SECURITY (COMP0141):

MATH MEETS CRYPTOGRAPHY

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MATH MEETS CRYPTOGRAPHY

Two interesting settings to consider from a cryptographic perspective:

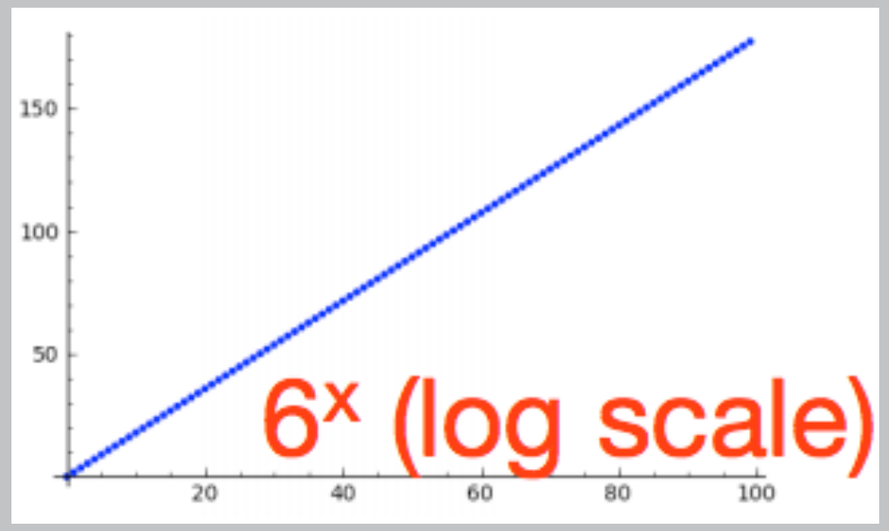
The finite field F_p for a very large prime p (1024 bits or more)

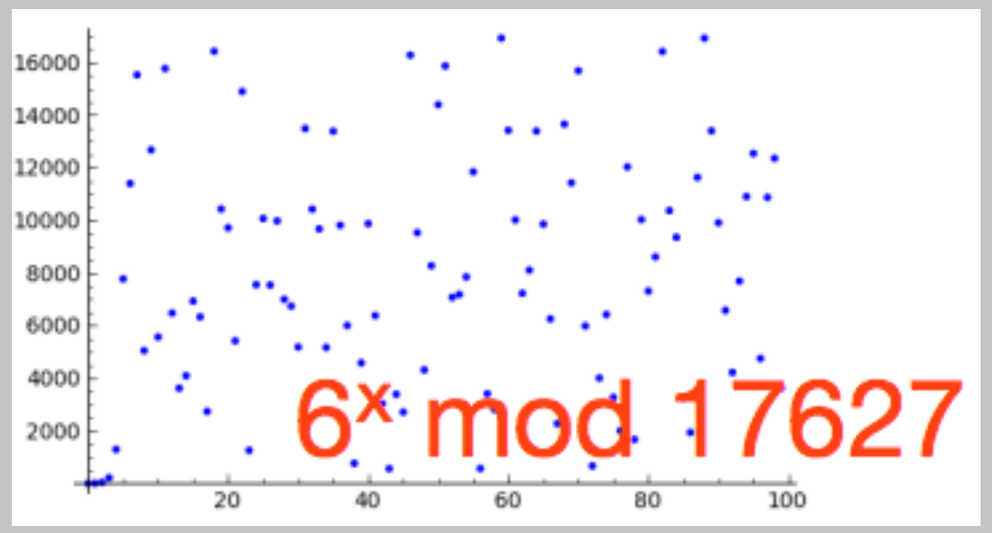
The ring (Z/NZ)* for N = pq for very large primes p,q (1024 bits or more)

DISCRETE LOGARITHM

Discrete logarithm problem: for a fixed prime p, given g and y, find x such that $g^x = y \bmod p$

Example: $6^x = 10000 \bmod 17627$





This problem seems to be very difficult to solve (like for modern computers and large enough p, until the heat death of the sun)

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THE RING (Z/NZ)*

Euler totient function φ is $\varphi(N) = \left|\{x \text{ in } \{0,\ldots,N-1\} \mid \gcd(x,N) = 1\}\right|$

Euler's theorem: $x^{\varphi(N)} = 1 \bmod N$ for $x \in (Z/NZ)^*$

Now let N = pq for p and q two different odd primes

RSA

RSA problem: given an integer N = pq, find p and q

Example: the RSA-1024 challenge is to find p and q for N =
135066410865995223349603216278805969938881475605667027524485143851526510604859533833940287150571909441798207282164471551373680419703964191743046496589274256239341020864383202110372958725762358509643110564073501508187510676594629205563685529475213500852879413773285339061097505443349998111500569772368909275636

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$$\begin{aligned}\varphi(N) = \varphi(pq) &= pq - \left|x : \gcd(x,pq) \neq 1\}\right| \\ &= pq - \left|\{x : p \mid x\}\right| - \left|\{x : q \mid x\}\right| + \left|\{0\}\right| \\ &= pq - q - p + 1 = (p-1)(q-1)\end{aligned}$$

This means that $(Z/NZ)^*$ has $\varphi(N) = (p-1)(q-1)$ elements

ONE-WAY FUNCTIONS

More generally, discrete log and RSA are examples of something called a **one-way function**

This is a function f( ) such that

-(1) it is easy to compute f(x) for all x, but

-(2) it is assumed to be very difficult to compute x given f(x), or in fact to compute any y such that f(y) = f(x)

Discrete log: $f(x) = g^x \bmod p$

RSA: f(p,q) = pq