P & C:
$P_r^{n+1} = P_r^n + rP_{r-1}^n$.

Circular permutation: $Q_r^n = \frac{P_r^n}{r}$.

$C_r^{n+1} = C_{r-1}^n + C_r^n$.

$H_r^n = C_r^{r+n-1}$.

Arrange r distinct objects around n identical circles such that no circle is empty: $s(r,n) = s(r-1,n-1) + (r-1)s(r-1,n)$.

$s(r,r-1) = C_2^r$.

Binomial & Multinomial:
$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$.
$\binom{n}{r} = \frac{n-r+1}{r}\binom{n}{r-1}$.
$\binom{n}{m}\binom{m}{r} = \binom{n}{r}\binom{n-r}{m-r}$.

Vandermonde's Identity:

$$\sum_{i=0}^{r}\left[\binom{m}{i}\binom{n}{r-i}\right] = \binom{m+n}{r}.$$

Chu Shih-Chieh Identity:

$$\sum_{i=0}^{n-r}\binom{r+i}{r} = \binom{n+1}{r+1}$$
$$\sum_{i=0}^{k}\binom{r+i}{i} = \binom{r+k+1}{k}.$$

Multinomial coefficient:

$$\binom{n}{n_1, n_2, \cdots, n_m} = \frac{n!}{\prod_{i=1}^m n_i!}.$$

Pigeonhole Principle: If at least $kn+1$ objects are distributed into n distinct sets, then there exists a set with at least $k+1$ objects.

Generalised PP: If at least $\sum_{i=1}^n k_i + (n-1)$ distinct objects are distributed into n distinct sets, then there exists at least one set (i-th) with at least k_i objects.

Ramsey Numbers: $R(p,q) \leq R(p-1,q) + R(p,q-1)$. (Bound lowered by 1 if both on RHS are even.)

$R(2,q) = q$, $R(1,q) = 1$.

Distribution Problems:
Distinct into distinct:

- Each box at most 1: P_r^n.
- Each box any number of objects: n^r.
- Each box any number of objects with internal ordering: $\frac{(n-1+r)!}{(n-1)!}$.

Identical into distinct:

- Each box any number of objects: $H_r^n = C_r^{n+r-1}$.
- No box empty: $H_{r-n}^n = C_{r-n}^{r-1}$.

Distinct into identical:

- No box empty: $S(r,n) = S(r-1,n-1) + nS(r-1,n)$

Number of partitions of A with $|A| = n$: $\sum_{i=1}^n S(n,i)$.

Number of surjective mapping from $[1,r]\cap\mathbb{N}$ to $[1,n]\cap\mathbb{N}$: $F(r,n) = \sum_{k=0}^n (-1)^k C_k^n (n-k)^r$.

$S(r,n) = \frac{1}{n!}F(r,n)$.

$D(n,r,k) = \frac{C_k^r}{(n-r)!}\sum_{i=0}^{r-k}(-1)^i C_i^{r-k}(n-k-i)!$.

$D_n = D(n,n,0) = n!\sum_{i=0}^n \frac{(-1)^i}{i!}$.

GPIE:

$$E(m) = \sum_{k=m}^{q}(-1)^{k-m}C_m^k\omega(k).$$