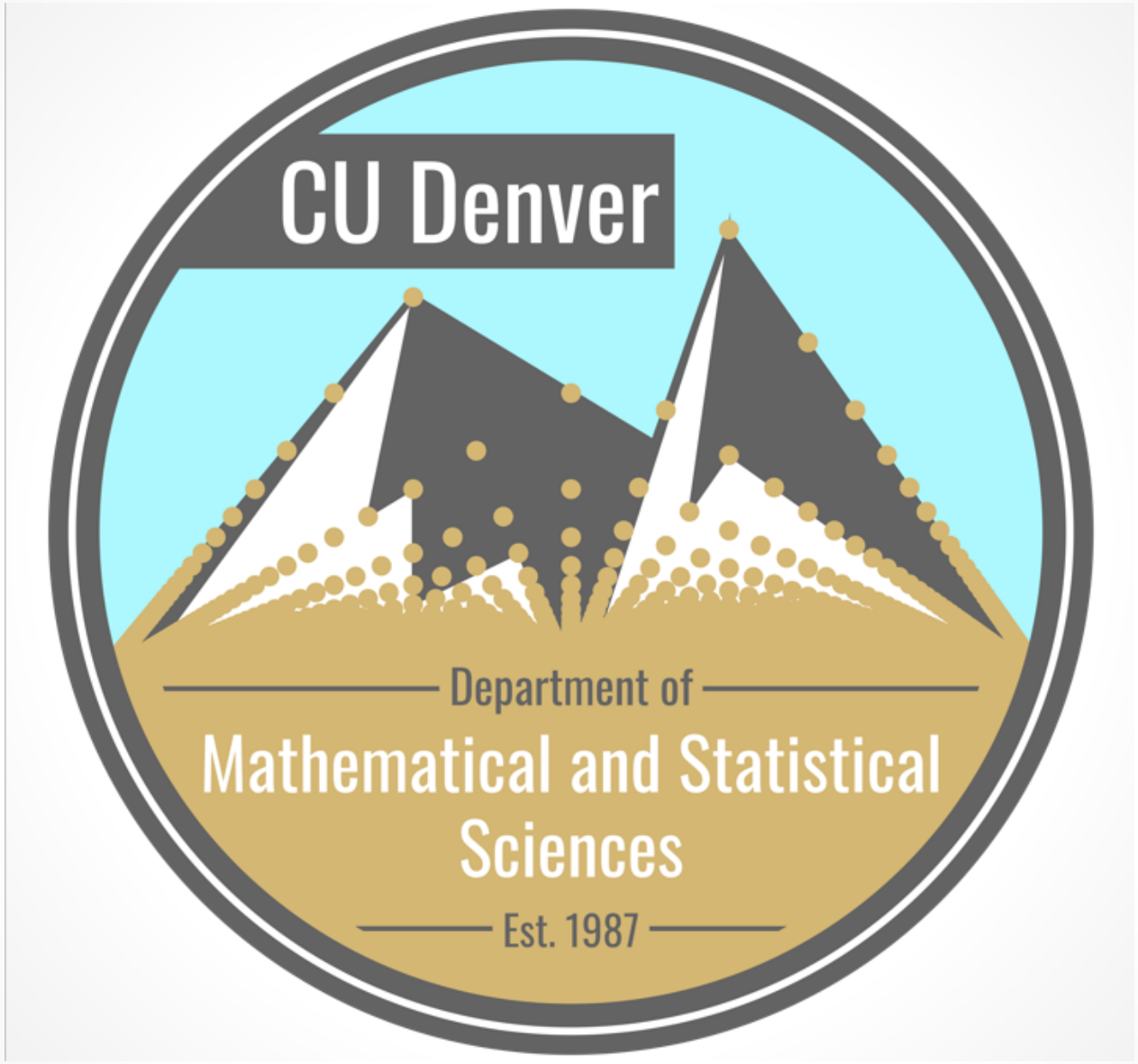


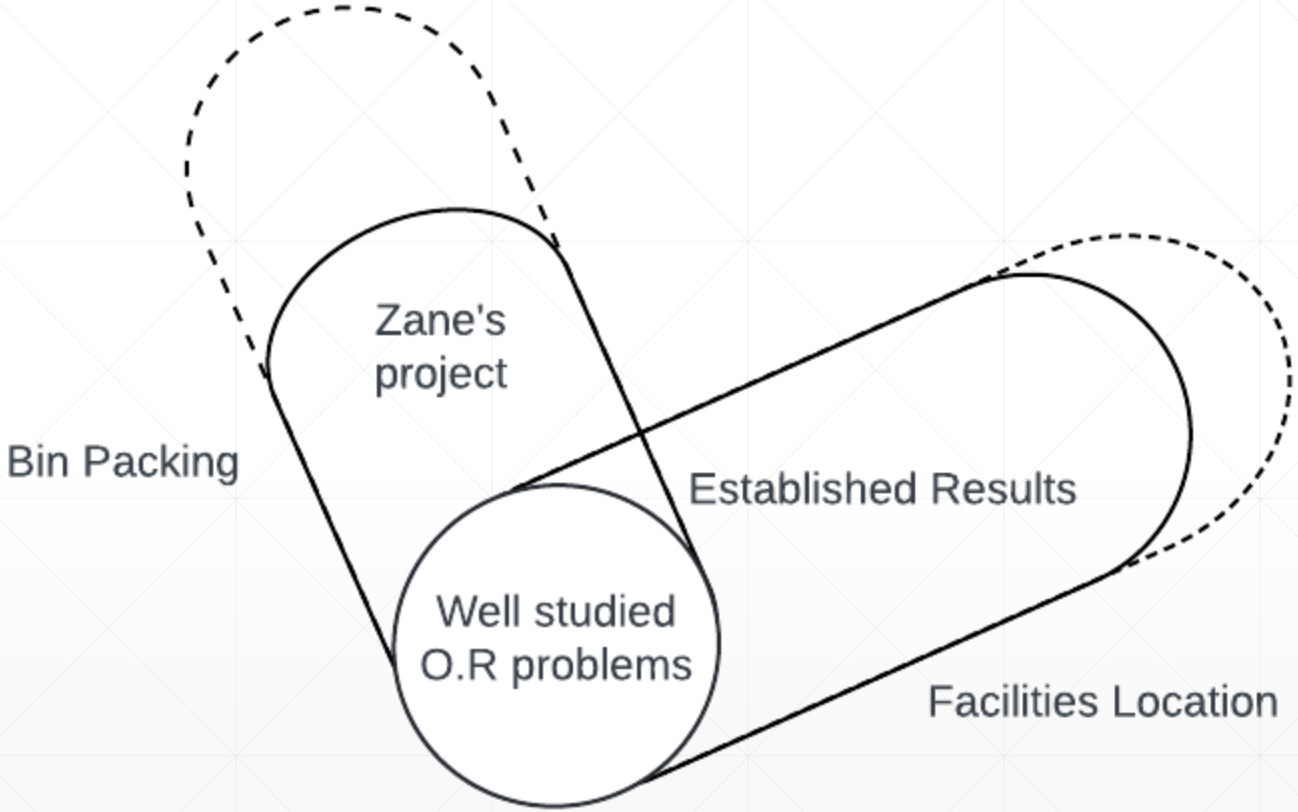
The bin packing problem

Further incorporating the mathematics of equity into operations research

Moral motivation

- “It has to start somewhere, it has to start sometime, what better place than here, what better time than now?”
  -Zack de la Rocha
- Equity is a distinctly human concern, it is neither considered nor required by mathematics, or syndicates.
- O.R used by factories, warehouses, and boardrooms to **maximize profit and minimize cost**
  - Weak benefits – HDHP
  - Acceptable losses – you!
- Incorporation of equity, of humanity is necessary

What is the context of this project?



What will this project achieve?

- Modification of the classic B.P problem into an equivalent formation – Syndicate model
- Incorporate equity via Kolm-Pollak Equally-Distributed Equivalent – Equity model
- Code both these models in python, run on Alderaan cluster, analyze results
- Juxtapose the output of these two models
  - Time to run
  - Total spend on bins
  - Inequality measure

What is operations research?

- Operations Research makes a ‘best’ decision
  - Optimization model – parameters, restrictions, defined objective

- There are many well studied problems in O.R. domain
  - Bin packing (B.P) – focus of this presentation
    - Items of different sizes are packed bins
    - Minimize bins used, pack all items

Bin Packing Problem (classic formulation)

Classic Bin Packing Model

Sets

B = $\{1, \ldots, b \ldots, m\}$ - Number of bins to fill, from 1 to m using index b

I = $\{1, \ldots, i, \ldots n\}$ - Number of items to pack, from 1 to n using index i

Parameters

C_b - Capacity of bin b, $\forall b \in B$

E_i - Size of item i, $\forall i \in I$

P_b - Price to purchase bin b, $\forall b \in B$

Decision Variables

$$X_b = \begin{cases} 1 & \text{if bin b is packed with any item} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B$$

$$Y_{i,b} = \begin{cases} 1 & \text{if item i is packed into bin b} \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, \forall b \in B$$

Model

(2.1) $$\min \quad \sum_{b \in B} P_b X_b$$

(2.2) $$\text{s.t} \quad \sum_{b \in B} Y_{i,b} = 1, \forall i \in I$$

(2.3) $$\sum_{i \in I} E_i \cdot Y_{i,b} \leq C_b \cdot X_b, \forall b \in B$$

(2.4) $$X_b \in \{0,1\} \, \forall b \in B$$

(2.5) $$Y_{i,b} \in \{0,1\} \, \forall i \in I, \, \forall b \in B$$

(2.6)

Reference

(2.1) The objective is to minimize the total number of bins used.

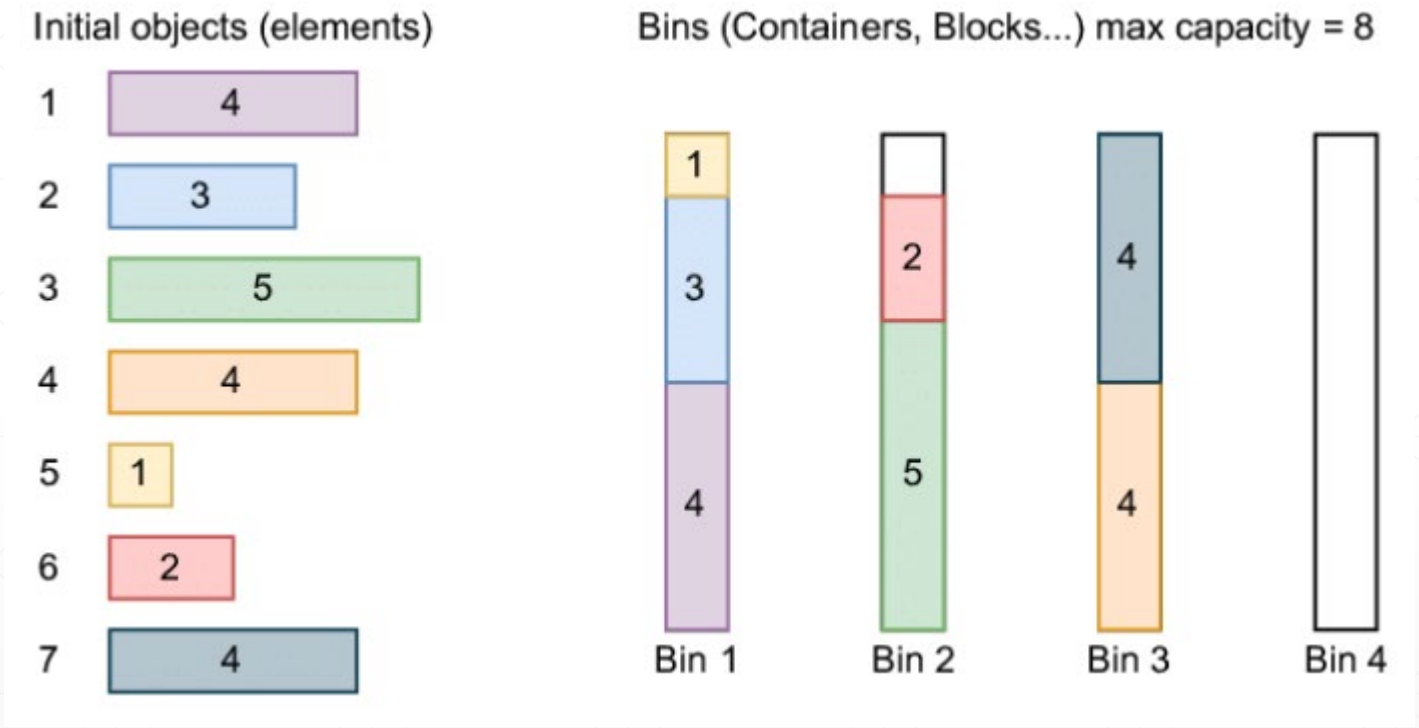
(2.2) Item may only be packed into one bin.

(2.3) Total size of items $(i_1, i_2, \ldots)$ in bin b must be be less than or equal to capacity of bin b.

(2.4) Binary choice, is the bin packed or no?

(2.5) Binary choice, is the bin packed with the specific item or no?

Bin Packing Problem (classic formulation)



Transform into Syndicate model

- Difficult to incorporate equity into classic model
- Transform into equivalent model – syndicate model
- Introduce new set: $S = \{1, \ldots, s, \ldots, n\}$ - Set of Syndicates selling bins.
- Corresponding parameter $A_{b,s} = \begin{cases} 1 & \text{if bin b is purchased from syndicate s} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B \, \forall s \in S$
- Combine pieces info new O.F $\min \sum_{s \in S} \sum_{b \in B} A_{b,s} \cdot P_b \cdot X_b, \forall s \in S$
- Simplify $D_s = \sum_{b \in B} A_{b,s} \cdot P_b \cdot X_b, \forall s \in S$

Syndicate model

Sets

B $= \{1, \ldots, b \ldots, m\}$ - Set of bins to fill, from 1 to m using index b

S $= \{1, \ldots, s, \ldots, n\}$ - Set of Syndicates selling bins, from 1 to n using index s

I $= \{1, \ldots, i, \ldots p\}$ - Set of items to pack, from 1 to p using index i

Parameters

C_b - Capacity of bin b, $\forall b \in B$

P_b - Price to purchase bin b, $\forall b \in B$

$$A_{b,s} = \begin{cases} 1 & \text{if bin b is available from syndicate s} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B \, \forall s \in S$$

E_i - Size of item i, $\forall i \in I$

Decision Variables

$$X_b = \begin{cases} 1 & \text{if bin b is packed with any item} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B$$

$$Y_{i,b} = \begin{cases} 1 & \text{if item i is packed into bin b} \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, \forall b \in B$$

D_s - Total money paid to syndicate s (for bins), $\forall s \in S$

Model

$$\text{(2.7)} \quad \min \quad \sum_{s \in S} D_s$$

$$\text{(2.8)} \quad \text{s.t} \quad \sum_{b \in B} Y_{i,b} = 1, \forall i \in I$$

$$\text{(2.9)} \quad \sum_{i \in I} E_i \cdot Y_{i,b} \leq C_b \cdot X_b, \forall b \in B$$

$$\text{(2.10)} \quad D_s = \sum_{b \in B} A_{b,s} \cdot P_b \cdot X_b, \forall s \in S$$

$$\text{(2.11)} \quad X_b \in \{0, 1\} \, \forall b \in B$$

$$\text{(2.12)} \quad Y_{i,b} \in \{0, 1\} \, \forall i \in I, \, \forall b \in B$$

2.7 Objective is to minimize the total amount paid, summed over all syndicates.

2.8 Item may only be packed into one bin.

2.9 Total size of items $(i_1, i_2, \ldots)$ in bin b must be be less than or equal to capacity of bin b.

2.10 Dark money paid to syndicate s equals sum of the bins purchased (from the syndicate) times the price of the bins.

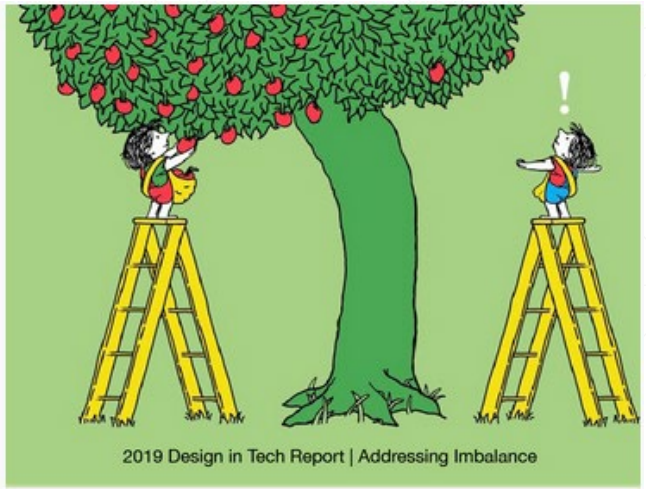
2.11 Binary choice, is the bin packed or no?

2.12 Binary choice, is the bin packed with the specific item or no?

What is equity?

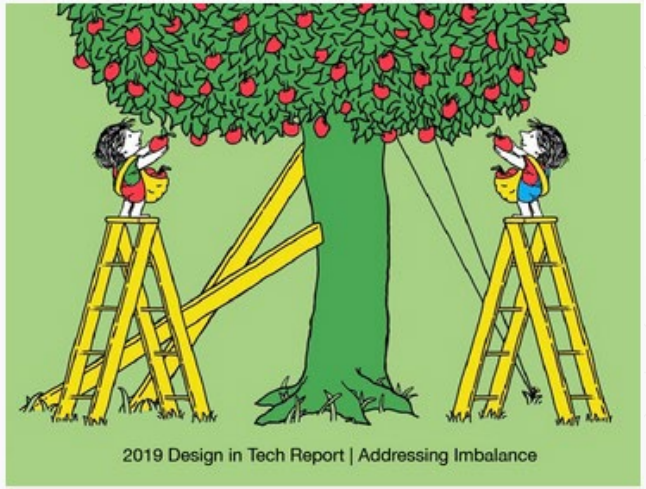
- "The consistent and systematic fair, just, and impartial treatment of all individuals, including individuals who belong to underserved communities that have been denied such treatment"
- An equitable distribution of amenities is not the same as an equal distribution

INEQUALITY



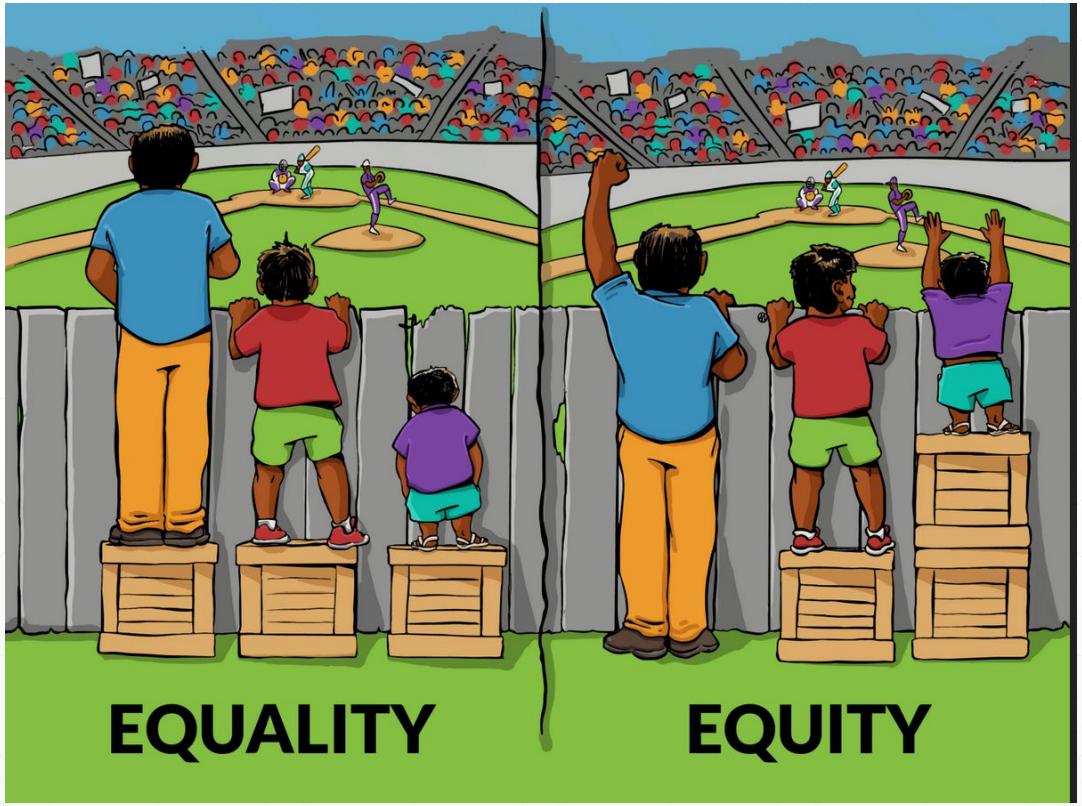
EQUALITY

EQUITY



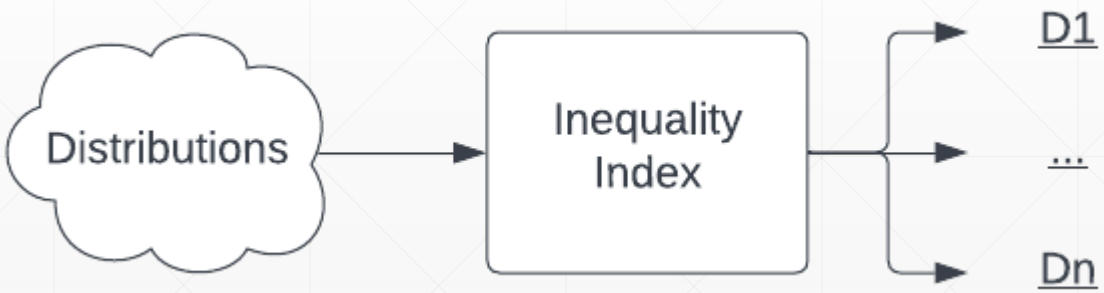
JUSTICE

What is equity?



Mathematics of equity - Inequality Indices

- Goal: Measure different distributions of both amenities (desirable) and dis-amenities.
  - Which of the distributions is ‘best’?
  - What does ‘best’ mean?
- Inequality Index



Ideal properties

- **Symmetry**: Inequality of a population is based solely on the distribution of the quantity in question and no other rankings.
- **Population independence**: The number of individuals does not influence the measure of inequality.
- **Principle of transfers**: If a quantity is redistributed from an advantaged individual to a disadvantaged individual, the inequality should decrease.
- **Scale dependence**: The inequality measure should reflect the total sum of the quantity measured. For instance, high exposure to burdens (for example hazards or health risks) would be penalized.

Ideal properties (continued)

- **Separable**: The measure can be used to examine the inequality between subgroups (e.g., different demographic groups), and therefore can incorporate consideration of need or vulnerability, and, subsequently, inequity.
- **Multivariate**: The measure can be used to evaluate multiple quantities and the correlation of advantages or disadvantages.
- **Mirror property**: The measure can be used for distributions of both amenities (desirable) and dis-amenities (undesirable) quantities.

Equally Distributed Equivalent (EDE)

- EDE's are an extension of Inequality Index
  - Derived from the same idea: ranking distributions
  - EDE's account for both quantity and dispersion of a distribution
- "Just because a distribution is less equal than another, does not necessarily mean that it is less desirable; total levels are also relevant"
- "What is the level of risk that would make an individual indifferent between a distribution in which everyone received that risk, and the actual un-equal risk."

Kolm-Pollak EDE

$$\Xi(X) = -\frac{1}{\kappa}\ln\left(\frac{1}{N}\sum_{i\in N} e^{-\kappa X_i}\right)$$

- X is a distribution, N is sample size
- Kappa is the inequality aversion parameter
  - Subjective constant, representing how averse to inequality the population is
  - Kappa is the key to satisfying the mirror property
    - Negative kappa is used for a distribution of dis-amenities
    - Positive kappa is used for a distribution of amenities

Incorporation of KP-EDE into Syndicate model

- Model specific formulation $\Xi(D_s) = -\frac{1}{\kappa}\ln\left(\frac{1}{|S|}\sum_{s\in|S|} e^{-\kappa D_s}\right)$

- Use techniques to simplify the formulation of the objective function
- Discard scalar multiple $\ln\left(\frac{1}{|S|}\sum_{s\in|S|} e^{-\kappa D_s}\right)$
- Discard scalar multiple |S|
  Log function is monotonically increasing $\min \sum_{s\in S} e^{-\kappa D_s}$

Equity model

Sets

B = $\{1, \ldots, b \ldots, m\}$ - Set of bins to fill, from 1 to m using index b
S = $\{1, \ldots, s, \ldots, n\}$ - Set of Syndicates selling bins, from 1 to n using index c
I = $\{1, \ldots, i, \ldots p\}$ - Set of items to pack, from 1 to p using index i

Parameters

C_b - Capacity of bin b, $\forall b \in B$
P_b - Price to purchase bin b, $\forall b \in B$
E_i - Size of item i, $\forall i \in I$
D_s - Total money paid to syndicate s (for bins), $\forall s \in S$

$$A_{b,s} = \begin{cases} 1 & \text{if bin b is purchased from syndicate s} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B \, \forall s \in S$$

Decision Variables

$$X_b = \begin{cases} 1 & \text{if bin b is packed with any item} \\ 0 & \text{otherwise} \end{cases}, \forall b \in B$$

$$Y_{i,b} = \begin{cases} 1 & \text{if item i is packed into bin b} \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, \forall b \in B$$

Model

$$\min \quad \sum_{s \in S} e^{-\kappa D_s} \tag{1}$$

$$\text{s.t} \quad \sum_{b \in B} Y_{i,b} = 1, \forall i \in I \tag{2}$$

$$\sum_{i \in I} E_i \cdot Y_{i,b} \leq C_b \cdot X_b, \forall b \in B \tag{3}$$

$$D_s = \sum_{b \in B} A_{b,s} \cdot P_b \cdot X_b, \forall s \in S \tag{4}$$

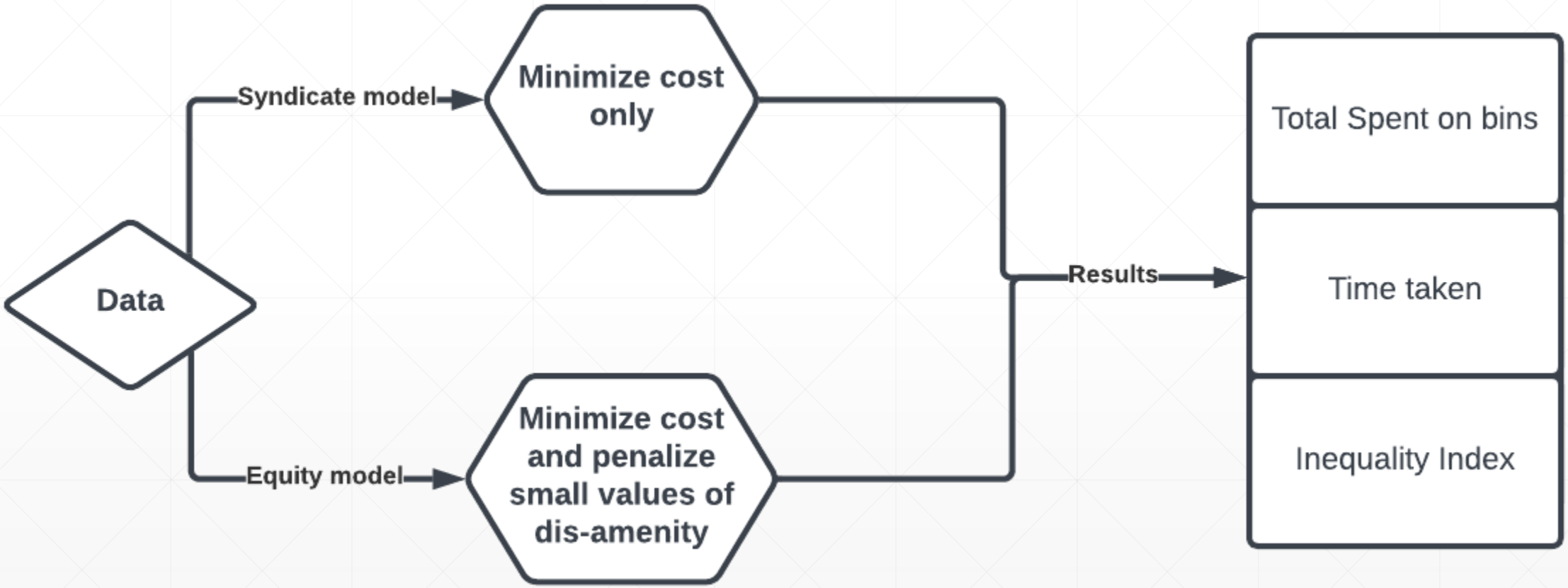
$$X_b \in \{0, 1\} \, \forall b \in B \tag{5}$$

$$Y_{i,b} \in \{0, 1\} \, \forall i \in I, \, \forall b \in B \tag{6}$$

Reference

1 Objective is to minimize the total amount paid, summed over all syndicates.
2 Item may only be packed into one bin.
3 Total size of items $(i_1, i_2, \ldots)$ in bin b must be be less than or equal to capacity of bin b.
4 Dark money paid to syndicate s equals sum of the bins purchased (from the syndicate) times the price of the bins.
5 Binary choice, is the bin packed or no?
6 Binary choice, is the bin packed with the specific item or no?

Outline


Data
Syndicate model
Minimize cost only
Equity model
Minimize cost and penalize small values of dis-amenity
Results
Total Spent on bins
Time taken
Inequality Index

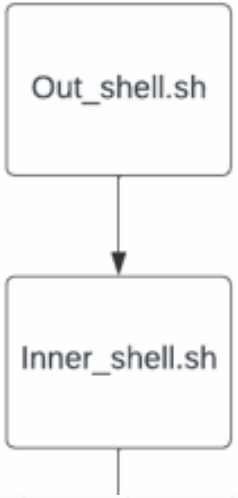
Implementing syndicate and equity models

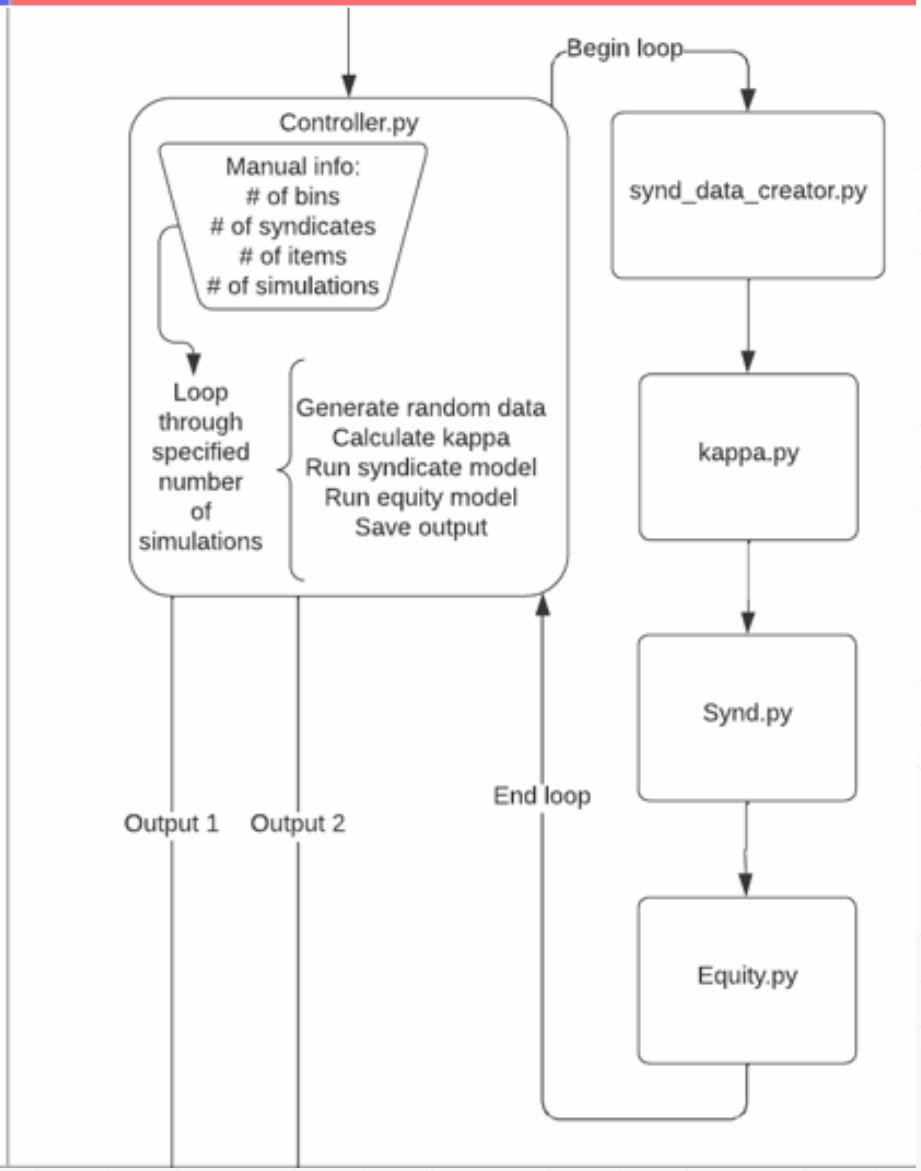
- Coded in python
- PySCIPOpt - Fastest non-commercial solvers for mixed integer programming (MIP) and mixed integer nonlinear programming (MINLP)
- Used CCM Alderaan cluster to run simulations
  - University supercomputer, 64 AMD cores, 512 GM memory per core
  - Funded by National Science Foundation OAC-2019089

Data creation

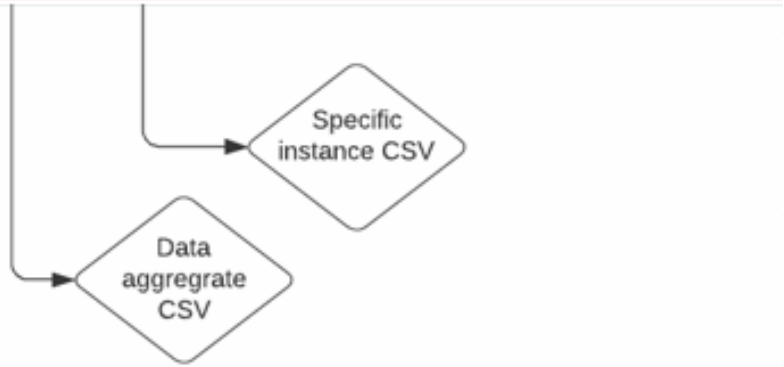
- No real-world data, all random integers, generated via numpy
- Simulation sized based on 3 parameters,
  - Number of bins
  - Number of items
  - Number of syndicates
  - [B,I,S] – [5,5,5], [10,10,10], [15,10,10]
- Created dataframe object in synd_data_creator.py – passed to models

Code flow

	Notes	Code outline
BASH	The outer shell provides parameters for code execution, and invokes the singularity container to run an isolated instance of the code. In addition, we specify the required .sif file which has the necessary solver PySCIOpt. The Inner shell provides further parameters, specifying the file we want to run, and the application we want to use to run that program.	

	Notes	Code outline
Python	The controller file drives the process. It requires 4 pieces of information to run each simulation: 1: How many bins 2: How many syndicates 3: How many items 4: How many simulations using the data above The syndicate data creator randomly creates the bin volumes, price to purchase bins, item sizes, and the availability matrix, of which bins are available from which syndicate. It passes that info along via a datframe object. The kappa file creates the kappa value. This is the inequality aversion parameter in the KP-EDE. As that parameter is a subjective measure, we create one here by calculating the cost to purchase every item it's own bin. The syndicate model is run, using both the kaapa value, and the dataframe object passed by the data creator file The equity model is run, using both the kaapa value, and the dataframe object passed by the data creator file	

Code flow

	Notes	Code outline
Output	There are two different outputs, one is a specific instance of the solved problem, including the dataframe created by the data creator file. The other file contais 6 data points aggegrated data over hundreds of simulations. Time required to run both models, the equity score of both models, and the total spend on bins for both models.	

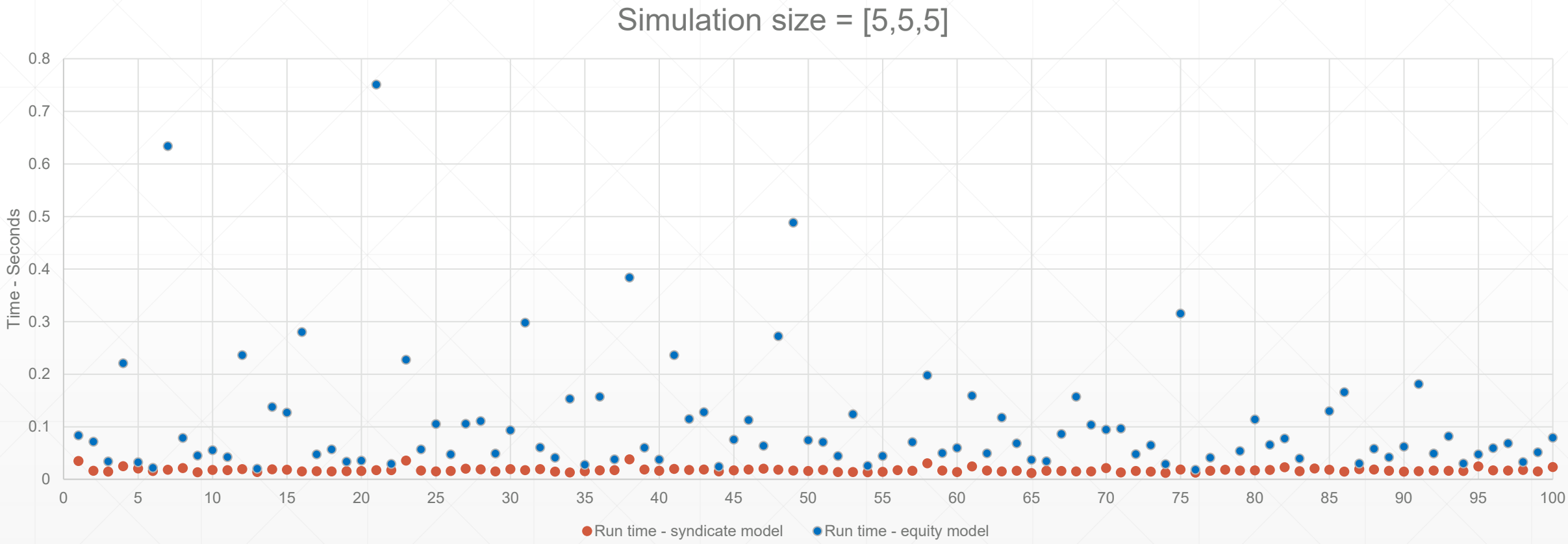
Results – aggregate data

Simulation Number	Simulation size	Run time - syndicate model	Run time - equity model	Total spent - syndicate model	Total spent - equity model	Inequality index, syndicate model	Inequality index, equity model
0	[5, 5, 5]	0.034822941	0.083558798	8	8	6.297442541	6.297442532
1	[5, 5, 5]	0.016434908	0.071810722	14	14	8.436563657	8.436562657
2	[5, 5, 5]	0.014610767	0.03406024	6	6	5.909982829	5.909982829
3	[5, 5, 5]	0.02485919	0.220913649	15	16	13.19116633	11.87312731
4	[5, 5, 5]	0.020640373	0.032467365	9	9	10.15484549	10.15484549
5	[5, 5, 5]	0.016018629	0.02188015	12	12	7.140252037	7.140252037
6	[5, 5, 5]	0.01821208	0.634047985	8	9	8.607342974	8.35100005
7	[5, 5, 5]	0.021677017	0.078855038	11	12	20.11019792	11.87312731
8	[5, 5, 5]	0.013413191	0.04517436	6	6	6.070126019	6.070125157
9	[5, 5, 5]	0.017848969	0.055671692	6	6	8.436563657	8.436563657
10	[5, 5, 5]	0.017596483	0.04254961	6	6	8.436563657	8.436563656

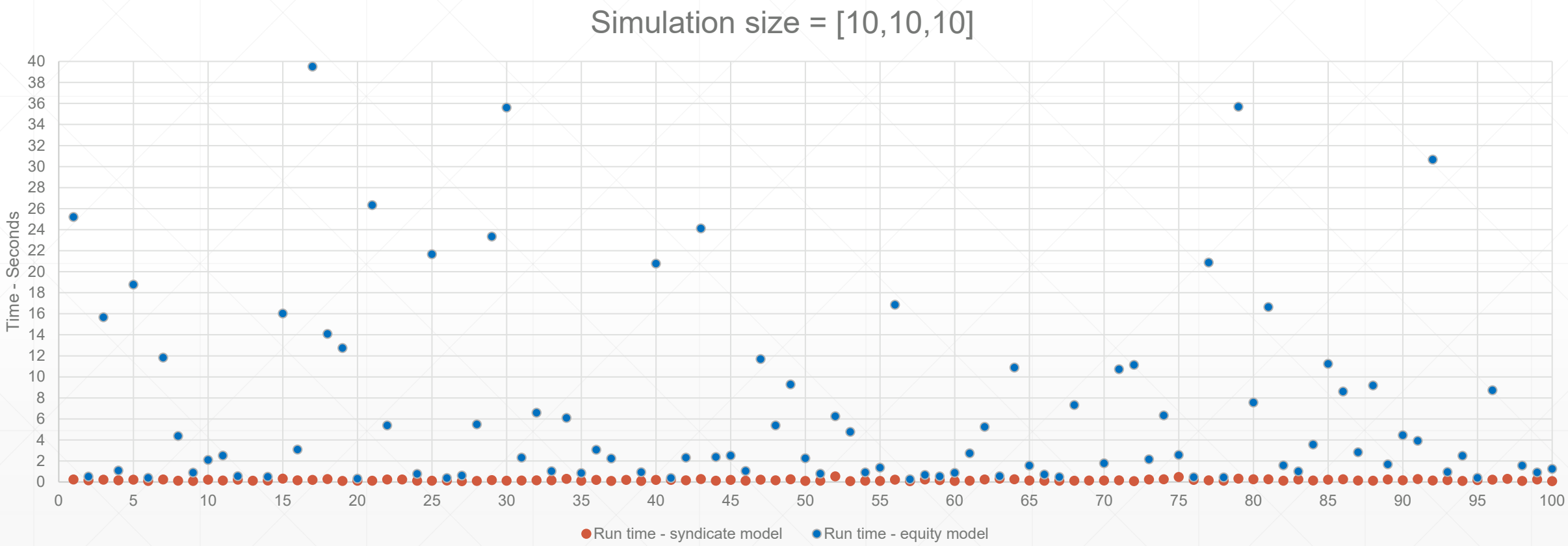
Results – specific simulation [5,5,5]

Unique Bins	Synd. Names	Item Size	Bin Volume	Price to buy bin	Avail 0?	Avail 1?	Avail 2?	Avail 3?	Avail 4?	bins filled	items packed in below bin	buy from synd 0?	list	B.F 1?	L 1	B.F 2?	L 2	B.F 3?	L 3	B.F 4?	L 4
0	0	3	6	8	1	0	0	0	0	0	3	0	[]	0	[3]	0	[]	0	[9]	0	[]
1	1	3	8	6	0	1	0	0	0	0	3	0		0		0		0		0	
2	2	2	8	7	0	1	0	0	0	0	3	0		0		0		0		0	
3	3	4	8	4	0	1	0	0	0	1	9	0		1		0		0		0	
4	4	3	6	9	0	1	0	0	0	0	9	0		0		0		0		0	
5			8	6	0	0	1	0	0	0		0		0		0		0		0	
6			8	7	0	0	1	0	0	0		0		0		0		0		0	
7			6	9	0	0	1	0	0	0		0		0		0		0		0	
8			8	6	0	0	0	1	0	0		0		0		0		0		0	
9			8	4	0	0	0	1	0	1		0		0		0		1		0	
10			6	9	0	0	0	1	0	0		0		0		0		0		0	
11			8	6	0	0	0	0	1	0		0		0		0		0		0	
12			8	7	0	0	0	0	1	0		0		0		0		0		0	
13			8	4	0	0	0	0	1	0		0		0		0		0		0	

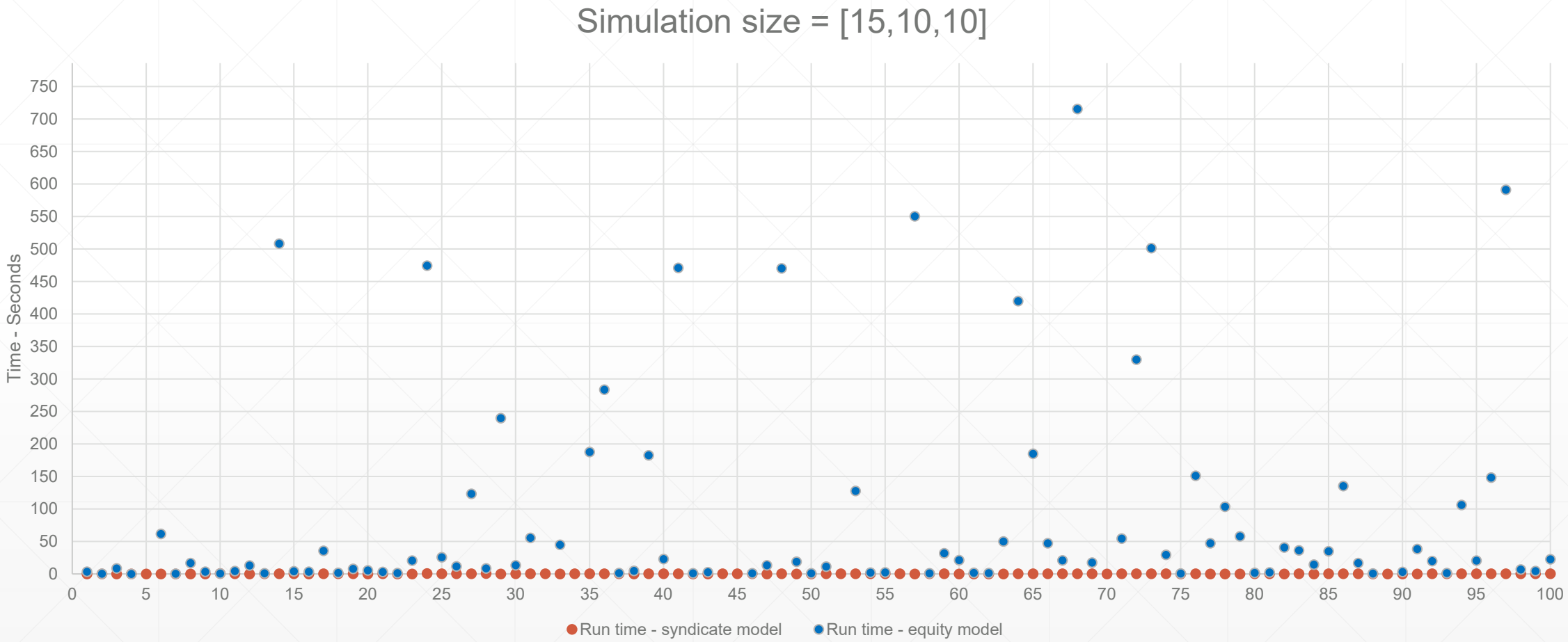
Time to run [5,5,5]



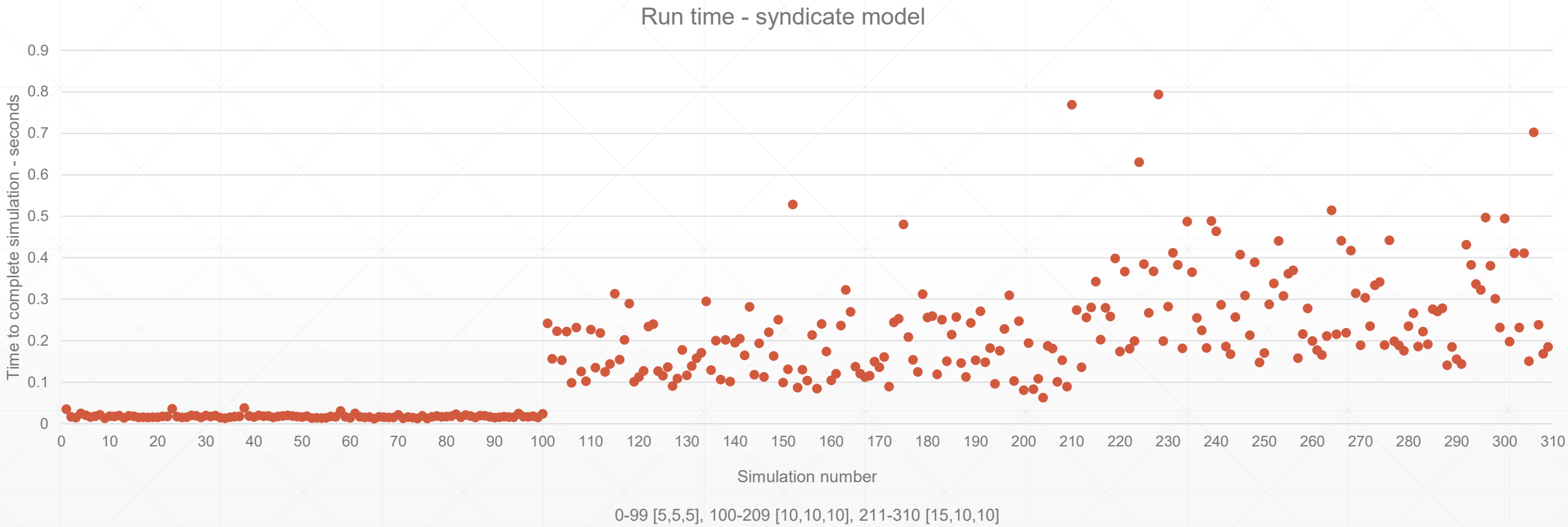
Time to run [10,10,10]



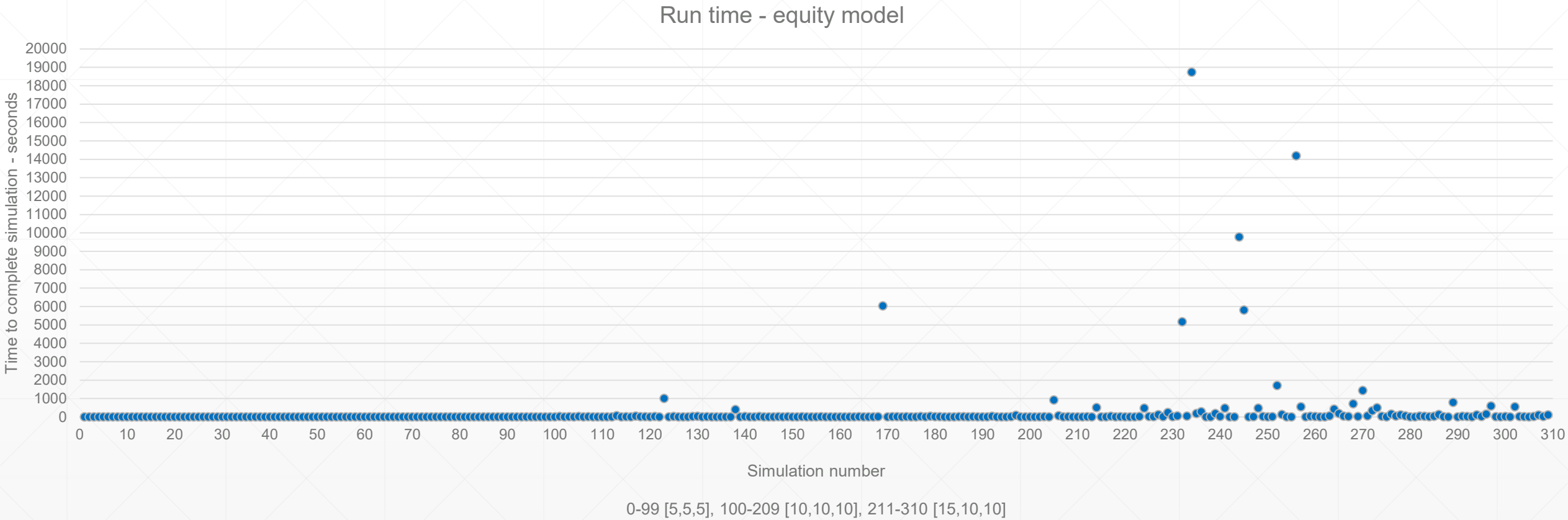
Time to run [15,10,10]



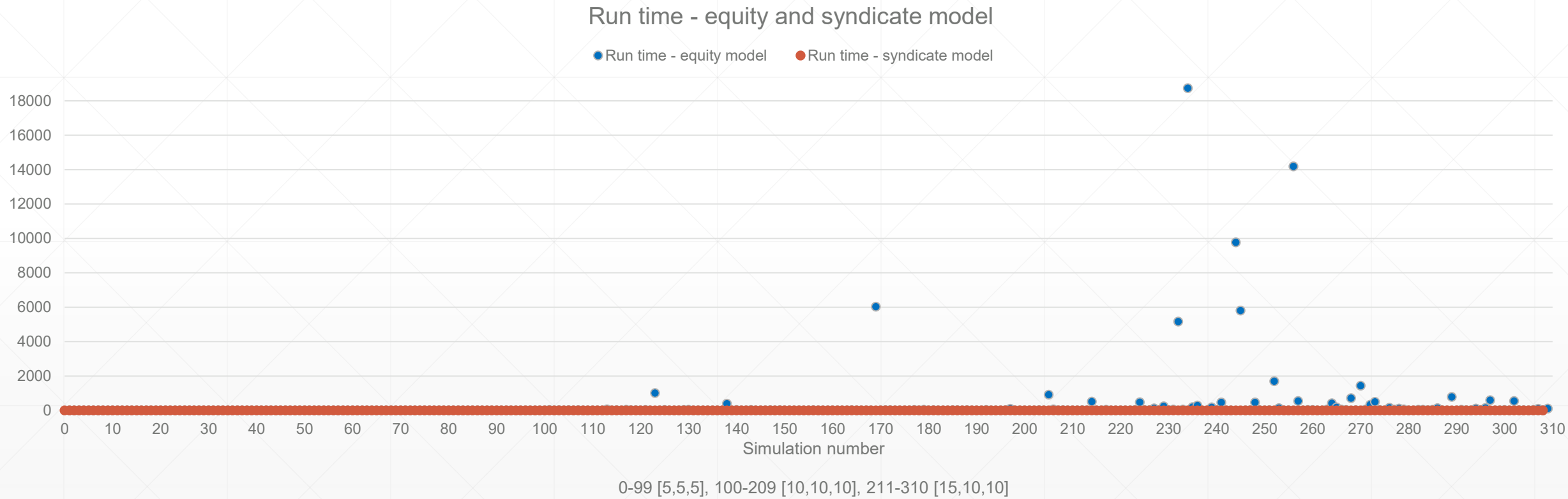
Time to run syndicate model



Time to run equity model

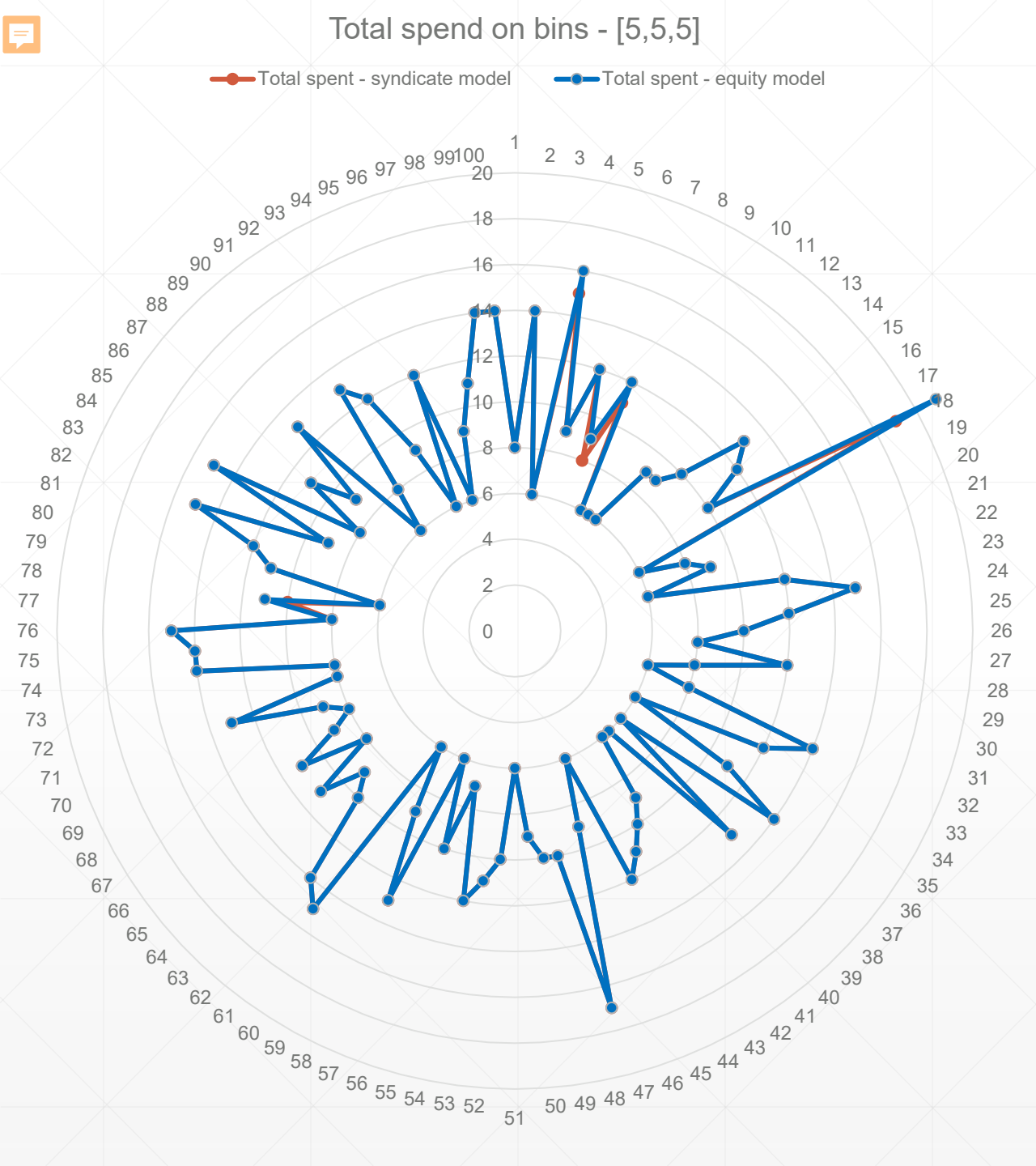


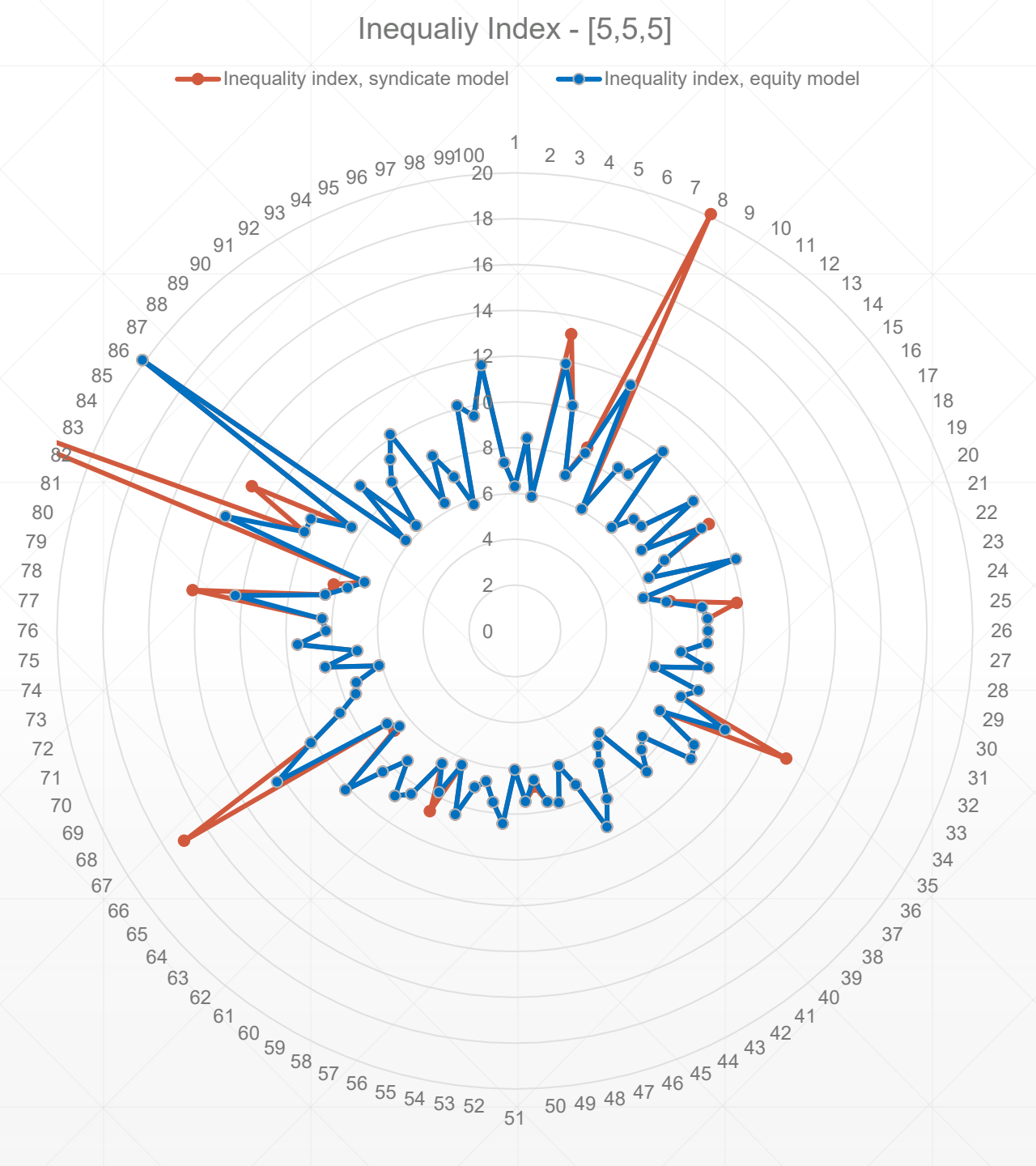
Time to run all

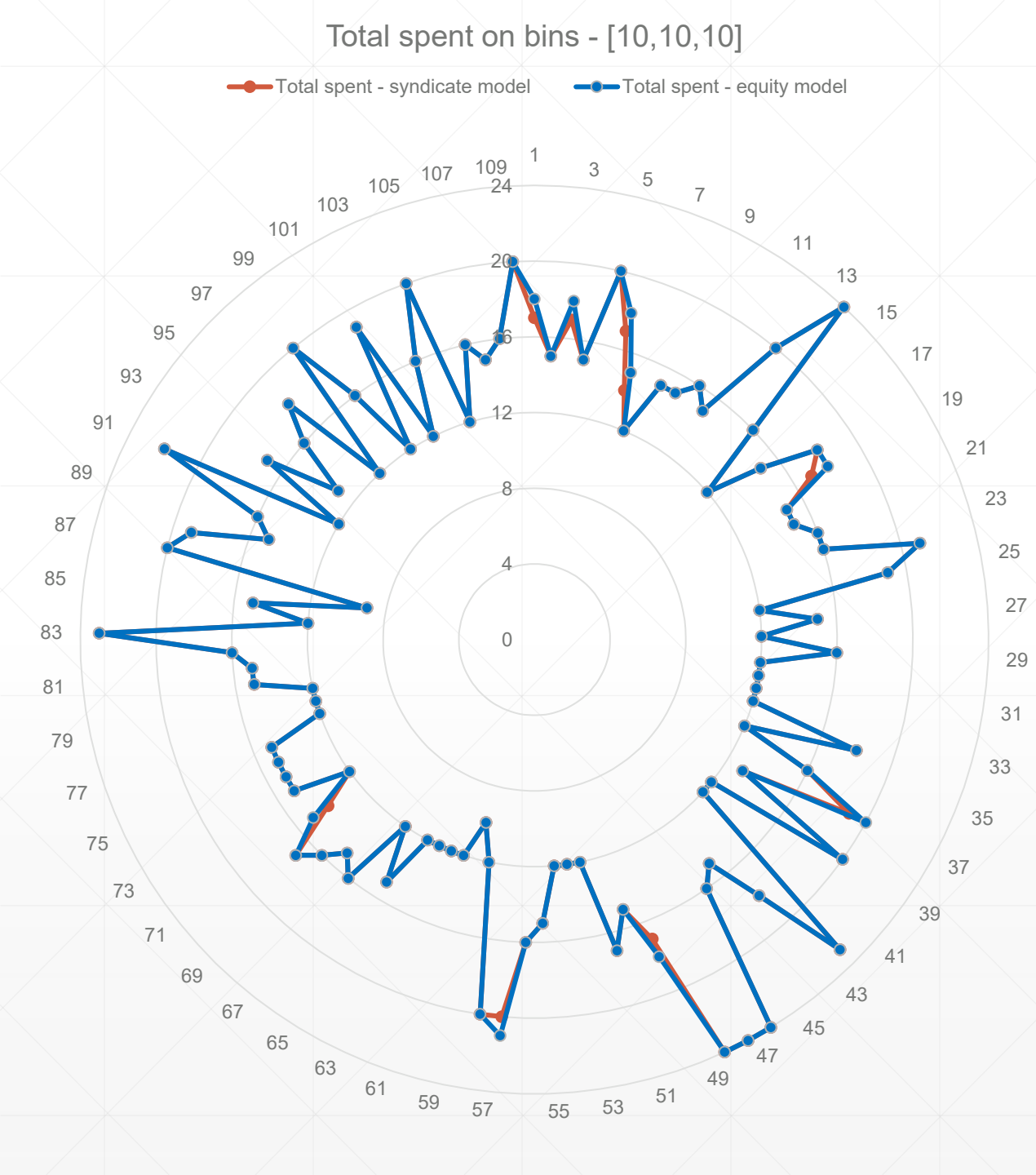


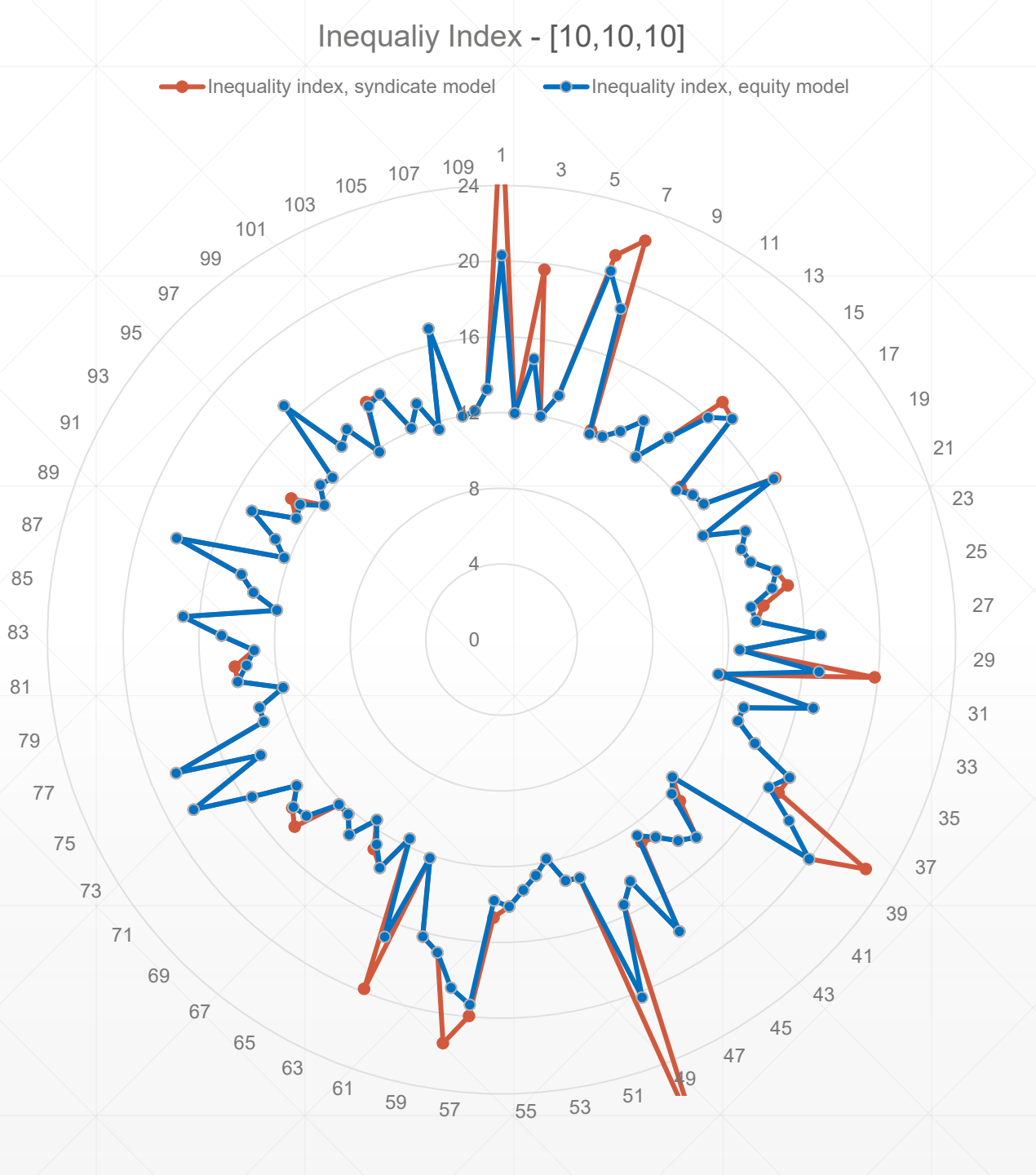
Total run times

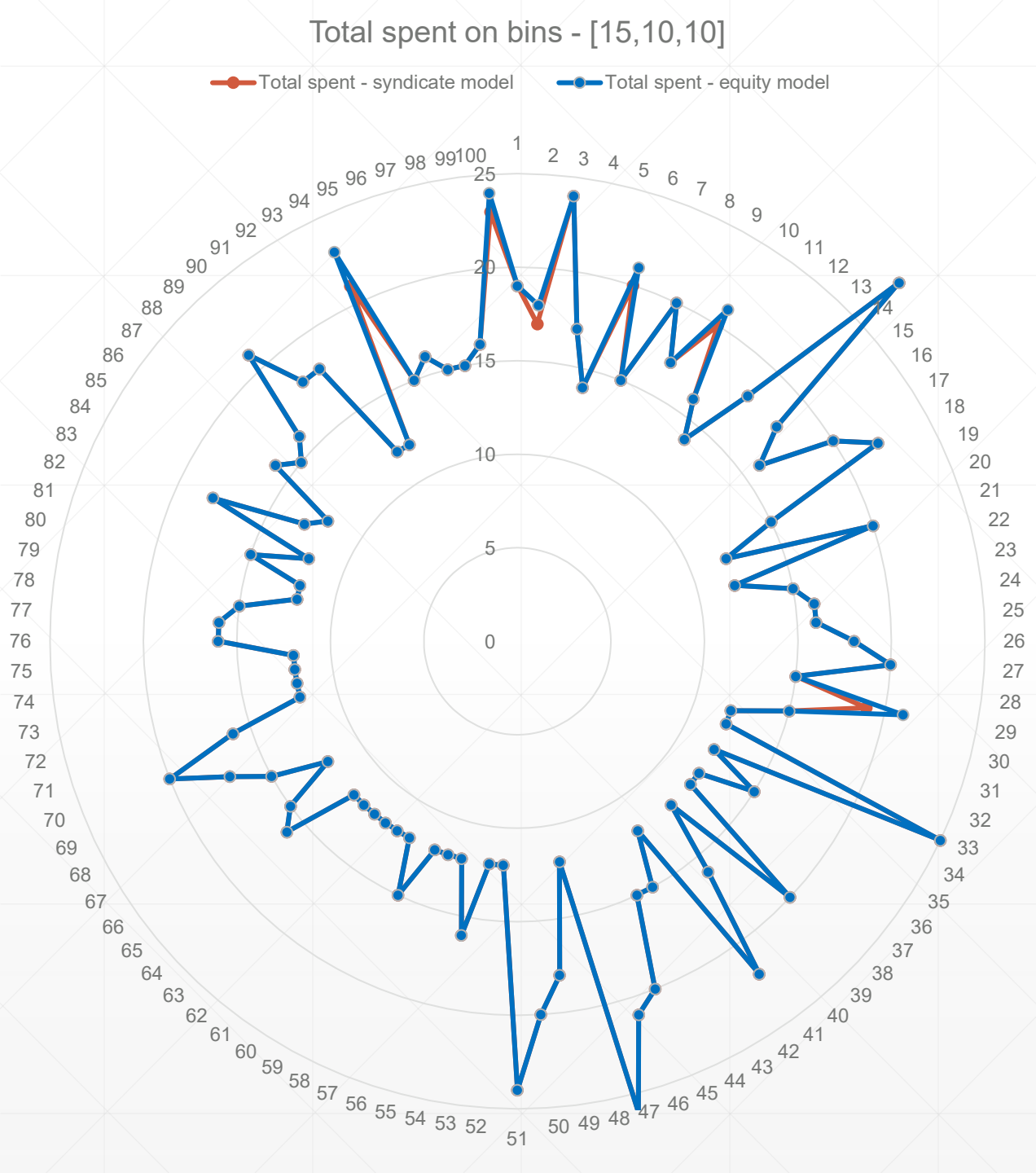
Sample Size	Average run time - syndicate model (sec)	Average run time - equity model (sec)	Delta	% Delta
5,5,5	0.017783685	0.155541496	0.137758	-88.57%
10,10,10	0.176804026	85.18156917	85.00477	-99.79%
15,10,10	0.298722172	664.2305109	663.9318	-99.96%
5,10,15*	0.10651021	1.05272975	0.94622	-89.88%
15,15,15*	0.84802413	629.6023943	628.7544	-99.87%
10,20,30*	1.121283054	2.657177448	1.535894	-57.80%
25,10,20*	0.974376917	1412.861234	1411.887	-99.93%
15,30,40*	178.2820818	245.6341405	67.35206	-27.42%
20,10,10*	0.377670765	44.89071059	44.51304	-99.16%
50,50,50**	1482.230456			

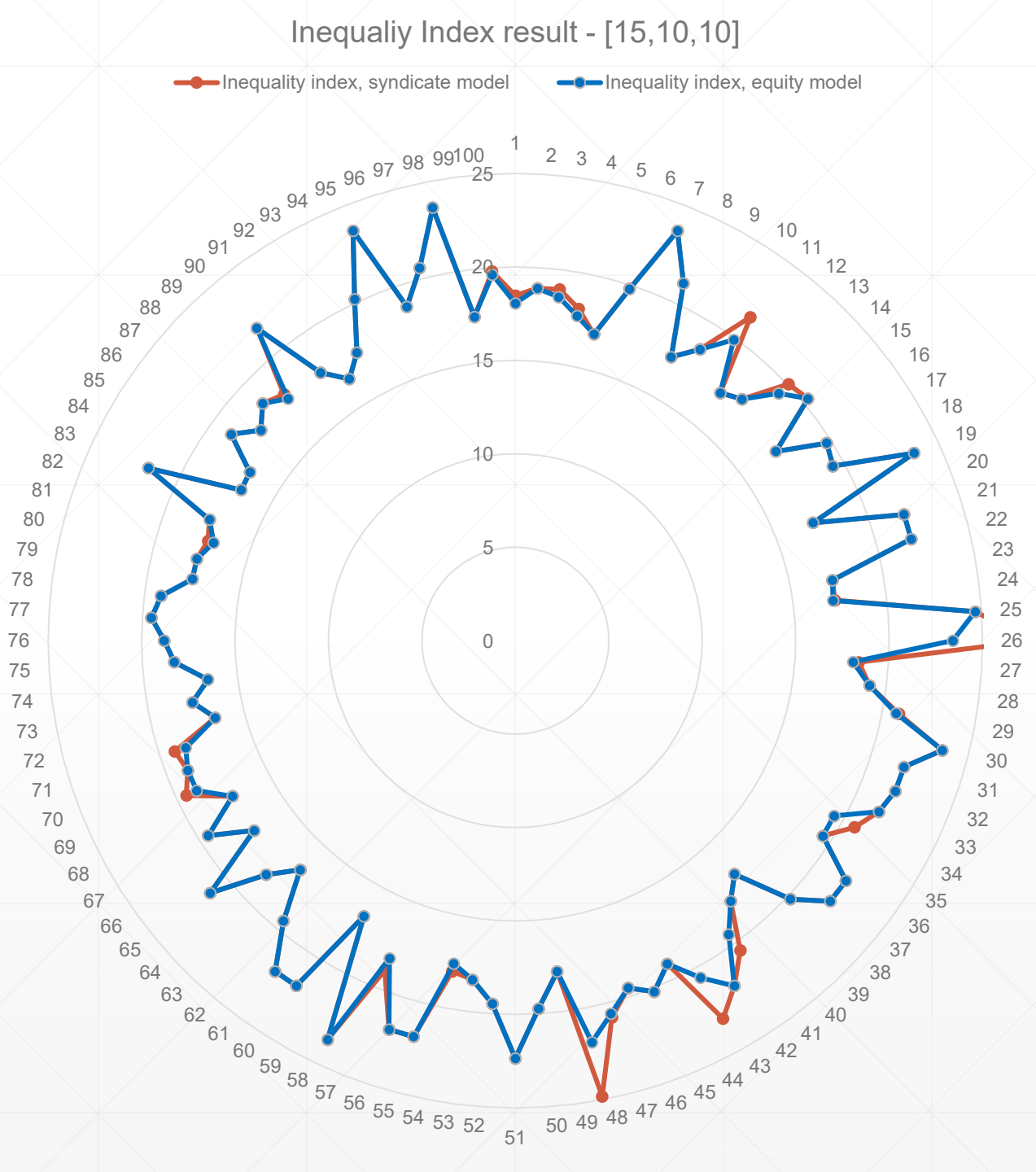

Total spend on bins - [5,5,5]
Total spent - syndicate model
Total spent - equity model


Inequaliy Index - [5,5,5]
Inequality index, syndicate model
Inequality index, equity model


Total spent on bins - [10,10,10]
Total spent - syndicate model
Total spent - equity model


Inequaliy Index - [10,10,10]
Inequality index, syndicate model
Inequality index, equity model


Total spent on bins - [15,10,10]
Total spent - syndicate model
Total spent - equity model


Inequaliy Index result - [15,10,10]
Inequality index, syndicate model
Inequality index, equity model

Total inequality index

Sample Size	Average inequality index - syndicate model	Average inequality index - equity model	Delta	% Delta
5,5,5	8.684478454	8.249869373	0.434609	5.27%
10,10,10	14.67870295	14.23254598	0.446157	3.13%
15,10,10	19.65028696	19.45660276	0.193684	1.00%
5,10,15*	11.75336921	10.11577305	1.637596	16.19%
15,15,15*	17.37367455	17.37367358	9.67E-07	0.00%
10,20,30*	18.07494853	17.80249123	0.272457	1.53%
25,10,20*	33.59140914	33.59140647	2.67E-06	0.00%
15,30,40*	20.21192275	20.00490556	0.207017	1.03%
20,10,10*	22.14025204	22.14025172	3.17E-07	0.00%

Total spend on bins

Sample Size	Average spend on bins - syndicate model	Average spend on bins - equity model	Delta	% Delta
5,5,5	10.07	10.13	-0.06	-0.59%
10,10,10	15.58715596	15.66972477	-0.08257	-0.53%
15,10,10	15.93	16.01	-0.08	-0.50%
5,10,15*	14.6	14.8	-0.2	-1.35%
15,15,15*	18	18	0	0.00%
10,20,30*	27	28	-1	-3.57%
25,10,20*	15	15	0	0.00%
15,30,40*	33	33	0	0.00%
20,10,10*	12	12	0	0.00%

Sources

- https://www.researchgate.net/figure/Illustration-of-the-bin-packing-problem_fig3_359599093
- https://designintech.report/2019/03/11/%F0%9F%93%B1design-in-tech-report-2019-section-6-addressing-imbalance/
- https://interactioninstitute.org/illustrating-equality-vs-equity/