6.924112550053068e-153, 1.5803119927319295e-44, 3.600214107782885e-13,  
 1.7182827602069164e-36, 3.134234742459258e-11, 1.7866481918469052e-74, 8.769993474269297e-  
 96, 9.220085212091515e-34, 2.9759392383986456e-47, 1.1627016105757434e-47,  
 1.9602226381397775e-183, 7.118913579096365e-267, 1.7661126405860535e-50, 2.196058548889562e-  
 12, 5.887044963940026e-272, 1.3693096567704002e-35, 2.553237459578462e-165,  
 7.607216312137444e-86, 0.0, 1.5980125035644086e-130, 5.458634712457478e-165,  
 1.0636288818886574e-292, 7.347572781132657e-254, 2.142345134917951e-234, 3.84985884658864e-  
 68, 4.6520005590962837e-169, 5.734951375027457e-234, 7.214680645205899e-83,  
 5.941629449347956e-171, 4.501172823565854e-127, 3.4055884334564824e-206, 1.3086807483484668e-  
 222, 1.2009371935201481e-12, 1.0391867283128223e-17, 3.6599887453125364e-  
 06, 6.891916270063163e-115, 1.7811540909480162e-28, 5.323601121674967e-23,

2.992814517535276e-143, 8.165505096945511e-127, 3.1117362955962846e-284, 4.686449726386893e-92, 1.0892693125145285e-77, 9.781751325217266e-114, 6.750922623197084e-108, 1.851135540494695e-129, 2.119871721017389e-228, 2.196058548889562e-12, 2.196058548889562e-12, 1.8836175905560171e-140, 2.663455185377118e-134, 4.325602003806717e-124, 0.0, 7.843050469889371e-131, 3.8292786821464873e-87, 2.9115610374998085e-194, 7.910848433139616e-46, 7.488210626685707e-36, 9.436018349801437e-228, 1.023986388761573e-84, 6.774462362276554e-51, 1.8967596900706457e-35, 2.83264468322401e-52, 1.3894668569246997e-46, 6.958851710626334e-203, 6.288895563621873e-42, 2.154687710782385e-97, 5.485471110504044e-47, 4.126626176555975e-226, 7.356522225715979e-82, 1.4169246838755459e-129, 9.545249142275234e-62, 1.1298067821773864e-146, 6.889153159041888e-218, 1.5793995725212948e-23, 6.133471276866304e-31, 1.5104823089435096e-91, 7.32547651765495e-12, 4.2917378994347505e-46, 5.596541709474626e-64, 3.33959005345087e-39, 3.4962476213223553e-35, 2.542459772111687e-92, 9.06950353277594e-41, 8.959199579337228e-121, 1.2587240010587098e-47, 2.7979882924580926e-104, 8.002832166103362e-70, 2.1812766983678233e-79, 3.43914719046827e-42, 2.196058548889562e-12, 3.2852861089946826e-71, 1.1451215272162276e-133, 2.791413360584576e-144, 3.4304915446279126e-194, 2.4930653291875996e-28, 1.1569854480499017e-34, 1.3551274577371497e-35, 4.709257458962993e-77, 6.000178420499581e-07, 5.011657932627437e-41, 2.0799348045711547e-23, 1.4117586858347147e-77, 3.8572760302686673e-72, 1.5116484692478989e-59, 7.533693480471016e-29, 2.8637194426359247e-30, 1.4492783e-317, 1.8648506243625745e-64, 1.7036406983704287e-18, 3.6599887453125364e-06, 1.1402834856730307e-48, 1.8384927844767414e-112, 6.059180493189739e-92, 1.5920557414510808e-23, 4.962616495459418e-62, 3.600214107782885e-13, 1.6017688355628475e-75, 4.542810210494395e-191, 3.596426150320837e-145, 5.110557974052797e-65, 3.7661935518303553e-44, 4.7727238124320445e-24, 1.2009371935201481e-12, 7.896200443832205e-64, 9.999061407657854e-141, 2.839316731899085e-12, 9.175668018693828e-105, 1.2479979930433857e-29, 9.119567976900508e-72, 3.4391471904682705e-42, 1.0035865647427719e-27, 7.131239260496651e-34, 3.136272566082179e-57, 3.911518668316007e-40, 3.945136495285533e-58, 3.942862757487305e-40, 1, 2.540807784130806e-46, 4.4818310869742946e-17, 1, 2.3712547351346142e-124, 1.2961541621878914e-25, 1.2067851004403128e-293, 1.1374337301569715e-23, 8.538064420335103e-59, 3.23973102674427e-106, 2.0458743034423095e-148, 4.772723812432045e-24, 7.954303152263033e-18, 2.196058548889562e-12, 4.0556492989671846e-181, 5.0859381753544156e-136, 2.40367876037254e-18, 1.510702707955499e-144, 4.5207994943089836e-119, 2.4110930844364257e-64, 4.567658608711424e-35, 5.515751385594769e-78, 7.974053008116437e-205, 5.766710759200328e-88, 6.631541554629624e-70, 1.3047802467564741e-39, 8.18983962556742e-74, 1.3734273445932792e-41, 3.7000565728713758e-146, 2.214825728393125e-136, 7.360238920392263e-78, 3.2043406782033154e-100, 0.0, 4.959751243781958e-41, 2.6959161540243607e-48, 1.3080492765090266e-64, 2.999786268865943e-47, 8.266006268808593e-35, 6.511194902688886e-113, 5.518789468240063e-40, 3.809985130557881e-68, 3.477399345284425e-88, 3.914061868107589e-86, 4.772723812432045e-24, 6.000178420499581e-07, 6.623413110345628e-06, 6.416240612838472e-62, 0.0, 1.3256777706954837e-11, 1.093838063472971e-268, 1.5805485019543208e-109, 2.479625210353182e-81, 6.696932104755856e-214, 1.1929399965137684e-104, 1.3376809225560248e-128, 3.361983520478162e-240, 1.3337656991601655e-63, 7.414934953069061e-57, 1, 5.474917429102598e-40, 1.30165353823064e-110, 3.2631725782744464e-138, 2.818576763977524e-58, 5.6409797271907875e-39, 2.198753615505768e-76, 5.30979268870743e-

135, 1.3444535354082012e-63, 6.938118101105994e-23, 4.999495112707294e-41,  
 1.3135991216307103e-149, 3.4137784302160565e-63, 8.215101788721782e-120, 3.6599887453125364e-  
 06, 1.7319263786597512e-11, 0.0, 3.5759807708281795e-80, 1.9544327694065702e-  
 59, 1, 6.623413110345628e-06, 7.32547651765495e-12, 1.2009371935201481e-12,  
 3.918581457401569e-125, 5.600180481244886e-110, 1.9117261371413202e-113, 1.9558451772980572e-  
 130, 1.6695049004280318e-260, 2.3113718638267954e-223, 5.927502127265861e-  
 68, 6.99774825338072e-41, 5.072860105128166e-86, 2.311215475943194e-213, 3.635353060361507e-  
 73, 2.1784933257405153e-115, 2.0978109530181388e-41, 8.031930798410777e-75,  
 5.6244748485804e-58, 2.6405649962477773e-88, 2.0015024710217814e-06, 1.2951555766192005e-  
 60, 3.2629362122723074e-113, 1.0253679123051472e-70, 2.093831194731592e-93,  
 1.5669591277002466e-51, 1.2411188860907132e-160, 1.995158340861958e-97, 7.441728861535444e-  
 224, 0.0, 9.85515615905827e-118, 3.646281278726888e-76, 0.0, 2.0076762527181855e-  
 52, 9.332680535477047e-79, 1.4406655731623715e-187, 7.979416770227333e-89,  
 6.262074177391724e-296, 5.701254680780138e-114, 5.710308038010835e-124, 4.711978000210057e-  
 98, 2.640373728462644e-63, 9.569173893033815e-90, 0.0, 3.395620705512919e-  
 141, 1.0812343519534155e-98, 3.119135759368842e-82, 2.196058548889562e-12,  
 1.8173594680248002e-85, 0.0, 5.707011093808175e-103, 1.2149949508504122e-  
 76, 1.0701195260614832e-141, 1.745426568125493e-226, 3.420193736714059e-176,  
 1.7077806216099915e-90, 3.343688495029043e-103, 1.8805967666484102e-17, 1.781154090948016e-  
 28, 3.974166041474985e-12, 7.152683801488381e-40, 4.234553718353022e-40, 7.13143098132968e-  
 93, 3.4098410675216864e-24, 6.413668209974341e-165, 1.4422501427800499e-24,  
 1.8863976058001725e-63, 7.487841173760655e-139, 1.211958479125027e-51, 5.415653662203378e-  
 121, 1.9977054196081954e-58, 8.4805535163016e-111, 1.2149949508504125e-76,  
 4.8551203823763636e-63, 7.954303152263033e-18, 5.6828994549095075e-18, 8.225787887366341e-  
 212, 3.249401464681705e-68, 4.5675560436412225e-197, 1.931968719402104e-35,  
 1.5680637962911182e-299, 1.6361727250258615e-115, 1.0799090563015077e-34,  
 1.729718168767626e-50, 1.7999252836560384e-53, 3.6599887453125364e-06, 1,  
 1.5236513569139234e-34, 7.205837432934923e-19, 9.812854973648187e-23, 1.0322666486970244e-  
 109, 2.0127063360750904e-77, 1.4422501427800499e-24, 2.0015024710217814e-  
 06, 7.612537975714257e-29, 4.999495112707294e-41, 5.330277697832306e-146,  
 1.8875291112865823e-128, 2.6050616244652974e-76, 5.485471110504043e-47, 8.522255057105349e-  
 77, 5.492203039428052e-111, 1.7319263786597512e-11, 8.648817e-318, 1.7015142961593972e-  
 116, 4.837175540027401e-30, 7.298237194650556e-141, 5.781009347881584e-57,  
 2.181276698367824e-79, 1.2765171712030438e-40, 1.095069475707386e-161, 2.9795944610264106e-  
 114, 1.1899963170371555e-153, 9.988241974440836e-124, 8.642098592486835e-  
 45, 1.3241190353843786e-134, 7.70550199453223e-74, 1.4904379367831685e-206,  
 3.8268088423041586e-56, 2.0774432796887176e-102, 2.3255257072594342e-113,  
 3.510810558399593e-80, 8.475469730958044e-46, 7.724599882894985e-40, 3.600214107782885e-  
 13, 2.196058548889562e-12, 4.709257458962993e-77, 1.3000894351222403e-177,  
 8.145907182104658e-186, 2.2528948611553366e-63, 5.326551483950764e-203, 1.039262006650022e-  
 42, 1.1886225841928206e-111, 5.2051670335970785e-168, 4.7877924312890484e-  
 95, 7.539719978182604e-140, 9.925293442102905e-159, 4.8224401681837015e-130,  
 7.828914280082316e-46, 2.1665921178320175e-143, 2.0015024710217814e-06, 6.938118101105994e-  
 23, 3.487431614155804e-172, 1.558397156400462e-76, 3.803411729903087e-23,  
 8.51653515660129e-196, 2.40217490125637e-103, 4.065092740283114e-82, 2.4930653291875996e-  
 28, 4.0106379062990896e-51, 6.000178420499581e-07, 7.002479118478765e-143,

1.3176743115200775e-18, 1.5404861515121465e-261, 1.4422501427800499e-24, 2.1130489532722968e-135, 1.4289606111105449e-63, 9.192645616587385e-72, 2.196058548889562e-12, 1.91527251925443e-95, 3.142087334227145e-36, 1.763906450117053e-145, 4.8519657206564837e-17, 1.3759072580186649e-126, 5.862880585087464e-106, 4.033073254645794e-110, 1.986738451478712e-27, 2.231180796347495e-146, 2.4050673672323836e-39, 3.6599887453125364e-06, 9.638403150693656e-119, 6.938118101105994e-23, 3.4704576615516543e-56, 2.196058548889562e-12, 1.6544431972501932e-40, 5.192409310988616e-37, 2.0799348045711547e-23, 4.097366435897727e-54, 1.4759918339067213e-87, 1, 1.2687186017179185e-22, 1.8967596900706457e-35, 2.114621296813388e-41, 1.8033059196346325e-163, 3.748050146046493e-115, 6.774462362276553e-51, 2.9579727244165055e-54, 5.676238751892439e-153, 2.0127063360750904e-77, 8.061719504042102e-24, 1.0481180929780174e-35, 1.7162705933469987e-174, 1.3953621438960168e-131, 2.1184943137354554e-182, 3.1672657556893195e-36, 1.1283916136905079e-23, 2.196058548889562e-12, 1.5545024662216928e-51, 4.674519770569343e-98, 2.4826682781841874e-145, 2.6422818445917496e-134, 2.088344040951002e-146, 4.294316427798226e-195, 9.185775570441195e-235, 7.92113061779213e-135, 3.642181013422862e-183, 6.76972656854773e-108, 0.0, 1.1195348236110226e-194, 0.0, 2.911266001408673e-23, 1.6685415590548762e-33, 3.6022939539292963e-34, 7.598646629364714e-106, 0.0, 1.3807790833333844e-268, 1.455486014583357e-63, 7.781649102062831e-53, 2.8017632319915663e-90, 6.156225087267534e-64, 6.000178420499581e-07, 2.196058548889562e-12, 1.2725113008625388e-205, 1.5030911385059114e-125, 2.8446495623146817e-122, 7.488788825662014e-76, 7.15732698939925e-83, 2.242268893701989e-305, 8.444574591215031e-203, 1.3135942005420414e-137, 9.361458249334181e-122, 1.47695149751635e-133, 1.0092503417577186e-64, 3.2314146390249705e-277, 2.190521049832829e-149, 7.061612848748297e-99, 7.419218566050496e-78, 1.1627016105757434e-47, 8.296107090563328e-264, 2.59244486919277e-91, 1.3411956982766397e-75, 5.151131855883577e-37, 6.1858869815801425e-155, 7.320022074092994e-103, 2.196058548889562e-12, 1.5824440896505152e-30, 2.5894090291245586e-229, 6.99774825338072e-41, 5.333213621342324e-108, 1.8065184535326142e-117, 9.906426652761162e-280, 3.986424649156244e-58, 0.0, 0.0, 2.7827027554309057e-132, 1.4661630122670481e-179, 2.9524986800179754e-75, 5.8482501936746385e-93, 7.843261327513779e-190, 1.9545836800957664e-99, 2.0961925980174284e-73, 1.2009371935201481e-12, 2.09377490433743e-34, 2.5086508493834598e-145, 4.090099612670189e-115, 8.335242873838365e-268, 8.818404596959608e-127, 4.833799007407454e-90, 9.552618504087235e-30, 2.3543909650565844e-117, 1.4402745938123518e-200, 1.0191272172028246e-180, 1.3120840507839659e-110, 8.733175776041997e-70, 3.945136495285532e-58, 2.196058548889562e-12, 7.50984990960555e-135, 1.4359164674700008e-51, 3.564984314303009e-34, 4.732053837263527e-06, 4.255474808863981e-53, 3.3460275878780023e-46, 1, 0.0, 3.998412333719408e-64, 4.959751243781957e-41, 0.0, 5.970090976729888e-119, 3.466454926515297e-17, 4.395416612110966e-18, 8.018043200168112e-18, 1.255578243262265e-22, 3.600214107782885e-13, 5.297551400338203e-57, 1.005597727315115e-71, 7.978838785828362e-64, 3.575980770828179e-80, 3.1304099926240984e-252, 3.6596414584337444e-143, 5.693160717515665e-103, 7.423497020240892e-96, 6.01868641249258e-53, 1.6005777633235214e-250, 0.0, 6.235305782092159e-24, 8.803157147341632e-70, 6.000178420499581e-07, 9.73e-322, 6.988826109168289e-80, 4.2186987169849335e-113, 3.2136573365143727e-22, 3.466393638995506e-194, 8.195568865994801e-17, 1.0629341634938946e-28, 1.743450794625009e-22, 7.706089047547392e-111, 5.992528179989407e-46, 2.160192699869702e-19, 3.267101774125917e-74, 1.80430415924273e-53, 1.918962505988673e-

99, 8.872404699645113e-17, 9.634021281817271e-23, 1.0481180929780174e-35,  
 1.238881913089678e-75, 6.891409971826086e-87, 2.2121086871229553e-69, 1.5422581282519522e-  
 79, 2.196058548889562e-12, 2.1758230926619067e-51, 2.7666724315001554e-111,  
 2.445309897943487e-259, 9.494946878368126e-37, 1.431715135762316e-70, 1.52900474279243e-  
 242, 5.2396817226014086e-51, 4.191244578718115e-198, 4.631322163344514e-78,  
 3.678173678076225e-162, 9.119992785726957e-41, 6.000178420499581e-07, 1.1374337301569714e-  
 23, 6.988826109168289e-80, 6.882963000404328e-23, 1.7562402133547855e-138,  
 4.732053837263527e-06, 1.7319263786597512e-11, 8.861092364394549e-56, 5.323601121674967e-  
 23, 4.7727238124320445e-24, 4.732053837263527e-06, 8.061719504042102e-24,  
 2.2082425302898306e-186, 7.677964477839754e-47, 5.665171565654987e-90, 1.1833709057454105e-  
 79, 5.61692162438898e-293, 4.323631026672422e-25, 5.333331538572701e-152,  
 5.787684711950297e-96, 1.0951510544134765e-192, 1.498767207284301e-41, 2.653347333825657e-  
 17, 3.6113192331992017e-59, 1.07790946087968e-52, 2.196058548889562e-12, 1.9953565124698523e-  
 181, 5.960222451810237e-99, 2.842801223576618e-76, 3.0253489912522312e-40,  
 6.000178420499581e-07, 5.492203039428052e-111, 5.1412267075780594e-33, 3.2080287302736773e-  
 127, 6.845856936806517e-76, 3.0963666194588233e-143, 4.070081539758277e-146,  
 3.600214107782885e-13, 1.2163933554190783e-103, 1.2009371935201481e-12, 2.8653738153117883e-  
 51, 3.3638445830274347e-130, 1.0395568394600357e-238, 1.2031056520709263e-  
 97, 1.931968719402104e-35, 4.6765201689033076e-135, 6.7837912876967876e-180,  
 1.3306315121334192e-103, 2.985118742029364e-93, 3.861734019577396e-114, 2.735604746789937e-  
 62, 6.273178581794522e-17, 3.134234742459258e-11, 1.3160896040954675e-82,  
 1.775813993024496e-22, 6.938118101105994e-23, 2.0799348045711544e-23, 9.321916466006809e-  
 40, 4.350009396457607e-77, 3.0253489912522312e-40, 4.697488251929093e-52,  
 1.2695434999625824e-68, 3.2831548383726164e-28, 3.903993166679116e-33, 1.2009371935201481e-  
 12, 5.474917429102599e-40, 1.2473075323906728e-139, 1.972226195311842e-73,  
 2.681116160866839e-17, 2.204937276915561e-135, 6.148089495864657e-134, 1.2946363221030703e-  
 163, 0.0, 7.153201939498058e-65, 3.115538229819499e-43, 7.205837432934923e-  
 19, 8.392533261257231e-67, 2.196058548889562e-12, 2.893689936465834e-30, 7.06623792171733e-  
 160, 7.833428994241183e-64, 1.401307218985542e-39, 7.886344787003525e-162,  
 4.520350504818058e-35, 2.839316731899085e-12, 6.000178420499581e-07, 1, 8.727501310087728e-  
 24, 1.009250341757719e-64, 6.277257290811305e-63, 6.942119118221057e-41, 6.362026511250893e-  
 69, 8.72750131008773e-24, 6.997748253380721e-41, 1.2009371935201481e-12, 1.1106040402085268e-  
 51, 1.5078706117083424e-34, 4.8519657206564837e-17, 1.059775859667773e-81,  
 1.5236513569139234e-34, 2.9345947872045e-23, 1.3272085382872078e-50, 6.193784069713306e-  
 116, 6.000178420499581e-07, 4.761268864608923e-83, 1.0228053496610365e-45,  
 1.2009371935201481e-12, 1.4644719782218193e-140, 3.096044317244768e-44, 6.761617207544796e-  
 187, 3.0284735603140387e-157, 2.909312040660337e-142, 1, 1.9484320189863575e-  
 28, 1.3435799636389604e-17, 6.770551010338483e-30, 6.000178420499581e-07,  
 2.1427613331182878e-100, 1, 4.6974882519290924e-52, 1.0979183982392436e-52,  
 3.974166041474985e-12, 1, 6.623413110345628e-06, 1.6056224676424093e-16, 1.2009371935201481e-  
 12, 1, 1.781154090948016e-28, 4.531675906416486e-60, 3.1208825075096375e-  
 280, 1.781154090948016e-28, 4.5063579240710246e-238, 1.8978554494377163e-  
 56, 5.916685631466405e-145, 1.8668598291044285e-63, 1.478879757979327e-240,  
 3.600214107782885e-13, 7.982831775502257e-45, 1.7036406983704286e-18, 1.64235821269968e-  
 86, 5.600612896782163e-150, 2.63225434617958e-17, 3.662286167710428e-214,  
 6.397004010375184e-56, 3.339831972231492e-64, 1.271897281704025e-47, 1.1293234579349012e-

190, 4.743909645797683e-31, 8.155094060222792e-166, 7.891649552010595e-46,  
 6.235305782092158e-24, 9.327967818344242e-83, 3.399204017667088e-96, 2.1326410729966465e-  
 34, 2.6373283905093596e-24, 1, 2.293215033596555e-86, 3.6599887453125364e-  
 06, 3.693226856523237e-182, 3.337751615756848e-77, 4.006012141506297e-12,  
 4.493059981039524e-42, 8.47443768328101e-124, 6.623413110345628e-06, 1.2118434795152653e-  
 188, 1.2009371935201481e-12, 0.0, 1.7223798033772845e-142, 1.8555064222685546e-  
 188, 5.735052771747654e-57, 7.992666699021476e-124, 2.024940339132141e-291,  
 5.648747764912133e-168, 1.36332919281297e-193, 9.705667409445573e-226, 6.883129061859177e-  
 154, 1.0565169553519755e-35, 2.3830786280809825e-103, 1.0147479829540914e-  
 70, 3.412058085469853e-70, 2.196058548889562e-12, 1.5014468928948105e-303,  
 7.10486213783953e-73, 4.1629946509133726e-29, 2.296142620421619e-47, 1.656473577829789e-  
 104, 2.0459654790676873e-30, 2.6959161540243607e-48, 2.2765764215327003e-  
 29, 3.8594006226490045e-34, 3.576239813672888e-105, 2.4112677432087943e-89,  
 3.73708376863018e-134, 6.296619952974719e-109, 1.0258353381092746e-169, 6.526531457104244e-  
 73, 3.053192279182994e-135, 1.933134801542963e-112, 1.6155353523879639e-93,  
 4.180543739996954e-261, 2.5704822548329977e-260, 3.420705602323395e-241, 1.7445843552886552e-  
 68, 4.015559877225788e-115, 1.7671701791851632e-112, 3.704952752158398e-76,  
 4.1299006064123103e-29, 2.8102019068286795e-111, 2.9131588547262327e-69, 1.2448235649859176e-  
 122, 3.5474735790964767e-242, 2.4225322229773908e-127, 1.6515915322054117e-  
 288, 3.909071430729855e-125, 2.5036430090225345e-113, 3.4773993452844246e-  
 88, 3.5328378913755657e-35, 2.8936899364658335e-30, 1.6384164116710215e-145,  
 2.5393434163697127e-28, 1.0159943557767999e-137, 2.0159846372648796e-38, 5.047920522549096e-  
 73, 8.332244080653573e-35, 1.2796226944865551e-40, 2.952498680017976e-75,  
 2.196058548889562e-12, 6.444541969967186e-35, 1.6677438322909446e-96, 1.0404620483451129e-  
 81, 4.709257458962991e-77, 3.552339462169905e-66, 7.205837432934923e-19, 2.8256384981846444e-  
 83, 5.809144017604815e-42, 1.0629341634938944e-28, 2.9056649193439274e-16,  
 2.0015024710217814e-06, 9.652592226957481e-30, 1.1921648721409977e-58, 8.974476025610344e-  
 119, 9.80034356225692e-62, 5.505543898647115e-99, 4.691755842300089e-293,  
 4.666224547451942e-156, 2.8637194426359247e-30, 1.5574989906403182e-58, 6.832685963598293e-  
 69, 1.3495802489730724e-192, 3.8994214594762887e-171, 3.519341985380836e-  
 152, 2.196058548889562e-12, 1.0818589817762412e-119, 1.5746638135773316e-  
 223, 1.3176743115200775e-18, 7.828914280082317e-46, 1.9613550600952476e-204,  
 4.190812619484205e-102, 1.6677007116584738e-40, 1.269450235968835e-40, 6.186185834457056e-  
 49, 2.3471510050149373e-71, 2.2419814038601763e-75, 4.688581277438742e-188,  
 1.8693785254804425e-49, 1, 1.3601241365859414e-230, 3.6599887453125364e-06,  
 4.006012141506297e-12, 1.2009371935201481e-12, 4.848709216247117e-136, 5.65552243633084e-  
 89, 5.442258144994689e-72, 5.2938480642861026e-114, 2.0788378207782654e-189,  
 2.0621092395088173e-108, 1.4661959351498418e-17, 8.037549572983177e-18, 1.0826408076784958e-  
 190, 1.1235741818533515e-56, 6.623413110345628e-06, 2.868682478602078e-90,  
 9.890825214966708e-303, 3.935769590189755e-173, 7.906280969229411e-25, 1.4422501427800499e-  
 24, 5.815695516028175e-22, 2.077282883180603e-62, 5.278619290500381e-30, 4.234553718353021e-  
 40, 2.259783327711717e-50, 3.83869778750125e-146, 2.4272019981104617e-134,  
 0.0, 1.3694088491015415e-60, 6.0110125742524425e-92, 4.5072456349798557e-  
 113, 2.02478469515038e-45, 1.749002301527881e-152, 2.6373283905093596e-24,  
 6.25812241341696e-76, 4.595402239199175e-28, 3.283154838372616e-28, 8.517343345312202e-  
 59, 5.110928181154144e-90, 2.774346657008098e-111, 4.810968978428211e-24,



4.006012141506297e-12, 8.834318719080597e-128, 2.9219429816691433e-308, 1.4012705036062163e-189, 6.062381765677784e-216, 1.6777143795137338e-226, 8.716373726973379e-63, 2.7742843600888186e-273, 3.2451073067771745e-138, 1.9710874967201572e-52, 3.0420491794138565e-71, 3.0587624656642217e-46, 2.308431226617315e-190, 9.610898801772941e-82, 1.419340700352755e-127, 4.107636306205382e-82, 4.547929055448108e-87, 2.911266001408673e-23, 2.1865728894256415e-64, 4.27106271608163e-117, 4.1782466553732233e-302, 6.000178420499581e-07, 3.7458020325886803e-256, 5.278619290500381e-30, 2.7350706952902747e-112, 3.600214107782885e-13, 2.8620183662751272e-189, 2.196058548889562e-12, 0.0, 2.1743605048813113e-167, 8.531733077981727e-240, 9.920281473647015e-163, 4.846283905733667e-155, 1.6585882814888327e-65, 7.8867416954910105e-143, 9.669274711150895e-75, 3.1115807681783906e-97, 2.1314191970223945e-234, 1.6923040132984827e-125, 2.280818272910527e-244, 1

## Discussion