

SECURITY (COMP0141): USES OF HASH FUNCTIONS



HASH FUNCTIONS

Two main security properties:

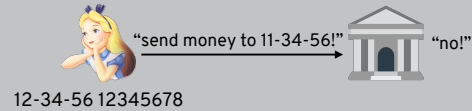
- **Pre-image resistance:** given $H(x)$ it's hard to find x
- **Collision resistance:** it's hard to find x and y so that $x \neq y$ but $H(x) = H(y)$

Applications:

- [File checksum](#)
- MACs
- Digital signatures
- Commitments
- Blockchains
- Virus scanning (next week)
- Password storage (Week 7)
- ...and many more!

CHECKSUM

Used to detect errors introduced by humans (replaced digits, transposition, phonetic, etc.)



Also useful for errors due to corruption (inevitable for big files)



Checksums are mostly for accidental errors so just need uniformity/ uniqueness

CHECKSUM

Example: validating an IBAN (International Bank Account Number)

GB82 WEST 123456 12345678
WEST12345612345678GB82
3214282912345612345678161182
3214282912345612345678161182 mod 97?

= 0 so we know this isn't a valid IBAN (need = 1 mod 97)

Feel free to check out some other simple examples of checksums at:
https://en.wikipedia.org/wiki/Check_digit

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HMAC

Uses a hash function to achieve a MAC

$$\text{HMAC}(K, m) = H((K \oplus \text{opad}) \parallel H((K \oplus \text{ipad}) \parallel m))$$

- so first we compute $h_{\text{inner}} = H((K \oplus \text{ipad}) \parallel m)$
- then we compute $H((K \oplus \text{opad}) \parallel h_{\text{inner}})$

opad and ipad are fixed strings

HMAC: Keyed-Hashing for Message Authentication. M. Bellare, R. Canetti, H. Krawczyk. RFC 2104.

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HMAC is a very famous standardised MAC

DANGEROUS HASH-BASED MAC

Why not do something simpler like $\text{MAC}(K,m) = H(K \parallel m)$?

This is subject to something called a **length extension attack**

(You should never design your own cryptography!)

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We won't cover length extension attacks because we're not covering how hash functions are built, but if you're interested you can find out more here: https://en.wikipedia.org/wiki/Length_extension_attack and https://en.wikipedia.org/wiki/Merkle%E2%80%93Damg%E2%80%93Schwartz%E2%80%93Zuccherato_construction

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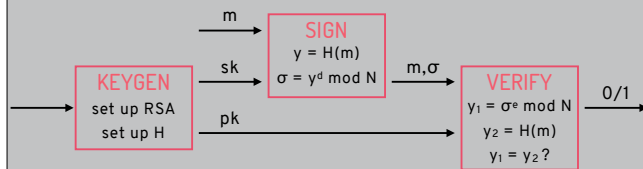
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FULL DOMAIN HASH (FDH)



Correctness: Relies on having range of H be $\mathbb{Z}/N\mathbb{Z}$

Unforgeability: we can prove EUF-CMA security assuming hash function behaves like a **random oracle**

Exemplifies the **"hash-and-sign" paradigm**

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Can also construct signatures from hash functions and RSA as long the hash function has the right range

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COMMITMENTS

Let's play a game: **will we be out of lockdown at the end of term?**

Need people to **commit** to their guesses in a way such that...

- **Hiding**: no one else can see who guessed what
- **Binding**: no one can pretend they guessed right if they didn't

Everyone can compute $H(\text{guess}||r)$ for some random r and store it on a public bulletin board

At the end, someone can reveal (guess, r) and everyone can check that $H(\text{guess}||r) = h_i$ for some h_i stored on the board

This forms a type of (binding) **prediction market**

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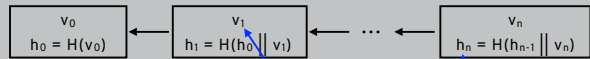
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BLOCKCHAINS / HASH CHAINS

More generally, hashes can commit to entire series of values to form a **tamper-evident data structure**



can't replace old values without breaking pre-image resistance
(need to find different h_{i-1}^* such that $H(h_{i-1}^* || v_i) = h_i$)

In a blockchain, values represent blocks (collections of transactions + “proof-of-work”), so if we wanted to erase an old transaction we would have to rewrite the entire history and redo all the “work” since then

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There are also more efficient tamper-evident data structures than hash chains, such as Merkle trees (https://en.wikipedia.org/wiki/Merkle_tree)

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