Contents

1 Optimisation Models

Definition 1.0.1 ▸ Optimal Solution

Consider an optimisation problem with respect to $f(\boldsymbol{x})$ subject to constraint $\boldsymbol{x} \in S \subseteq \boldsymbol{R}^n$. A feasible solution $\boldsymbol{x}^*$ is called an **optimal solution** if $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in S$.

1.1 Unconstrained Non-linear Programs

Definition 1.1.1 ▸ Open Set

Let $S \subseteq \boldsymbol{R}^n$ be a set. S is called **open** if for all $\boldsymbol{x} \in S$ there exists $\epsilon > 0$ such that the ball

$$B(\boldsymbol{x}, \epsilon) = \{\, \boldsymbol{y} \in \boldsymbol{R}^n : \ \|\boldsymbol{y} - \boldsymbol{x}\| < \epsilon \,\}$$

is a subset of S.

Definition 1.1.2 ▸ Closed Set

Let $S \subseteq \boldsymbol{R}^n$ be a non-empty set. S is said to be **closed** if for all convergent sequences $\{\boldsymbol{x}_i\}_{i=1}^{\infty}$ with $\boldsymbol{x}_i \in S$ for $i = 1, 2, \cdots$, the limit $\lim_{i\to\infty} \boldsymbol{x}_i \in S$.

The empty set is both open and closed.

Remark. A set is open if and only if its complement is closed.

Theorem 1.1.3 ▸ Intersection of Closed Sets

If C_1 and C_2 are both closed, then $C_1 \cap C_2$ is closed.

Theorem 1.1.4