(1) Find the smallest IEEE 64bit floating point number $x_1 > 16$ (i.e., the next double precision number after 16), and the largest IEEE 64bit floating point number $x_2 < 16$ (i.e., the previous double precision number before 16). In both cases, find all bits of x_1 or x_2 and the difference from 16, and write the difference as a multiple of the machine epsilon eps. You can use the Matlab function num2bitchar provided to find the bits of IEEE 64bit floating point numbers.

I foresee a potential error in my understanding, so I present two answers for x_1 and x_2.

(a) We can add the subnormal, smallest representational number (SRT), $SRT = 2^{-52} \times 2^{-1022} = 2^{-1074}$. While technically a number, observe that due to technical limits, $x_1 = SRT + 16 = 16$. In fact, $\forall r \in \mathbb{R}$, $r + SRT = r$

(b) Alternatively Choose $\epsilon_{mach}(16) = 3.552713678800501e^{-15}$.
Choose $x_1 = 16 + \epsilon_{mach}(16) > 16$, and observe due to finite memory that $\nexists\, \epsilon < \epsilon_{mach}$ s.t $16 + \epsilon > 16$. In fact, due to limitations, $\forall \epsilon < \epsilon_{mach}(16)$, $16 + \epsilon = 16$. WLOG, x_2 is chosen similarly. Results of MATLAB code are below.

```
Q1
The smallest representable change from 16 is eps(16) As a direct consequence
this means x1 = 16+eps(16) and x2 = 16-eps(16), are the only two additional
real numbers (that can be represented) in the range [16-eps(16),16+eps(16)]

The binary representation of 16 is
0100000000110000000000000000000000000000000000000000000000000000

The binary representation of x1 > 16 is
0100000000110000000000000000000000000000000000000000000000000001

The binary representation of x2 < 16 is
0100000000101111111111111111111111111111111111111111111111111110

The value of x1 is 16.000000000000004

The value of x2 is 15.999999999999996

Observe eps(16) == 16*eps. The following variable (multipleEPS) is set to the
logical condition eps(16) == 16*eps
multipleEPS =

  logical

   1
```

(2) Rearrange the expression $1.0000001 - \sqrt{1.0000002}$ to compute without loss of significance. Compute in MATLAB the values of the original and rearranged expressions.

Let $a = 1.0000001$ and let $b = 1.000002$ Observe
$a - \sqrt{b} = a - \sqrt{b} \times \frac{a+\sqrt{b}}{a+\sqrt{b}} = \frac{a^2-b}{a+\sqrt{b}}$. Results of MATLAB code are below.

```
>> hw2

originalExpressionEvaluation =

     5.107025913275720e-15


reworkedExpressionEvaluation =

     5.107025402573192e-15
```