Heat Transfer - Zak Olech - 9/26/2019

General Information

Conversion Factors

Process

0 - Known 1 - Find 2 - Schematic 3 - Assumptions 3 - Analysis

Metric Prefix

SI Prefixes

| Multiplication Factor | Prefix [†] | Symbol |
|---|---------------------|--------|
| $1\ 000\ 000\ 000\ 000 = 10^{12}$ | tera | Т |
| $1\ 000\ 000\ 000 = 10^9$ | giga | G |
| $1\ 000\ 000 = 10^6$ | mega | M |
| $1\ 000 = 10^3$ | kilo | k |
| $100 = 10^2$ | hecto‡ | h |
| $10 = 10^{1}$ | deka‡ | da |
| $0.1 = 10^{-1}$ | deci‡ | d |
| $0.01 = 10^{-2}$ | centi‡ | С |
| $0.001 = 10^{-3}$ | milli | m |
| $0.000\ 001 = 10^{-6}$ | micro | μ |
| $0.000\ 000\ 001 = 10^{-9}$ | nano | n |
| $0.000\ 000\ 000\ 001 = 10^{-12}$ | pico | р |
| $0.000\ 000\ 000\ 000\ 001 = 10^{-15}$ | femto | p f |
| $0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$ | atto | a |

Geometric Relationships

Circle

(1)

(2)

| Variab | les (Alphbetical By Variable) |
|--------------------|---|
| | |
| | |
| | |
| | |
| | |
| | |
| A | area, m ² |
| A_b | area of prime (unfinned) surface, m ² |
| A_c | cross-sectional area, m ² |
| A_p | fin profile area, m ² |
| A_r | nozzle area ratio |
| <i>a</i> | acceleration, m/s ² ; speed of sound, m/s |
| Bi | Biot number |
| Во С | Bond number |
| C | molar concentration, kmol/m³; heat capacity rate, W/K |
| C | drag coefficient |
| C_D | friction coefficient |
| $C_f \\ C_t$ | thermal capacitance, J/K |
| C_t | Confinement number |
| c | specific heat, J/kg · K; speed of light, m/s |
| c_p | specific heat at constant pressure, J/kg·K |
| c_p c_v | specific heat at constant volume, J/kg·K |
| D | diameter, m |
| $D_{ m AB}$ | binary mass diffusivity, m ² /s |
| D_b | bubble diameter, m |
| D_h | hydraulic diameter, m |
| $d^{''}$ | diameter of gas molecule, nm |
| E | thermal plus mechanical energy, J; electric |
| | potential, V; emissive power, W/m ² |
| E^{tot} | total energy, J |
| Ec | Eckert number |
| \dot{E}_{g} | rate of energy generation, W |
| $\dot{E}_{ m in}$ | rate of energy transfer into a control volume, W |
| $\dot{E}_{ m out}$ | rate of energy transfer out of control volume, W |
| $\dot{E}_{ m st}$ | rate of increase of energy stored within a control |
| | volume, W |
| e | thermal internal energy per unit mass, J/kg; |
| | surface roughness, m |
| F | force, N; fraction of blackbody radiation in a |
| | wavelength band; view factor |

| E. | E-mi-a-mi-a- |
|-------------------------|--|
| Fo | Fourier number |
| Fr | Froude number |
| f | friction factor; similarity variable |
| G | irradiation, W/m ² ; mass velocity, kg/s·m ² |
| Gr | Grashof number |
| Gz | Graetz number |
| g | gravitational acceleration, m/s ² |
| H | nozzle height, m; Henry's constant, bars |
| h | convection heat transfer coefficient, $W/m^2 \cdot K$; |
| | Planck's constant, J·s |
| h_{fg} | latent heat of vaporization, J/kg |
| $h_{fg}^{\prime\prime}$ | modified heat of vaporization, J/kg |
| h_{sf} | latent heat of fusion, J/kg |
| h_m | convection mass transfer coefficient, m/s |
| $h_{ m rad}^m$ | radiation heat transfer coefficient, W/m ² ·K |
| I | electric current, A; radiation intensity, W/m ² ·sr |
| i | electric current density, A/m ² ; enthalpy per unit |
| | mass, J/kg |
| J | radiosity, W/m ² |
| Ja | Jakob number |
| J_i^* | diffusive molar flux of species i relative to the |
| o _i | mixture molar average velocity, kmol/s · m ² |
| \dot{J}_i | diffusive mass flux of species <i>i</i> relative to the |
| Ji | mixture mass average velocity, kg/s·m ² |
| ; | Colburn <i>j</i> factor for heat transfer |
| j_H | Colburn <i>j</i> factor for mass transfer |
| j _m k | thermal conductivity, W/m·K |
| | • • |
| k_B | Boltzmann's constant, J/K |
| k_0 | zero-order, homogeneous reaction rate |
| | constant, kmol/s·m³ |
| k_1 | first-order, homogeneous reaction rate |
| | constant, s ⁻¹ |
| k_1'' | first-order, surface reaction rate constant, m/s |
| L | length, m |
| Le | Lewis number |
| | |

| M | mass, kg |
|-------------------|--|
| \dot{M}_i | rate of transfer of mass for species, i, kg/s |
| $M_{i,g}$ | rate of increase of mass of species i due to |
| | chemical reactions, kg/s |
| $\dot{M}_{ m in}$ | rate at which mass enters a control volume, kg/s |
| $M_{ m out}$ | rate at which mass leaves a control |
| | volume, kg/s |
| $\dot{M}_{ m st}$ | rate of increase of mass stored within a |
| | control volume, kg/s |
| \mathcal{M}_i | molecular weight of species i, kg/kmol |
| Ма | Mach number |
| m | mass, kg |
| \dot{m} | mass flow rate, kg/s |
| m_i | mass fraction of species i , ρ_i/ρ |
| N | integer number |
| N_L, N_T | number of tubes in longitudinal and |
| | transverse directions |
| Nu | Nusselt number |
| NTU | number of transfer units |
| N_i | molar transfer rate of species i relative to |
| | fixed coordinates, kmol/s |
| N_i'' | molar flux of species i relative to fixed |
| | coordinates, kmol/s · m ² |
| \dot{N}_i | molar rate of increase of species i per unit |
| | volume due to chemical reactions, |
| | kmol/s⋅m ³ |
| N_i'' | surface reaction rate of species i, |
| | kmol/s ⋅ m ² |
| \mathcal{N} | Avogadro's number |
| n_i'' | mass flux of species i relative to fixed |
| | coordinates, kg/s·m ² |
| \dot{n}_i | mass rate of increase of species i per unit |
| | volume due to chemical reactions, |
| | $kg/s \cdot m^3$ |
| | |

| P | power, W; perimeter, m |
|-------------------------|--|
| P_L, P_T | dimensionless longitudinal and transverse |
| 2. | pitch of a tube bank |
| Pe | Peclet number |
| Pr | Prandtl number |
| p | pressure, N/m ² |
| Q | energy transfer, J |
| | heat transfer rate, W |
| $\stackrel{q}{\dot{q}}$ | rate of energy generation per unit |
| | volume, W/m ³ |
| q' | heat transfer rate per unit length, W/m |
| q'' | heat flux, W/m ² |
| q^* | dimensionless conduction heat rate |
| R | cylinder radius, m; gas constant, J/kg·K |
| ${\mathcal R}$ | universal gas constant, J/kmol·K |
| Ra | Rayleigh number |
| Re | Reynolds number |
| R_e | electric resistance, Ω |
| R_f | fouling factor, m ² · K/W |
| R_m | mass transfer resistance, s/m ³ |
| $R_{m,n}$ | residual for the m , n nodal point |
| R_t | thermal resistance, K/W |
| $R_{t,c}$ | thermal contact resistance, K/W |
| $R_{t,f}$ | fin thermal resistance, K/W |
| | |

| 1 | | |
|---|--------------------------------|--|
| | $R_{t,o}$ | thermal resistance of fin array, K/W |
| | r_o | cylinder or sphere radius, m |
| | r, ϕ, z | cylindrical coordinates |
| | r, θ, ϕ | spherical coordinates |
| | S | solubility, kmol/m ³ · atm; shape factor for |
| | | two-dimensional conduction, m; nozzle |
| | | pitch, m; plate spacing, m; Seebeck |
| | | coefficient, V/K |
| | S_c | solar constant, W/m ² |
| | S_D, S_L, S_T | diagonal, longitudinal, and transverse pitch of a tube bank, m |
| | Sc | Schmidt number |
| | Sh | Sherwood number |
| | St | Stanton number |
| | T | temperature, K |
| | t | time, s |
| | U | overall heat transfer coefficient, W/m ² · K; |
| | | internal energy, J |
| | u, v, w | mass average fluid velocity components, m/s |
| | u*, v*, w* | molar average velocity components, m/s |
| | V | volume, m ³ ; fluid velocity, m/s |
| | v | specific volume, m ³ /kg |
| | W | width of a slot nozzle, m |
| | \dot{W} | rate at which work is performed, W |
| | We | Weber number |
| | X | vapor quality |
| | X_{tt} | Martinelli parameter |
| | <i>X</i> , <i>Y</i> , <i>Z</i> | components of the body force per unit volume, N/m ³ |
| | <i>x</i> , <i>y</i> , <i>z</i> | rectangular coordinates, m |
| | X_c | critical location for transition to turbulence, m |
| | $x_{\mathrm{fd},c}$ | concentration entry length, m |
| | $x_{\mathrm{fd},h}$ | hydrodynamic entry length, m |
| | $x_{\mathrm{fd},t}$ | thermal entry length, m |
| | x_i | mole fraction of species i , C_i/C |
| | Ž | thermoelectric material property, K ⁻¹ |
| | | |

| Greek Let | ters | | |
|---------------------------------|--|------------------------------------|---|
| α | thermal diffusivity, m ² /s; accommodation | μ | viscosity, kg/s·m |
| β Γ | coefficient; absorptivity volumetric thermal expansion coefficient, K ⁻¹ mass flow rate per unit width in film condensation, kg/s • m ratio of specific heats | $ u $ $ ho $ $ ho_e $ $ \sigma $ | kinematic viscosity, radiation, s ⁻¹ mass density, kg/m ³ ; electric resistivity, Ω Stefan–Boltzmann co |
| δ | hydrodynamic boundary layer thickness, m | | conductivity, $1/\Omega$ |
| δ_c | concentration boundary layer thickness, m | | N/m ² ; surface tens |
| δ_p^c | thermal penetration depth, m | Φ | viscous dissipation f |
| δ_t^{ν} | thermal boundary layer thickness, m | φ | volume fraction |
| $oldsymbol{arepsilon}$ | emissivity; porosity; heat exchanger | ϕ | azimuthal angle, rad |
| | effectiveness | ψ | stream function, m ² / |
| $oldsymbol{arepsilon}_f$ | fin effectiveness | au | shear stress, N/m ² ; tr |
| $\stackrel{j}{oldsymbol{\eta}}$ | thermodynamic efficiency; similarity variable | ω | solid angle, sr; perfu |
| η_f | fin efficiency | Subscri | pts |
| η_o | overall efficiency of fin array | A, B | species in a binary m |
| θ | zenith angle, rad; temperature difference, K | abs | absorbed |
| K | absorption coefficient, m ⁻¹ | am | arithmetic mean |
| λ | wavelength, μ m | atm | atmospheric |
| $\lambda_{	ext{mfp}}$ | mean free path length, nm | b | base of an extended |
| | | C | carnot |
| | | c | cross-sectional; conc |
| | | cr | critical insulation thi |
| | | cond | conduction |
| | | conv | convection |
| | | CE | 4 0 |

| μ | viscosity, kg/s·m |
|-----------|--|
| ν | kinematic viscosity, m ² /s; frequency of |
| | radiation, s ⁻¹ |
| ho | mass density, kg/m ³ ; reflectivity |
| $ ho_e$ | electric resistivity, Ω/m |
| σ | Stefan–Boltzmann constant, W/m ² · K ⁴ ; electrical conductivity, $1/\Omega$ · m; normal viscous stress, |
| | N/m ² ; surface tension, N/m |
| Φ | viscous dissipation function, s ⁻² |
| φ | volume fraction |
| ϕ | azimuthal angle, rad |
| ψ | stream function, m ² /s |
| au | shear stress, N/m ² ; transmissivity |
| ω | solid angle, sr; perfusion rate, s ⁻¹ |
| Subscr | ipts |
| A, B | species in a binary mixture |
| abs | absorbed |
| am | arithmetic mean |
| atm | atmospheric |
| b | base of an extended surface; blackbody |
| C | carnot |
| c | cross-sectional; concentration; cold fluid; critical |
| cr | critical insulation thickness |
| cond | conduction |
| conv | convection |
| CF | counterflow |
| D | diameter; drag |
| dif | diffusion |
| e | excess; emission; electron |
| evap | evaporation |
| f | fluid properties; fin conditions; saturated liquid |
| C | conditions |
| fc | forced convection |
| fd | fully developed conditions |
| g 11 | saturated vapor conditions |
| Н | heat transfer conditions |

| h | hydrodynamic; hot fluid; helical |
|------------------|--|
| i | general species designation; inner surface of an |
| | annulus; initial condition; tube inlet |
| | condition; incident radiation |
| L | based on characteristic length |
| l | saturated liquid conditions |
| lat | latent energy |
| lm | log mean condition |
| m | mean value over a tube cross section |
| max | maximum |
| 0 | center or midplane condition; tube outlet |
| | condition; outer |
| p | momentum |
| ph | phonon |
| R | reradiating surface |
| r, ref | reflected radiation |
| rad | radiation |
| S | solar conditions |
| S | surface conditions; solid properties; |
| | saturated solid conditions |
| sat | saturated conditions |
| sens | sensible energy |
| sky | sky conditions |
| SS | steady state |
| sur | surroundings |
| t | thermal |
| tr | transmitted |
| \boldsymbol{v} | saturated vapor conditions |
| \mathcal{X} | local conditions on a surface |
| λ | spectral |
| ∞ | free stream conditions |
| Supersc | ripts |
| * | molar average: dimensionless quantity |

* molar average; dimensionless quantity

Overbar

surface average conditions; time mean

Chapter 1 - Introduction

- 1 What and how
- 2 Physical Origins and Rate Equations
- 3 Relationship to Thermodynamics

Thermal and Mechanical Energy Equation at an Instat (t)

$$\delta E_{\rm st} = E_{\rm in} - E_{\rm out} + E_{\rm gen} \tag{3}$$

$$\dot{E}_{\rm st} = \frac{dE_{\rm st}}{dt} = \dot{E}_{\rm in} - \dot{E}_{\rm out} + \dot{E}_{\rm gen} \tag{4}$$

Simplified Steady-Flow Thermal Energy Equation

$$q = \dot{(}m)c_p(T_{\text{out}} - T_{\text{in}}) \tag{5}$$

General Heat Transfer Relationships

Multiple equations 1.1, 1.3a, and 1.7 by A if you need to calculate per unit area.

TABLE 1.5 Summary of heat transfer processes

| Mode | Mechanism(s) | Rate Equation | Equation Number | Property or Coeftiient |
|------------|--|--|--------------------|--|
| Conduction | Diffusion of energy due to random molecular motion | $q_x''(W/m^2) = -k\frac{dT}{dx}$ | (1.1) | $k\left(\mathbf{W}/\mathbf{m}\cdot\mathbf{K}\right)$ |
| Convection | Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection) | $q''(W/m^2) = h(T_s - T_\infty)$ | (1.3a) | h (W/m²⋅K) |
| Radiation | Energy transfer by electromagnetic waves | $q''(W/m^2) = \varepsilon \sigma(T_s^4 - T_{sur}^4)$ or $q(W) = h_r A(T_s - T_{sur})$ | (1.7) (1.8) | $e h_r (W/m^2 \cdot K)$ |

Chapter 2 - Introduction to Conduction

- 1 The Conduction Rate Equation
- 2 The Thermal Properties of Matter
- 3 The Heat Diffusion Equation
- 4 Boundary and Initial Conditions

Chapter 3 - One-Dimensional **Steady-State Conduction**

- 1 The Plane Wall
- 2 An Alternative Conduction Analysis
- 3 Radial Systems
- 4 Summary of One-Dimensional Conduction Results
- 5 Conduction with Thermal Energy Generation

Ohmic heating

$$\dot{q} = \frac{\dot{E}_g}{\forall} = \frac{I^2 R_e}{\forall} \tag{6}$$

Absorption of Radiation

The Plane Wall

Heat Equation

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + \dot{q} = 0 \to \frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \tag{7}$$

General Solution Symmetric Surface Conditions

- Temperature Distribution
- Overall Energy Balance

Radial Systems

Cylindrical (Tube) Wall Solid Cylinder (Circular Rod)

- Heat Equation (Cylindrical)

Spherical Wall (Shell) Solid Sphere

- Heat Equation (Spherical)

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

| equation with | no generation | | |
|-------------------------------------|----------------------------------|--|---|
| | Plane Wall | Cylindrical Wall ^a | Spherical Wall ^a |
| Heat equation | $\frac{d^2T}{dx^2} = 0$ | $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$ | $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$ |
| Temperature distribution | $T_{s,1} - \Delta T \frac{x}{L}$ | $T_{s,2} + \Delta T \frac{\ln{(r/r_2)}}{\ln{(r_1/r_2)}}$ | $T_{s,1} - \Delta T \left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$ |
| Heat flux (q'') | $k\frac{\Delta T}{L}$ | $\frac{k \Delta T}{r \ln \left(r_2 / r_1 \right)}$ | $\frac{k \Delta T}{r^2[(1/r_1) - (1/r_2)]}$ |
| Heat rate (q) | $kA\frac{\Delta T}{L}$ | $\frac{2\pi Lk \Delta T}{\ln\left(r_2/r_1\right)}$ | $\frac{4\pi k \Delta T}{(1/r_1) - (1/r_2)}$ |
| Thermal resistance ($R_{t,cond}$) | $\frac{L}{kA}$ | $\frac{\ln\left(r_2/r_1\right)}{2\pi Lk}$ | $\frac{(1/r_1) - (1/r_2)}{4 \pi k}$ |

The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

| $\frac{\cosh m(L-x) + (hlmk) \sinh}{\cosh mL + (hlmk) \sinh}$ $\frac{\cosh m(L-x)}{\cosh mL}$ | | $M \frac{\sinh mL + (h/mk)}{\cosh mL + (h/mk)}$ $M \tanh mL$ | (3.77) |
|---|---------|--|--|
| | | M tanh mL | |
| | | | (3.81) |
| $\frac{(\theta_L/\theta_b)\sinh mx + \sinh m}{\sinh mL}$ | (L-x) | $M \frac{(\cosh mL - \epsilon)}{\sinh mL}$ | θ_L/θ_b |
| | (3.82) | | (3.83) |
| e^{-mx} | (3.84) | M | (3.85) |
| | sinh mL | $\sinh mL$ (3.82) | $\frac{1}{\sinh mL} \frac{M}{\sinh mL}$ (3.82) |

Chapter 4

Appendix

Appendix C.