EMTH171 S2-19
Case study 1

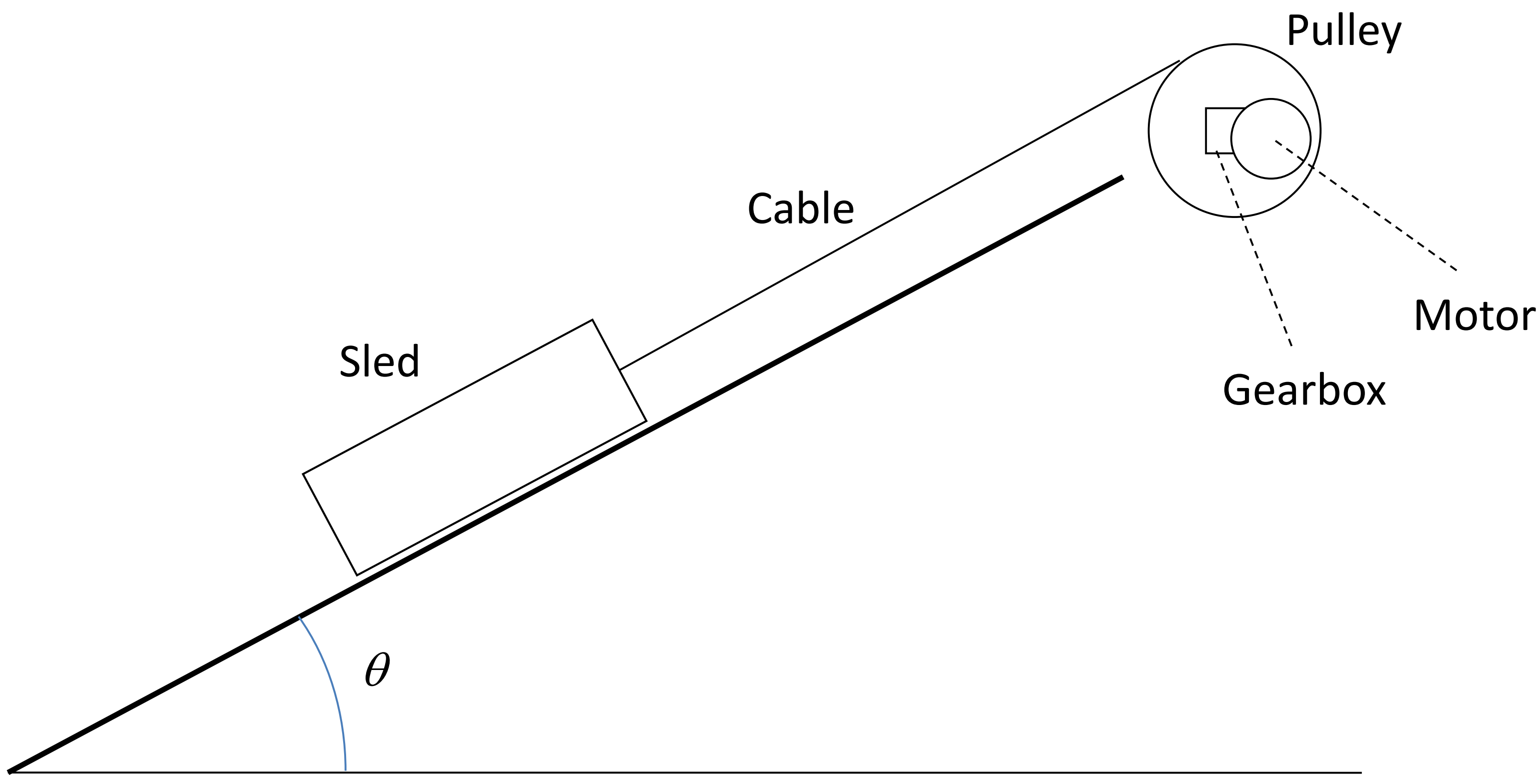
Using Newton's Method to match power supply to demand

Assoc. Prof. Digby Symons

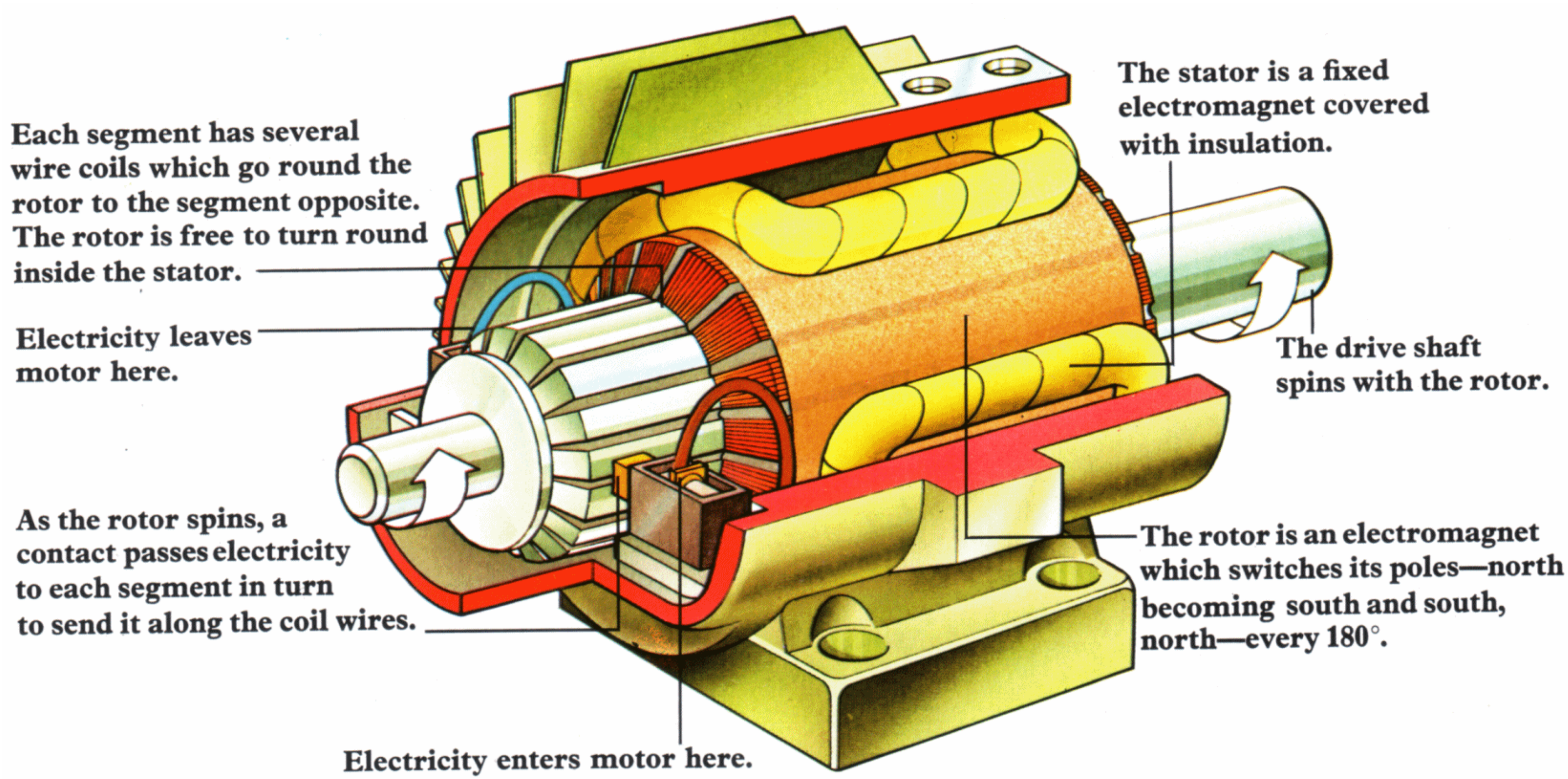
Department of Mechanical Engineering

digby.symons@canterbury.ac.nz

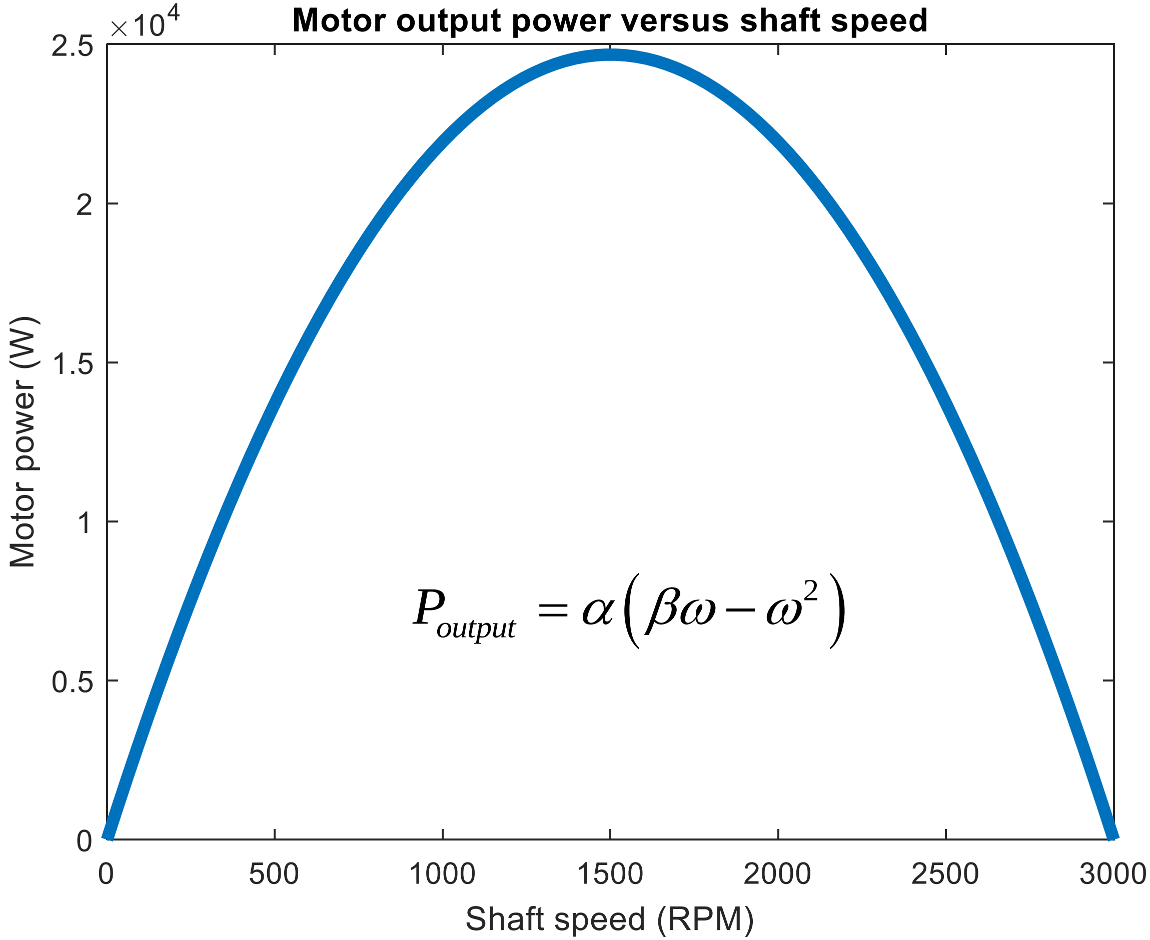
Exercise 1: hauling a sled up an incline with a cable



Electric motor



Electric motors give peak power output at a certain speed



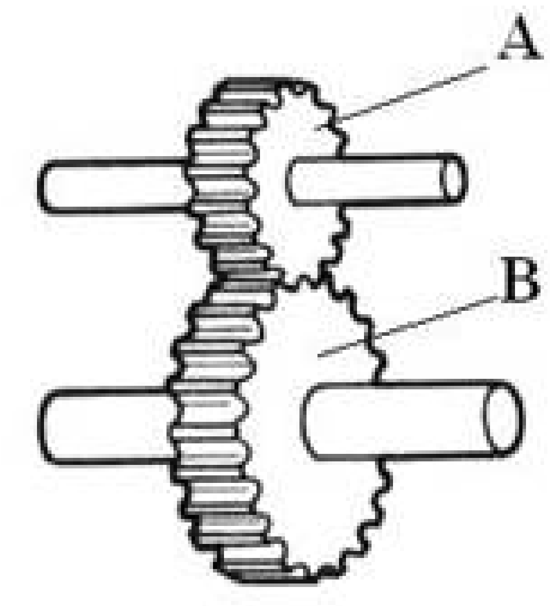
α = 1 W.s^2/rad^2

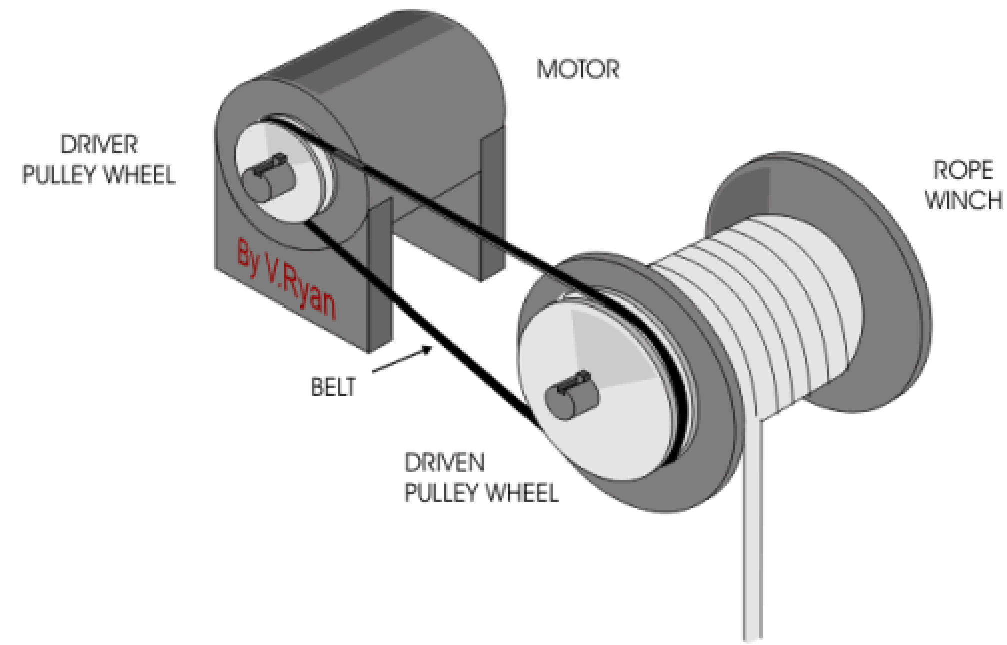
β = 314.16 rad/s

1 rev/min = 1 rpm = 2π rad/60s

rad/s x 30/π = rpm

A gearbox or two-pulley system is used to reduce the shaft speed





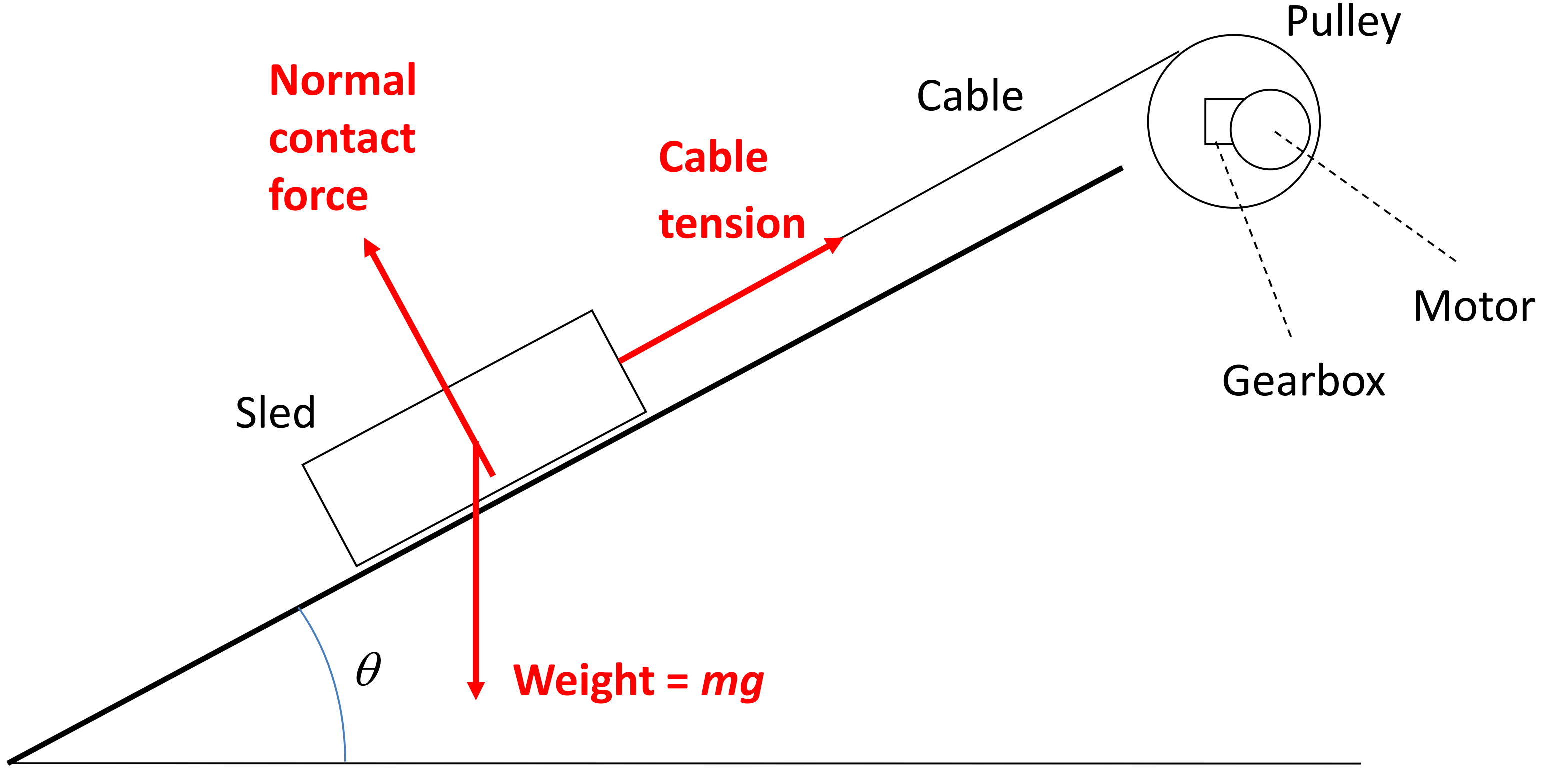
Tangential speed of winch drum $$V = \frac{R\omega}{r_{gb}}$$

R is radius of winch drum (m)
ω is speed of motor (rad/s)

Gearbox ratio r_{gb} = $\frac{\text{diameter of driven pulley}}{\text{diameter of driver pulley}}$ or $\frac{\text{number of teeth on driven gearwheel}}{\text{number of teeth on driver gearwheel}}$

If $r_{gb}>1$ the system **reduces** shaft speed: output shaft is slower than input

Equilibrium of forces on sled



Work

A force does **mechanical work** when it acts on a body such that there is a displacement of the point of application in the direction of the force.

Thus a force does work when it results in movement.

$$\text{mechanical work} = \text{force} \times \text{distance}$$

Power

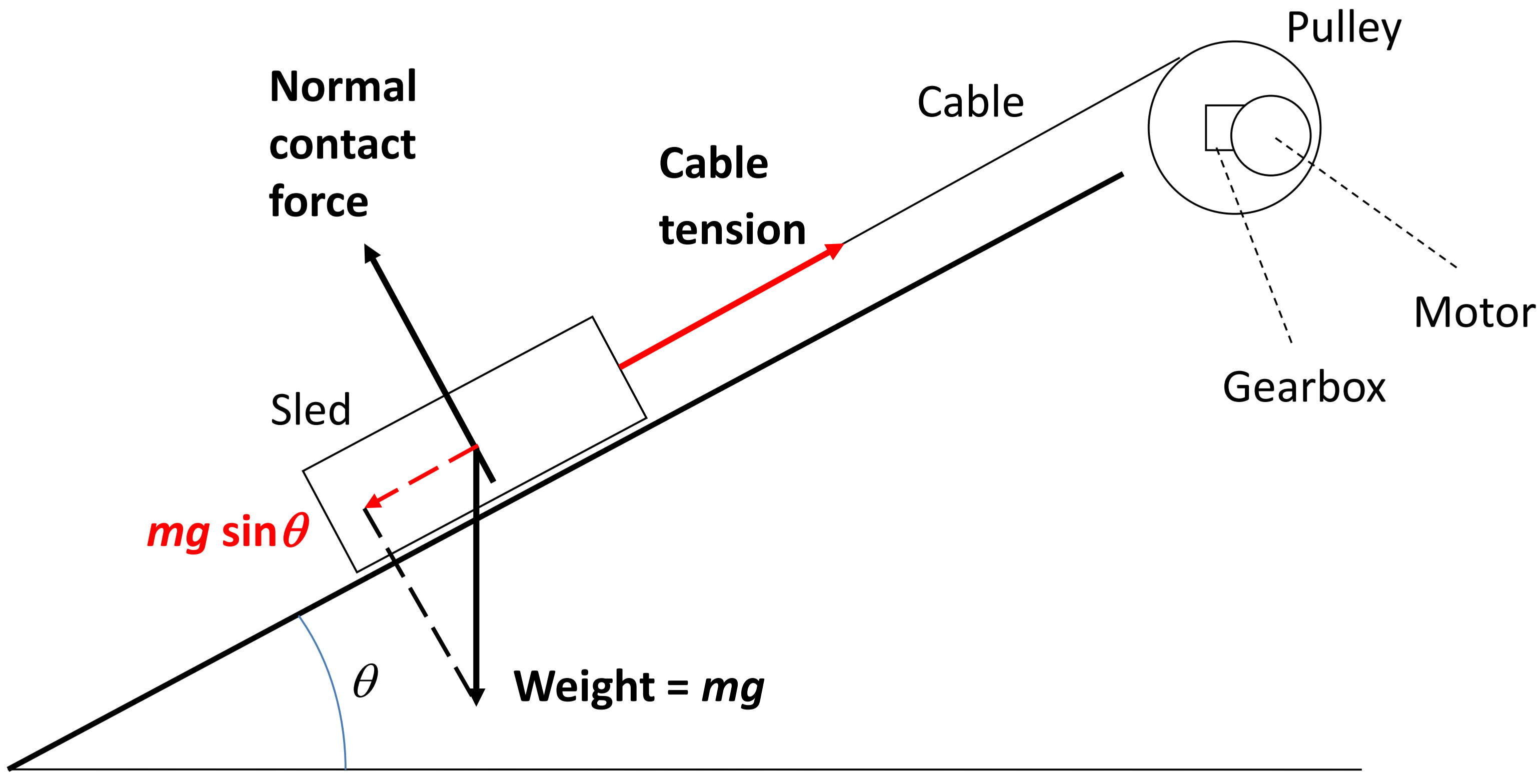
Mechanical power *P* = rate of doing work

$$\text{work} = Fd$$

$$P = \frac{\partial}{\partial t}\left(Fd\right) = F\,\frac{\partial d}{\partial t} + d\,\frac{\partial F}{\partial t}$$

$$= F\,\frac{\partial d}{\partial t}\,(\text{if force is constant}) = Fv$$

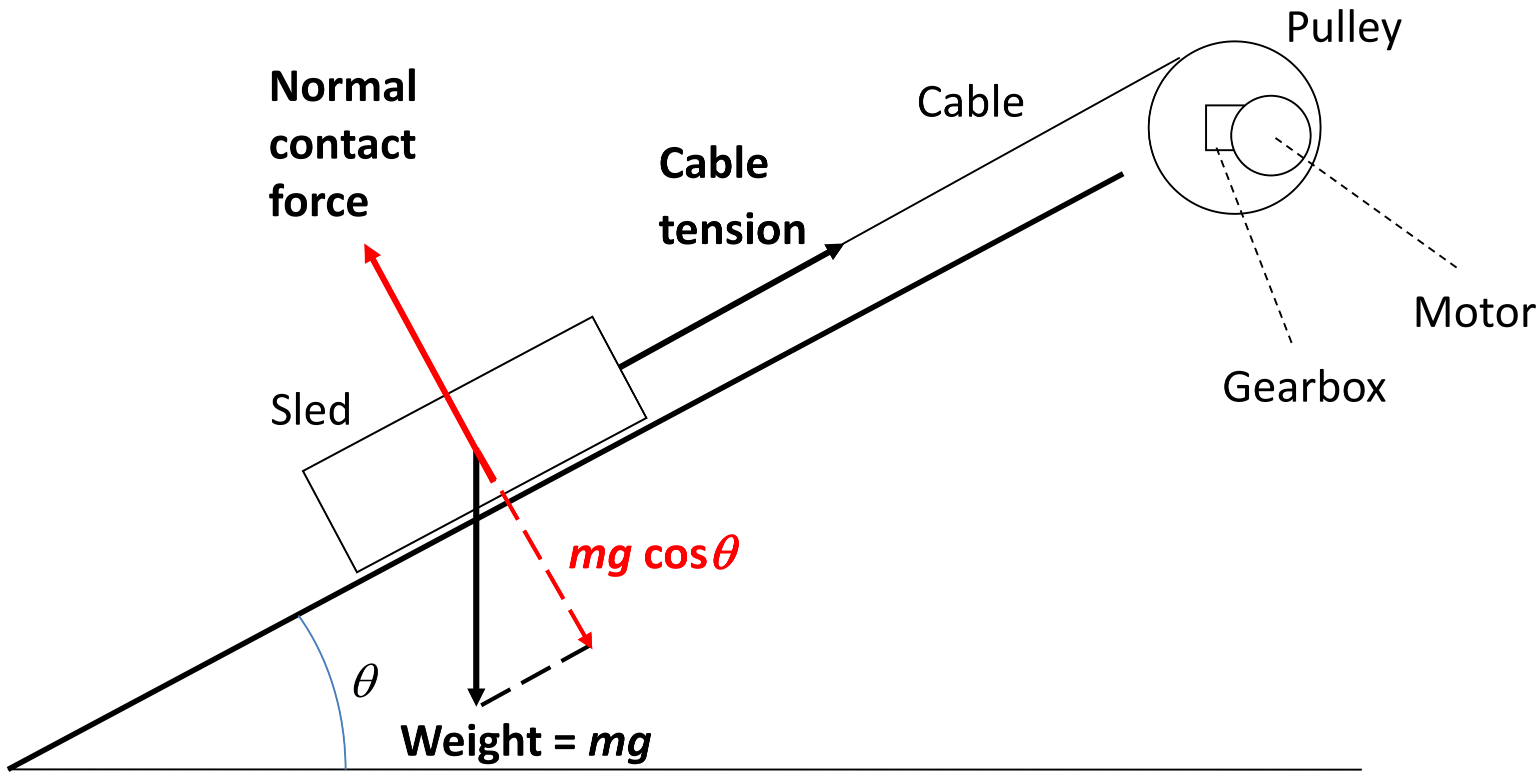
Force and power required to climb a gradient



$$F = mg\sin(\theta)$$

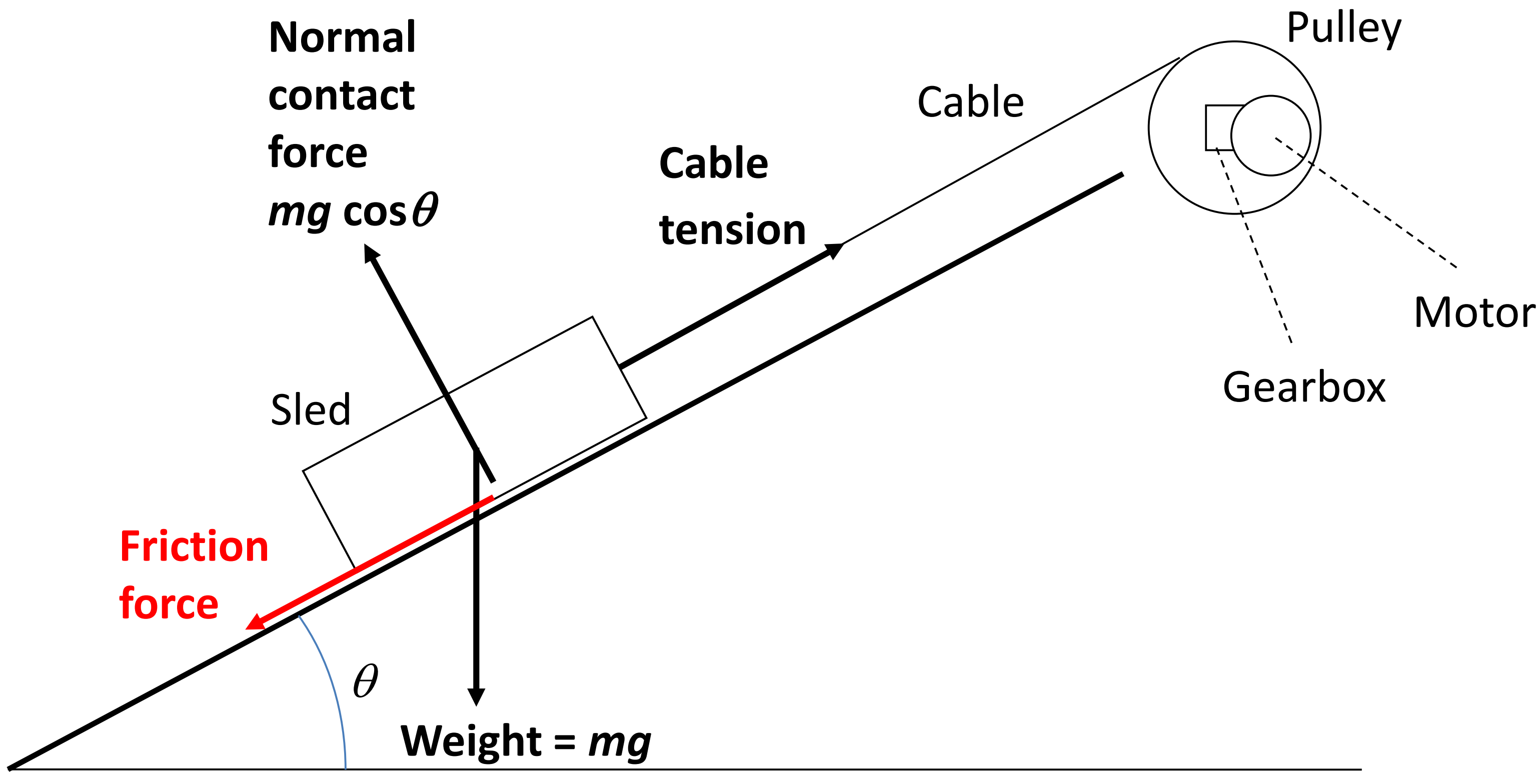
$$\textbf{Power} = \boldsymbol{mg}\,\textbf{sin}(\theta)\,\textbf{V}$$

Equilibrium requires a contact force



Normal contact force = $mg \cos(\theta)$

Contact force can result in friction



Friction force = $C_f\, mg \cos(\theta)$

Power to overcome friction = $C_f\, mg \cos(\theta)\, V$

Friction is a resistive force which dissipates power

Power to overcome friction = $C_f\, mg \cos(\theta)\, V$

coefficient of static friction (stiction) = μ_s

coefficient of kinetic friction (when moving) = μ_k = C_f

Materials	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.3	0.25
Wood on wood	0.25-0.5	0.2
Glass on glass	0.94	0.4
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	0.10	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003
Very rough surfaces		1.5

Power supply

$$P_{output} = \alpha\left(\beta\omega - \omega^2\right)$$

Power demand

$$P_{demand} = mg\sin\left(\theta\right)V + C_f\, mg\cos\left(\theta\right)V$$

Sled moves at constant speed when: $P_{demand} = P_{output}$

V = const., ω = const.

$$mg\sin\left(\theta\right)V + C_f\, mg\cos\left(\theta\right)V = \alpha\left(\beta\omega - \omega^2\right)$$

Substitute so there is only one independent variable:

$$mg\sin\left(\theta\right)V + C_f\, mg\cos\left(\theta\right)V = \alpha\left(\beta\frac{r_{gb}V}{R} - \frac{{r_{gb}}^2 V^2}{R^2}\right)$$

Rearrange as an expression which evaluates to zero when solved:

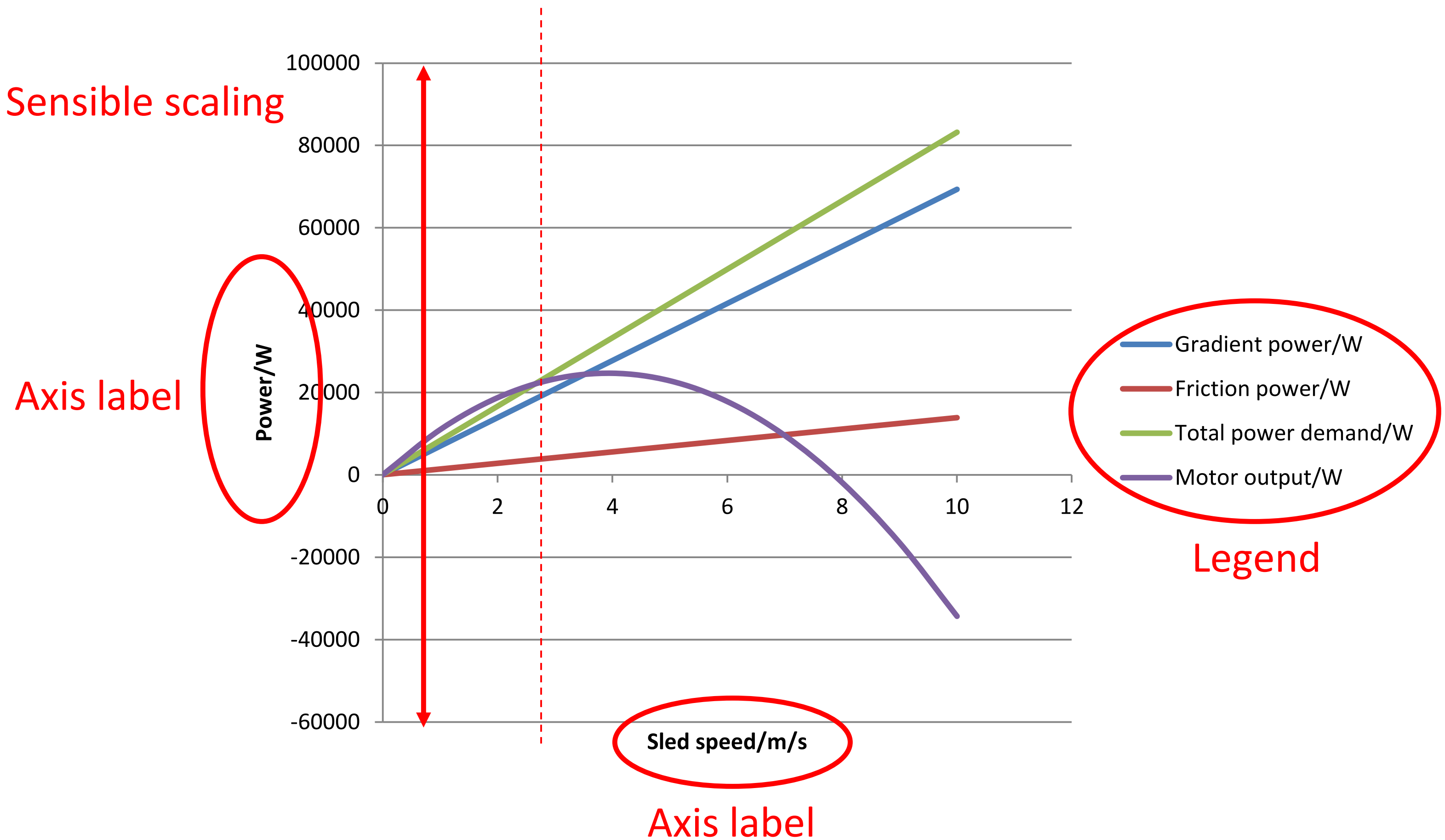
$$f\left(V\right) = mg\sin\left(\theta\right)V + C_f\, mg\cos\left(\theta\right)V - \alpha\left(\beta\frac{r_{gb}V}{R} - \frac{{r_{gb}}^2 V^2}{R^2}\right) = 0$$

Task: Find the value of *V* that satisfies this equation: *a root-finding problem*

Graphical solution

Sled mass = 1000 kg
Gear ratio = 20
Pulley radius = 0.5 m
Gradient = 45°
C_f = 0.2

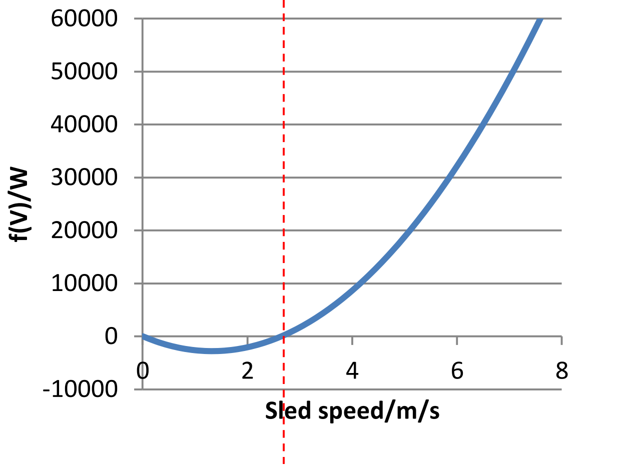
Total power demand = motor output
(occurs at 2 < V < 3 m/s)



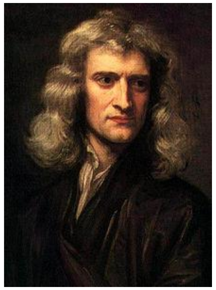
In your report, each figure must have a caption.

Problem: find the root of:

$$f\left(V\right) = mg\sin\left(\theta\right)V + C_f mg\cos\left(\theta\right)V - \alpha\left(\beta\frac{r_{gb}V}{R} - \frac{r_{gb}{}^2V^2}{R^2}\right) = 0$$



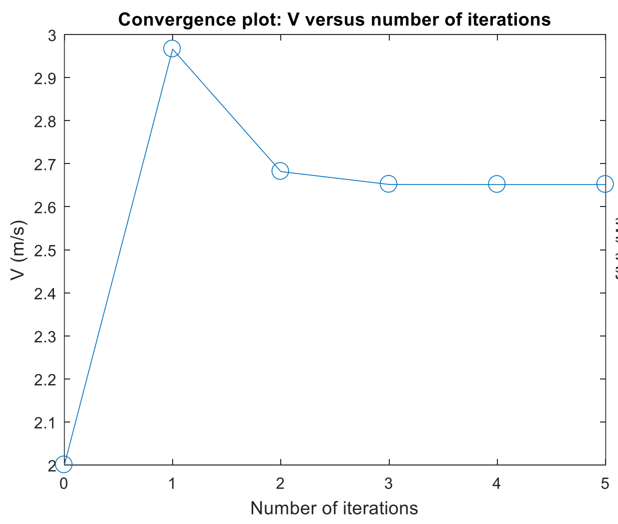
Newton's method: $$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$$

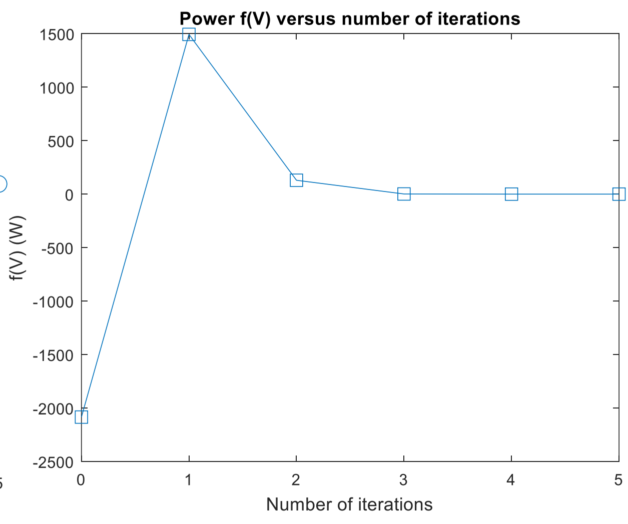


Where:

$$f'\left(V\right) = mg\sin\left(\theta\right) + C_f mg\cos\left(\theta\right) - \alpha\left(\beta\frac{r_{gb}}{R} - \frac{2r_{gb}{}^2V}{R^2}\right) = \frac{\partial f}{\partial V}$$

	n	V_n (m/s)	$f(V_n)$ (Watts)
initial guess	0	2.0000	-2084.61917
1st iteration	1	2.9662	1493.46542
2nd iteration	2	2.6817	129.51023
3rd iteration	3	2.6518	1.42550
4th iteration	4	2.6515	0.00018
5th iteration	5	2.6515	-3e-12





Write a MATLAB script to use Newton's method to solve for V.

- Check your iterated solution against your graph. Does it lie at the speed that the power output and power demand curves cross?
- Experiment with different initial guesses.
- Take g = 9.81m/s^2

Your report section for Exercise 1 must include:

- Introduction describing the problem solved, and the methods used
- Plot of the motor power and power demand as a function of *V*
- Convergence plot (*V* versus iteration number)
- Discussion of whether there is more than one solution
- Clear statement saying at what constant velocity *V* the sled travels
- Copy of your script, suitably commented (in an Appendix)

All plots must:

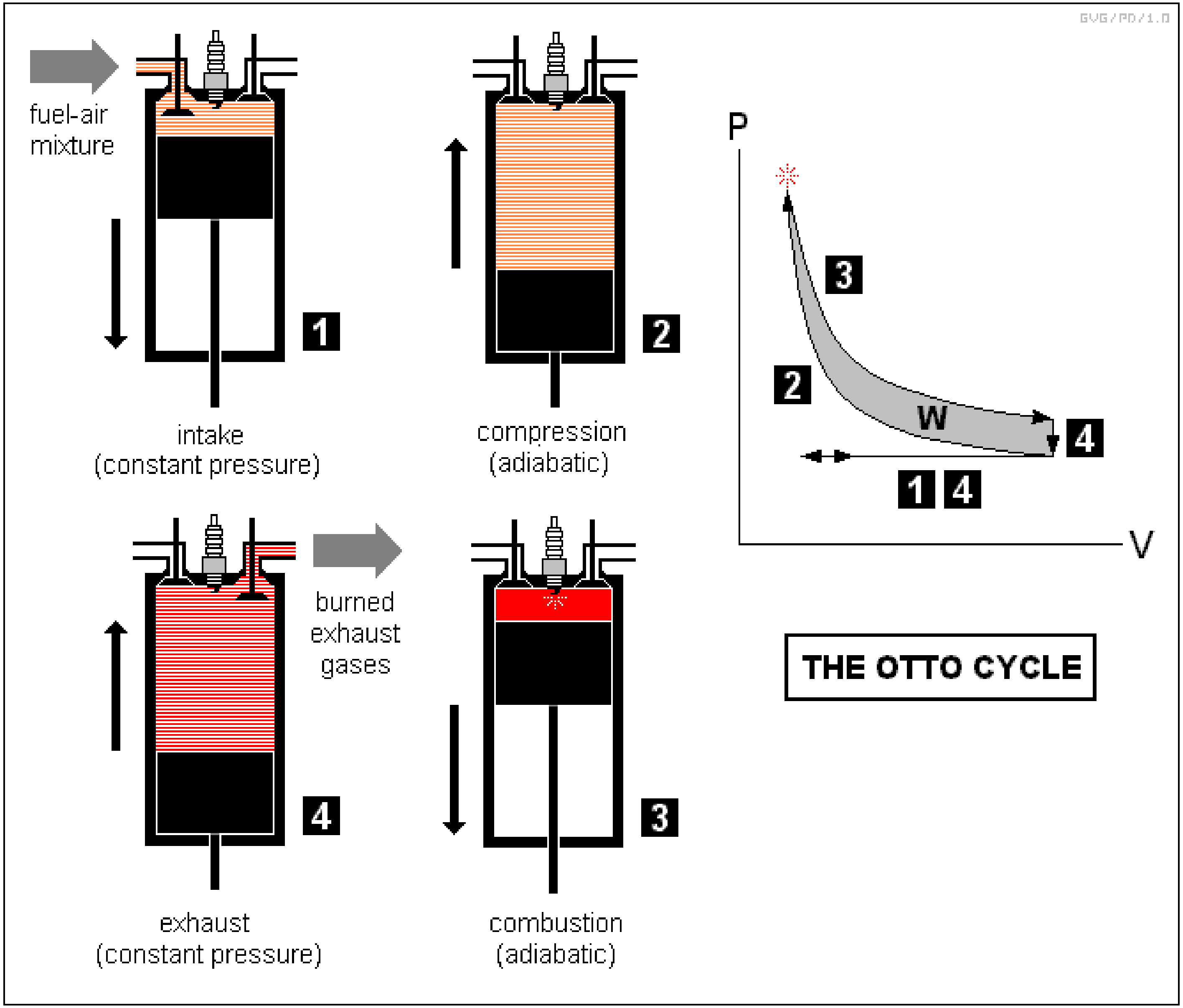
  - be scaled properly
  - have both axes labelled
  - have a caption
  - have a legend (or defining caption) if there is more than one curve on a plot

Exercise 2: Road car power

Medium size family road car

Toyota 2ZR-FE engine

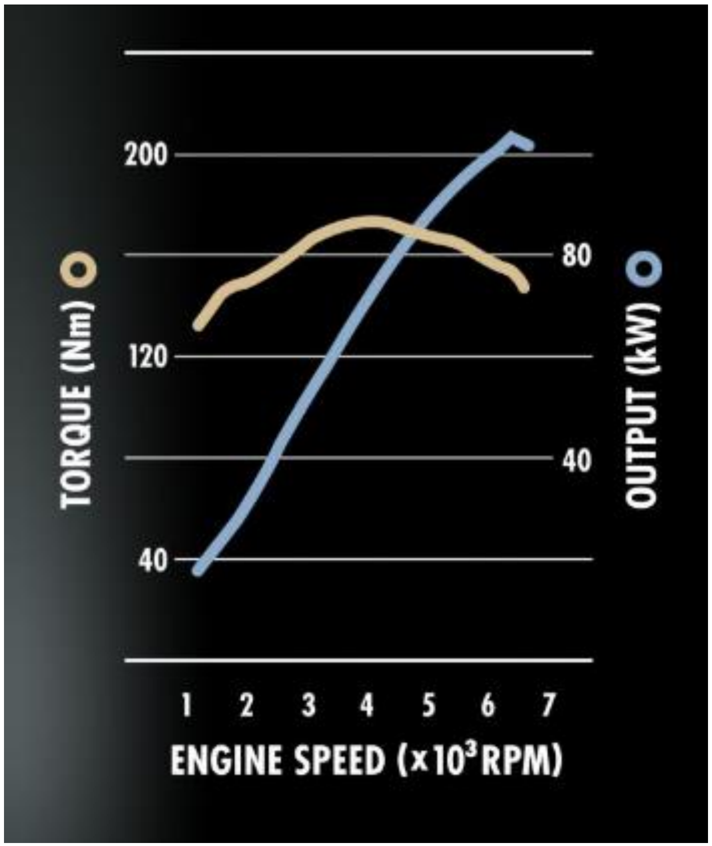
Otto cycle engines



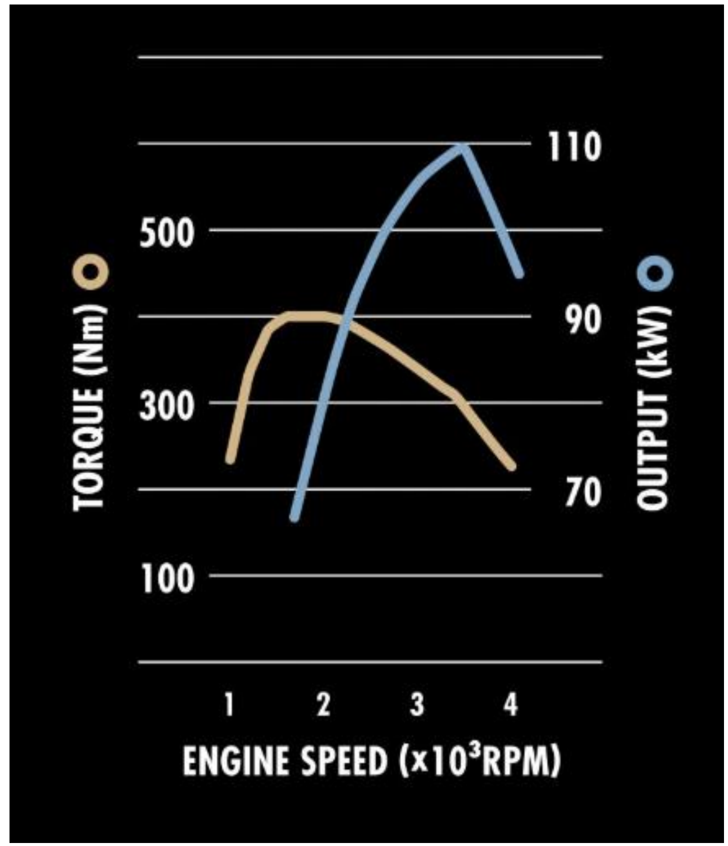
Torque and Power

Power = Torque × angular velocity

$$P = T\omega$$

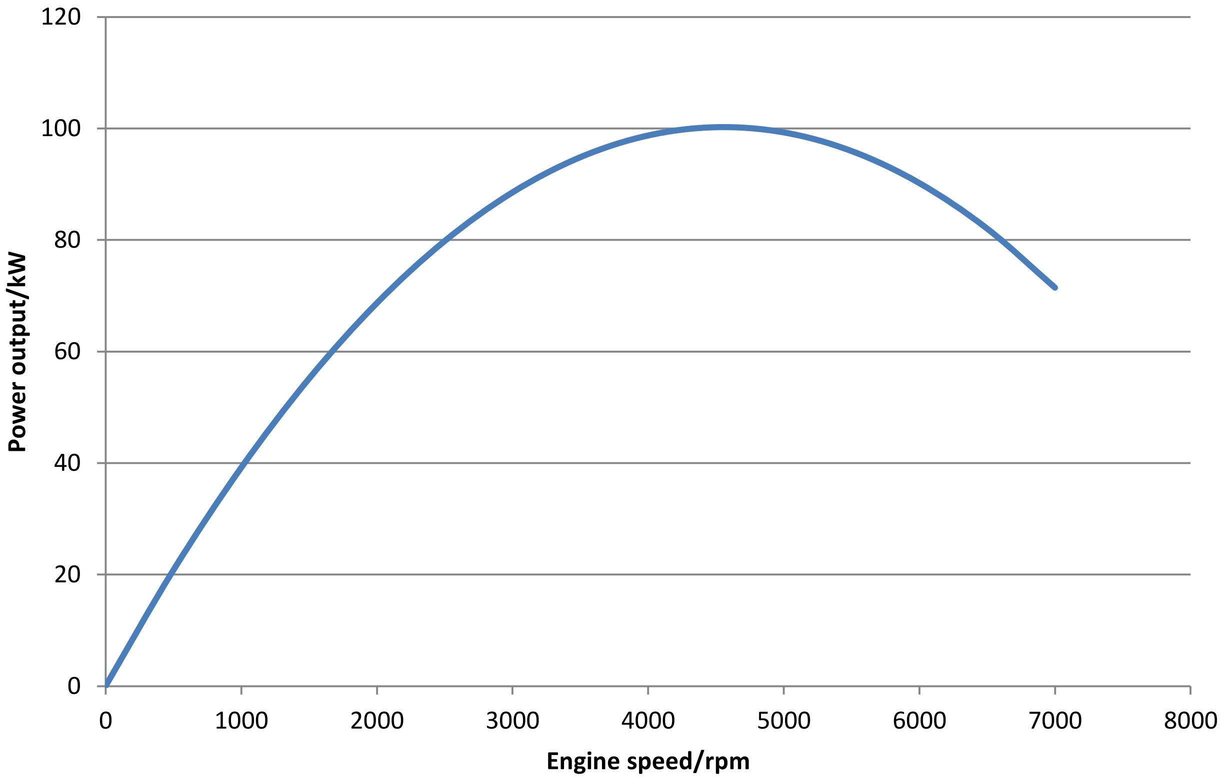


Toyota 2ZR-FE-1.8L petrol



Toyota 2GD-FTV-2.4L turbo diesel

Mathematical model of engine power supply



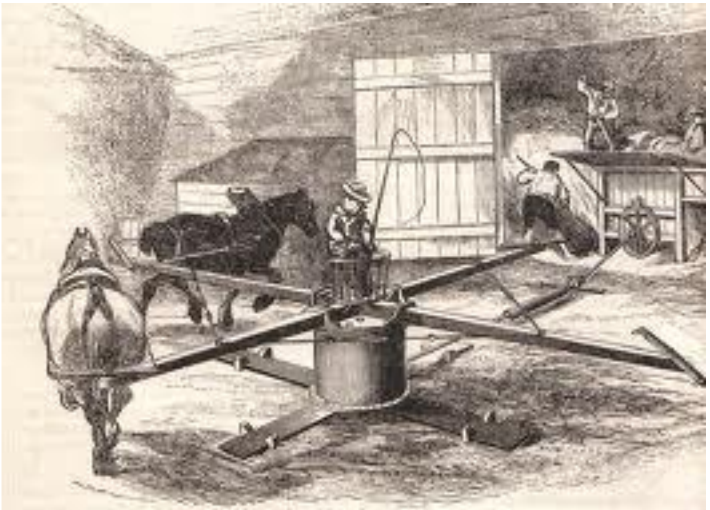
$$P_{output} = \alpha\omega - \beta\omega^2$$

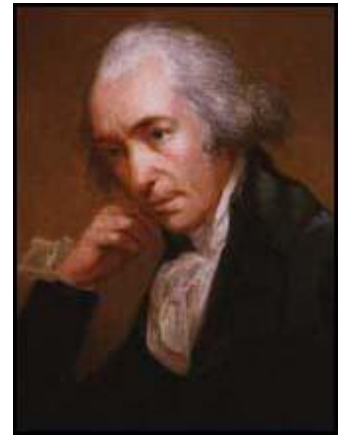
α = 420W.s/rad

β = 0.44W.s^2/rad^2

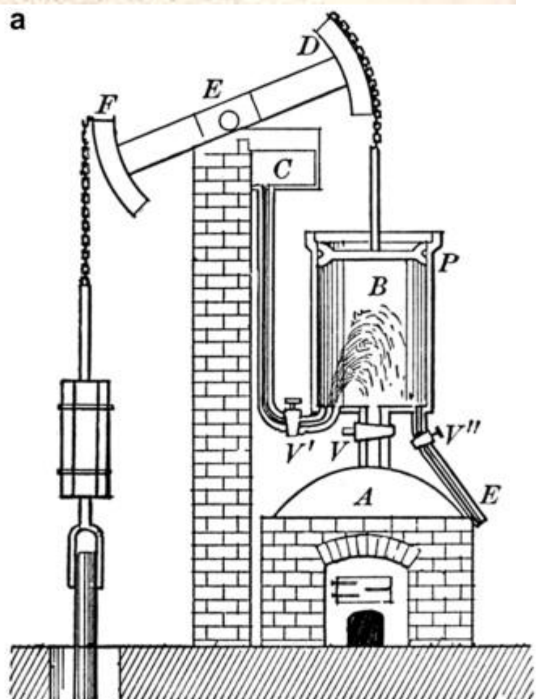
Max power ≈ 100kW ≈ 134hp

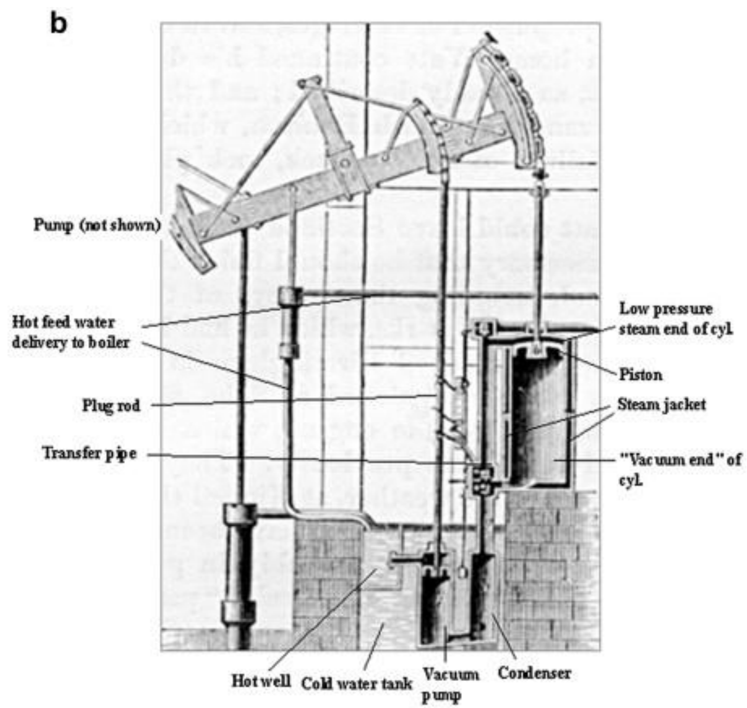
1 horsepower = 746 W





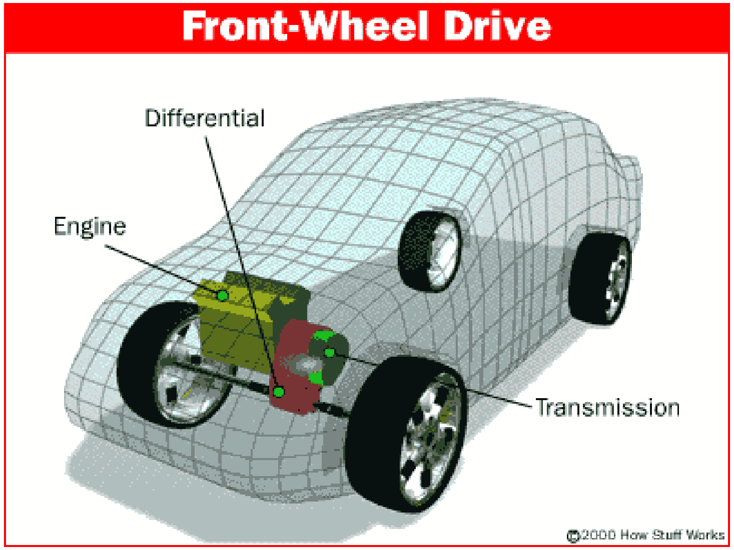
James Watt 1736-1819

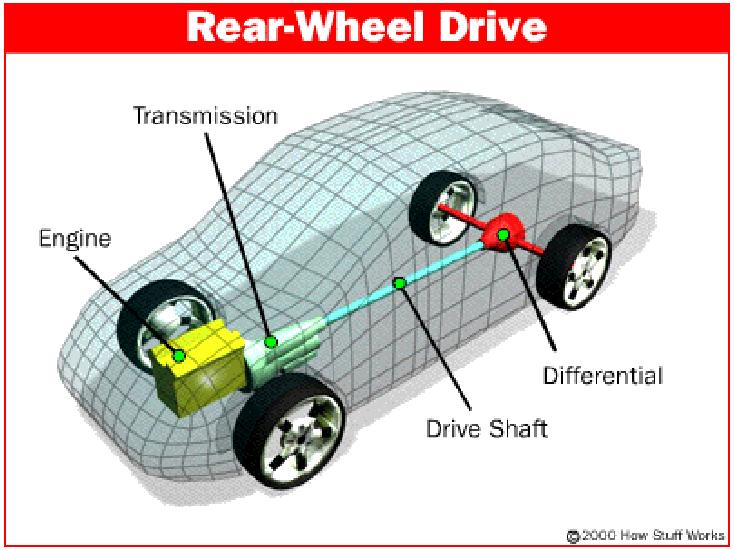


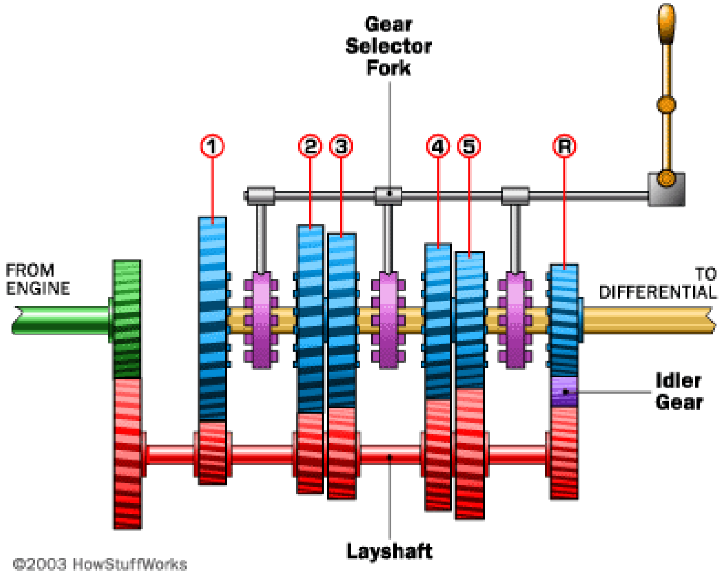


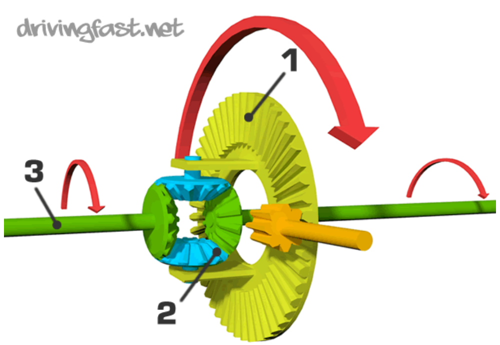
Horsepower is **not** an SI unit

Transmission gear ratios









$$\omega_w = \frac{\omega}{r_{fd} r_{gb}}$$

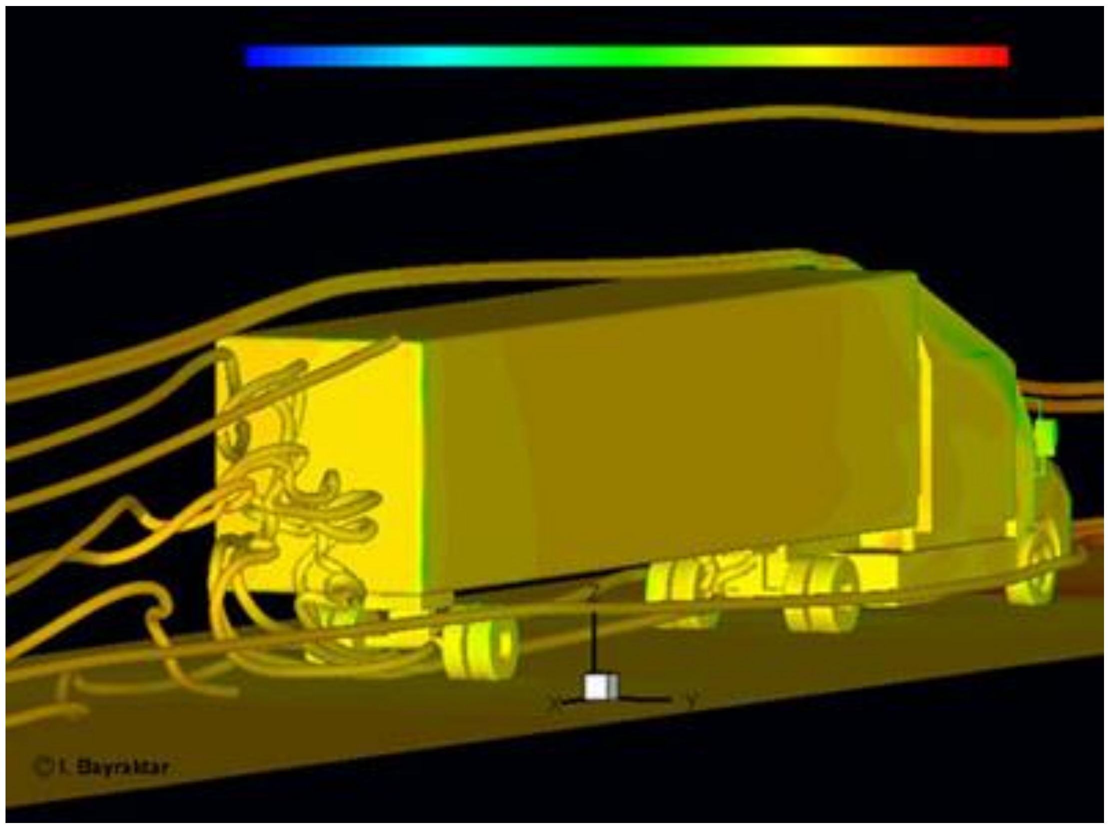
$$V = \frac{R\omega}{r_{fd} r_{gb}}$$

Gearbox: in first gear, r_{gb}~ 3 to 3.5. In top gear (4^{th} or 5^{th}) ~ 0.7 to 1
Final drive 3.5 < r_{fd} < 4.2

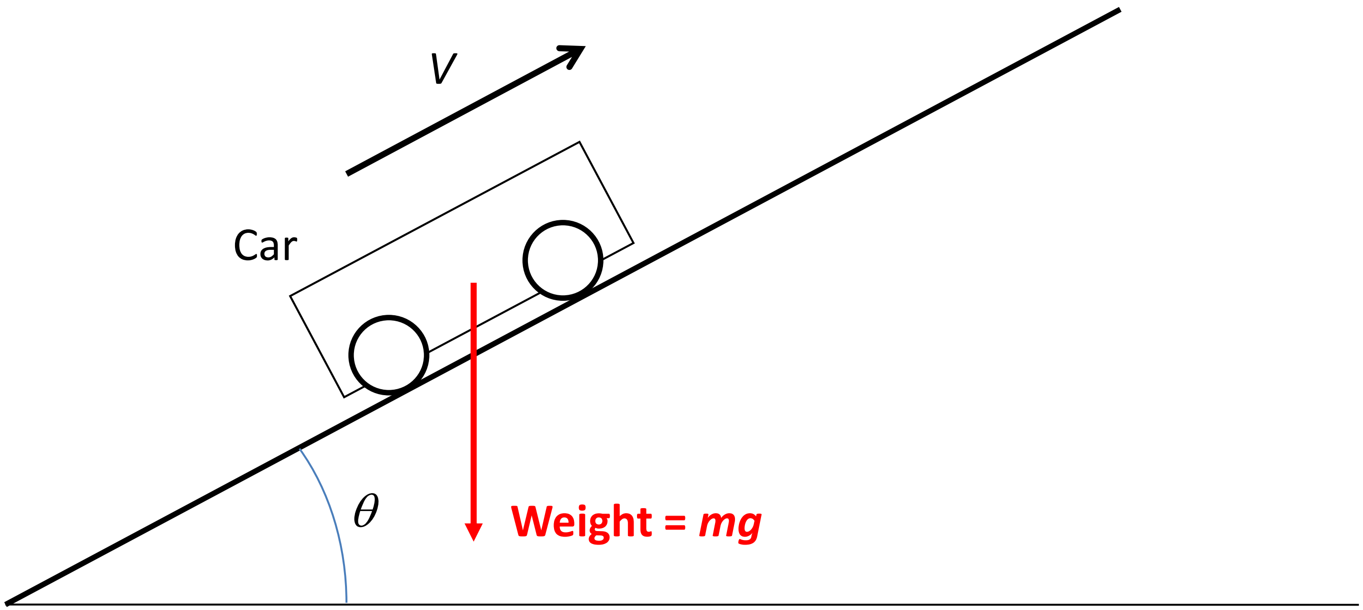
Power losses: Aerodynamic drag

$$\text{resisting force due to drag } F_D = C_D A \frac{1}{2} \rho V^2$$

$$\text{power required to overcome drag } P_D = C_D A \frac{1}{2} \rho V^3$$

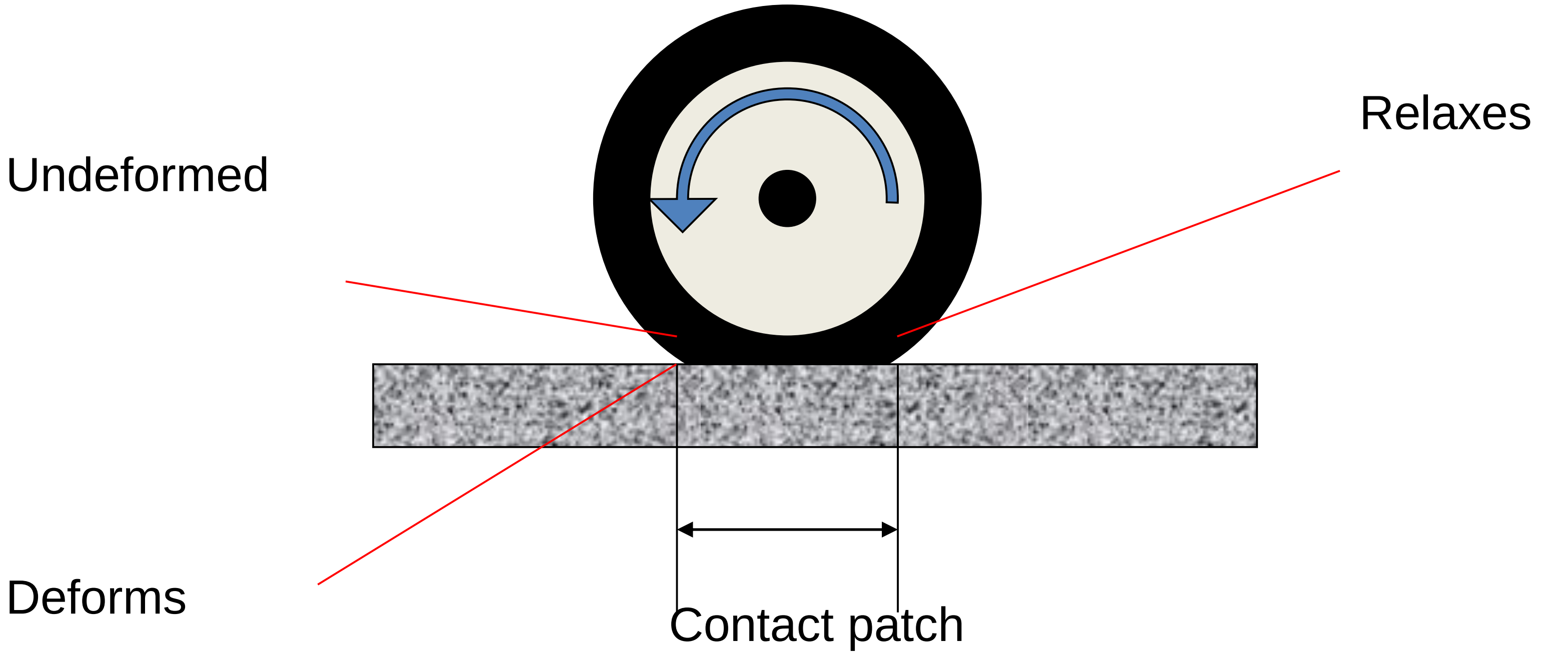


Power required to climb a gradient

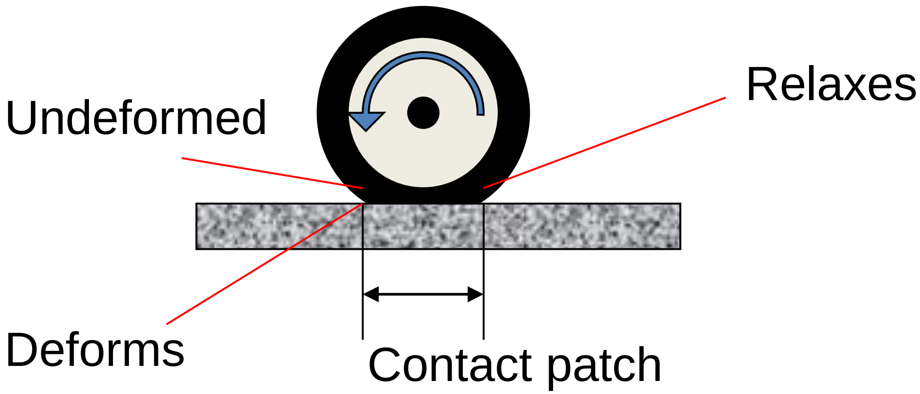


Power to climb gradient = $mg\sin(\theta)\,V$

Rolling resistance



Power losses:
Rolling resistance



$$F_{rr} = C_{rr} mg \cos(\theta)$$

$$P_{rr} = C_{rr} mg \cos(\theta) V$$

C_{rr}	
0.0002-0.001	Steel railway wheel on steel track
0.001	19mm smooth bicycle tyre on wooden track
0.002	As above on smooth concrete
0.004	As above on asphalt
0.008	As above on rough sealed road
0.0025	Michelin eco-marathon tyres
0.006-0.01	Low resistance car and truck tyres on smooth road
0.010-0.015	Ordinary car tyres on concrete
0.030-0.035	Ordinary car tyres on asphalt
0.05-0.065	Ordinary car tyres on grass and mud
0.3	Ordinary car tyres on sand

Acceleration power

Force required to accelerate at rate a

$$F = ma$$

Power required when travelling at speed V to accelerate at rate a

$$P = maV$$

Total power demand = Engine power

$$C_d A \frac{1}{2} \rho V^3 + mg\sin(\theta)V + C_{rr} mg\cos(\theta)V + maV = \left(\frac{\alpha V}{K} - \frac{\beta V^2}{K^2} \right)$$

<u>Rearrange into a form which can be solved by Newton's method.</u>

<u>Write a MATLAB script to solve for *V* with the following parameters:</u>

Mass 1500 kg
Drag coefficient C_d = 0.30
Frontal area 2.0 m^2
C_{rr} = 0.010
Wheel radius 0.205m
Engine parameters: α = 420 W.s/rad, β = 0.440 $W.s^2/rad^2$
Final drive ratio of 3.50
Top gear with a gearbox ratio r_{gb} = 0.80
g = 9.81 m/s^2 and air density is 1.2 kg/m^3
Throttle is wide open at all times
Assume there is no wind

Total power demand = Engine power

$$C_d A \frac{1}{2} \rho V^3 + mg\sin(\theta)V + C_{rr} mg\cos(\theta)V + maV = \left(\frac{\alpha V}{K} - \frac{\beta V^2}{K^2} \right)$$

- Case 1: No gradient, no acceleration
- Case 2: A gradient of 10°, no acceleration
- Case 3: A gradient of -5°, no acceleration
- Case 4: No gradient, an acceleration of 1 m/s^2

Follow the instructions in the notes

Reports must have a title, date and the names and email addresses of all authors.