P & C: $P_r^{n+1} = P_r^n + rP_{r-1}^n$.
Circular permutation: $Q_r^n = \frac{P_r^n}{r}$.
$C_r^{n+1} = C_{r-1}^n + C_r^n$.
$H_r^n = C_r^{r+n-1}$.

Binomial & Multinomial:
$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$.
$\binom{n}{r} = \frac{n-r+1}{r}\binom{n}{r-1}$.
$\binom{n}{m}\binom{m}{r} = \binom{n}{r}\binom{n-r}{m-r}$.
Multinomial coefficient:

$$\binom{n}{n_1, n_2, \cdots, n_m} = \frac{n!}{\prod_{i=1}^m n_i!}.$$

Boole's Inequality: $P\left(\bigcup E_i\right) \leq \sum P(E_i)$.

Conditional Probability

- $P(E \mid F) = \frac{P(EF)}{P(F)}$.
- $P(E_1 E_2 \cdots E_n) = P(E_1)P(E_2 \mid E_1)P(E_3 \mid E_1 E_2) \cdots P(E_n \mid E_1 E_2 \cdots E_{n-1})$.
- $P(E) = P(E \mid F)P(F) + P(E \mid F^c)P(F^c)$.
- $P(F_j \mid E) = \frac{P(E|F_j)P(F_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum P(E|F_i)P(F_i)}$.
- E and F are independent iff E and F^c are independent.

DRV & CRV:

- $\mathrm{Var}(X) = E[X^2] - (E[X])^2 = E[(X - \mu_X)^2]$.
- Binomial: $p(x) = C_x^n p^x (1-p)^{n-x}, \mu = np, \sigma^2 = np(1-p)$.
- Poisson: $p(x) = \mathrm{e}^{-\lambda}\frac{\lambda^x}{x!}, \mu = \sigma^2 = \lambda, N(t) \sim \mathrm{Po}(\lambda t)$.
- Geo (no. of trials up to the 1st success): $p(x) = (1-p)^{x-1}p, \mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$.
- Neg. B (up to the r-th success): $p(x) = C_{r-1}^{x-1}p^r(1-p)^{x-r}, \mu = \frac{r}{p}, \sigma^2 = \frac{r(1-p)}{p^2}$.
- Hypergeo (choose n from N with m type 1): $p(x) = \frac{C_x^m C_{n-x}^{N-m}}{C_n^N}, \mu = np, \sigma^2 = np(1-p)\left(1 - \frac{n-1}{N-1}\right)$.
- Uniform: $f(x) = \frac{1}{b-a}, F(x) = \frac{x-a}{b-a}, \mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$.
- Normal: $\phi(z) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)$.
- Exponential: $f(x) = \lambda\mathrm{e}^{-\lambda x}, F(x) = 1 - \mathrm{e}^{-\lambda x}, \mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$.
- Hazard rate: $\lambda(x) = \frac{f(x)}{1-F(x)}$.
- $\Gamma(\alpha) = \int_0^\infty \lambda\mathrm{e}^{-\lambda t}(\lambda t)^{\alpha-1}\,\mathrm{d}t = \int_0^\infty \lambda\mathrm{e}^{-\lambda t}x^{\alpha-1}\,\mathrm{d}x$.
- $\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \Gamma(n) = (n-1)!$.
- Gamma (time till n-th occurrence): $f(t) = \frac{\lambda\mathrm{e}^{-\lambda t}(\lambda t)^{\alpha-1}}{\Gamma(\alpha)}, \mu = \frac{\alpha}{\lambda}, \sigma^2 = \frac{\alpha}{\lambda^2}$.
- $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 x^{a-1}(1-x)^{b-1}\,\mathrm{d}x, \frac{B(a+1,b)}{B(a,b)} = \frac{a}{a+b}$.
- Beta (probability of success if a successes and b failures): $f(x) = \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}, \mu = \frac{a}{a+b}, \sigma^2 = \frac{ab}{(a+b)^2(a+b+1)}$.

Joint Distribution:

- $p_X(x) = \sum_y p(x,y), f_X(x) = \int_{-\infty}^\infty f(x,y)\,\mathrm{d}y$.
- $f(x,y) = \frac{\partial^2}{\partial a \partial b}F(x,y)$.
- Jacobian: $J(x,y) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$.
- $f_{U,V}(u,v) = f_{X,Y}(x,y)\left|J(x,y)\right|^{-1}$.
- X, Y independent iff $F(x,y) = F_X(x)F_Y(y)$ or $f(x,y) = f_X(x)f_Y(y)$ or $p(x,y) = p_X(x)p_Y(y)$.
- X, Y independent discrete: $p_{X+Y}(n) = \sum_{i+j=n} p_X(i)p_Y(j)$.
- X, Y independent continuous, $f_{X+Y}(n) = \int_{-\infty}^\infty\int_{-\infty}^{n-y} f_X(x)f_Y(y)\,\mathrm{d}x\,\mathrm{d}y = \int_{-\infty}^\infty F_X(n-y)f_Y(y)\,\mathrm{d}y = \int_{-\infty}^\infty F_X(n-y)\,\mathrm{d}F_Y(y)$.
- Sum of independent uniform is triangular:

$$f(N) = \begin{cases} \frac{n-2a}{(b-a)^2} & 2a < n \leq a+b \\ \frac{2b-n}{(b-a)^2} & a+b < n < 2b \end{cases}.$$

- Sum of Gamma(α, λ) and Gamma(β, λ) is Gamma$(\alpha+\beta, \lambda)$.

Conditional Distribution:

- $p_{X|Y}(x \mid y) = \frac{p(x,y)}{p_Y(y)}, f_{X|Y}(x \mid y) = \frac{f(x,y)}{f_Y(y)}, f_{X|X\in A} = \frac{f(x)}{\int_A f(x)\,\mathrm{d}x}$.
- X continuous, N discrete: $f_{X|N}(x \mid n) = \frac{P(N=n|X=x)}{P(N=n)}f(x)$.
- $E[X] = \sum\big(E[X \mid E]P(E)\big)$.

Expectation:

- Deviation: $X_i - \overline{X}$.
- Sample variance: $\sum \frac{(X_i - \overline{X})^2}{n-1}$.
- MGF: $M_X(t) = E[\mathrm{e}^{tX}], E[X^n] = M_X^{(n)}(0)$.
- $M_{X,Y}(s,t) = E[\mathrm{e}^{sX+tY}], M_X(s) = M_{X,Y}(s,0), M_Y(t) = M_{X,Y}(0,t)$. X, Y independent iff $M_{X,Y}(s,t) = M_X(s)M_Y(t)$.
- X is sum of independent r.v.: $M_X(t) = \prod M_{X_i}(t)$.
- X and Y are identically distributed iff $M_X = M_Y$ near 0.
- $\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$.
  - $\mathrm{Cov}(X,X) = \mathrm{Var}(X)$.
  - $\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X)$.
  - $\mathrm{Cov}(aX,Y) = a\mathrm{Cov}(X,Y)$.
  - $\mathrm{Cov}(X_1 + X_2, Y) = \mathrm{Cov}(X_1,Y) + \mathrm{Cov}(X_2,Y)$.
- $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y)$.
- $\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}} = \mathrm{Cov}\left(\frac{X}{\sigma_X}, \frac{Y}{\sigma_Y}\right)$.
- $E\big[E[X \mid Y]\big] = E[X]$.
- $\mathrm{Var}(X) = E[\mathrm{Var}(X \mid Y)] - \mathrm{Var}(E[X \mid Y])$.
- Given X, $E[Y \mid X]$ is the best predictor for Y.
- Given X, the best linear predictor for Y:

$$g(X) = E\left[\left(Y - \mu_Y - \rho(X,Y)\frac{\sigma_Y}{\sigma_X}(X - \mu_X)\right)\right].$$

Limit Theorems:

- Markov: $P(X \geq a) \leq \frac{\mu}{a}$.
- Chebyshev: $P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a}$.
- $\sigma^2 = 0 \implies P(X = \mu) = 1$.
- Weak: independent identical $X \implies \lim_{n\to\infty} P\left(\left|\frac{\overline{X}}{n}\right| \geq \epsilon\right) = 0$.
- Strong: $P\left(\lim_{n\to\infty} \frac{\overline{X}}{n} = \mu\right) = 1$.
- CLT: $\frac{\sum X_i - n\mu}{\sigma\sqrt{n}}$ tends to standard normal.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997