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1 Linear Programming

1.1 Linear Programming

Recall that in general, an optimisation problem can be formulated as

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^n} & f(\boldsymbol{x}) \\ \text{s.t. } & \boldsymbol{x} \in P, \end{aligned}$$

where $P \subseteq \mathbb{R}^n$ is called the *feasible set* (or *feasible region*).

Definition 1.1.1 ▸ Linear Programming Problem

A **linear programming** (LP) problem is an optimisation problem where the objective function f is linear and the feasible set P is a polyhedron.

We can therefore formulate a linear programming problem as

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^n} & \boldsymbol{c}^{\mathrm{T}} \boldsymbol{x} \\ \text{s.t. } & \boldsymbol{a}_i^{\mathrm{T}} \boldsymbol{x} \leq b_i \quad \text{for } i = 1, 2, \cdots, p \\ & \boldsymbol{a}_j^{\mathrm{T}} \boldsymbol{x} = b_j \quad \text{for } i = 1, 2, \cdots, m, \end{aligned}$$

where $\boldsymbol{c} \in \mathbb{R}^n$ is called the *cost* or *profit* vector, $\boldsymbol{a}_i^{\mathrm{T}} \boldsymbol{x}$ and $\boldsymbol{a}_j^{\mathrm{T}} \boldsymbol{x}$ are called the *constraints* and $\boldsymbol{x}$ is known as *decision variables*.