Solid Mechanics - Zak Olech - October 3, 2019

Variables (Alphbetical By Variable)

a	Constant; distance
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Forces; reactions
$A, B, C, \ldots$	Points
$A, \mathcal{A}$	Area
b	Distance; width
c	Constant; distance; radius
C	Centroid
$C_1, C_2, \ldots$	Constants of integration
C_P	Column stability factor
d	Distance; diameter; depth
D	Diameter
e	Distance; eccentricity; dilatation
E	Modulus of elasticity
f	Frequency; function
$\mathbf{F}$	Force
$F.S.$	Factor of safety
G	Modulus of rigidity; shear modulus
h	Distance; height
$\mathbf{H}$	Force
H, J, K	Points
$I, I_x, \ldots$	Moment of inertia
$I_{xy}, \ldots$	Product of inertia
J	Polar moment of inertia
k	Spring constant; shape factor; bulk modulus; constant
K	Stress concentration factor; torsional spring constant
l	Length; span
L	Length; span
L_e	Effective length
m	Mass
$\mathbf{M}$	Couple
$M, M_x, \ldots$	Bending moment
M_D	Bending moment, dead load (LRFD)
M_L	Bending moment, live load (LRFD)
M_U	Bending moment, ultimate load (LRFD)
n	Number; ratio of moduli of elasticity; normal direction
p	Pressure
$\mathbf{P}$	Force; concentrated load
P_D	Dead load (LRFD)
P_L	Live load (LRFD)
P_U	Ultimate load (LRFD)
q	Shearing force per unit length; shear flow
$\mathbf{Q}$	Force
Q	First moment of area
r	Radius; radius of gyration
$\mathbf{R}$	Force; reaction
R	Radius; modulus of rupture
s	Length
S	Elastic section modulus
t	Thickness; distance; tangential deviation
$\mathbf{T}$	Torque
T	Temperature
u, v	Rectangular coordinates
u	Strain-energy density
U	Strain energy; work
$\mathbf{v}$	Velocity
$\mathbf{V}$	Shearing force
V	Volume; shear
w	Width; distance; load per unit length
$\mathbf{W}, W$	Weight, load

$\mathbf{V}$	Shearing force
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w	Width; distance; load per unit length
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x, y, z	Rectangular coordinates; distance; displacements; deflections
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of centroid
Z	Plastic section modulus
α, β, γ	Angles
α	Coefficient of thermal expansion; influence coefficient
γ	Shearing strain; specific weight
γ_D	Load factor, dead load (LRFD)
γ_L	Load factor, live load (LRFD)
δ	Deformation; displacement
ϵ	Normal strain
θ	Angle; slope
λ	Direction cosine
ν	Poisson's ratio
ρ	Radius of curvature; distance; density
σ	Normal stress
τ	Shearing stress
ϕ	Angle; angle of twist; resistance factor
ω	Angular velocity

Conversion Factors

$$1\text{hp} = 550\text{ft*lb/s} = 6600 \text{ in*lb/s} \tag{1}$$

Freedom Unit BS

Force P – pounds (lb) or kilopound (kip)
Stress σ – pounds per square inch (psi) or kilopounds per square inch (ksi)

General

SI Prefixes

Multiplication Factor	Prefix†	Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto‡	h
10 = 10^{1}	deka‡	da
0.1 = 10^{-1}	deci‡	d
0.01 = 10^{-2}	centi‡	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

Moments

Moment of Inertia

Area Moment of Inertia for Rectangular Section

$$I_x = bh^3/12 \tag{2}$$

Parallel Axis Theorm (2-Dimensional)

$$I = I_{\text{Original}} + Ad^2 \tag{3}$$

Chapter 1 - Concept of stress

Axial Loading: Normal Stress

$$\sigma = \frac{P}{A} \tag{4}$$

Transverse Forces and Shearing Stress

$$\tau_{\text{ave}} = \frac{P}{A} \tag{5}$$

Single and Double Shear

Single Shear

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{F}{A} \tag{6}$$

Double Shear

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{7}$$

Bearing Stress

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{8}$$

Method of Solution

1. Clear and precise statement of problem
2. Draw one or several free-body diagrams; used to write equilibrium equations
3. Think SMART. Strategy, Modeling, Analysis, and Reflect & Think

Stresses on an Oblique Section

$$\sigma = \frac{P}{A_0} cos^2\theta \tag{9}$$

$$\tau = \frac{P}{A_0} sin\theta cos\theta \tag{10}$$

Stress Under General Loading

Factor of Safety

$$\text{Factor of safety} = \text{F.S.} = \frac{\text{ultimate load}}{\text{allowable load}} \tag{11}$$

Chapter 2 - Stress and Strain - Axial Loading

Normal Strain

$$\epsilon = \frac{\delta}{L} \tag{12}$$

Hooke's Law and Modulus of Elasticity

$$\sigma = E\epsilon \tag{13}$$

Elastic Deformation Under Axial Loading

$$\delta = \frac{PL}{AE} \tag{14}$$

$$\delta = \Sigma = \frac{P_i L_i}{A_i E_i} \tag{15}$$

Problems with Temperature Change

$$\delta_T = \alpha(\Delta T)L \tag{16}$$

$$\epsilon_T = \alpha\Delta T \tag{17}$$

Lateral Strain and Poisson's Ratio

$$v = -\frac{\text{lateral strain}}{\text{axial strain}} \tag{18}$$

Multiaxial Loading

$$\epsilon_x = \frac{\sigma_x}{E} \tag{19}$$

$$\sigma_y = \sigma_x = -\frac{v\sigma_x}{E} \tag{20}$$

Generalized Hooke's law for multiaxial loading

$$\sigma_x = +\frac{\sigma_x}{E} - \frac{v\sigma_y}{E} - \frac{v\sigma_z}{E} \tag{21}$$

$$\sigma_y = -\frac{\sigma_x}{E} - \frac{v\sigma_y}{E} - \frac{v\sigma_z}{E} \tag{22}$$

$$\sigma_z = -\frac{\sigma_x}{E} - \frac{v\sigma_y}{E} + \frac{v\sigma_z}{E} \tag{23}$$

Dilation

$$e = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) \tag{24}$$

Bulk Modulus

p: Hydrostatic Pressure

$$e = -\frac{p}{k} \tag{25}$$

k: bulk modulus of the material

$$k = \frac{E}{3(1-2v)} \tag{26}$$

Shearing Strain: Modulus of Rigidity

$$\tau_{xy} = G\gamma_{xy} \tag{27}$$

$$\tau_{yz} = G\gamma yz \tag{28}$$

$$\tau_{zx} = G\gamma_{zx} \tag{29}$$

$$\frac{E}{2G} = 1 + v \tag{30}$$

Stress Concentrations

$$K = \frac{\sigma_{\max}}{\sigma_{\text{avg}}} \tag{31}$$

Chapter 3 - Torsion

General

Deformation in Circular Shafts

$$\gamma = \frac{\rho\phi}{L} \tag{32}$$

$$\gamma_{max} = \frac{c\phi}{L} \tag{33}$$

$$\gamma = \frac{\rho}{c} * \gamma_{max} \tag{34}$$

Shearing Stresses in Elastic Range

$$\tau = \frac{\rho}{c}\tau_{max} \tag{35}$$

$$\tau_{max} = \frac{Tc}{J} \tag{36}$$

$$\tau = \frac{T\rho}{J} \tag{37}$$

Polar Moment of Inertia Solid Shaft

$$J = \frac{1}{2}\pi c^4 \tag{38}$$

c = radius

Polar Moment of Inertia of a Hollow Shaft inner radius c1, outer radius c2

$$J = \frac{1}{2}\pi(c_2^4 - c_2^4) \tag{39}$$

Angle of Twist

$$\phi = \frac{TL}{JG} \tag{40}$$

$$\phi = \Sigma\frac{TL}{JG} \tag{41}$$

Statically Indeterminante Shafts

Transmission Shafts

Power P is transmitted as:

$$P = 2\pi fT \tag{42}$$

T is the torque exerted at each end of the shaft
f the frequency (hz or s^{-1})

Stress Concentrations

$$\tau_{\max} = K\frac{Tc}{J} \tag{43}$$

K = Stress concentration factor
stress $\frac{Tc}{J}$ is computed for the smaller-diameter shaft

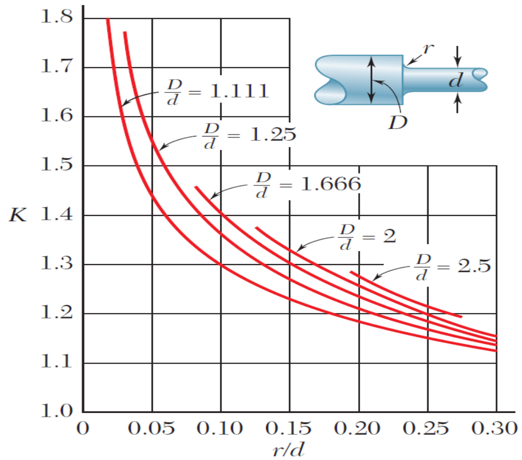

Fig. 3.28 Plot of stress concentration factors for fillets in circular shafts. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

Plastic Deformations

$$T = \int_0^c \rho\tau(2\pi d\rho) = 2\pi\int_0^c \rho^2\tau d\rho \tag{44}$$

Modulus of Rupture

This is a ficticious value.

$$R_t = \frac{T_u c}{j} \tag{45}$$

Solid Shaft of Elastoplastic Material

Maximum Elastic Torque; Solid Circular Shaft, Radius c

$$\tau_y = \frac{1}{2}\pi c^3\tau Y \tag{46}$$

Torque Related to ρ_y

$$T = \frac{4}{3}T_y(1 - \frac{1}{4}\rho\rho^3 yc^3) \tag{47}$$

Plastic Torque

$$T_p = \frac{4}{3}T_y \tag{48}$$

Plastic Torque Vs. Angle of Twist

$$T = \frac{4}{3}T_y(1 - \frac{1}{4}\frac{\phi^3 y}{\phi^3}) \tag{49}$$

Torsional Loading or Shaft Cross-Section Changes Along Length

$$\phi = \Sigma_i\frac{T_i L_i}{J_i G_i} \tag{50}$$

Thin-Walled Hollow Shafts

Shear Flow

$$q = \tau t \tag{51}$$

Average Shearing Stress τ at any given point in cross section

$$\tau = \frac{T}{2tA} \tag{52}$$

Chapter 4 - Pure Bending

1 - Symmetric Members in Pure Bending

$$\epsilon_x = -\frac{y}{\rho} \tag{53}$$

ρ - Radius of curvature of the neutral surface
y - Distance from neutral surface

2 - Stresses and Deformatoins in the Elastic Range

$$\sigma_x = -\frac{y}{c}\sigma_m \tag{54}$$

c - largest distance from the neutral axis to a point in the section

Elastic Flexture formula

$$\sigma_m = \frac{Mc}{I} \tag{55}$$

Equation To Use When Finding Normal Internal Stress

$$\sigma_x = -\frac{My}{I} \tag{56}$$

$-\sigma$ – Compression
σ – Tension

Eleastic Section Modulus

$$S = \frac{I}{c} \tag{57}$$

$$\sigma_m = \frac{M}{S} \tag{58}$$

Curvature of Member

$$\frac{1}{\rho} = \frac{M}{EI} \tag{59}$$

Eccentric Axial Loading

$$\sigma_x = \frac{P}{A} - \frac{M_y}{I} \tag{60}$$

Unsymmetric Bending

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \tag{61}$$

General Eccentric Axial Loading

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y Z}{I_y} \tag{62}$$

Curved Members

$$R = \frac{A}{\int\frac{dA}{r}} \tag{63}$$

$$\sigma_x = -\frac{My}{Ae(R-y)} \tag{64}$$

Factor of Safety

$$\sigma_m = K\frac{Mc}{I} \tag{65}$$

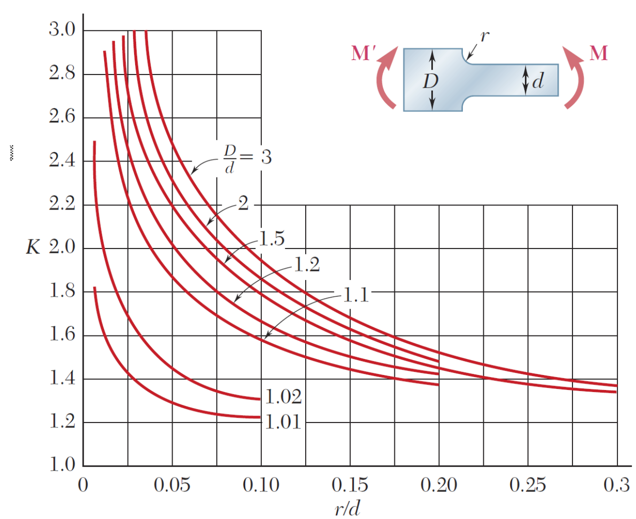

Fig. 4.24 Stress-concentration factors for *flat bars* with fillets under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

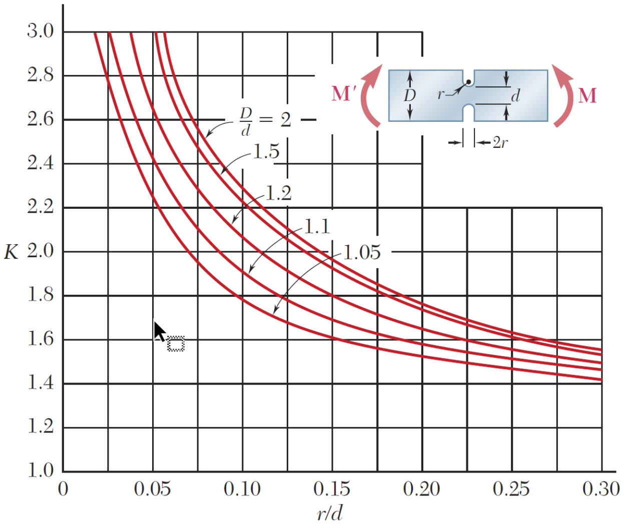

Fig. 4.25 Stress-concentration factors for *flat bars* with grooves (notches) under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

Equation for Force on a Single Area Within a T Section

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y} * A* \qquad (66)$$

Additional Notes

Area, width, and moment of inertia for W shapes should be given on a test or found in appendix c of textbook.
c in this section is the largest distance from the neutral axis to a point in the section.

Chapter 5 - Analysis and Design of Beams for Bending

Only statically determinate beams are considered in this chapter.

1 - Shear and Bending-Moment Diagrams

2 - Relationships Between Load, Shear, and Bending Moment

3 - Design of Prismatic Beams for Bending

Review and Summary

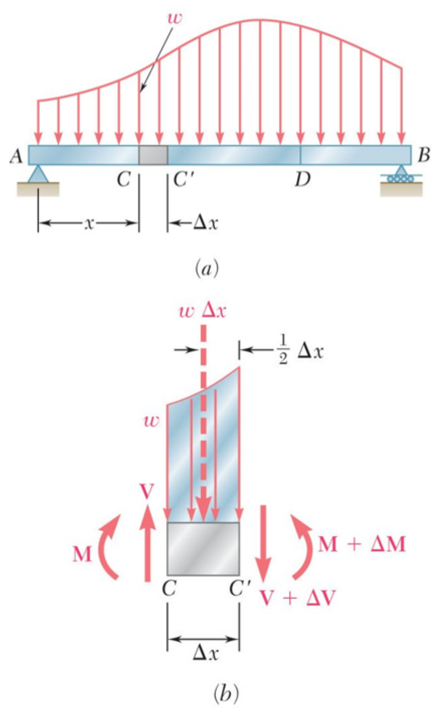

Fig. 5.9 (a) Simply supported beam subjected to a distributed load, with a small element between C and C′, (b) Free-body diagram of the element.

Design of Prismatic Beams

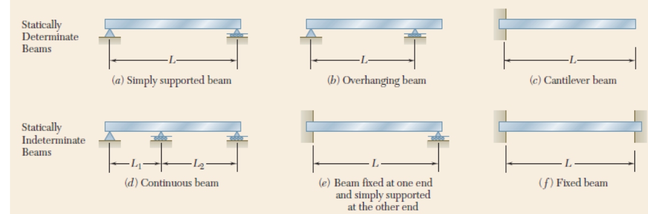

Fig. 5.20 Common beam support configurations.

Normal Stresses Due to Bending

Shear and Bending-Moment Diagrams

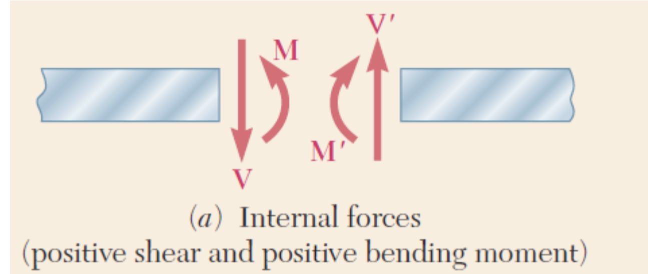

Fig. 5.22 Positive sign convention for internal shear and bending moment.

Relationships Between Load, Shear, and Bending Moment

Design of Prismatic Beams