# Chapter 3: System Modelling and Assessment

# 3.1 Water Resources Systems and Models

## 3.1.1 A Typical Water Resources System

A watershed is expected to supply water for a city, an agricultural area, and the downstream aquatic flora and fauna (see **Figure A** below for a layout of the watershed, river, and water supply intakes and returns). Approximately 45% of the city water withdrawals return to the river through the sewer system. Water returns from irrigation are assumed negligible.

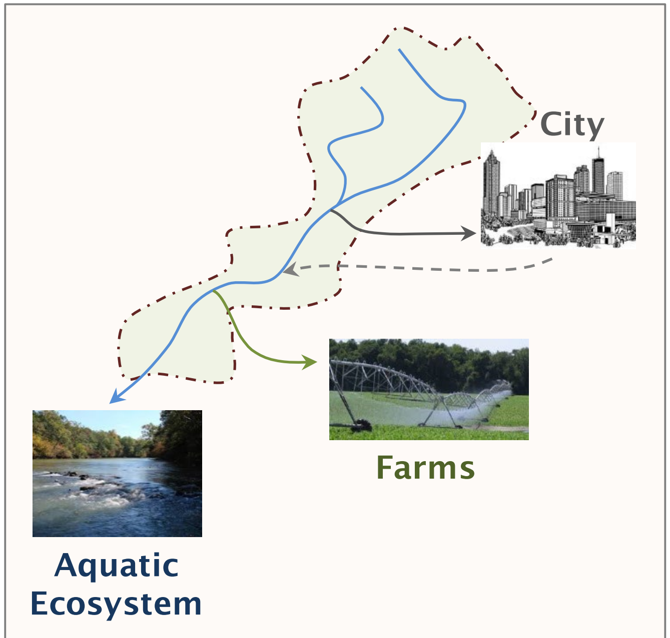



**Figure A:** River and Water Uses Layout

The annual river flow (natural, or unimpaired flow) at the watershed outlet for the period 1939 to 2008 is shown in **Figure B** and has a mean of 2028 cfs and standard deviation of 536 cfs.

The value of water supplied to the city is reflected in its water demand function (**Figure C**) which provides the value of one acre-foot of water as a function of the total annual water supply. Likewise, the farmer demand function is depicted in **Figure D**. The demand functions show that the marginal value of water *increases* as the total water allocation *decreases*. If the total water supply is Q, the price a user group is willing to pay is the value of its demand function at Q, and a measure of its economic welfare is the *consumer surplus (*shaded area in the figures).

The watershed Commission is mandated to allocate certain water supply targets for the city and the farms. If the annual flow is large enough to meet these targets, stakeholder welfare is measured by the consumer surplus. If, however, the targets cannot be met, the city and/or the farmers purchase the deficit from other sources at a price equal to the value of their demand function at the water amount actually supplied by the watershed. In these deficit years, the stakeholder welfare is measured by the consumer surplus less the cost of purchasing the deficit.

At least 50% of the annual flow is reserved in the river to sustain the downstream aquatic ecosystems.

Derive the stakeholder water share tradeoff as well as their economic welfare tradeoff.

**Figure B:** Natural Inflow Record

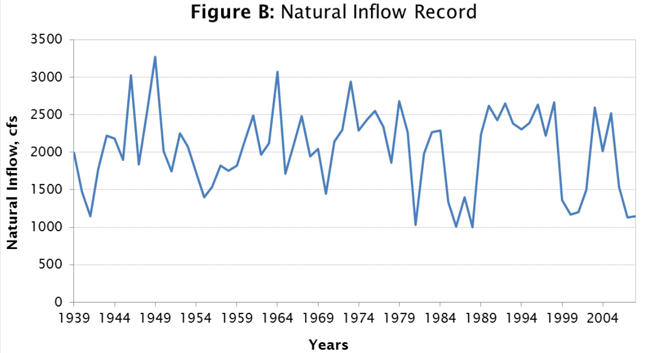



**Figure C:** City Water Demand Function

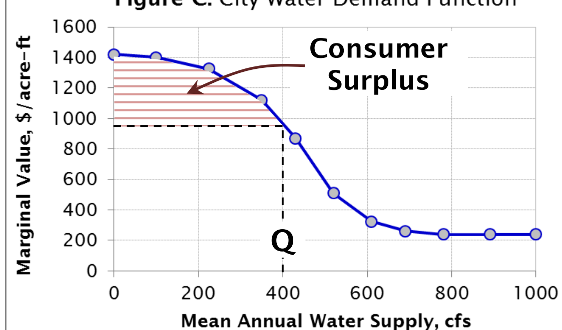
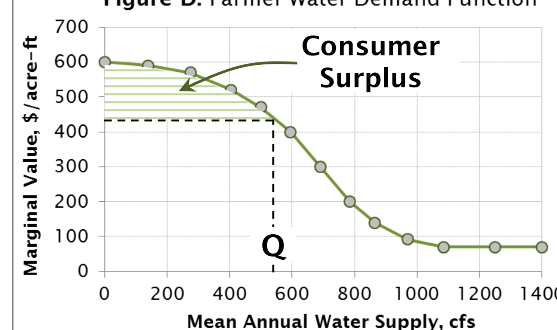



**Figure D:** Farmer Water Demand Function



# 3.1.2 Developing System Models

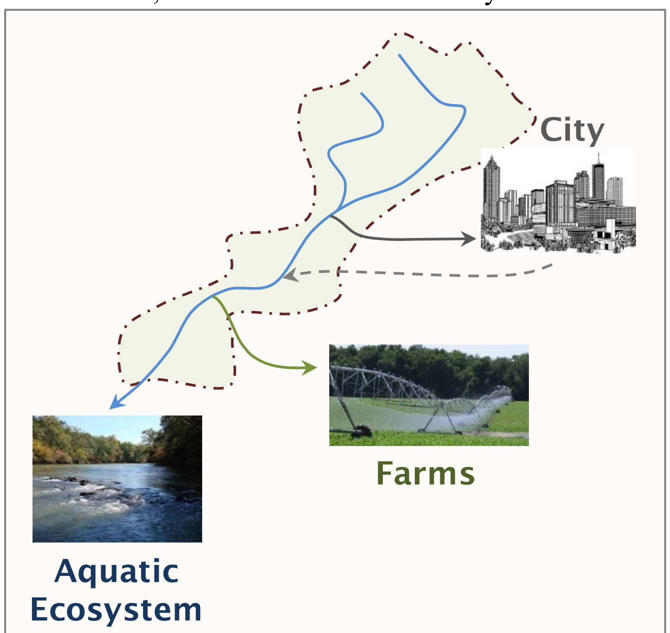
## System Context and Model Development

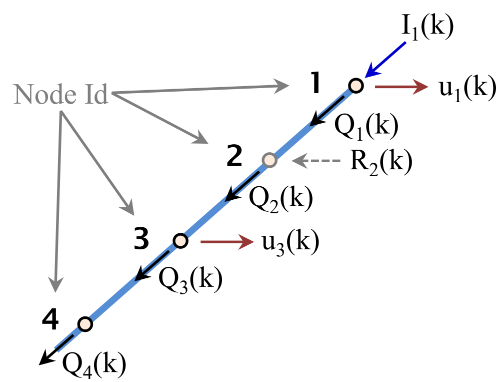
The above narrative describes (in quantitative detail) the system water resources (i.e., watershed surface runoff), stakeholder interests (i.e., urban and agricultural water supply, and environmental and ecological sustainability), alternative management decisions (i.e., water shares for each system stakeholder), and general criteria for assessing the system response to management decisions. These criteria call for meeting minimum environmental and ecological flow requirements, and maximizing urban and agricultural user welfare. The system is sufficiently complex to require the development of an analytical model to carry out the necessary assessments.

Model development generally involves the consistent mathematical definition and representation of four system elements: *(i)* Water resources system network and associated variables; *(ii)* flow relationships; *(iii)* water uses; *(iv)* management alternatives/decisions; and *(v)* system performance metrics (indices). These elements discussed next.

## i. System Network and Variable Definitions

The system network is conceptualized as illustrated below. The network includes four river nodes (1, 2, 3, and 4) each of which is associated with key system variables representing tributary inflows (I), water withdrawals (u), river flows (Q), and water returns (R). These variables are indexed by the node to which they correspond, and by a time index (k) indicating that they may take on different values in different years. The time index may range from 1 to N, where N is the number of years in the management horizon.




**Variable Definitions:**

k: Time index (years), k = 1, 2, …, N.

$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k. $I_1(k)$ observations are available for the period 1939 - 2008.

$u_1(k)$: City water withdrawal, node 1, year k.

$Q_1(k)$: Flow downstream of node 1, year k.

$R_2(k)$: City return flow, node 2, year k.

$Q_2(k)$: Flow downstream of node 2, year k.

$u_3(k)$: Irrigation water withdrawal, node 3, year k.

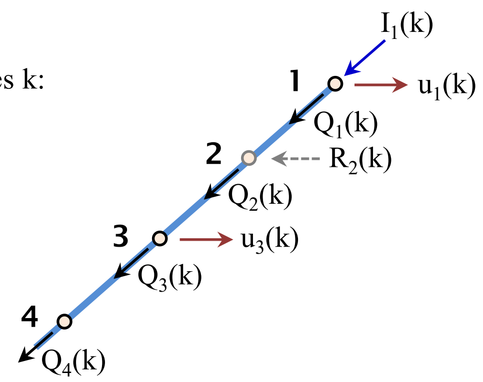
$Q_3(k)$: Flow downstream of node 3, year k.

$Q_4(k)$: Flow downstream of node 4, year k.

All variables represent fluxes and are expressed in units of *volume per year*.

# 3.1.2 Developing System Models[2]

## ii. Flow Relationships (System Model)




Flow relationships are based on mass balance and are stated for each system node and all times k:

**Node 1:** $Q_1(k) = I_1(k) - u_1(k)$; $k = 1, 2, \ldots, N$.

**Node 2:** $Q_2(k) = Q_1(k) + R_2(k)$; $k = 1, 2, \ldots, N$.

**Node 3:** $Q_3(k) = Q_2(k) - u_3(k)$; $k = 1, 2, \ldots, N$.

**Node 4:** $Q_4(k) = Q_3(k)$; $k = 1, 2, \ldots, N$.

Since there is no process contributing to or detracting from the flow between nodes 3 and 4, node 4 is unnecessary. Namely, the system model can comprise nodes 1, 2, and 3 without any information loss. It is generally advisable to reduce a system model to its simplest (or minimal) form, namely, a form that retains all essential features but no redundant relationships and variables. The mass balance relationships at nodes 1, 2, and 3 will henceforth be considered to comprise the system model.

The system variables can be distinguished in three categories: *(a) Natural inputs* determined by physical processes alone [i.e., $I_1(k)$]; *(b) regulated inputs* subject to management decisions [i.e., $u_1(k)$, $R_2(k)$, and $u_3(k)$]; and *(c) system outputs* determined by the flow relationships and the natural and regulated inputs [i.e., $Q_1(k)$, $Q_2(k)$, and $Q_3(k)$].

The flow relationships are mathematically incomplete without specification of the variable ranges within which they apply. Generally, all system variables are characterized by their own applicable ranges. The ranges of natural inputs are inherent in the processes that generate them, and are already incorporated in their values and sequences. The ranges of the regulated inputs are specified in the next section as part of the representation of the water uses and/or management decisions. The variable ranges that need to be specified in this section are those pertaining to *physically permissible* conditions for the output variables $Q_1(k)$, $Q_2(k)$, and $Q_3(k)$. In this system example, such conditions require that all river flows be non-negative at all times:

$$Q_1(k) \geq 0, \;\; Q_2(k) \geq 0, \;\; Q_3(k) \geq 0; \;\; \text{all } k = 1, 2, \ldots, N.$$

The system model consisting of the mass balance relationships for nodes 1, 2, and 3 and the non-negativity constraints stated above is now complete.

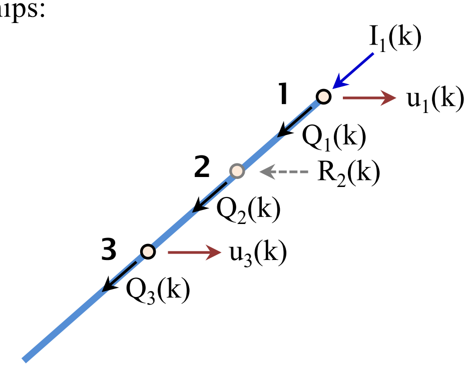
A last note pertains to the possibility of reducing the number of flow relationships (and associated variables) even further. This could potentially be achieved by substituting $Q_1(k)$ into the equation for $Q_2(k)$, and the resulting expression for $Q_2(k)$ into the equation for $Q_3(k)$, leading to $Q_3(k) = I_1(k) - u_1(k) + R_2(k) - u_3(k)$; $k = 1, 2, \ldots, N$. While these are legitimate mathematical operations, the final water balance relationship (and variable set) would not accurately represent the functioning of the physical system. This is because the flows $Q_1(k)$ and $Q_2(k)$ would become "invisible" in the aggregate water balance relationship, allowing the mathematical model to consider unrealistic management decisions for which the flows may become negative.

# 3.1.2 Developing System Models[3]

## iii. Water Uses

The water uses are modelled by the variables $u_1(k)$ (urban water supply), $u_3(k)$ (agricultural water supply), and $Q_3(k)$ (water for the environment and ecosystems). Additionally, $R_2(k)$ describes the return flow associated with $u_1(k)$.

Modeling of the water uses also includes the following operational requirements and relationships:




**Environmental and Ecological Flow:**

$Q_3(k) \geq 0.5\ I_1(k)$; all $k = 1, 2, \ldots, N$.

**Urban Water Supply:**

$u_1(k) \geq 0$, $R_2(k) = 0.45\ u_1(k)$; all $k = 1, 2, \ldots, N$.

**Agricultural Water Supply:**

$u_3(k) \geq 0$, all $k = 1, 2, \ldots, N$.

*Note 1:* The environmental and ecological flow constraint *also* reflects an already made management decision to retain at least 50% of the tributary inflow for sustaining the aquatic ecosystem. If this minimum inflow percent threshold were free to be determined by the decision process, it would be included in the management alternatives and decisions of the next section.
*Note 2:* It is not necessary to impose upper $u_1(k)$ and $u_3(k)$ limits, although these variables cannot have arbitrary large values. This is because infeasibly large values of these variables would trigger violation of other stated constraints and would automatically be excluded.

## iv. Management Alternatives and Decisions

The system management alternatives and decisions are related to the urban and agricultural user allocations [$u_1(k)$ and $u_3(k)$]. (Other management alternatives can also be envisioned and will be explored in subsequent examples.) However, the variables $u_1(k)$ and $u_3(k)$ defined earlier represent *actual* water use levels that stakeholders receive in year k, and can only be determined *after* inflows $I_1(k)$ are realized and nodal flows computed. Management decisions, on the other hand, are made at the *beginning* of each year. Accordingly, the only plausible decision option the Commission has is to set water sharing t*argets*. Furthermore, since inflows, water allocations, and resultant flows are independent from year to year, there is no basis to assume that these targets be time varying.

In view of the above, the decision space consists of time invariant water share targets $\{U_1, U_3\}$ for the city and the farmers, and the management problem is to determine their numerical values leading to desirable actual water share sequences $\{u_1(k), u_3(k);\ k = 1, 2, \ldots, N\}$ and system performance.

# 3.1.2 Developing System Models[4]

## v. Performance Metrics (Indices)

The general stakeholder performance criteria in the system narrative refer to *(i)* stakeholder water shares and *(ii)* economic welfare levels. However, the metrics by which to measure these criteria remain to be defined.

### Water Share Metrics:

Water shares metrics are intended to assess the absolute and relative magnitudes of the annual water volumes allocated to the system stakeholders. In any given year, however, the actual water shares [$u_1(k)$ and $u_3(k)$] depend on the annual inflow volume [$I_1(k)$] as well as on the water share targets $\{U_1, U_3\}$, and vary randomly. Because of this, they cannot explicitly serve as metrics. On the other hand, a statistic (such as the mean) of the actual annual water shares would depend uniquely on the decisions $\{U_1, U_3\}$ and would be a legitimate metric.

There are many different water shares statistics that can be adopted as performance metrics. These include the water shares mean (or expected value), standard deviation, minimum value, maximum value, and/or some critical percentile of the water share distribution such as the value exceeded 95%, 75%, 50%, 25%, or 5% of the time.

The water share performance metrics for the city and the farms will be denoted $J_{xyz}(U_1)$ and $J_{xyz}(U_3)$ respectively, where the subscript xyz may represent “mean”, “stdev”, “min”, “95%”, or any of the metrics mentioned above.

### Economic Welfare Metrics:

Let $D[u_1(k)]$ and $D[u_3(k)]$ denote respectively the marginal value of the water shares $u_1(k)$ and $u_3(k)$ for the city and the farmers in year k based on their individual demand curves. Let also $CS[u_1(k)]$ and $CS[u_3(k)]$ denote the corresponding *consumer surpluses* in year k, computed as detailed in the system narrative. Then, in any given year k, the economic welfare of the system stakeholders is calculated as follows:

$$EW[u_1(k)] = CS[u_1(k)] - D[u_1(k)] \max\{U_1 - u_1(k), 0\}$$

$$EW[u_3(k)] = CS[u_3(k)] - D[u_3(k)] \max\{U_3 - u_3(k), 0\}$$

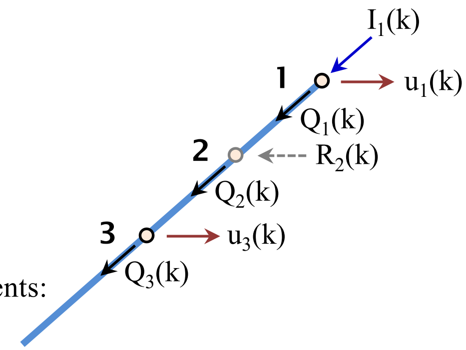
As these expressions indicate, $EW[u_1(k)]$ and $EW[u_3(k)]$ depend on $I_1(k)$ and vary from year to year. For this reason, economic welfare performance metrics need to based on some statistics of the underlying distributions, much like the water share metrics. The economic welfare metrics will be denoted $V_{xyz}(U_1)$ and $V_{xyz}(U_3)$, where the subscript xyz may represent “mean”, “stdev”, “min”, “95%”, or any other appropriate statistics.

### Environmental and Ecosystem Flow Metrics:

In this system, $\{U_1, U_3\}$ do not impact the flow reserved for the aquatic ecosystems. This flow is 0.5 $I_1(k)$ for all years k.

# 3.1.2 Developing System Models[5]

## Management Model Summary




### Management Objective:

Determine the city and farmer water shares, $U_1$ and $U_3$, such that

$$\{J_{xyz}(U_1), J_{xyz}(U_3)\} \text{ or } \{V_{xyz}(U_1), V_{xyz}(U_3)\}$$

are optimized, subject to the following system model and associated physical and operational requirements:

### System Model:

$$Q_1(k) = I_1(k) - u_1(k); \quad u_1(k) = \min\{U_1, I_1(k)\}; \quad k = 1, 2, \ldots, N.$$
$$Q_2(k) = Q_1(k) + R_2(k); \quad R_2(k) = 0.45\, u_1(k); \quad k = 1, 2, \ldots, N.$$
$$Q_3(k) = Q_2(k) - u_3(k); \quad u_3(k) = \min\{U_3, Q_2(k)\}; \quad k = 1, 2, \ldots, N.$$

### Physical Requirements:

The physical non-negativity requirements,

$$Q_1(k) \geq 0, \;\; Q_2(k) \geq 0, \;\; Q_3(k) \geq 0; \;\; \text{all } k = 1, 2, \ldots, N,$$

have already been incorporated in the system model equations at each node.

### Operational Requirements:

$$Q_3(k) \geq 0.5\, I_1(k); \quad \text{all } k = 1, 2, \ldots, N.$$
$$U_1 \geq 0; \quad \text{all } k = 1, 2, \ldots, N.$$
$$U_3 \geq 0; \quad \text{all } k = 1, 2, \ldots, N.$$

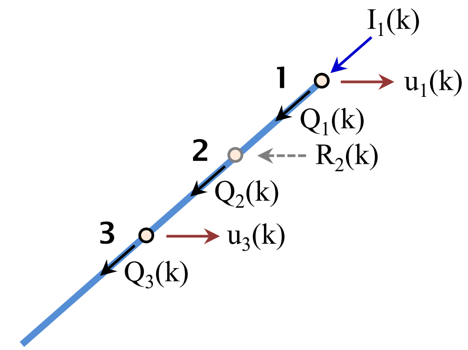
### Variable Definitions:

k: Time index (years), k = 1, 2, …, N.
$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k. $I_1(k)$ observations are available for the period 1939 - 2008.
$u_1(k)$: City water withdrawal, node 1, year k.
$Q_1(k)$: Flow downstream of node 1, year k.
$R_2(k)$: City return flow, node 2, year k.
$Q_2(k)$: Flow downstream of node 2, year k.
$u_3(k)$: Irrigation water withdrawal, node 3, year k.
$Q_3(k)$: Flow downstream of node 3, year k.
All variables represent fluxes and are expressed in units of *volume per year*.

# 3.1.3 System Simulation Using Models

## System Simulation/Response for Given Water Shares

Let the city and farmer water shares be some specific annual quantities $U_1$ and $U_3$. The purpose of the simulation process is to determine the values of the system variables, $\{u_1(k), Q_1(k), R_2(k), Q_2(k), u_3(k)$, and $Q_3(k)$; $k = 1, 2, \ldots, N\}$, and the values of the performance metrics, $\{J_{xyz}(U_1), J_{xyz}(U_3)\}$ or $\{V_{xyz}(U_1), V_{xyz}(U_3)\}$, associated with these shares over the 1939 – 2008 hydrologic period (N = 70 years).




### Simulation Logic:

Starting from k = 1 (i.e., year 1939), system simulation can proceed from upstream to downstream (node 1 to node 3) using the flow relationships to compute the unknown variables. Namely,

<u>k = 1:</u> $I_1(k)$ = Inflow of 1939

**Node 1:**
$u_1(k) = \min\{U_1, I_1(k)\}$;
$Q_1(k) = I_1(k) - u_1(k)$;

**Node 2:**
$R_2(k) = 0.45\, u_1(k)$;
$Q_2(k) = Q_1(k) + R_2(k)$;

**Node 3:**
$u_3(k) = \min\{U_3, Q_2(k)\}$;
$Q_3(k) = Q_2(k) - u_3(k)$;

This calculation sequence guarantees that the values of the system variables meet the flow equations as well as all physical and operational requirements *except* $Q_3(k) \geq 0.5\, I_1(k)$. If the latter constraint is violated, a way to enforce it is to reduce $U_1$ and $U_3$, and repeat the calculations until it is also satisfied. **But how are $U_1$ and $U_3$ to be reduced?**

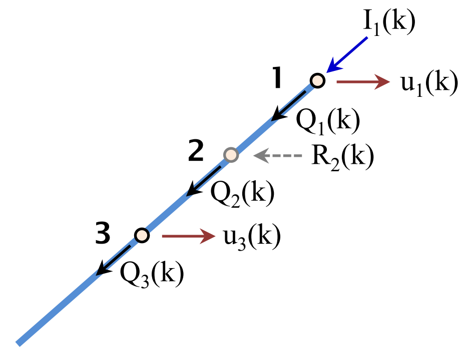
The $U_1$ and $U_3$ reduction rule in dry years constitutes the **drought management plan**, and it is an important decision that must be made by the system Commission, not by the technical analyst.

**Variable Definitions:**

k: Time index (years), k = 1, 2, …, N.
$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k. $I_1(k)$ observations are available for the period 1939 - 2008.
$u_1(k)$: City water withdrawal, node 1, year k.
$Q_1(k)$: Flow downstream of node 1, year k.
$R_2(k)$: City return flow, node 2, year k.
$Q_2(k)$: Flow downstream of node 2, year k.
$u_3(k)$: Irrigation water withdrawal, node 3, year k.
$Q_3(k)$: Flow downstream of node 3, year k.
All variables represent fluxes and are expressed in units of *volume per year*.

# 3.1.3 System Simulation Using Models[2]

## System Simulation/Response for Given Water Shares[2]




### Simulation Logic including a Drought Management Rule:

Let us assume that the system Commission decides that if a deficit occurs in any given year, it is to be shared by the city and the farmers *in proportion* to water shares $U_1$ and $U_3$. (Other reduction rules can also be envisioned, depending on stakeholder water rights and equity considerations.) Then, system simulation can proceed as follows:

k = 1: $I_1(k)$ = Inflow of 1939

**Node 1:**
$u_1(k) = \min\{U_1, I_1(k)\}$;
$Q_1(k) = I_1(k) - u_1(k)$;

**Node 2:**
$R_2(k) = 0.45\ u_1(k)$;
$Q_2(k) = Q_1(k) + R_2(k)$;

**Node 3:**
$u_3(k) = \min\{U_3, Q_2(k)\}$;
$Q_3(k) = Q_2(k) - u_3(k)$;

**Test for Environmental/Ecosystem Flow Deficit**:
If $Q_3(k) \geq 0.5\ I_1(k)$, then advance simulation to k+1.

If $Q_3(k) < 0.5\ I_1(k)$, then:
Compute the deficit: $D_{fct} = 0.5\ I_1(k) - Q_3(k)$;
Compute the water share reduction factors: $F_1 = U_1/(U_1+U_3)$ and $F_3 = U_3/(U_1+U_3)$;
Compute new water share targets: $U_j = \max\{U_j - F_j D_{fct}, 0\}$, j = 1, 3;
Repeat simulations for k until $D_{fct} \sim 0$, regardless if $Q_3(k) \geq 0.5\ I_1(k)$; then, advance to k+1.

k = 2: $I_1(k)$ = Inflow of 1940
Repeat flow simulations using the process outlined at time k.

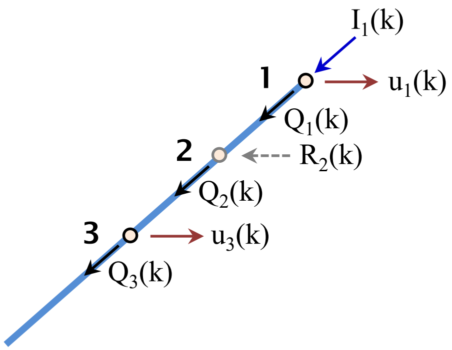
k = …

**Variable Definitions:**

k: Time index (years), k = 1, 2, …, N.
$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k. $I_1(k)$ observations are available for the period 1939 - 2008.
$u_1(k)$: City water withdrawal, node 1, year k.
$Q_1(k)$: Flow downstream of node 1, year k.
$R_2(k)$: City return flow, node 2, year k.
$Q_2(k)$: Flow downstream of node 2, year k.
$u_3(k)$: Irrigation water withdrawal, node 3, year k.
$Q_3(k)$: Flow downstream of node 3, year k.
All variables represent fluxes and are expressed in units of *volume per year*.

# 3.1.3 System Simulation Using Models[3]

## Performance Metrics for Given Water Shares




### Water Share Metrics:

Water share metrics are evaluated based on the *actual* sequences $\{u_1(k), u_3(k); k=1, 2, \ldots, N\}$ resulting from the system simulation. For example, if the metrics are the annual water share means over the simulation horizon, then,

$$J_{mean}(U_1) = 1/N\,[u_1(1) + u_1(2) + \ldots + u_1(N)]$$

$$J_{mean}(U_3) = 1/N\,[u_3(1) + u_3(2) + \ldots + u_3(N)]$$

Likewise, if the metrics are based on a particular water share percentile, x%, then,

$$J_{x\%}(U_1) = u_{1,x\%}$$

$$J_{x\%}(U_3) = u_{3,x\%}$$

where $u_{1,x\%}$ and $u_{3,x\%}$ are the actual water share values exceeded x% of the time.

### Welfare Metrics:

Welfare metrics are computed in a similar manner based on the actual economic welfare sequences $\{EW_1(k), EW_3(k); k=1, 2, \ldots, N\}$ computed as described previously. Thus, the mean welfare metric can be computed from:

$$V_{mean}(U_1) = 1/N\,[EW_1(1) + EW_1(2) + \ldots + EW_1(N)]$$

$$V_{mean}(U_3) = 1/N\,[EW_3(1) + EW_3(2) + \ldots + EW_3(N)]$$

And, the welfare x percentile metric from:

$$V_{x\%}(U_1) = EW_{1,x\%}$$

$$V_{x\%}(U_3) = EW_{3,x\%}$$

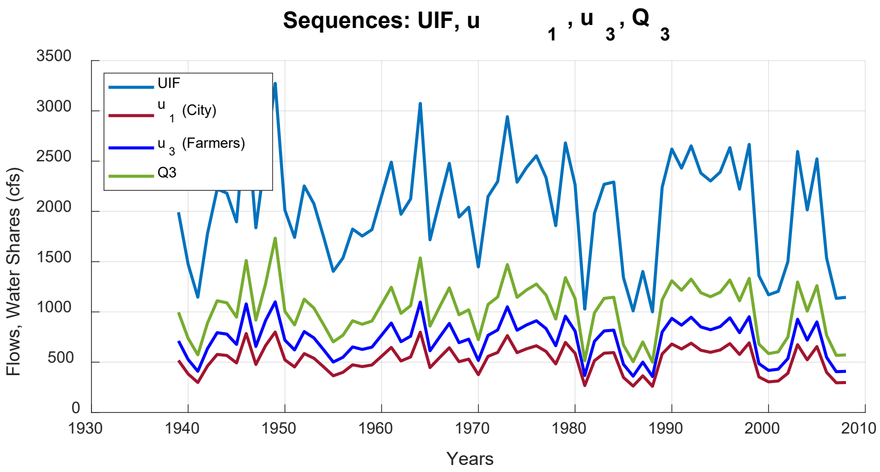
where $EW_{1,x\%}$ and $EW_{3,x\%}$ are the actual EW values exceeded x% of the time.

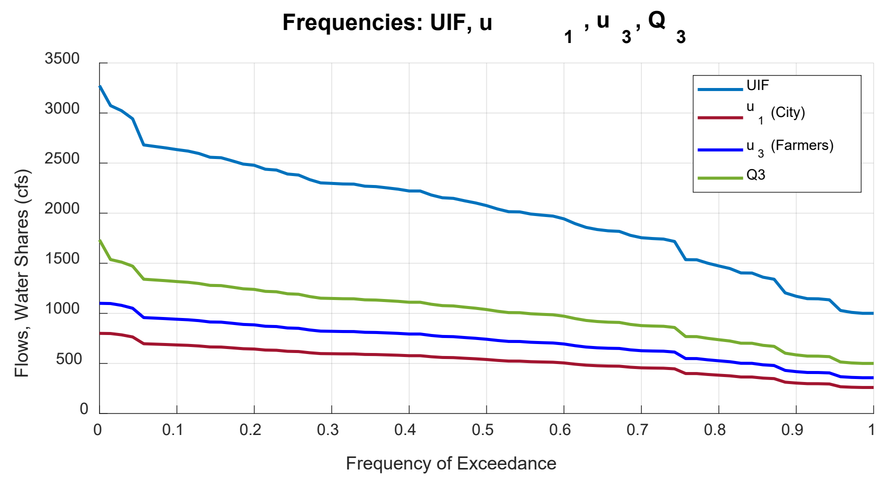
**Variable Definitions:**

k: Time index (years), k = 1, 2, …, N.
$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k. $I_1(k)$ observations are available for the period 1939 - 2008.
$u_1(k)$: City water withdrawal, node 1, year k.
$Q_1(k)$: Flow downstream of node 1, year k.
$R_2(k)$: City return flow, node 2, year k.
$Q_2(k)$: Flow downstream of node 2, year k.
$u_3(k)$: Irrigation water withdrawal, node 3, year k.
$Q_3(k)$: Flow downstream of node 3, year k.
All variables represent fluxes and are expressed in units of *volume per year*.

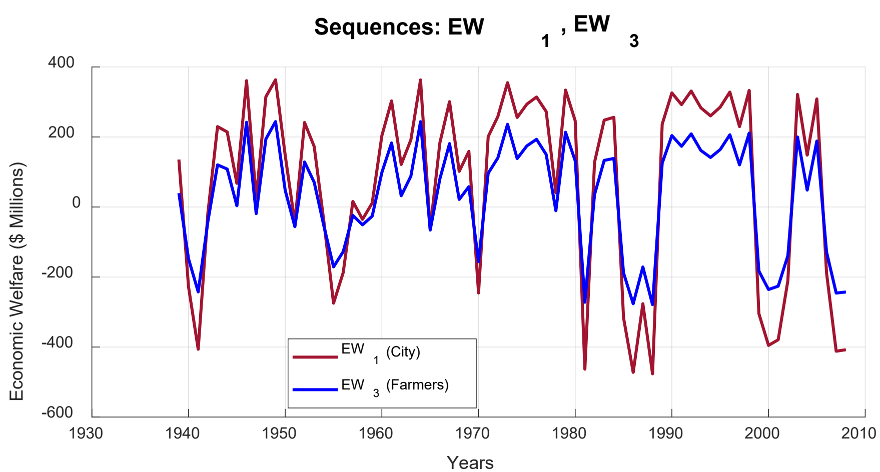
# 3.1.3 System Simulation Using Models[4]

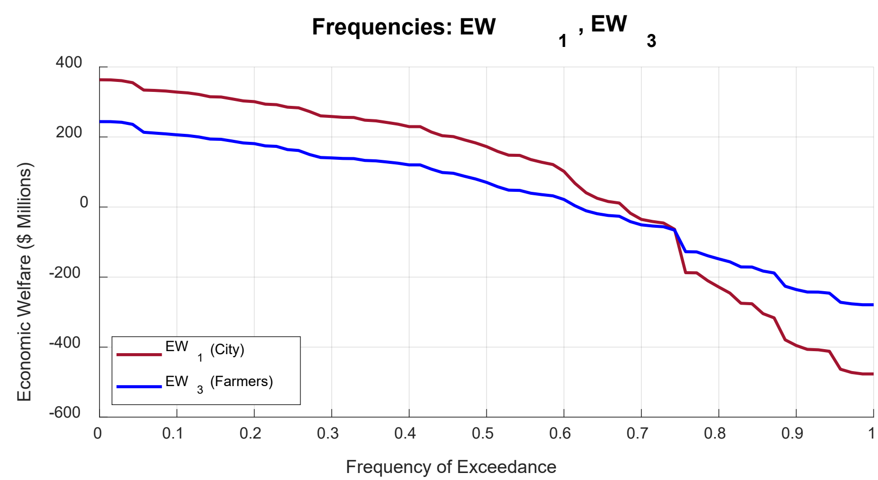
**Results:** Simulated variable sequences for $U_1$ = 800 cfs and $U_3$ = 1100 cfs:



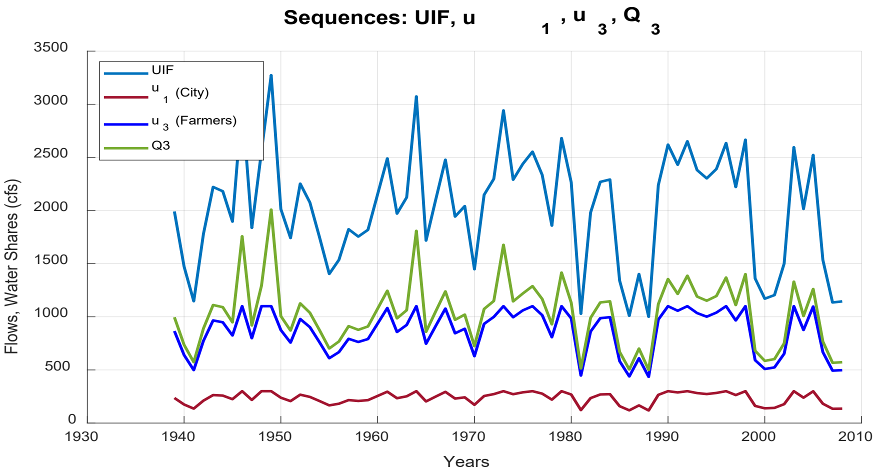

| $u_1$ mean | $u_3$ mean | $EW_1$ mean | $EW_3$ mean |
|---|---|---|---|
| cfs | cfs | million $ | million $ |
| 526 | 723 | 75 | 33 |

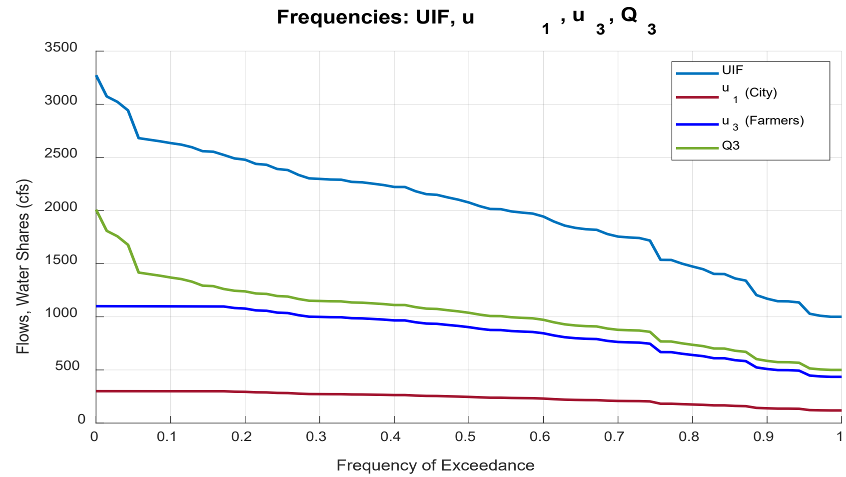


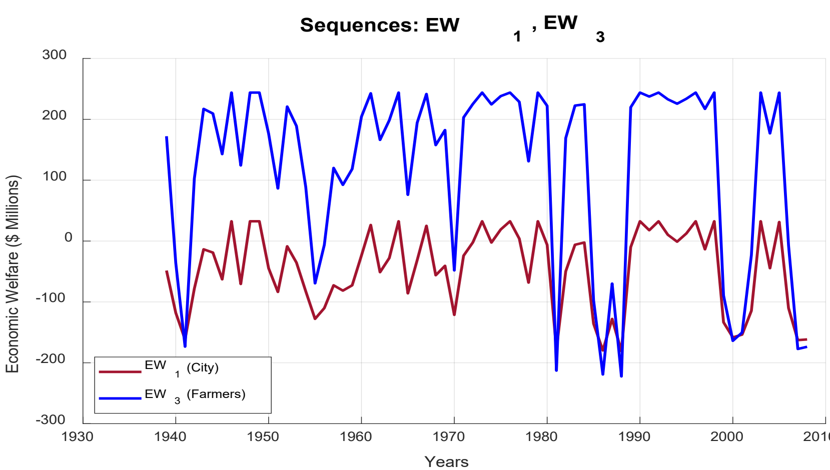
# 3.1.3 System Simulation Using Models[5]

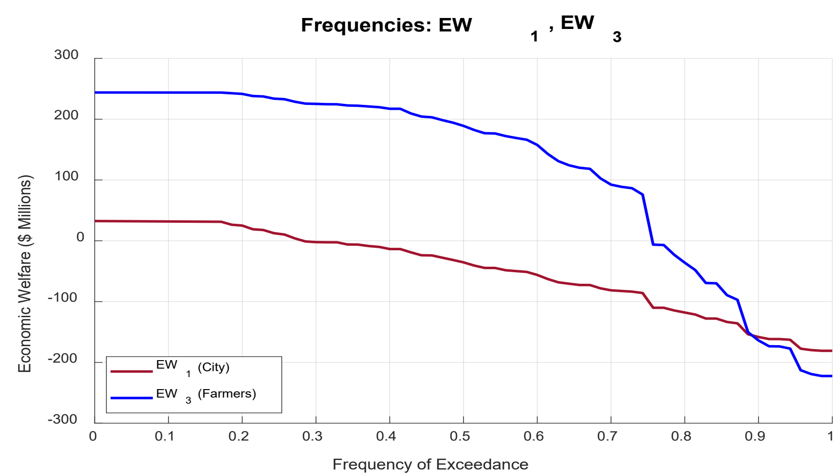
**Results:** Simulated variable sequences for $U_1$ = 300 cfs and $U_3$ = 1100 cfs:




| $u_1$ mean | $u_3$ mean | $EW_1$ mean | $EW_3$ mean |
|---|---|---|---|
| cfs | cfs | million $ | million $ |
| 236 | 864 | -46 | 123 |



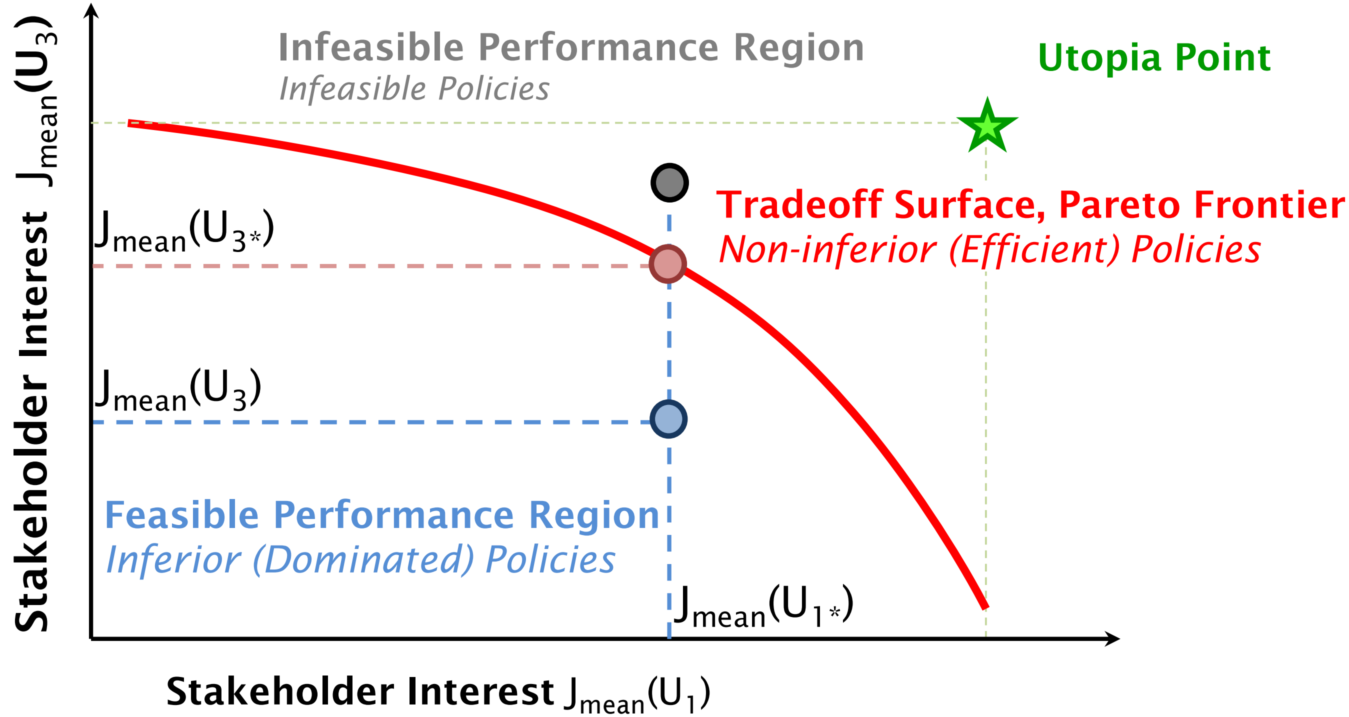
# 3.2 Deriving Tradeoffs

## Tradeoff Derivation

The system simulation and performance metrics calculations result in the derivation of a *single* performance metrics point $\{J_{mean}(U_1), J_{mean}(U_3)\}$ corresponding to given targets $\{U_1, U_3\}$. This point is feasible but not necessarily efficient (see earlier definition). Tradeoff derivation requires the *systematic* exploration of the feasible metrics space, and the determination of the Tradeoff Surface where each stakeholder metric cannot be improved without worsening the other.

A popular method to derive tradeoffs is to require that one stakeholder metric is higher or equal than a certain threshold, and maximize the other. [See illustration below where $J_{mean}(U_1)$ is constrained to be higher or equal than $J_{mean}(U_{1*})$, and $J_{mean}(U_3)$ is optimized to attain $J_{mean}(U_{3*})$]. Then, shift the constraint to an adjacent point and repeat the computations. This tradeoff derivation approach is not directly applicable to this system, however, because the $J_{mean}(U_{1*})$ constraint cannot be explicitly incorporated in the previous calculations. Alternatively, the performance metrics can be calculated over a grid of likely feasible $\{U_1, U_3\}$ values, and the Tradeoff generated based on the constraint method.

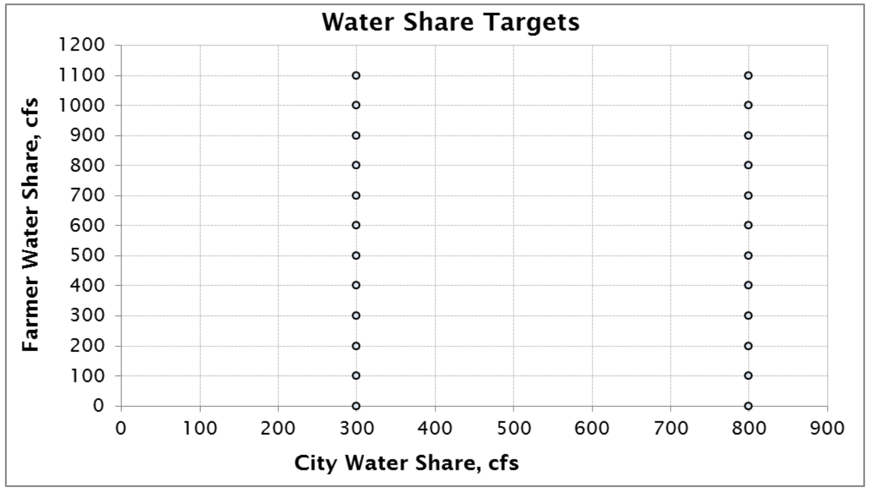
A third possibility could be to apply the constraint derivation approach in the decision space $\{U_1, U_3\}$. However, it should be noted that at each constraint threshold, this approach may not lead to an efficient point on the Tradeoff curve, and additional comparisons of the results at neighboring constraint thresholds may be necessary to determine the truly efficient water share policies.



# 3.2 Deriving Tradeoffs[2]

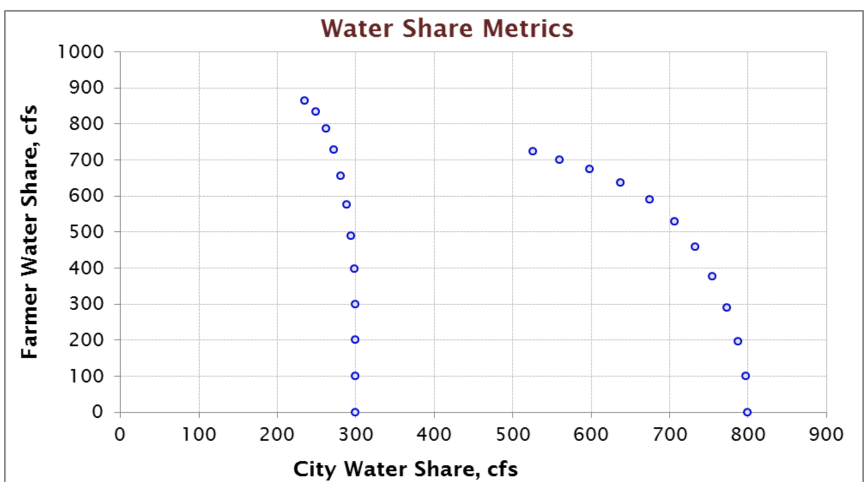
**Results:** Performance metrics for selected water share target ranges:

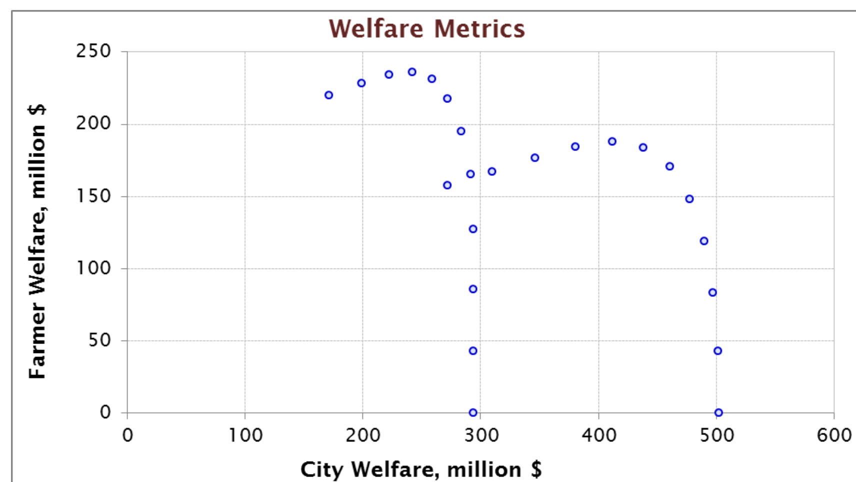
$U_1$ = {300, 800} cfs
$U_3$ = {0 – 1100, $\Delta U_3$=100} cfs




$J_{mean}(U_i)$ = 1/N [$u_i(1)$ + $u_i(2)$ +…+ $u_i(70)$], i=1, 3

$V_{mean}(U_i)$ = 1/N [$EW_i(1)$ + $EW_i(2)$ +…+ $EW_i(70)$], i=1, 3

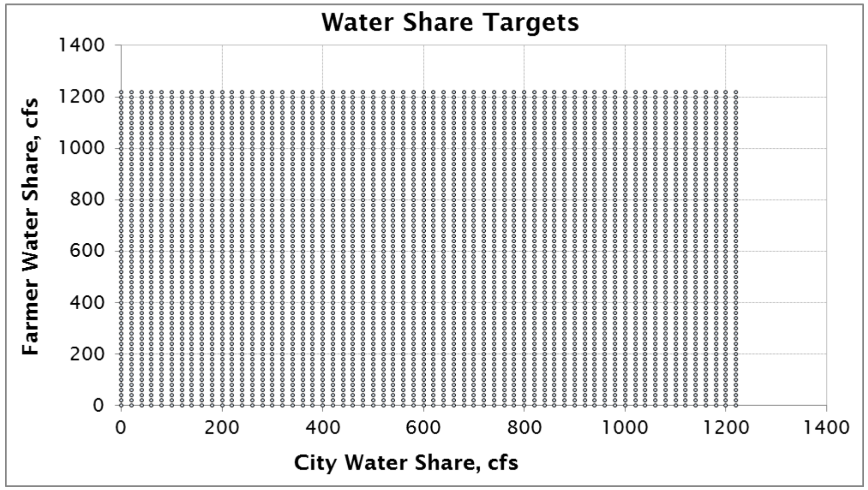


# 3.2 Deriving Tradeoffs[3]

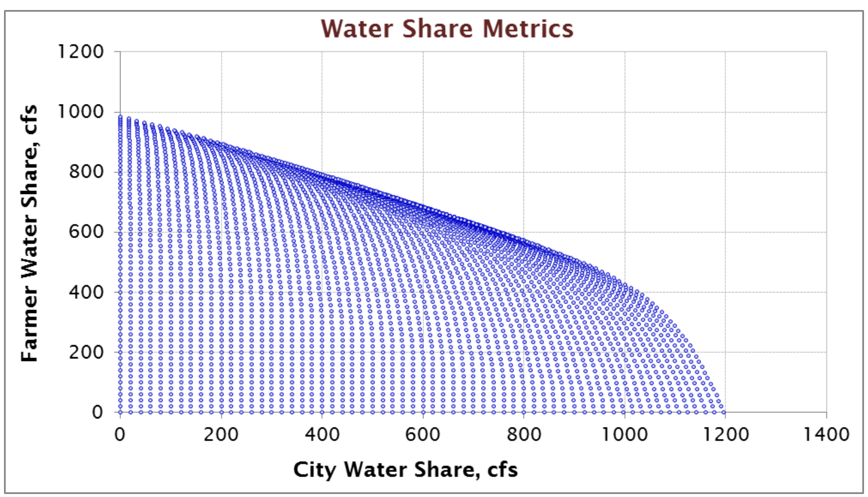
**Results:** Performance metrics for feasible water share targets:

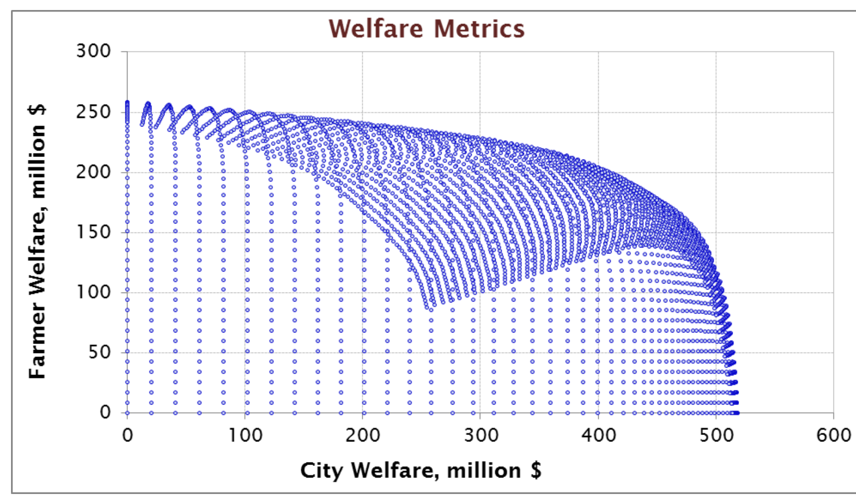
$U_1 = \{0 - 1220, \Delta U_1=20\}$ cfs
$U_3 = \{0 - 1220, \Delta U_3=20\}$ cfs




$$J_{mean}(U_i) = 1/N\,[u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$$

$$V_{mean}(U_i) = 1/N\,[EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$$

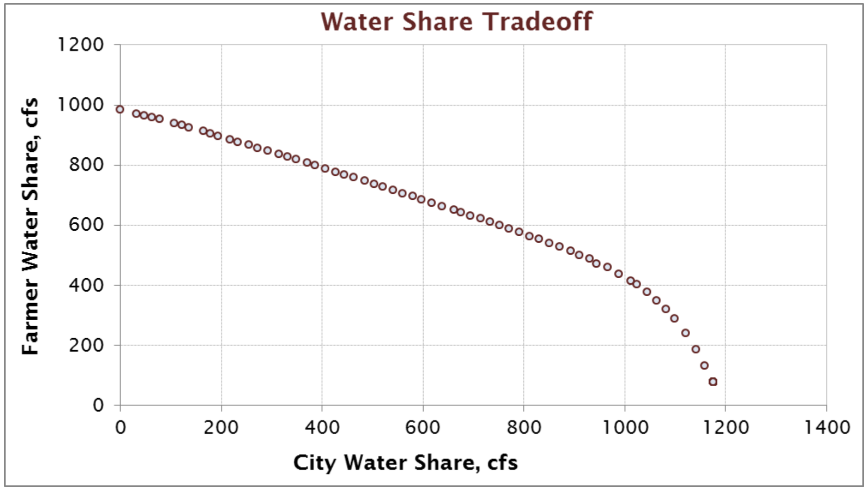


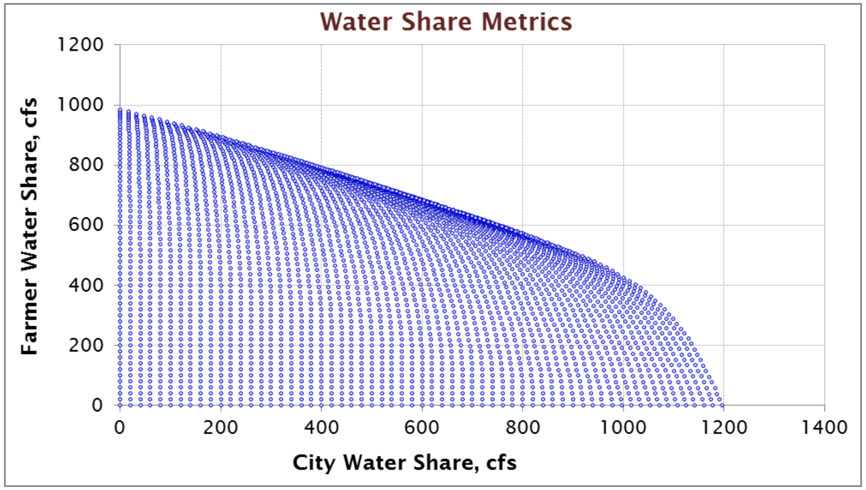



# 3.2 Deriving Tradeoffs[4]

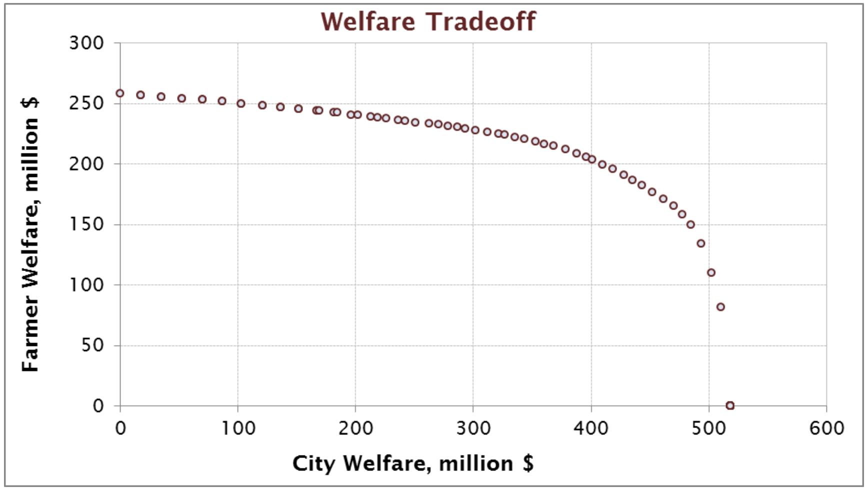
**Results:** Tradeoff Curves (Pareto Frontiers):

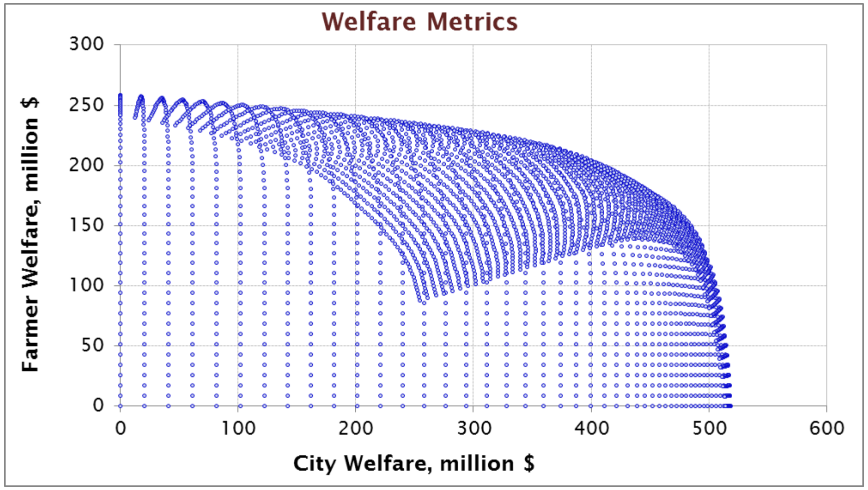
$$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$$




$$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$$

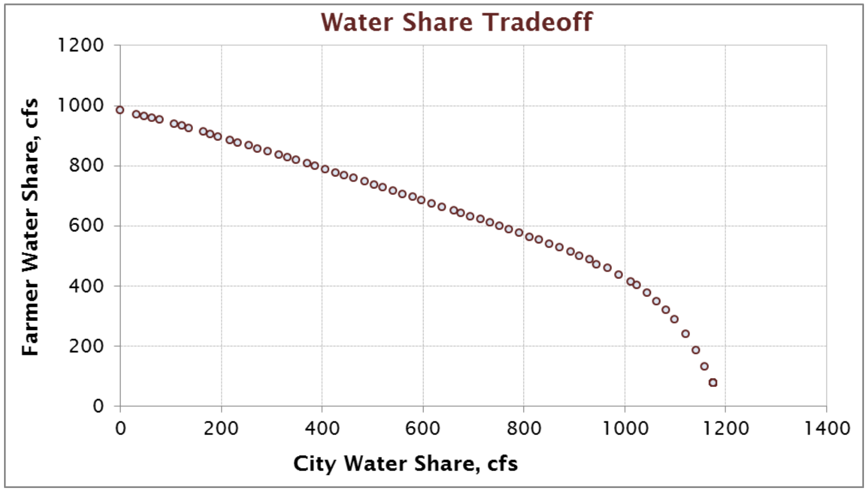


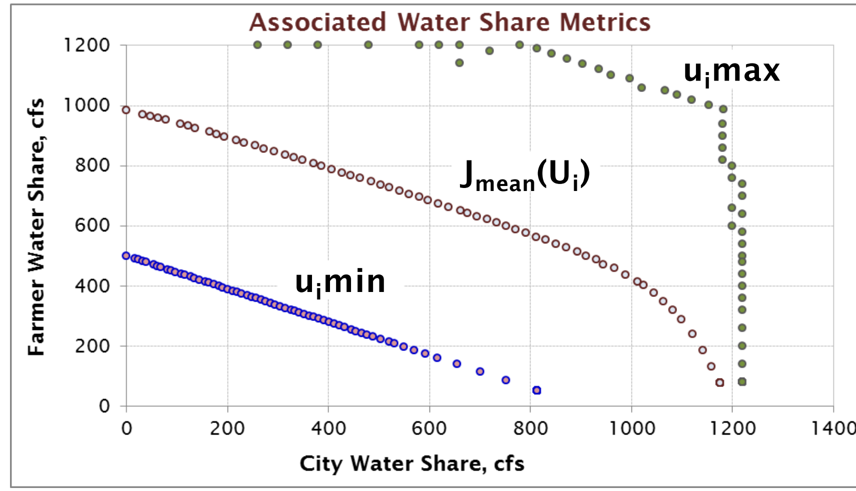
# 3.2 Deriving Tradeoffs[5]

**Results:** Tradeoff Curves (Pareto Frontiers) vs. Alternative Metrics:

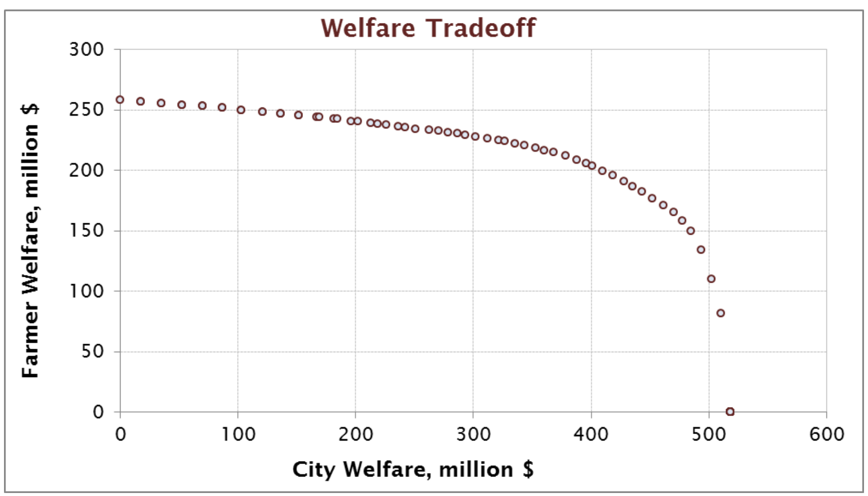
$J_{mean}(U_i) = 1/N\,[u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1,3$



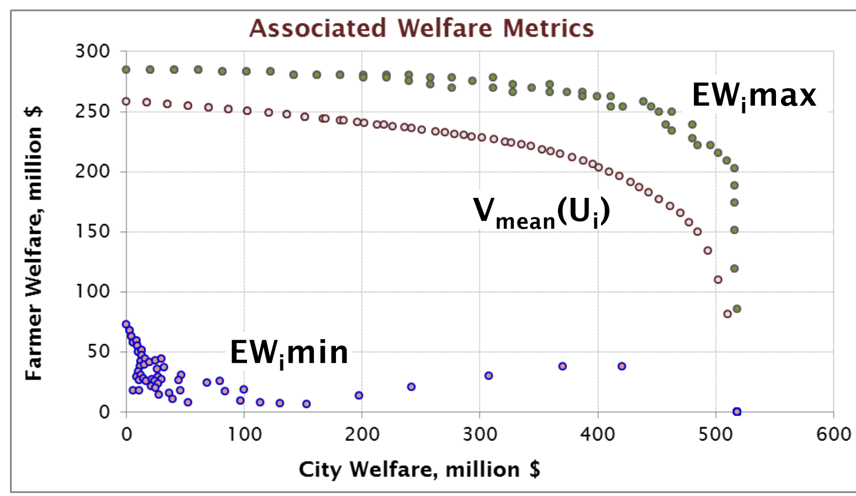

$u_i min(max) = min(max)\{u_i(1), \ldots, u_i(70)\},\ i=1,3$




$V_{mean}(U_i) = 1/N\,[EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1,3$



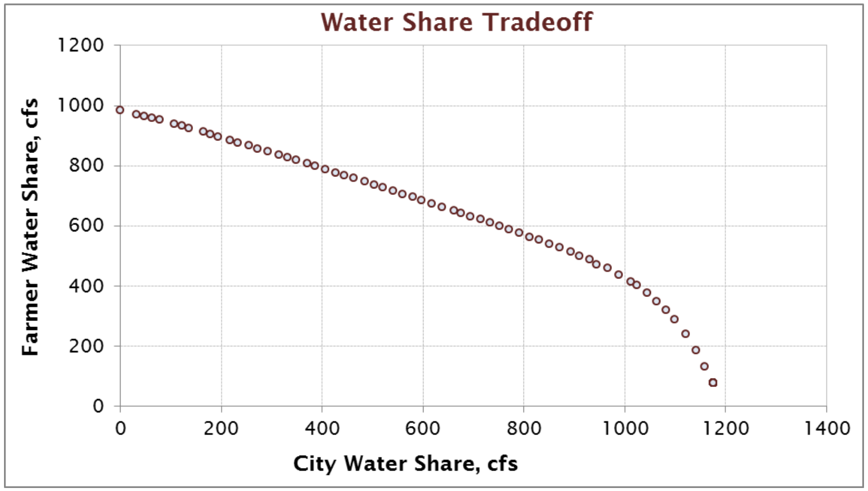

$EW_i min(max) = min(max)\{EW_i(1), \ldots, EW_i(70)\},\ i=1,3$



# 3.2 Deriving Tradeoffs[6]

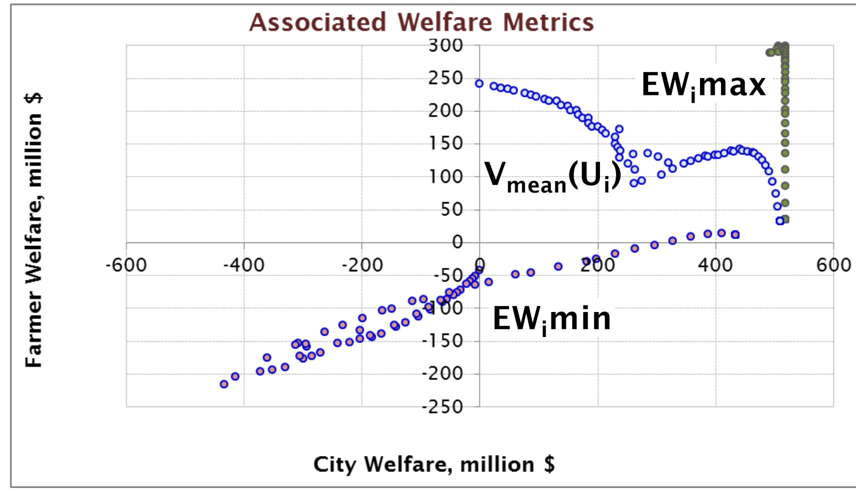
**Results:** Tradeoff Curves (Pareto Frontiers) vs. Alternative Metrics[2]:

$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1,\ 3$

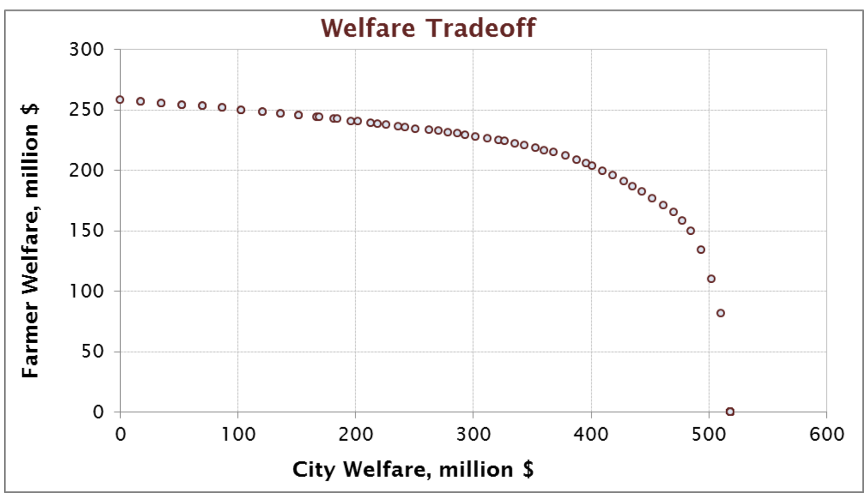



↔

$EW_i min(max) = min(max)\{EW_i(1), \ldots, EW_i(70)\},\ i=1,\ 3$

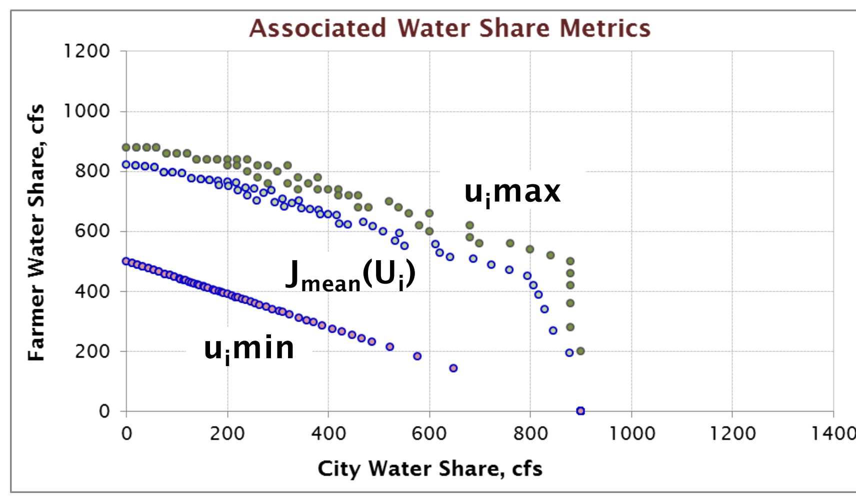



$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1,\ 3$




↔

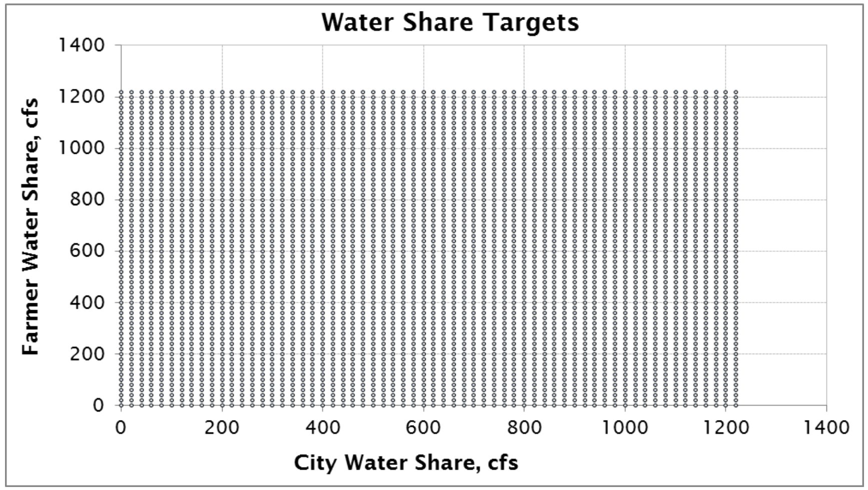
$u_i min(max) = min(max)\{u_i(1), \ldots, u_i(70)\},\ i=1,\ 3$



# 3.2 Deriving Tradeoffs[7]

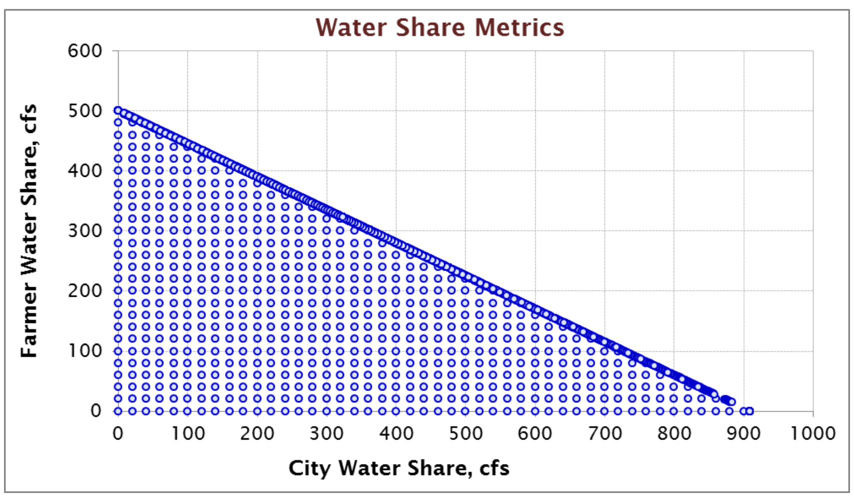
**Results:** Max/min performance metrics for feasible water share targets:

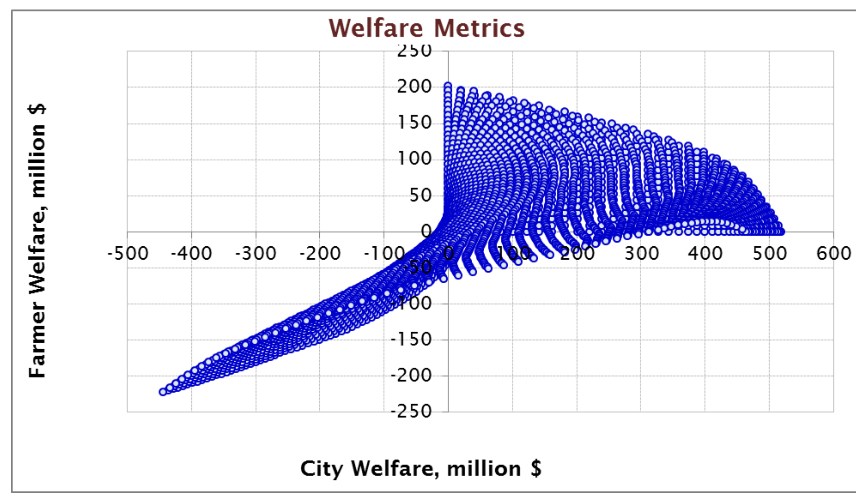
$U_1 = \{0 - 1220, \Delta U_1 = 20\}$ cfs
$U_3 = \{0 - 1220, \Delta U_3 = 20\}$ cfs




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$

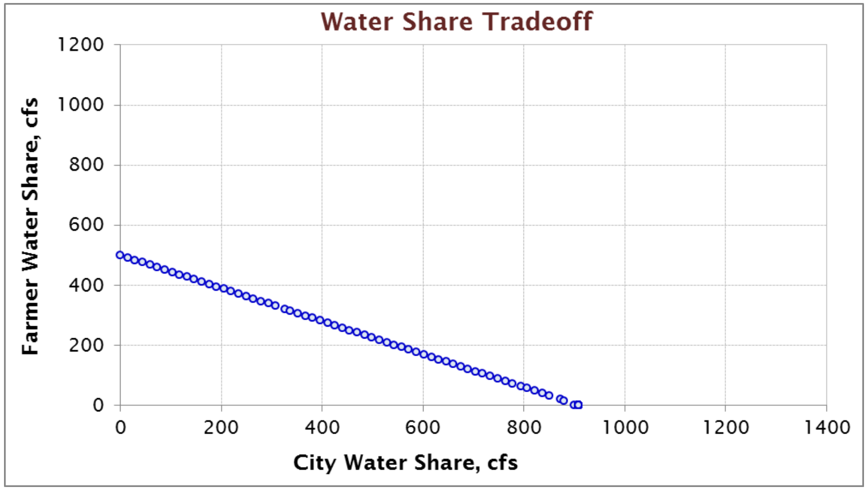


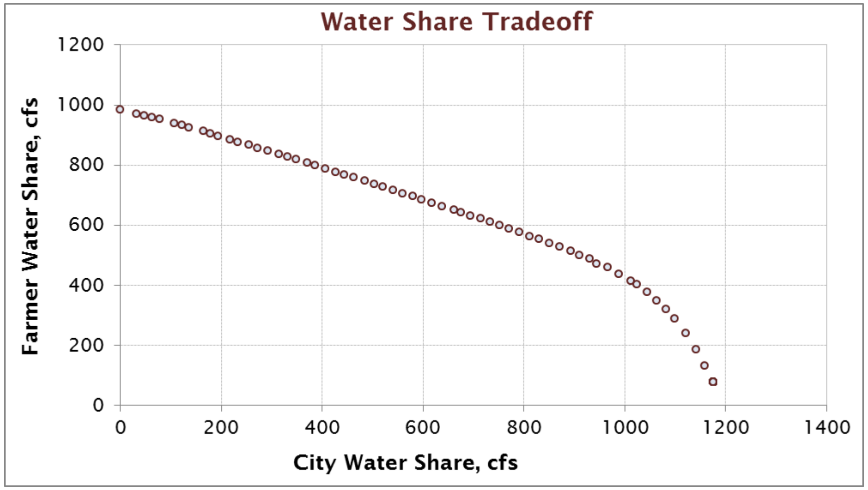
# 3.2 Deriving Tradeoffs[8]

**Results:** Max/min Metrics and Tradeoff Curves[2]:

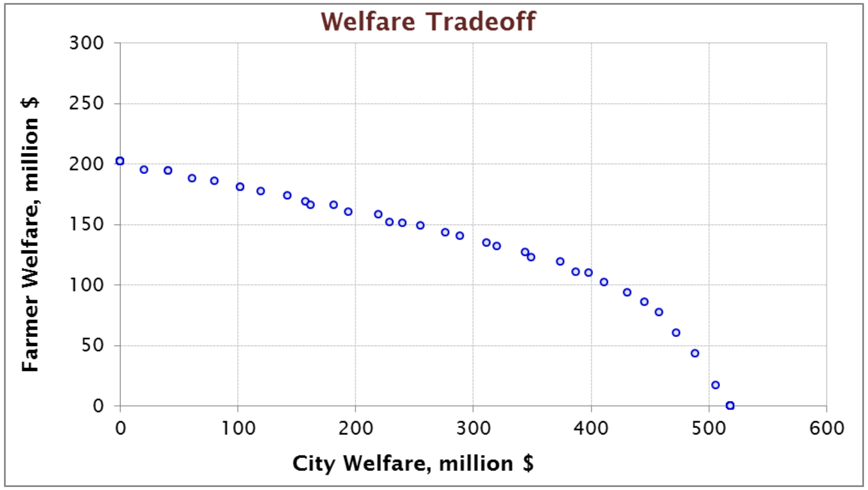
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



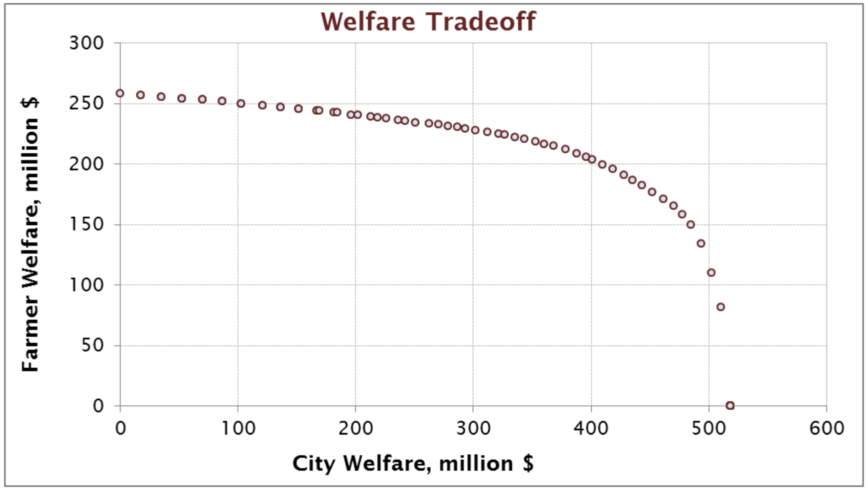

$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)], i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



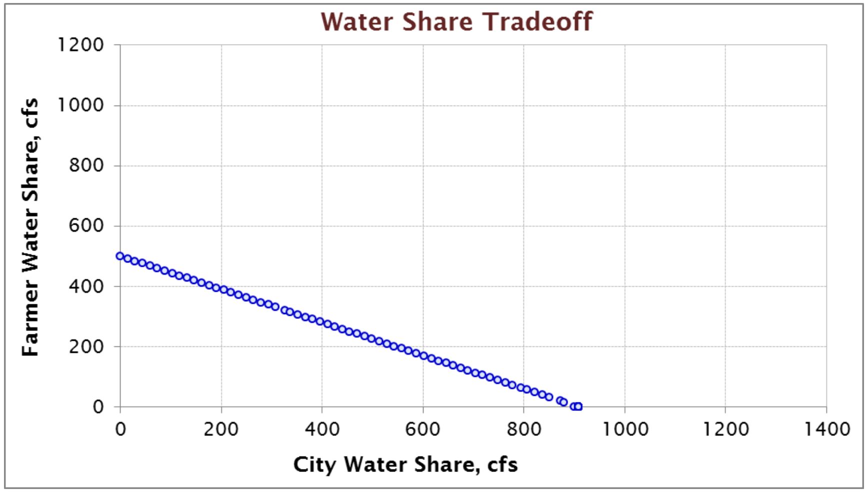

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)], i=1, 3$



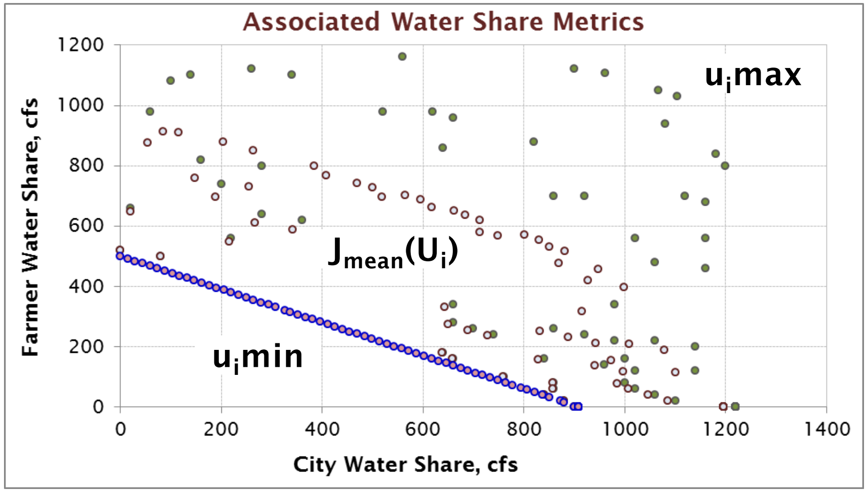
# 3.2 Deriving Tradeoffs[9]

**Results:** Max/min Metrics and Tradeoff Curves[3]:

$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$

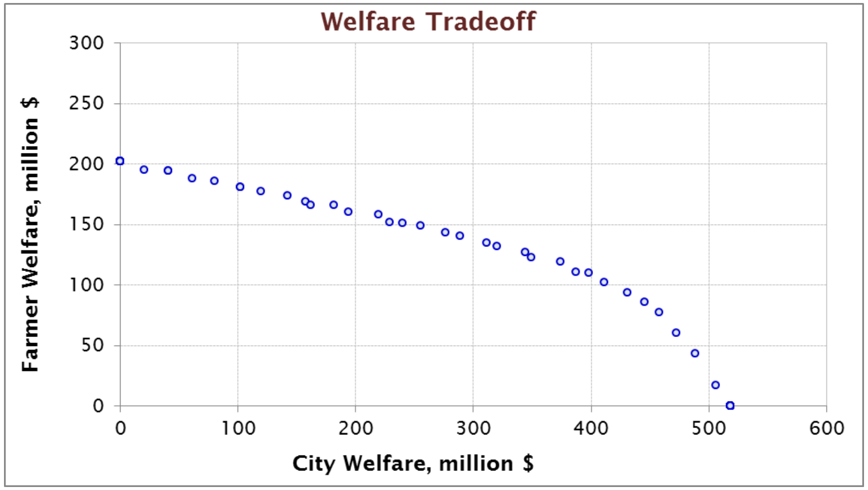



$u_i\min(\max) = \min(\max)\{u_i(1), \ldots, u_i(70)\}, i=1, 3$

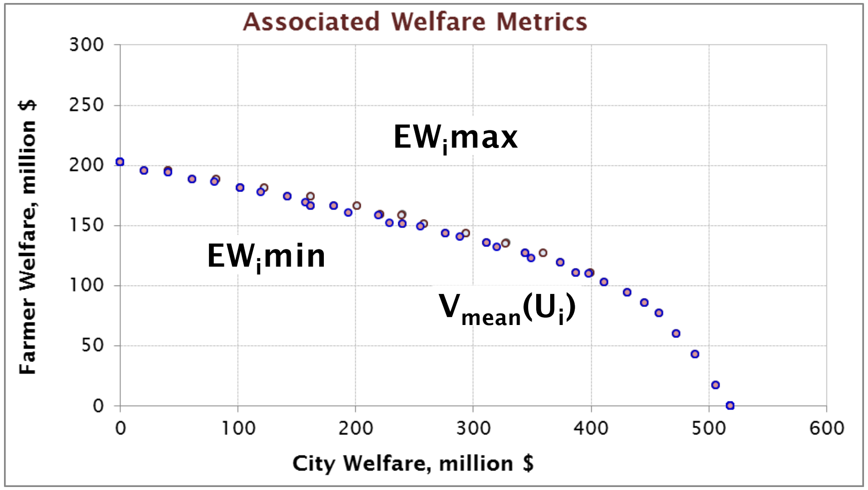



↔

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$EW_i\min(\max) = \min(\max)\{EW_i(1), \ldots, EW_i(70)\}, i=1, 3$

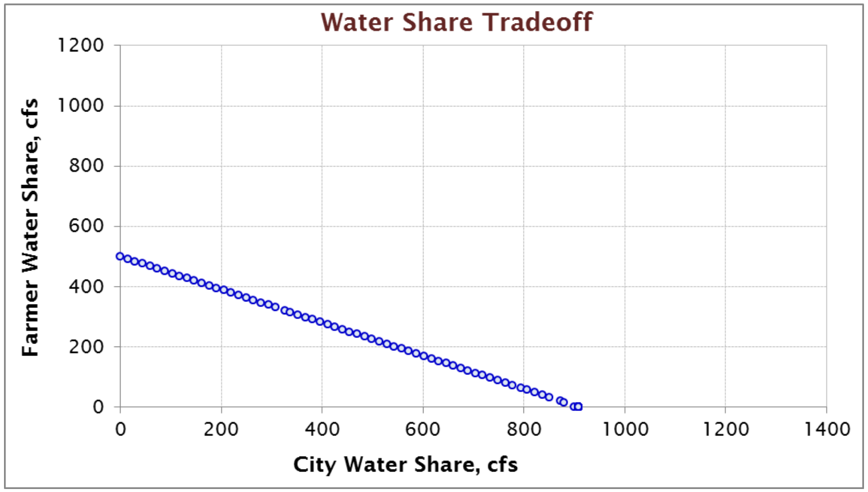



↔

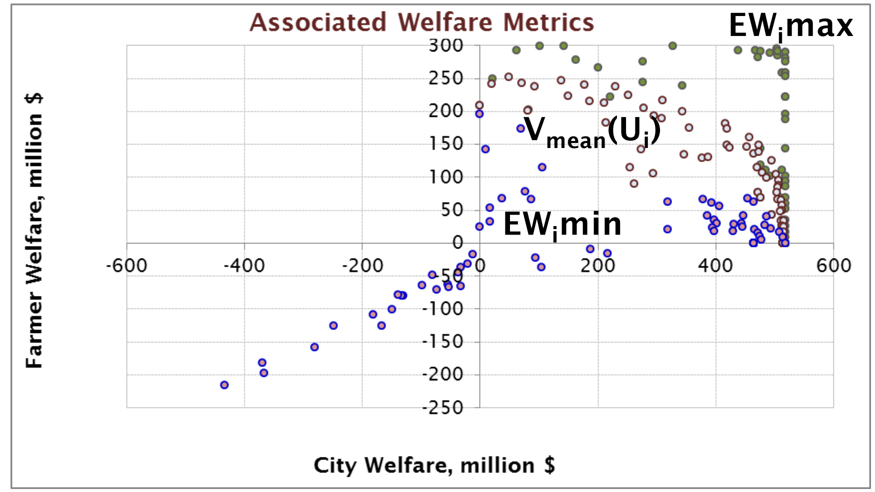
# 3.2 Deriving Tradeoffs[10]

**Results:** Max/min Metrics and Tradeoff Curves[4]:

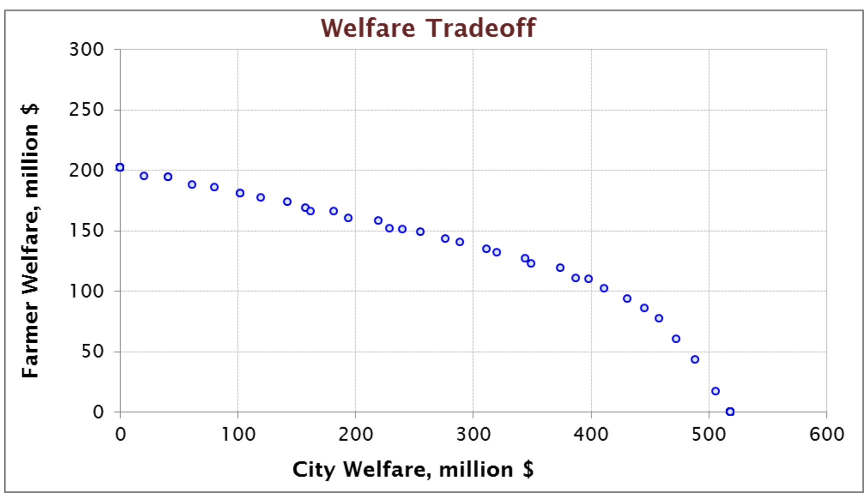
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



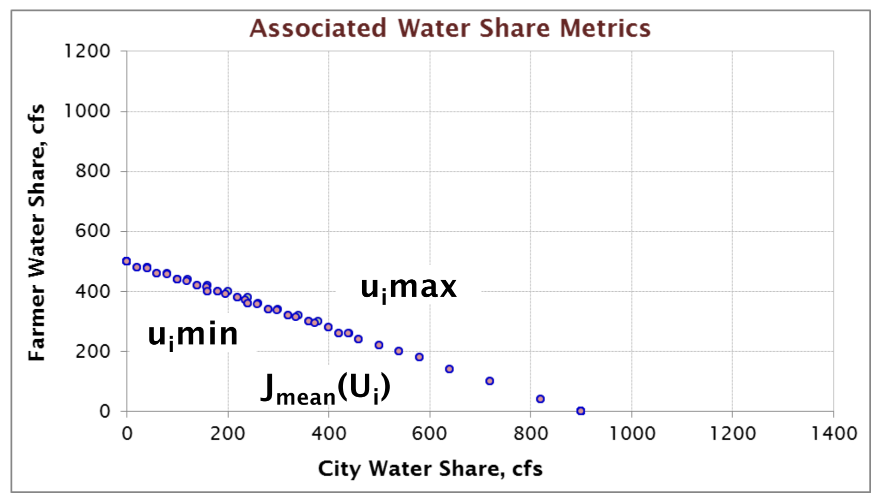

$EW_i\min(\max) = \min(\max)\{EW_i(1), \ldots, EW_i(70)\}, i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$u_i\min(\max) = \min(\max)\{u_i(1), \ldots, u_i(70)\}, i=1, 3$



# 3.3 Sensitivity Analysis

## 3.3.1 Sensitivity to Environmental Flow

For the system described in Example 3.1.1, you are asked to assess the benefits or impacts of water management alternatives associated with the environmental flow requirements.

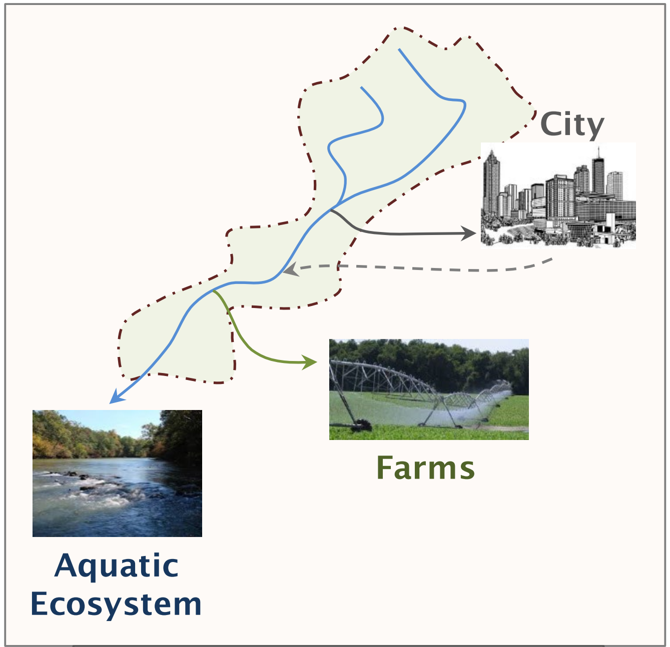



**Figure A:** River and Water Uses Layout

## Approach:

The benefits and impacts associated with the environmental flow requirements can be quantified by re-deriving the stakeholder tradeoffs for different environmental flow thresholds.

Each threshold implies a possible management alternative. For example, setting the environmental flow requirement at a particular fraction of the UIF, or some other appropriate minimum flow level, represents a regulatory policy change (institutional intervention). Such thresholds would need to reflect expert knowledge on aquatic flora and fauna habitats.

**Figure B:** Natural Inflow Record

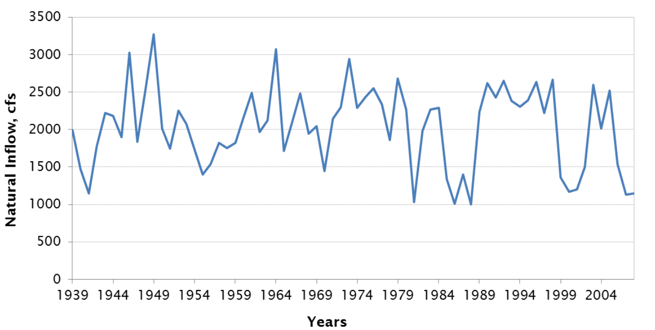



**Figure C:** City Water Demand Function

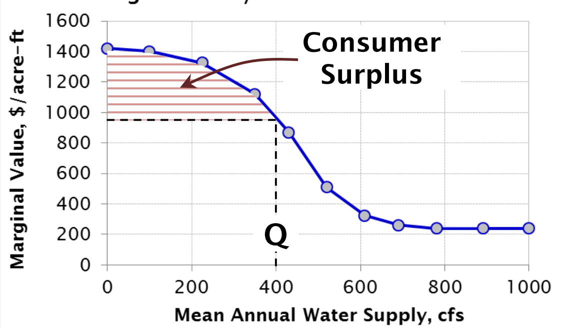
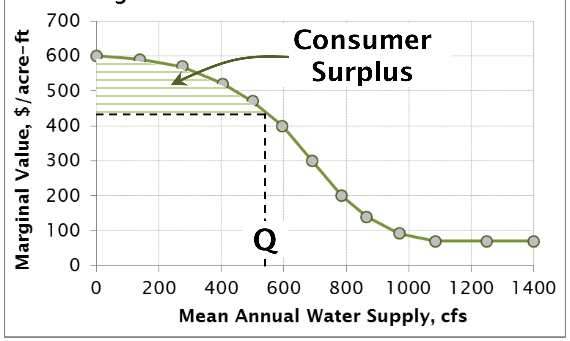


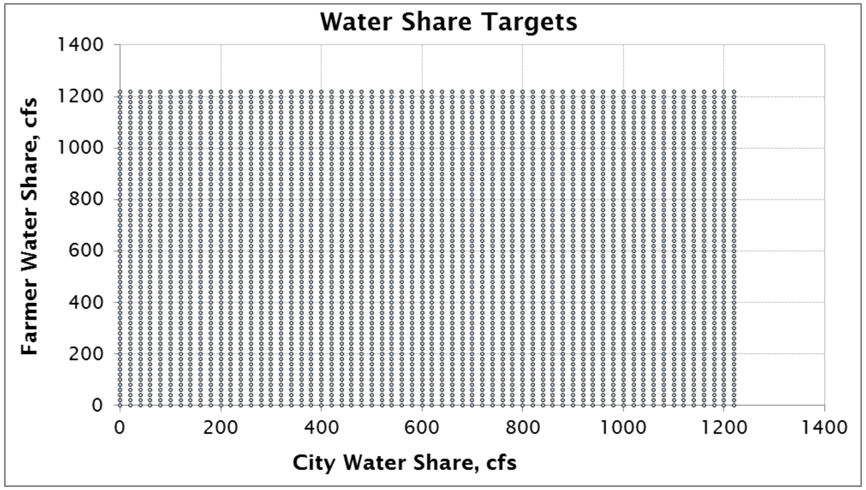

**Figure D:** Farmer Water Demand Function



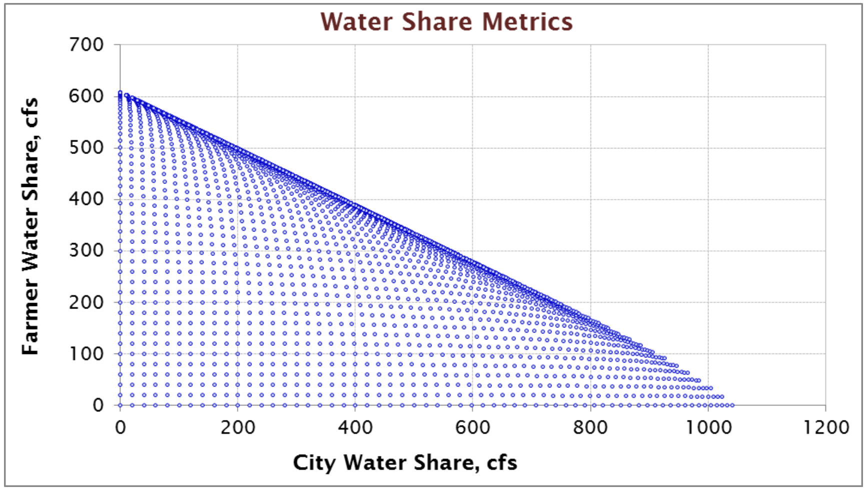
# 3.3.1 Sensitivity to Environmental Flow[2]

**Results:** Performance metrics for $Q_3(k) = 0.7\ I_1(k)$ environmental flow requirement:

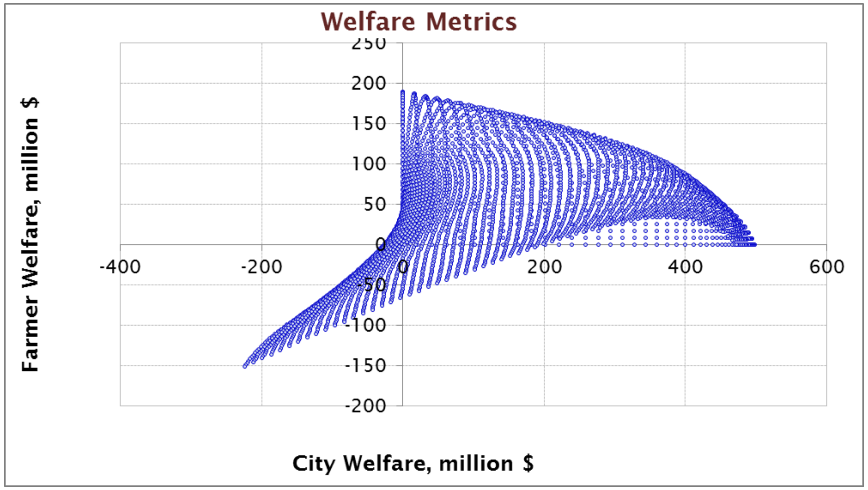
$U_1 = \{0 - 1220, \Delta U_1=20\}$ cfs
$U_3 = \{0 - 1220, \Delta U_3=20\}$ cfs




$$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$$



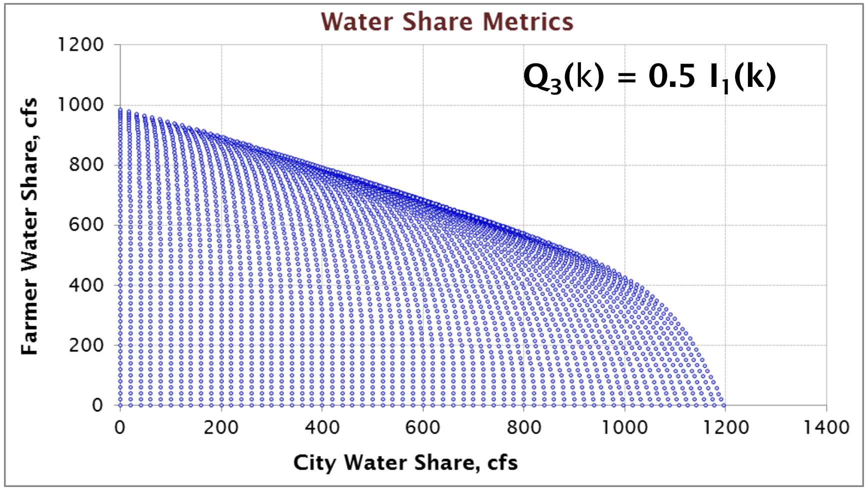

$$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$$



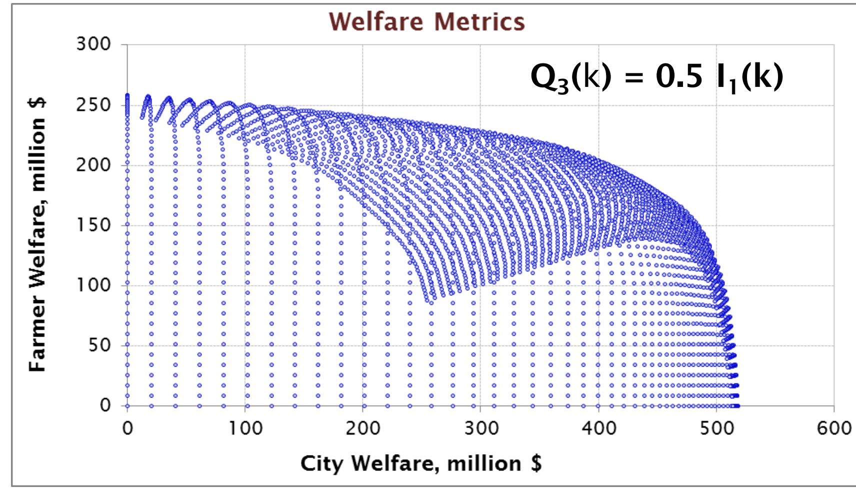
# 3.3.1 Sensitivity to Environmental Flow[3]

**Results:** Metrics Comparison for $Q_3(k) = 0.5\ I_1(k)$ [**Top**] and $Q_3(k) = 0.7\ I_1(k)$ [**Bottom**]:

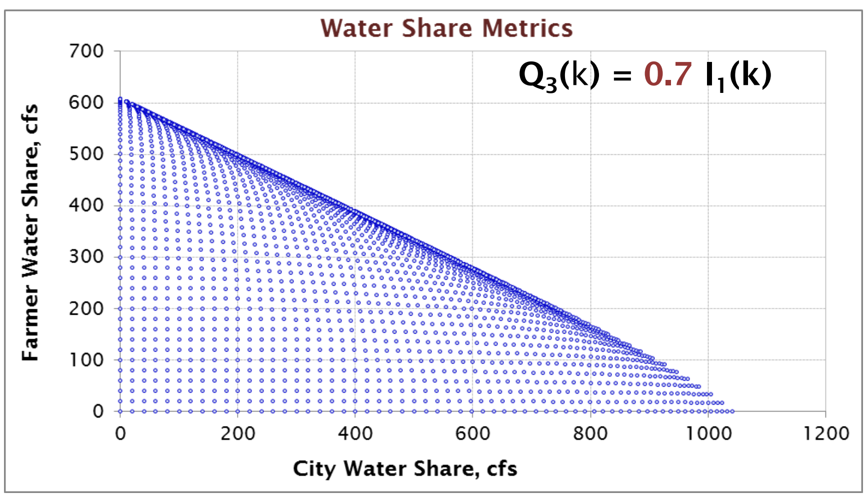
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



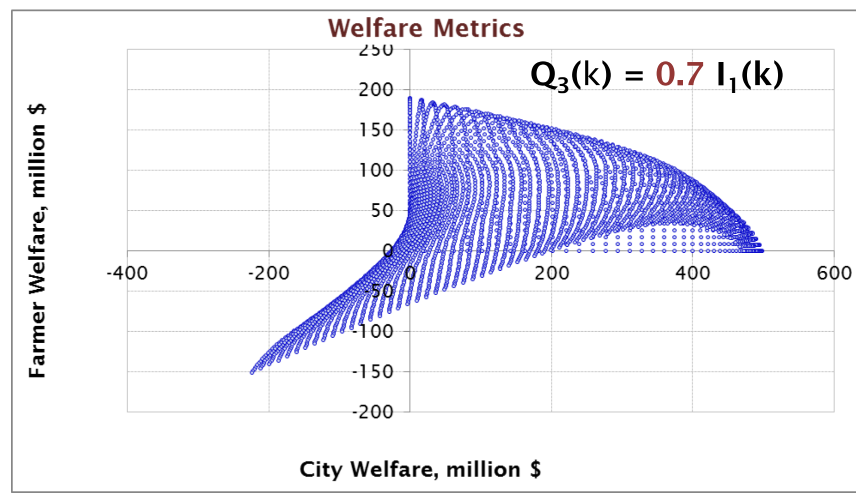

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



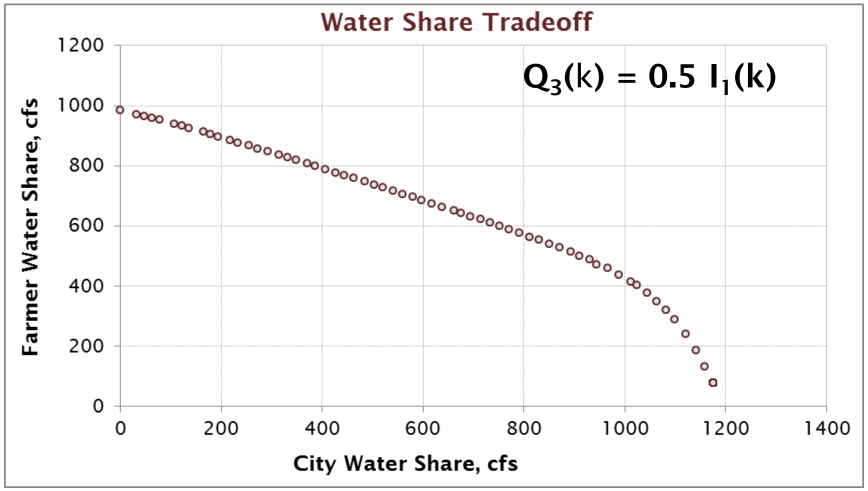

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



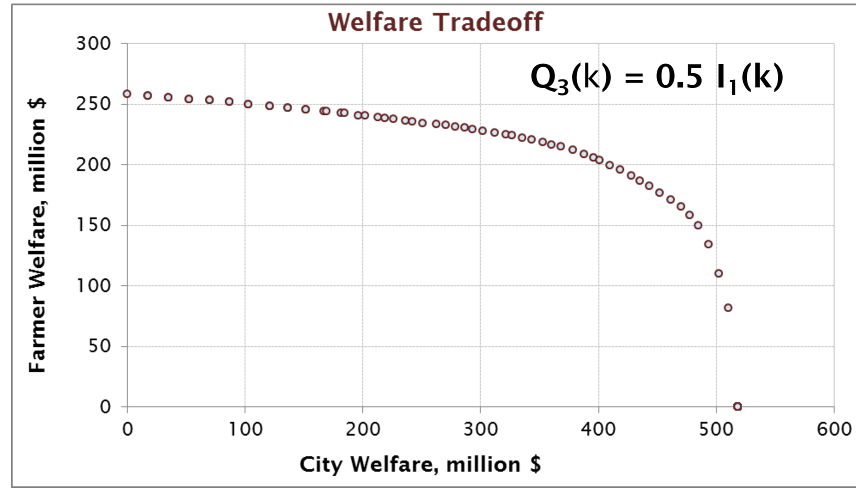
# 3.3.1 Sensitivity to Environmental Flow[3]

**Results:** Tradeoff Comparison for $Q_3(k) = 0.5\ I_1(k)$ [**Top**] and $Q_3(k) = 0.7\ I_1(k)$ [**Bottom**]:

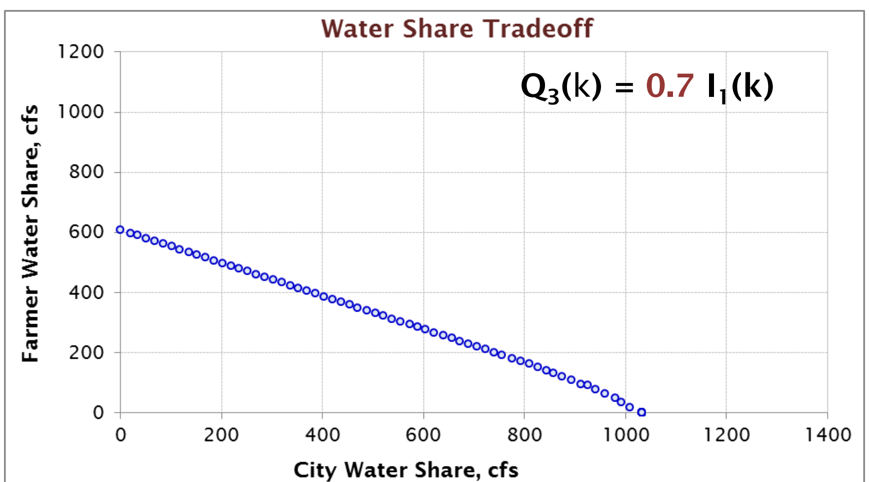
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



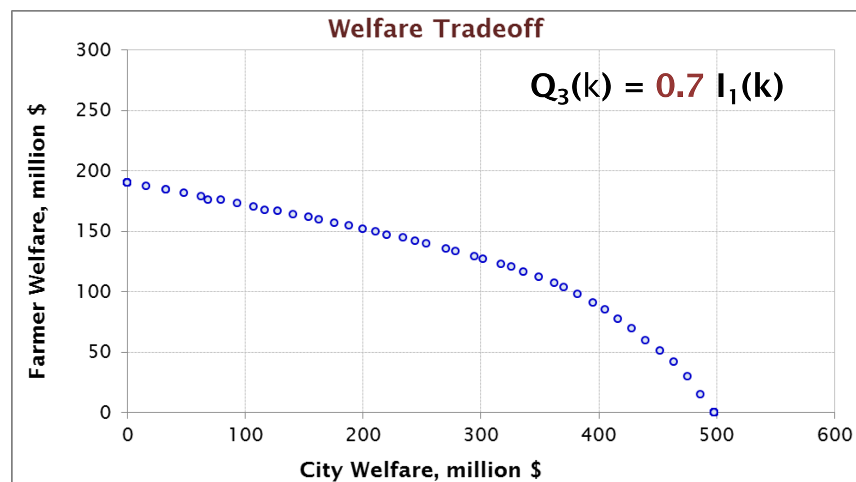

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



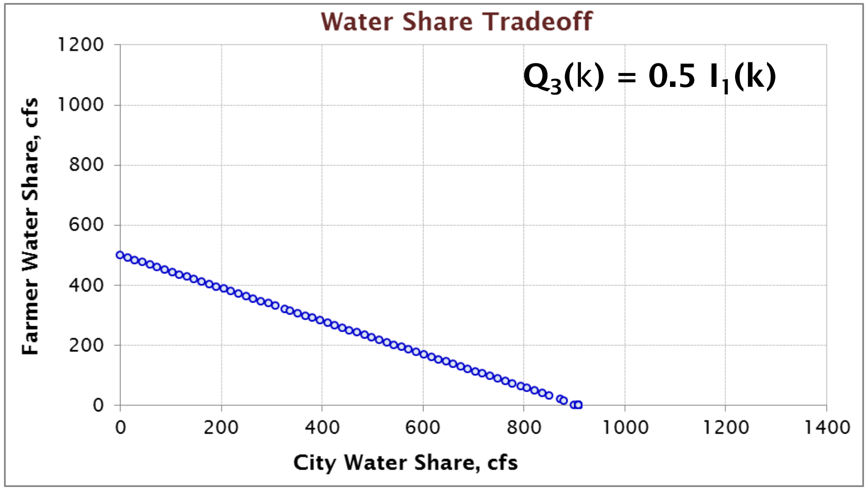

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



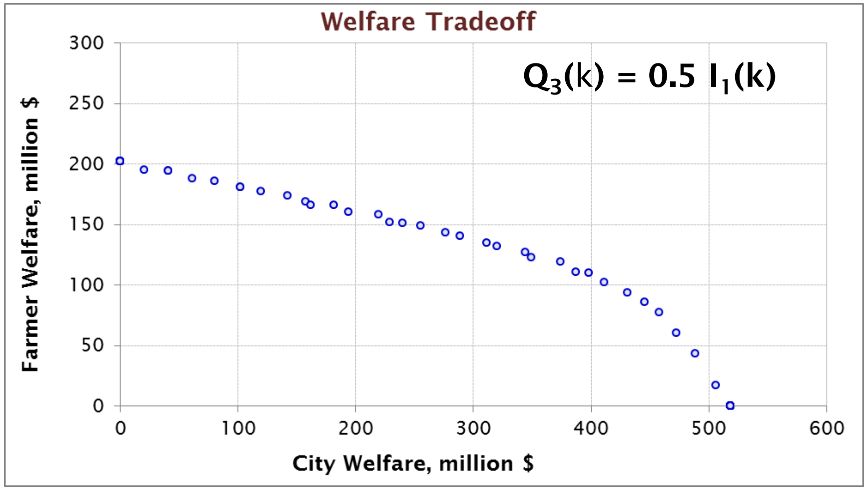
# 3.3.1 Sensitivity to Environmental Flow[4]

**Results:** Tradeoff Comparison for $Q_3(k) = 0.5\ I_1(k)$ [**Top**] and $Q_3(k) = 0.7\ I_1(k)$ [**Bottom**]:

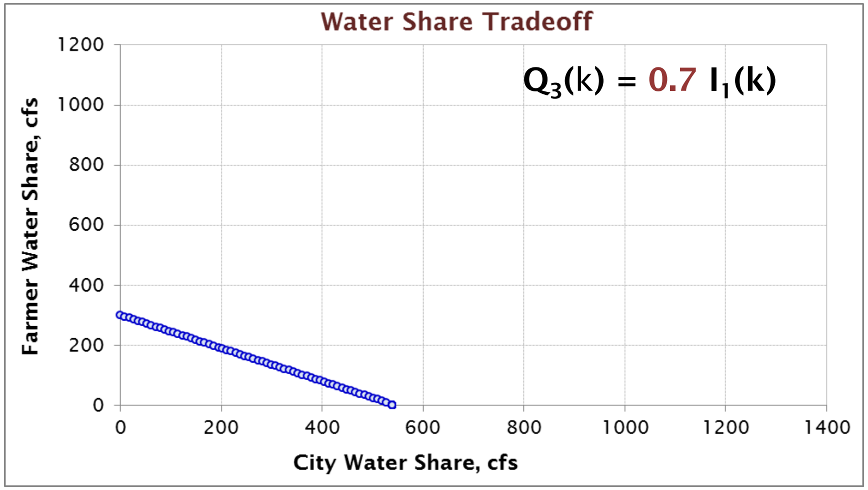
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



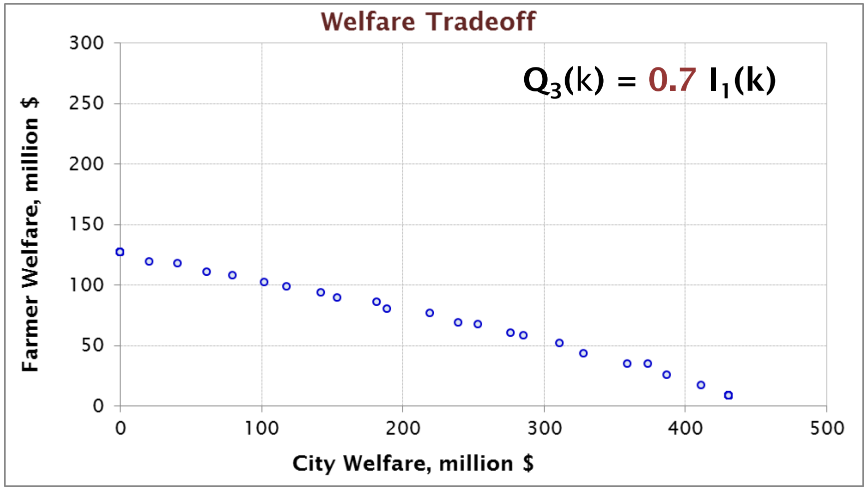

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



# 3.3.2 Sensitivity to Return Flow

For the system described in Example 3.1.1, you are asked to assess the benefits or impacts of water management alternatives associated with the city return flow.

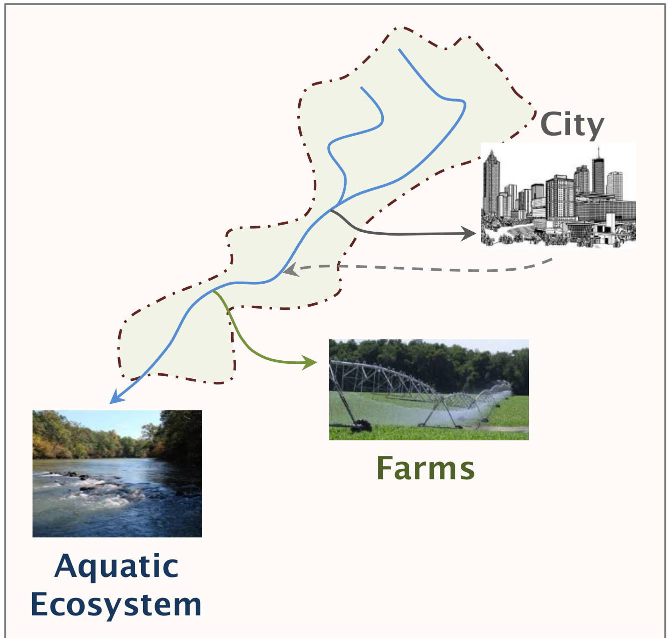



**Figure A:** River and Water Uses Layout

## Approach:

The benefits and impacts associated with the city return flow rate can be quantified by re-deriving the stakeholder tradeoffs for different return flow rates.

Each rate implies a possible management alternative. For example, increasing the city return flow rate may be accomplished by connecting more city residents to the sewer drainage system, or by rehabilitating the existing sewer system to reduce pipe leakage (structural intervention).

**Figure B:** Natural Inflow Record

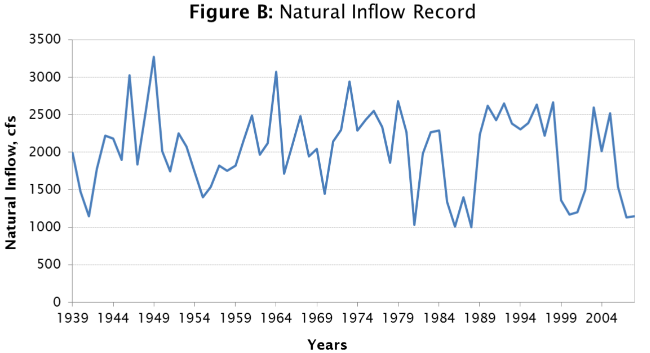



**Figure C:** City Water Demand Function

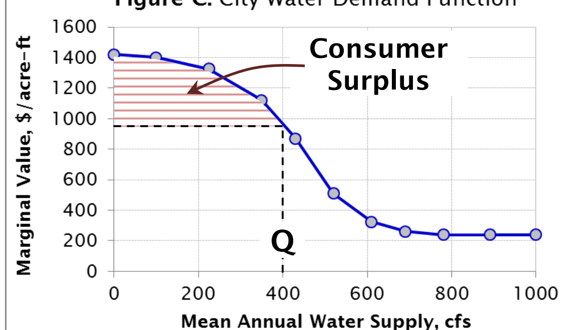
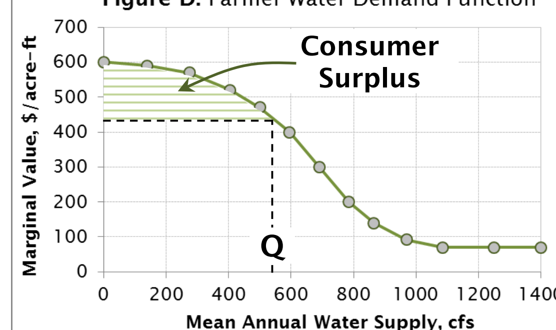


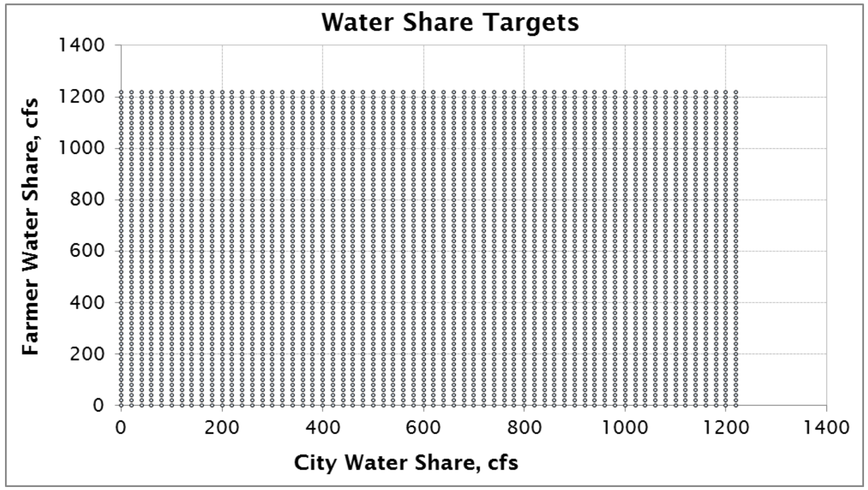

**Figure D:** Farmer Water Demand Function



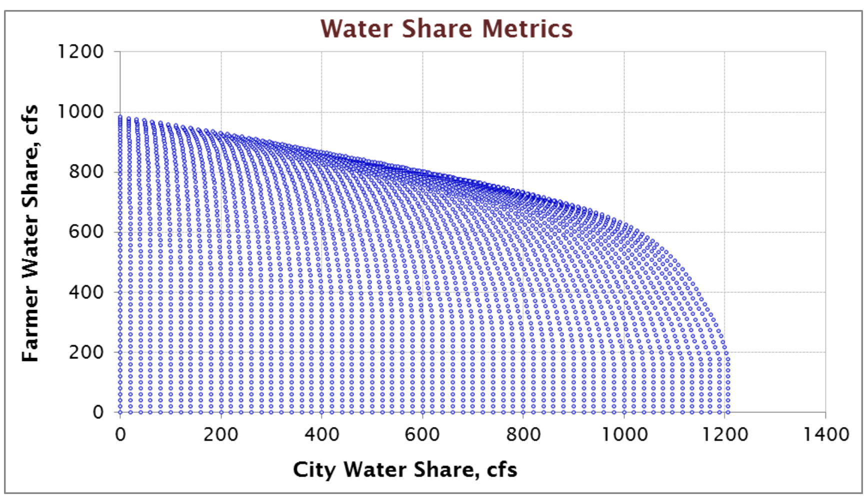
# 3.3.2 Sensitivity to Return Flow[2]

**Results:** Performance metrics for $R_2(k) = 0.65\ u_1(k)$ return flow:

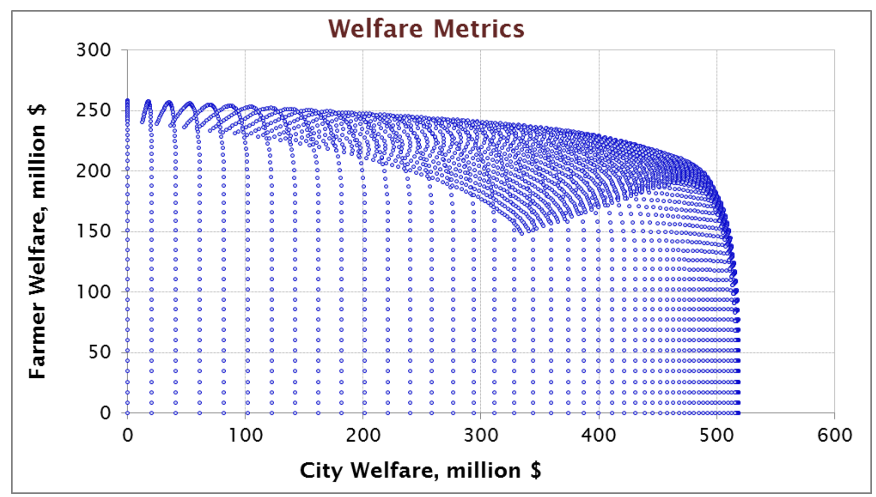
$U_1 = \{0 - 1220, \Delta U_1 = 20\}$ cfs
$U_3 = \{0 - 1220, \Delta U_3 = 20\}$ cfs




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



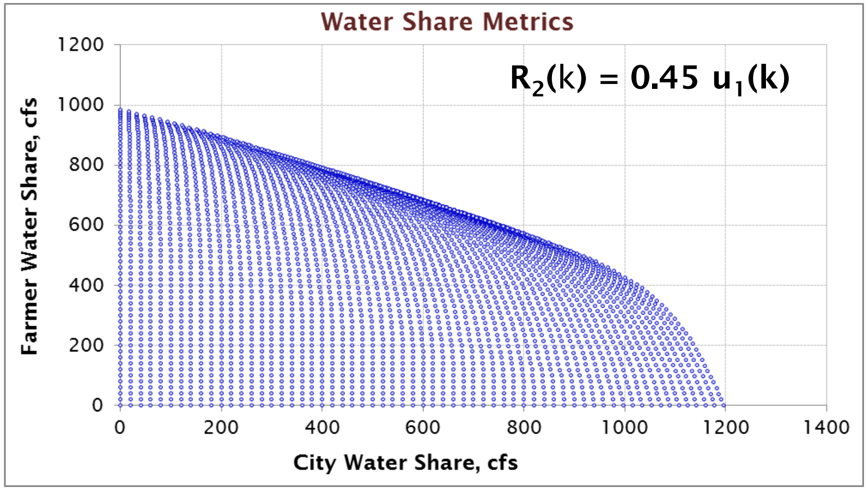

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



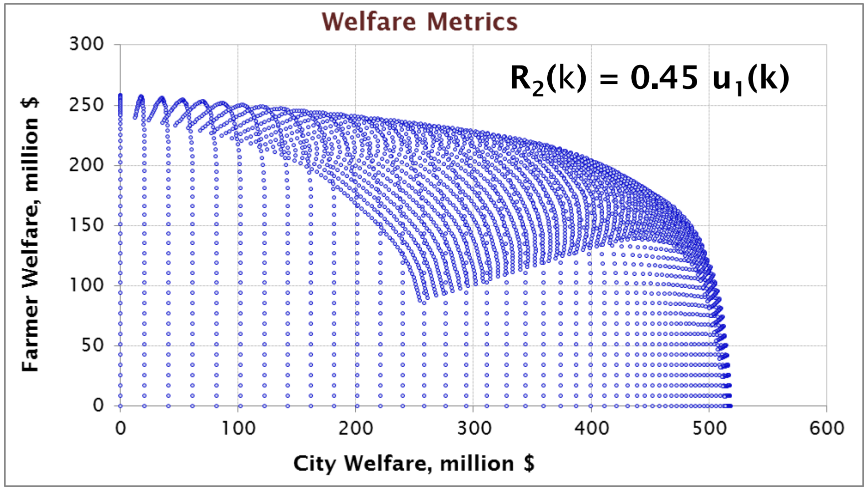
# 3.3.2 Sensitivity to Return Flow[3]

**Results:** Metrics Comparison for $R_2(k) = 0.45\ u_1(k)$ [**Top**], $R_2(k) = 0.65\ u_1(k)$ [**Bottom**]:

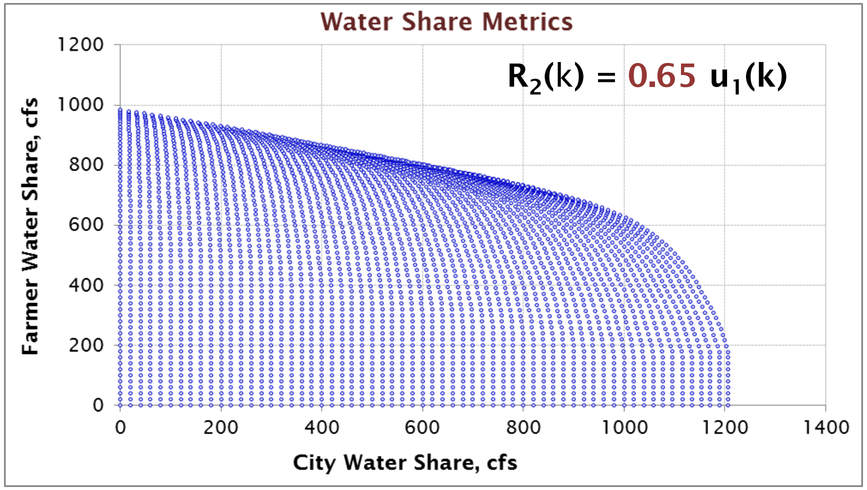
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



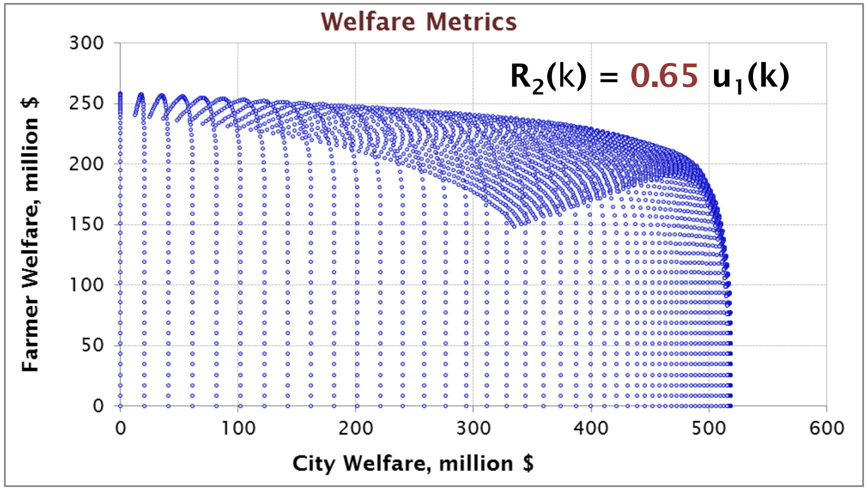

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



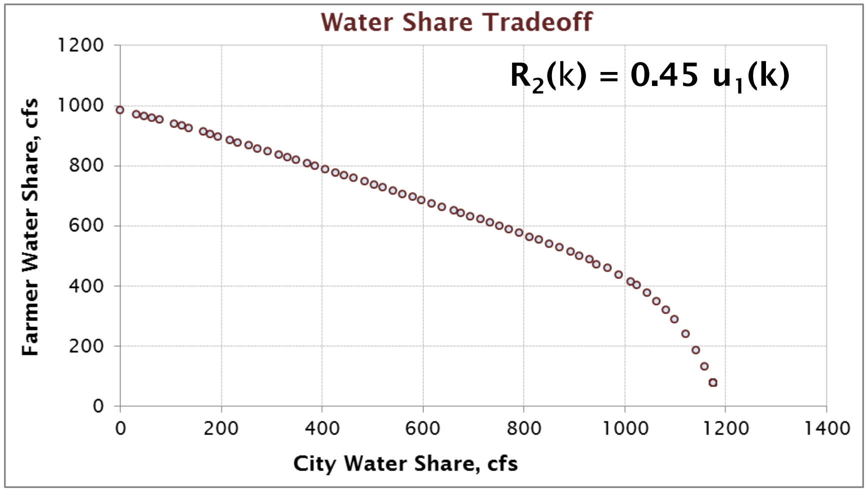

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



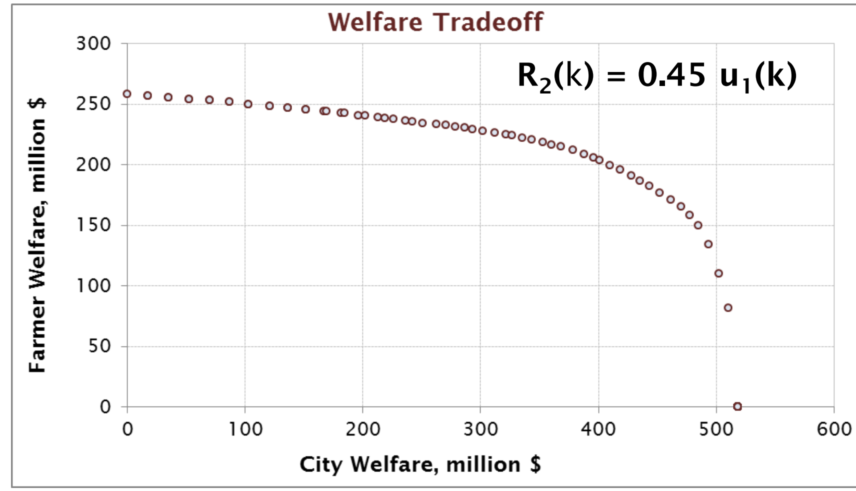
# 3.3.2 Sensitivity to Return Flow[4]

**Results:** Tradeoff Comparison for $R_2(k) = 0.45\ u_1(k)$ [**Top**], $R_2(k) = 0.65\ u_1(k)$ [**Bottom**]:

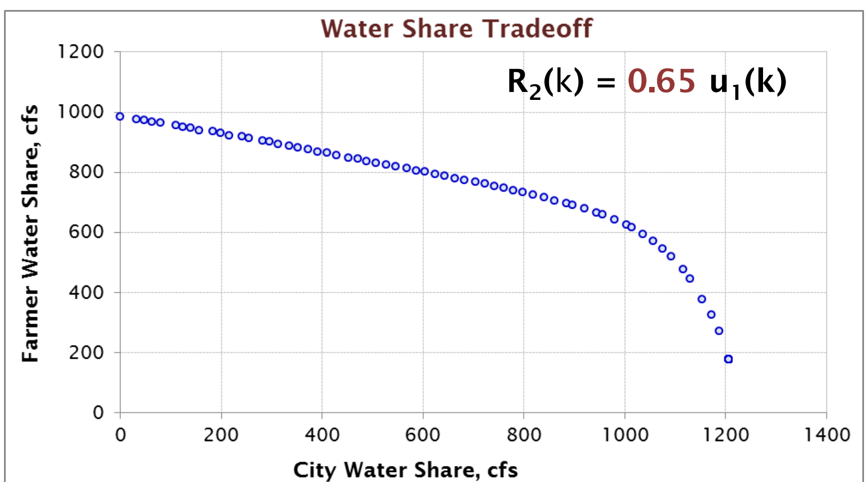
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)]$, i=1, 3



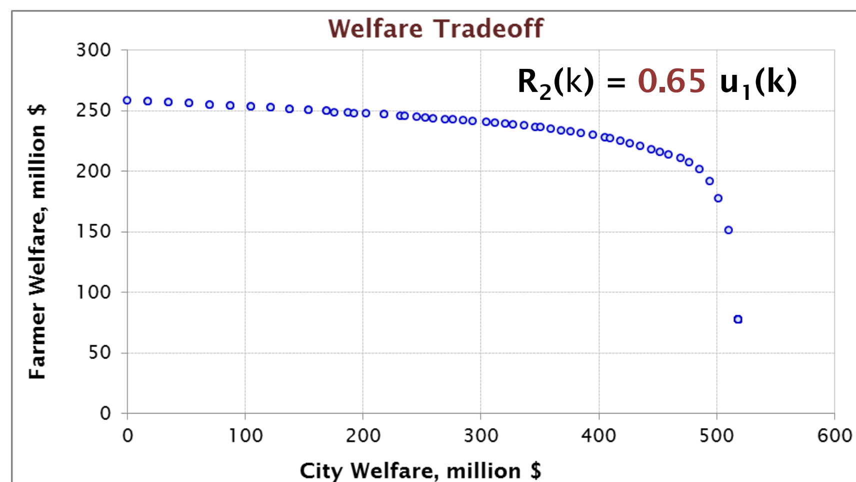

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)]$, i=1, 3




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)]$, i=1, 3



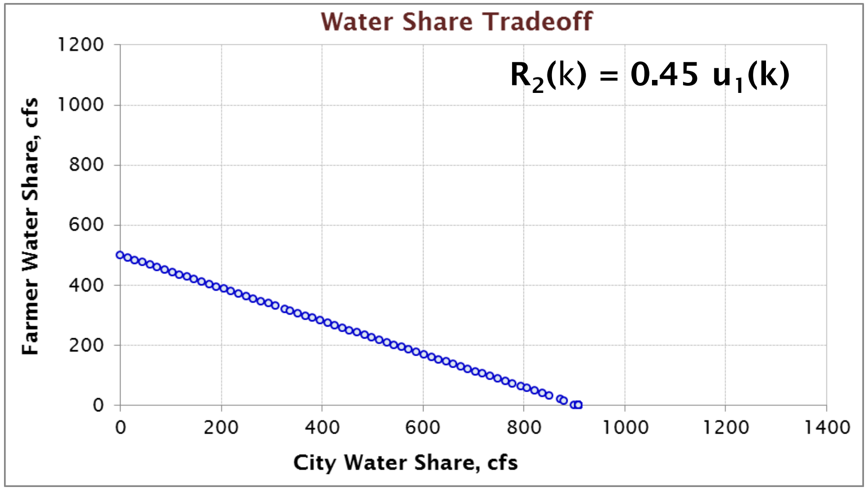

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)]$, i=1, 3



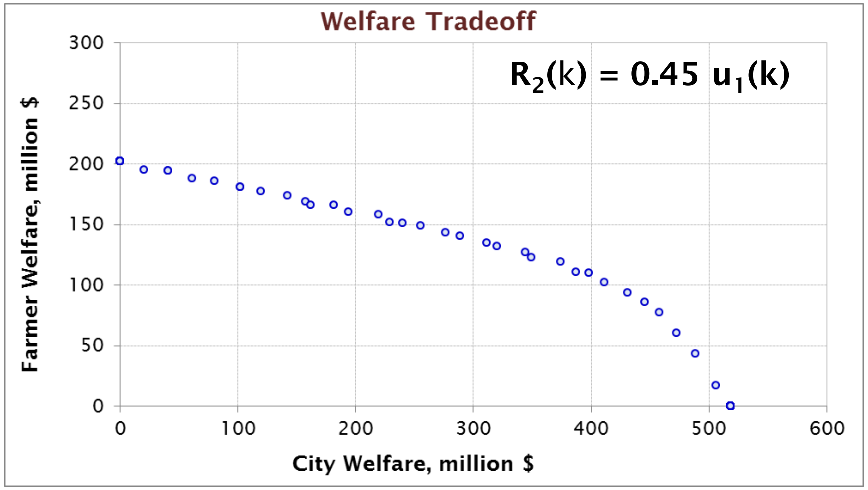
# 3.3.2 Sensitivity to Return Flow[5]

**Results:** Tradeoff Comparison for $R_2(k) = 0.45\ u_1(k)$ [**Top**], $R_2(k) = 0.65\ u_1(k)$ [**Bottom**]:

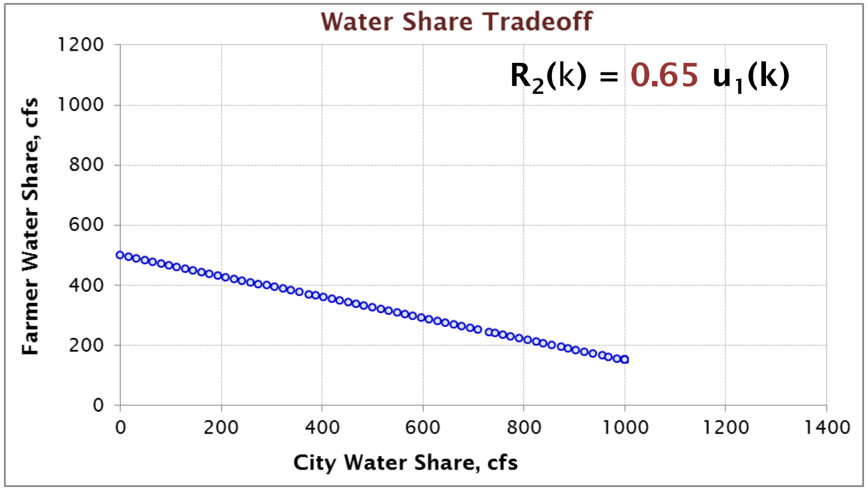
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



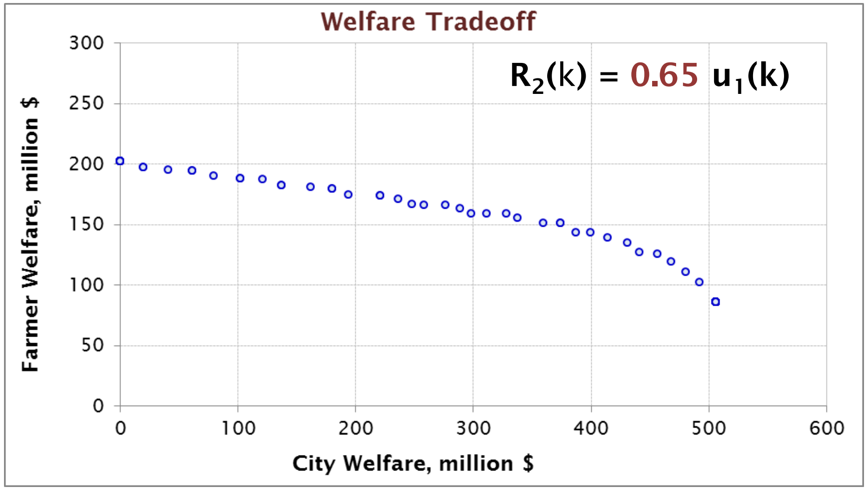

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



# 3.3.3 Sensitivity to Reservoir-related Interventions

For the system described in Example 3.1.1, you are asked to assess the benefits or impacts of water management alternatives associated with the construction of a dam with various reservoir capacities. The reservoir location, net evaporation rate, and storage – elevation curve are shown below.

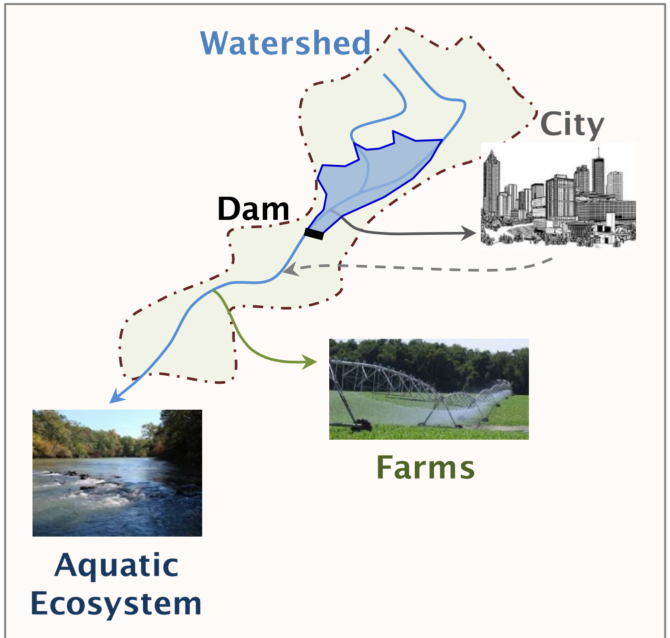



Figure A: River and Water Uses Layout

## Approach:

We assume that the reservoir location is as shown in the adjacent figure. The benefits and impacts that the dam implies for each basin stakeholder can be quantified by re-deriving the stakeholder tradeoffs for different reservoir capacities.

**Figure E:** Net Evaporation Rate

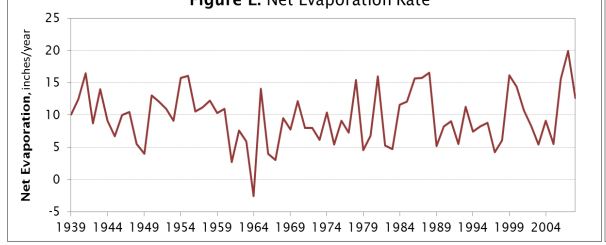



**Figure F:** Reservoir Level vs. Storage Curve

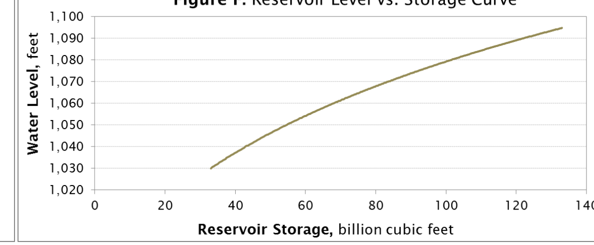



**Figure B:** Natural Inflow Record

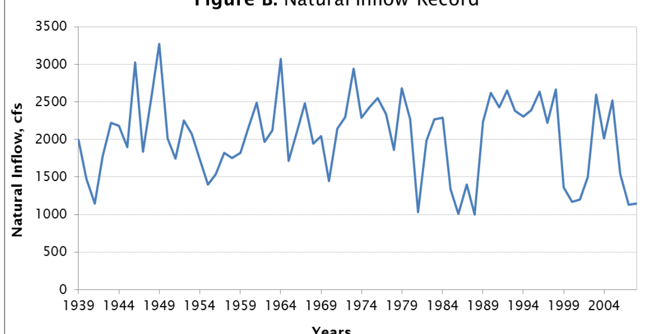



**Figure C:** City Water Demand Function

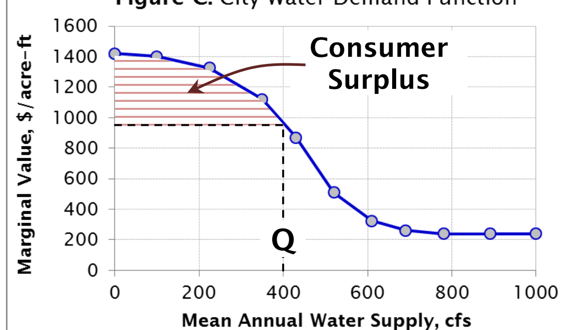
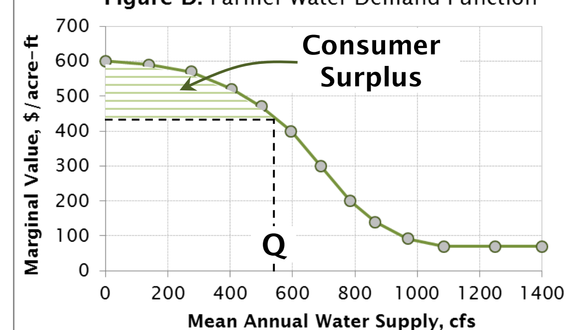



**Figure D:** Farmer Water Demand Function



# 3.3.3 Sensitivity to Reservoir-related Interventions[2]

## System Model

**Time Step k:**

**Node 1** (Reservoir Water Balance)**:**

$S_1(k+1) = S_1(k) + I_1(k) – E_1(k) [A_1(k) + A_1(k+1)]/2 – u_1(k) – Q_1(k) – Sp_1(k)$

$u_1(k) = U_1$ (*at first iteration*)

$u_3(k) = U_3$ (*at first iteration*)

$R_2(k) = 0.45\ u_1(k)$;

$Q_1(k) = u_3(k) – R_2(k) + 0.5\ I_1(k)$

$Sp_1(k) = 0$ (*at first iteration*)

**Node 2:**

$R_2(k) = 0.45\ u_1(k)$

$Q_2(k) = Q_1(k) + R_2(k) + Sp_1(k)$

**Node 3:**

$u_3(k) = \min\{U_3, Q_2(k)\}$

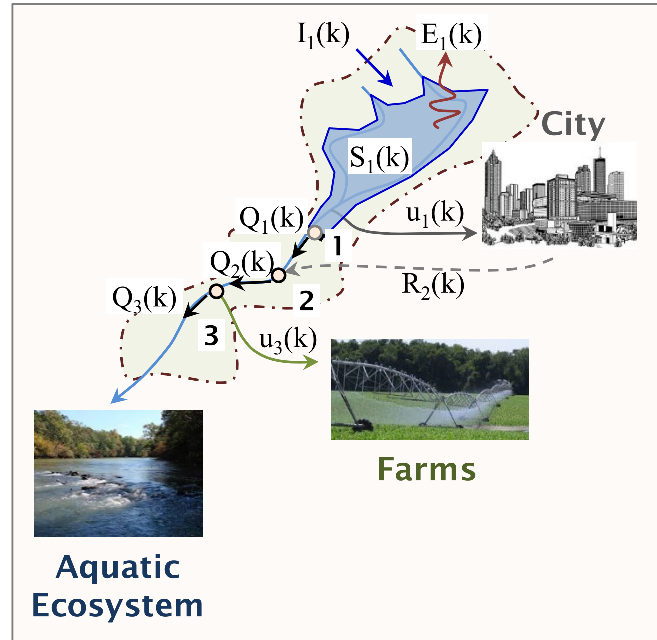
$Q_3(k) = Q_2(k) - u_3(k)$

**Check for storage bounds violations:**

- If $S_1(k+1) > S_{1mx}$, then:
  - $Sp_1(k) = S_1(k+1) - S_{1mx}$
  - $S_1(k+1) = S_{1mx}$
  
  And, repeat calculations at Nodes 1, 2, and 3 until $S_1(k+1) \approx S_{1mx}$.

- If $S_1(k+1) < S_{1mn}$, then:
  - $D_{fct}(k) = S_{1mn} - S_1(k+1)$
  - $S_1(k+1) = S_{1mn}$
  - $F_1 = U_1/(U_1+U_3)$ and $F_3 = U_3/(U_1+U_3)$
  - $u_j = \max\{U_j – F_jD_{fct}, 0\}, j = 1, 3$
  
  And, repeat calculations at Nodes 1, 2, and 3 until $S_1(k+1) \approx S_{1mn}$ or $\{u_j = 0, j = 1, 3\}$.

**Advance to Step k+1:**




**Variable Definitions:**

k: Time index (years), k = 1, 2, …, N.

$I_1(k)$: Tributary inflow (runoff from the entire watershed) in year k (1939 – 2008).

$S_1(k)$: Reservoir storage at beginning of year k.

$S_{1mx}$, $S_{1mn}$: Maximum, minimum storage.

$A_1(k)$: Reservoir area at beginning of year k.

$E_1(k)$: Reservoir net evaporation, year k.

$Q_1(k)$: Reservoir release, node 1, year k.

$Sp_1(k)$: Reservoir spillage, year k.

$u_1(k)$: City water withdrawal, year k.

$R_2(k)$: City return flow, node 2, year k.

$Q_2(k)$: Flow downstream of node 2, year k.

$u_3(k)$: Irrigation water withdrawal, node 3, year k.

$Q_3(k)$: Flow downstream of node 3, year k.

Fluxes are expressed in units of *volume per year*.

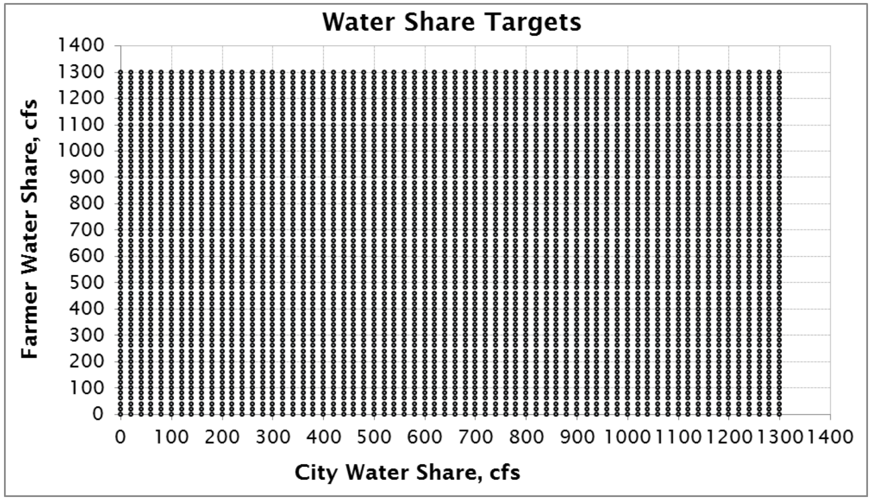
$E_1(k)$ is expressed in units of *depth per year*.

$S_1(k)$ is expressed in units of *volume*.

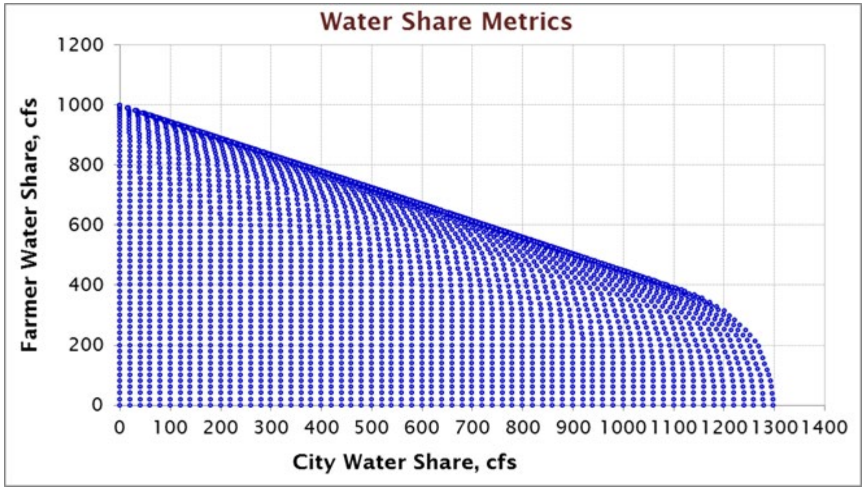
# 3.3.3 Sensitivity to Reservoir-related Interventions[3]

**Results:** Performance metrics for $S_{1mx}$ = 60 bcf, $S_{1mn}$ = 37.75 bcf:

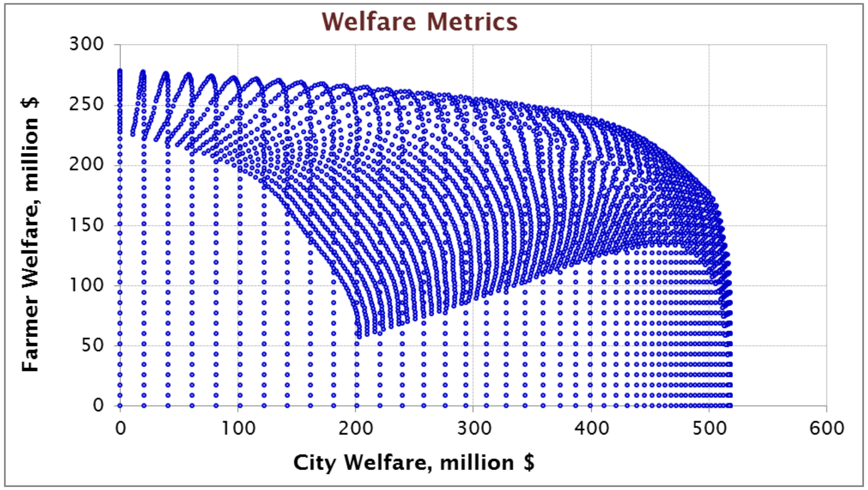
$U_1 = \{0 - 1300, \Delta U_1 = 20\}$ cfs
$U_3 = \{0 - 1300, \Delta U_3 = 20\}$ cfs




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)]$, i=1, 3




$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)]$, i=1, 3

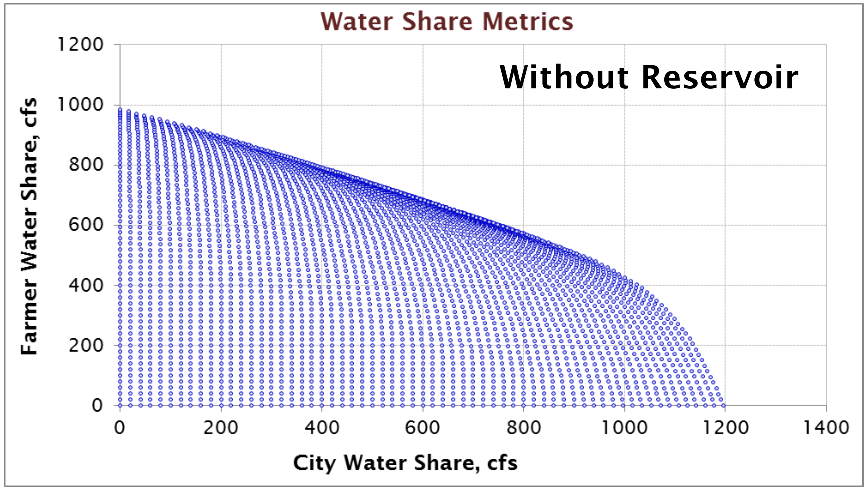


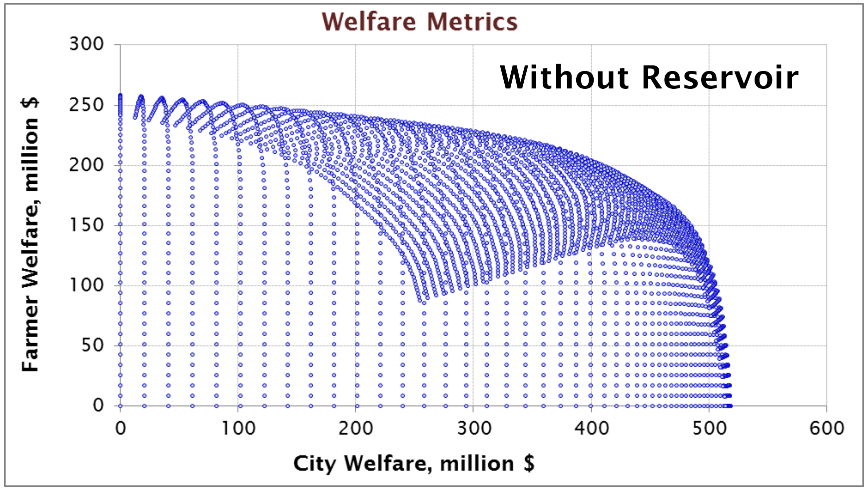
# 3.3.3 Sensitivity to Reservoir-related Interventions[4]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **60 bcf** [**Bottom**]:

$$J_{mean}(U_i) = 1/N\,[u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$$

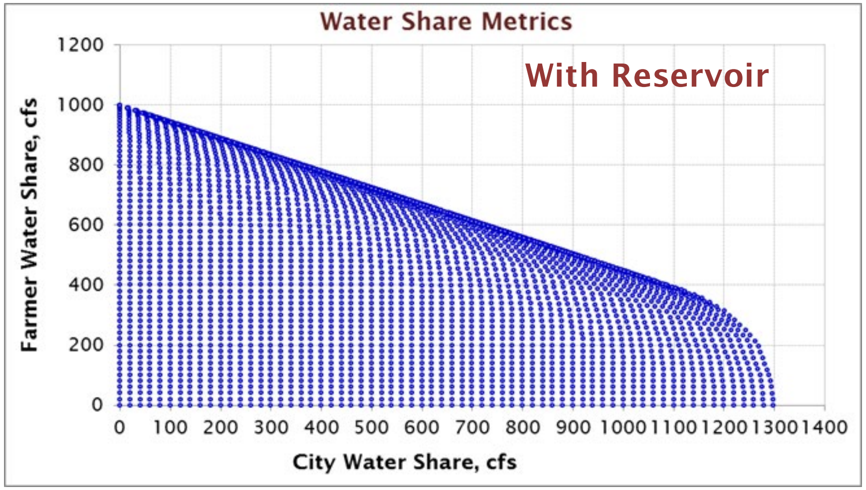
$$V_{mean}(U_i) = 1/N\,[EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$$

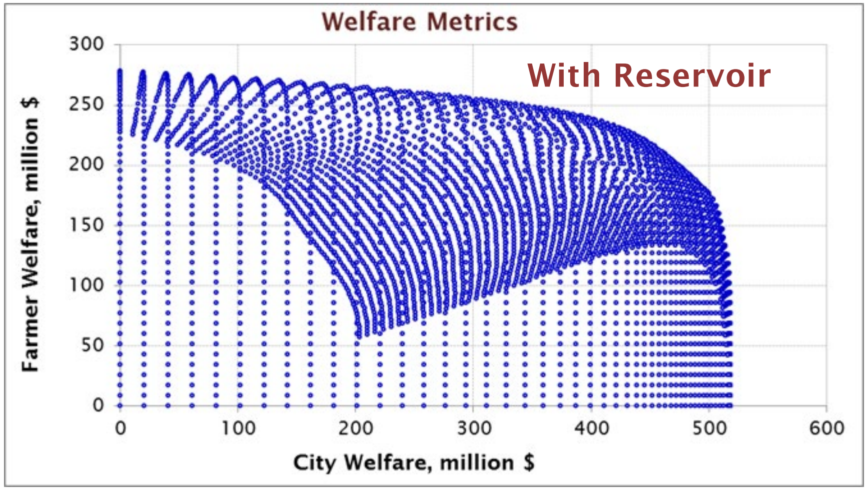



$$J_{mean}(U_i) = 1/N\,[u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$$

$$V_{mean}(U_i) = 1/N\,[EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$$

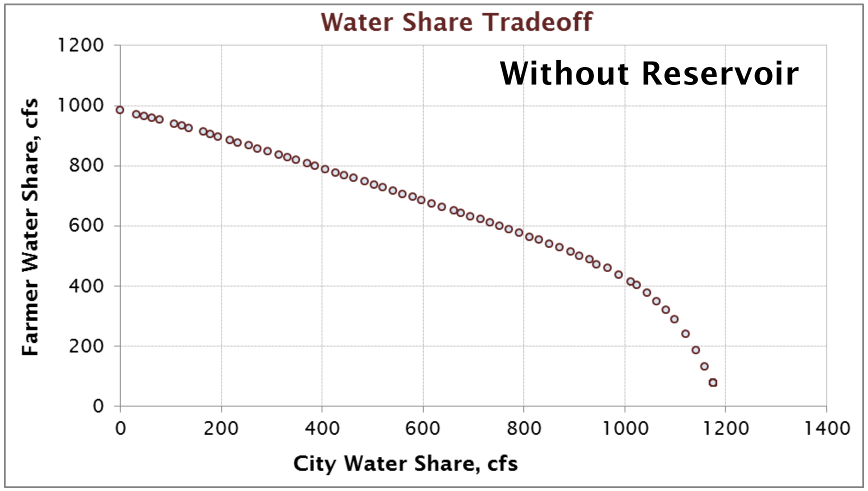


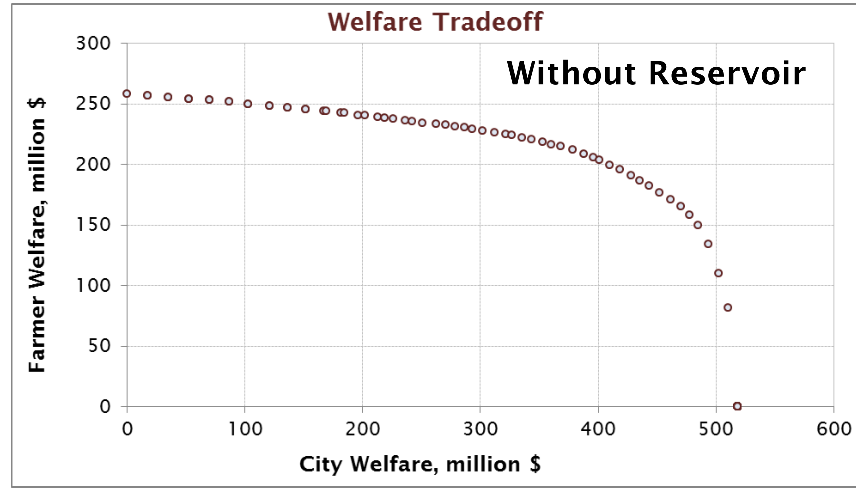
# 3.3.3 Sensitivity to Reservoir-related Interventions[5]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **60 bcf** [**Bottom**]:

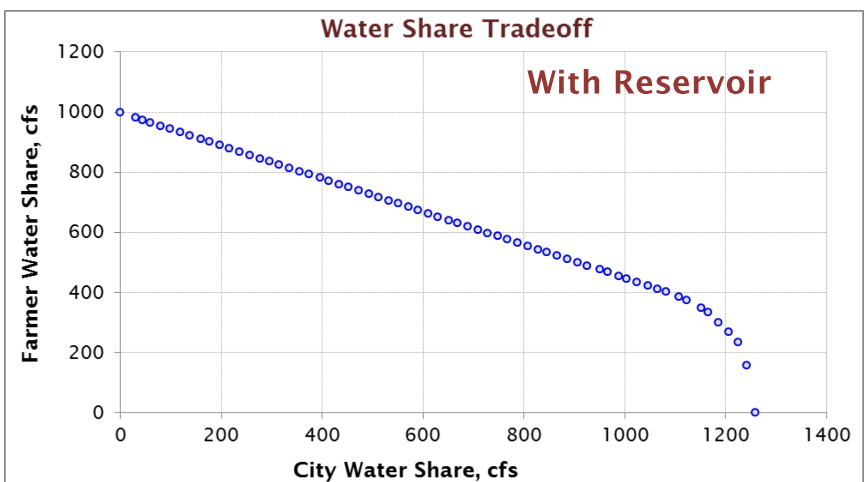
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



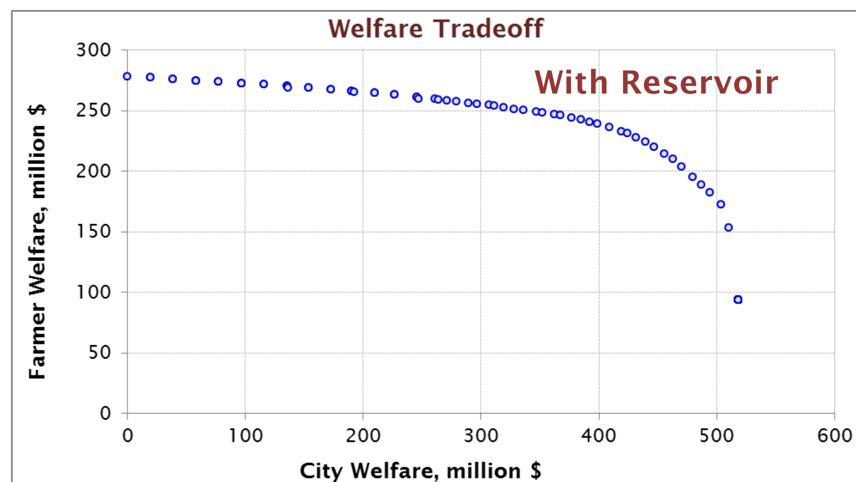

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



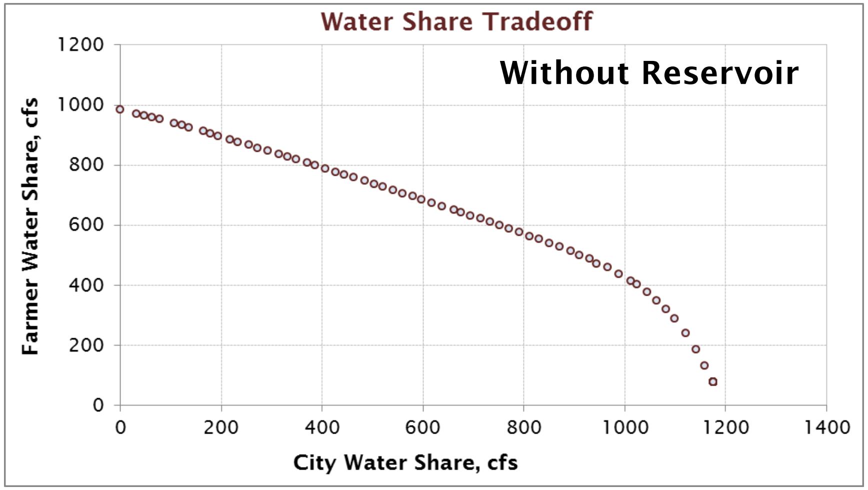

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



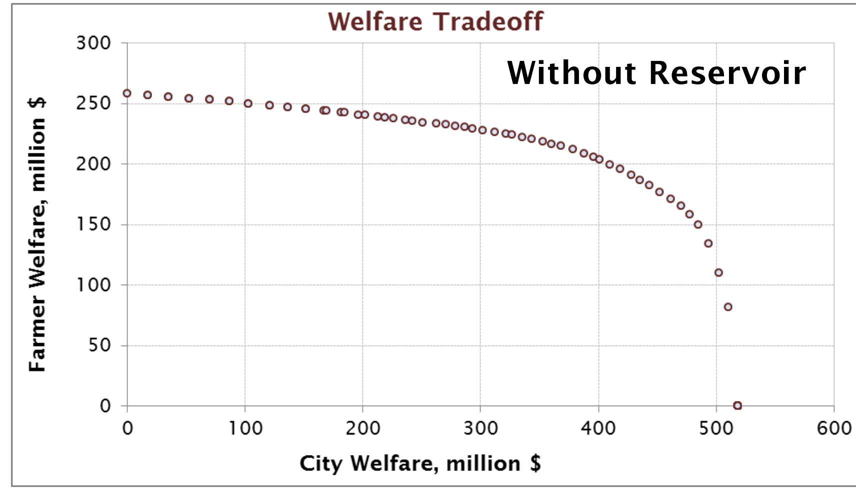
# 3.3.3 Sensitivity to Reservoir-related Interventions[6]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **70 bcf** [**Bottom**]:

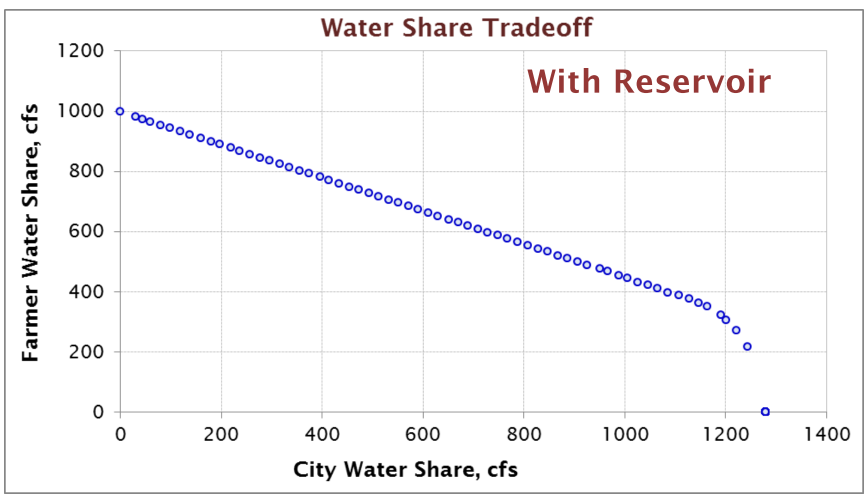
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



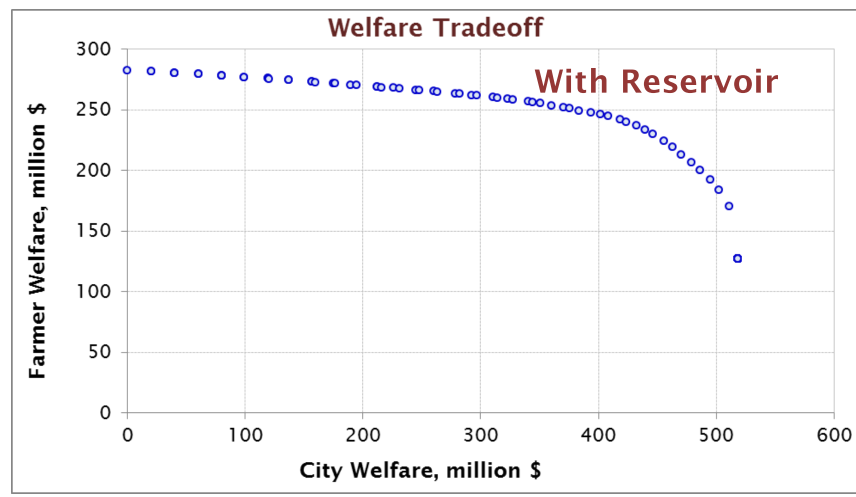

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



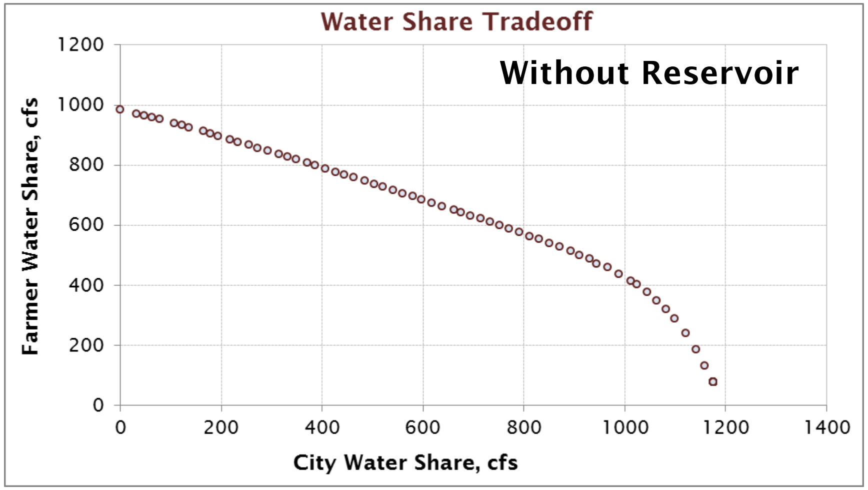

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



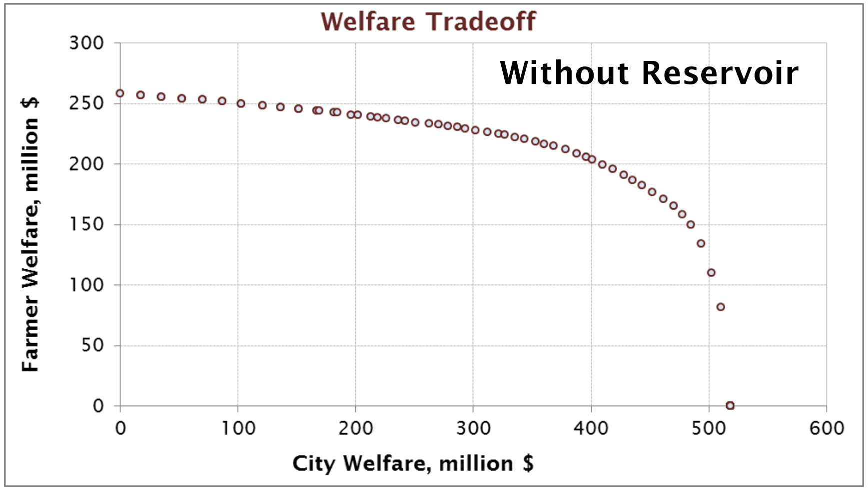
# 3.3.3 Sensitivity to Reservoir-related Interventions[7]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **83.5 bcf** [**Bottom**]:

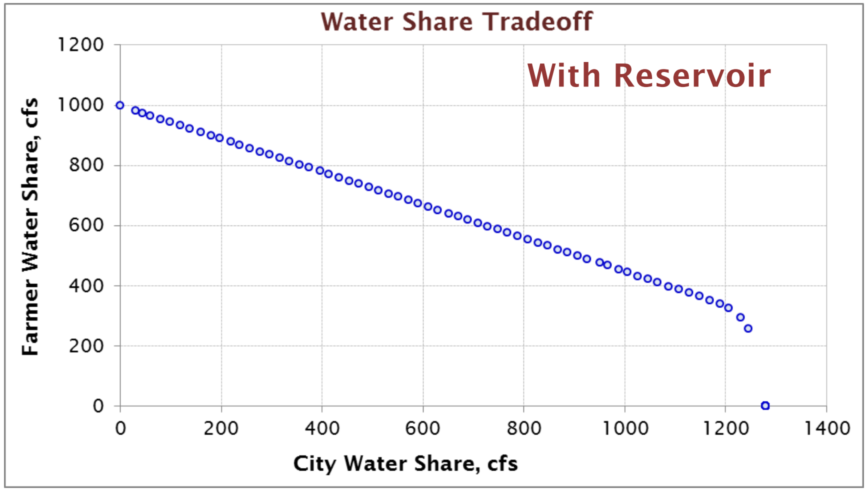
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



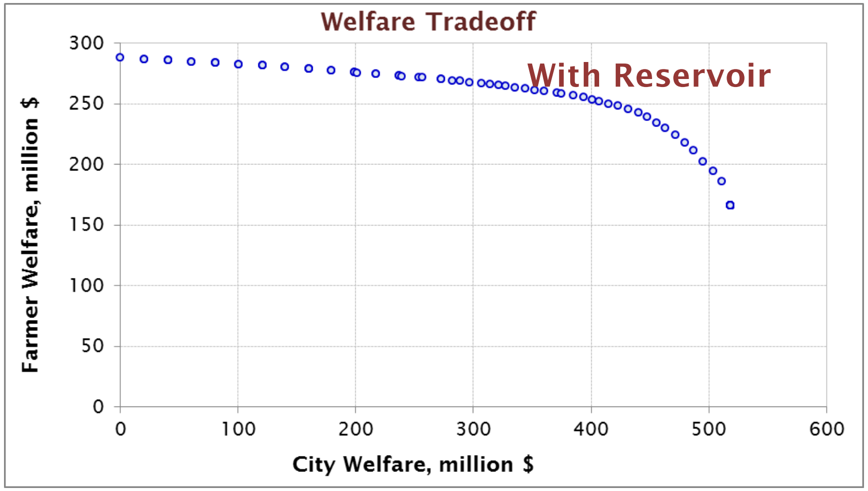

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



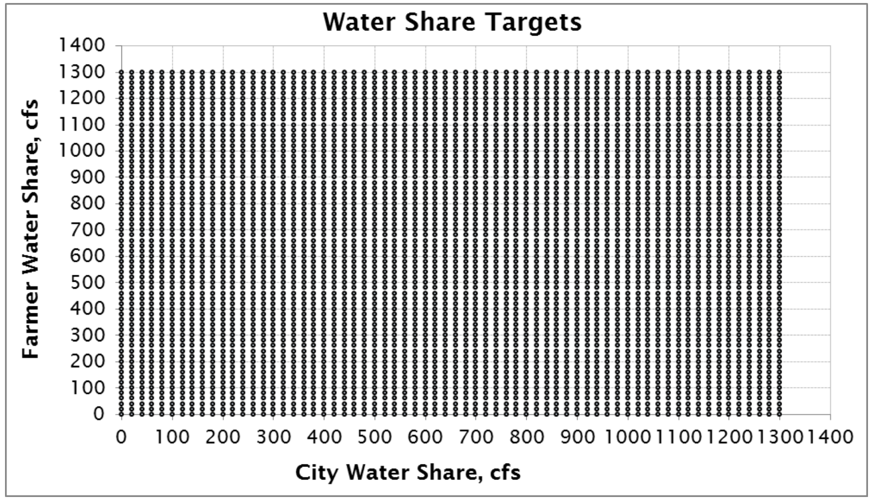

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



# 3.3.3 Sensitivity to Reservoir-related Interventions[8]

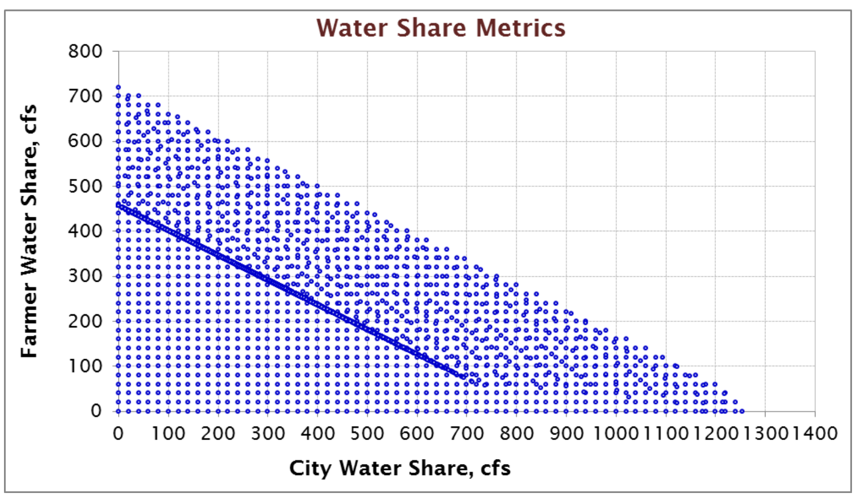
**Results:** Max/min Performance metrics for $S_{1mx}$ = 60 bcf, $S_{1mn}$ = 37.75 bcf:

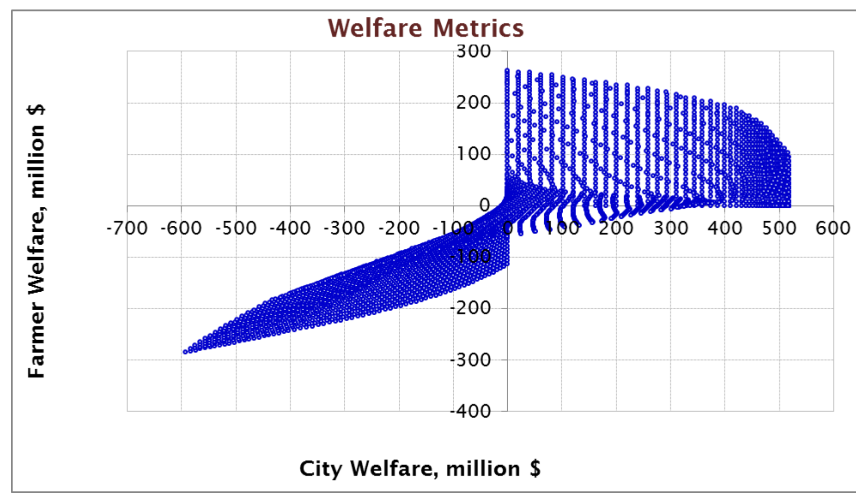
$U_1$ = {0 - 1300, $\Delta U_1$=20} cfs
$U_3$ = {0 – 1300, $\Delta U_3$=20} cfs




$J_{max/min}(U_i)$ = max{min{$u_i(1)$, …, $u_i(70)$}}, i=1, 3

$V_{max/min}(U_i)$ = max{min{$EW_i(1)$, …, $EW_i(70)$}}, i=1, 3

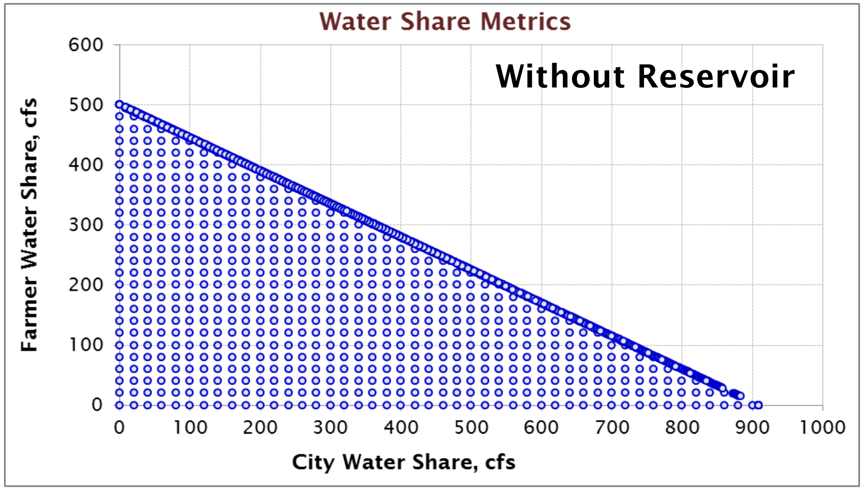


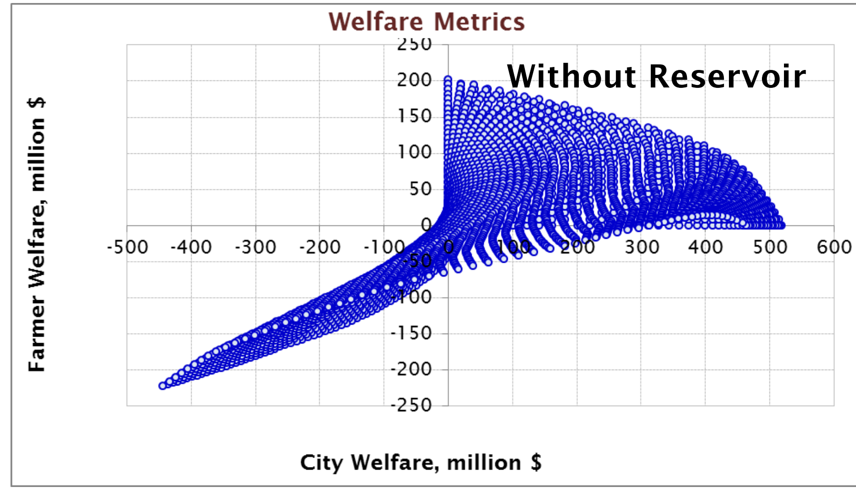
# 3.3.3 Sensitivity to Reservoir-related Interventions[9]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. <u>**60 bcf**</u> [**Bottom**]:

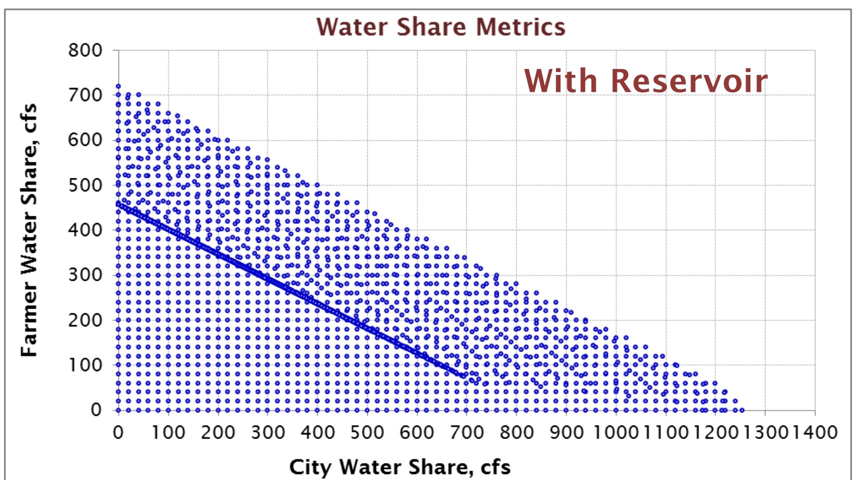
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



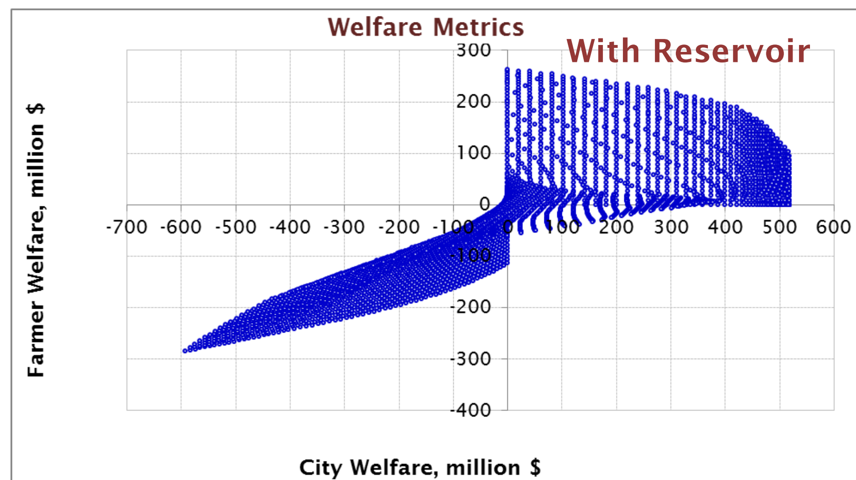

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



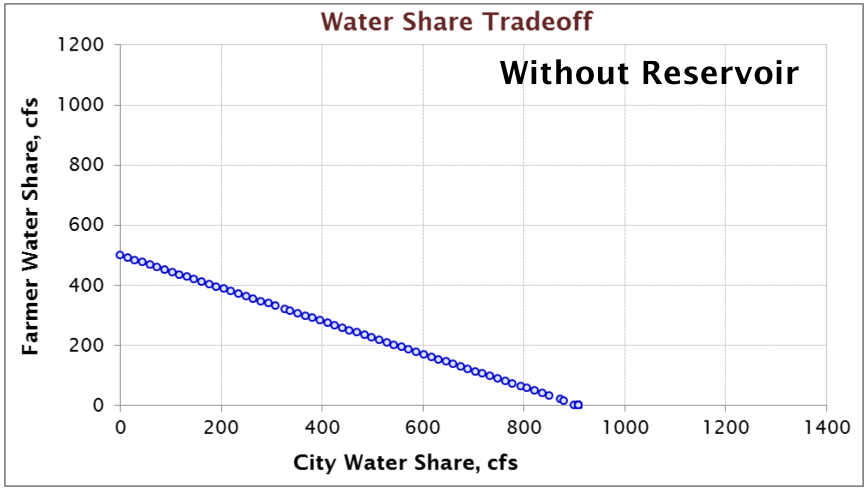

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



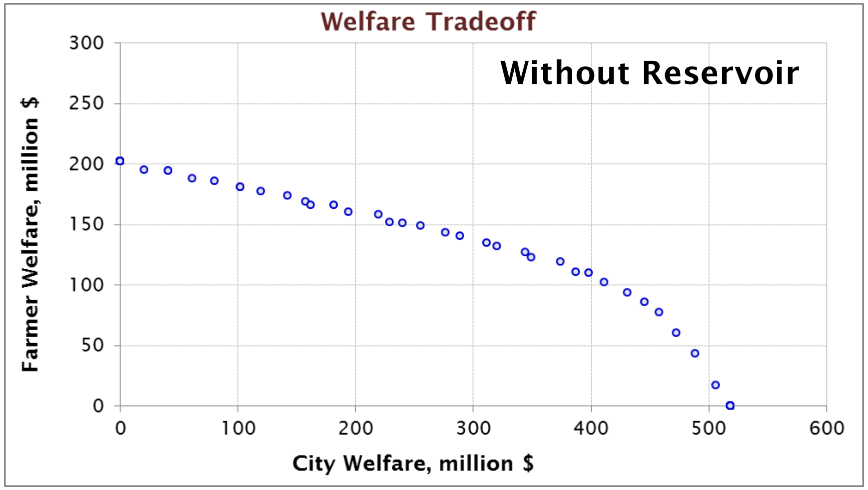
# 3.3.3 Sensitivity to Reservoir-related Interventions[10]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **60 bcf** [**Bottom**]:

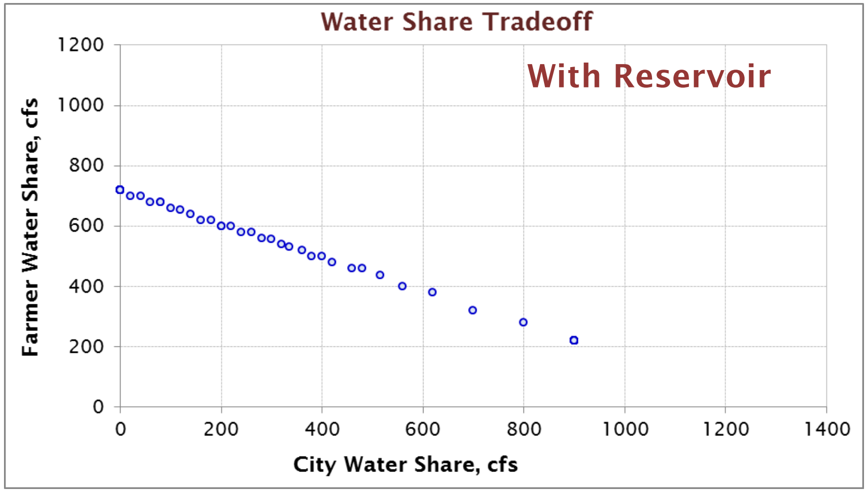
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



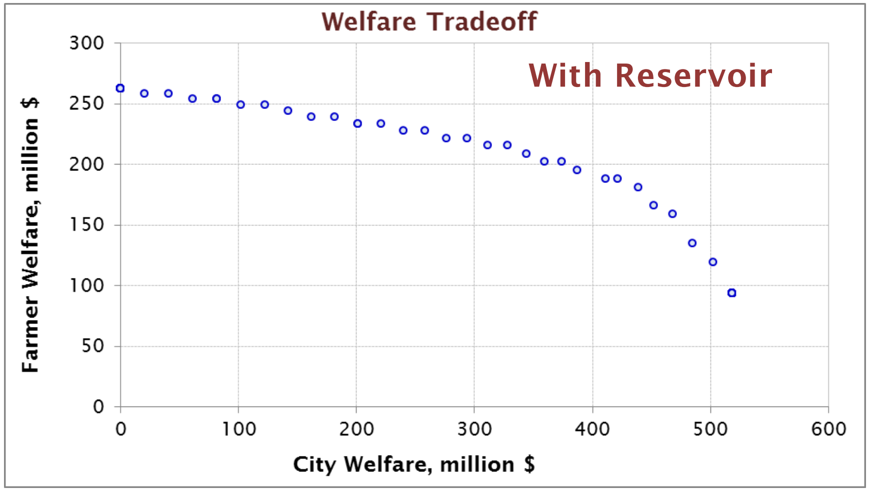

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



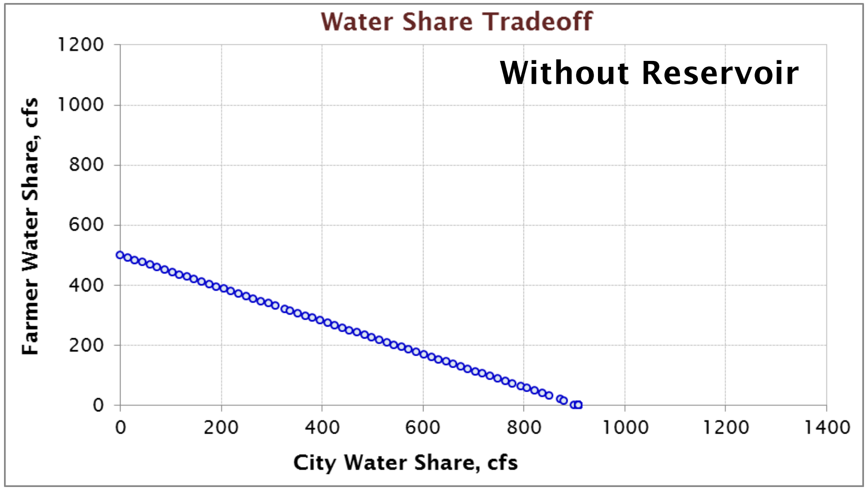

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



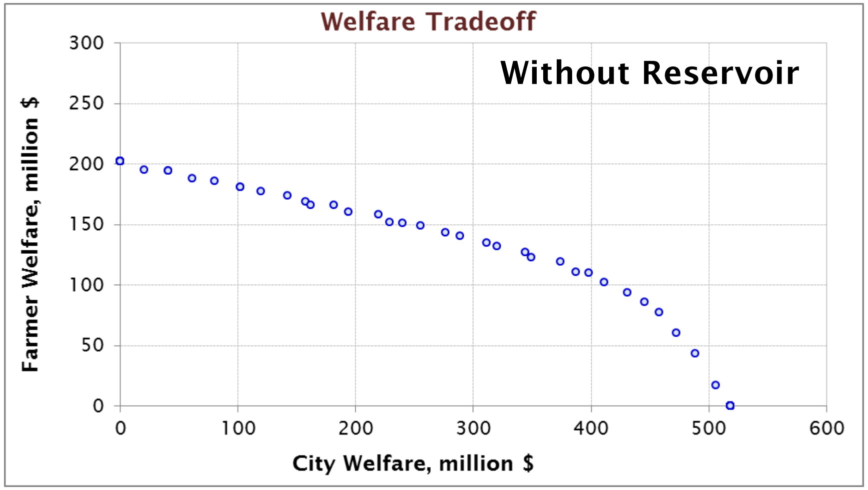
# 3.3.3 Sensitivity to Reservoir-related Interventions[11]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **70 bcf** [**Bottom**]:

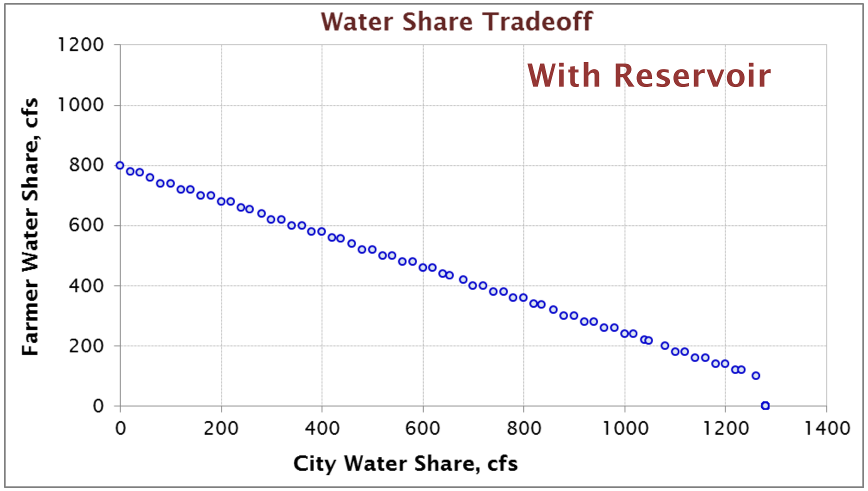
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



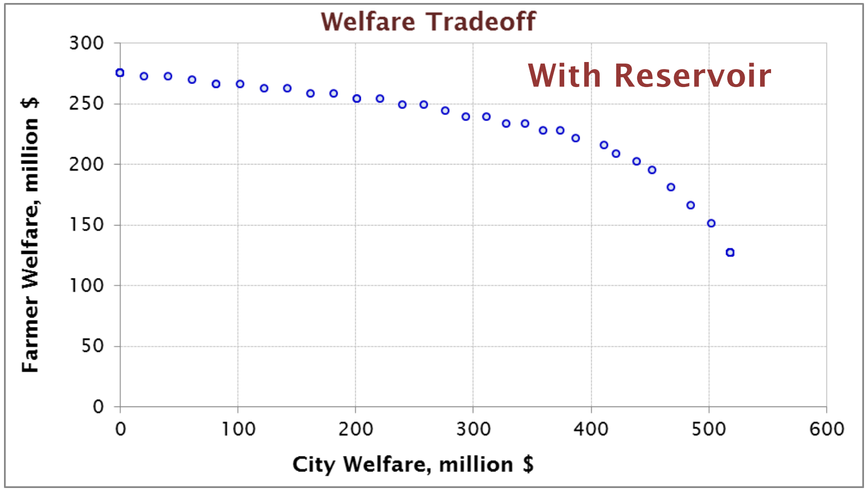

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



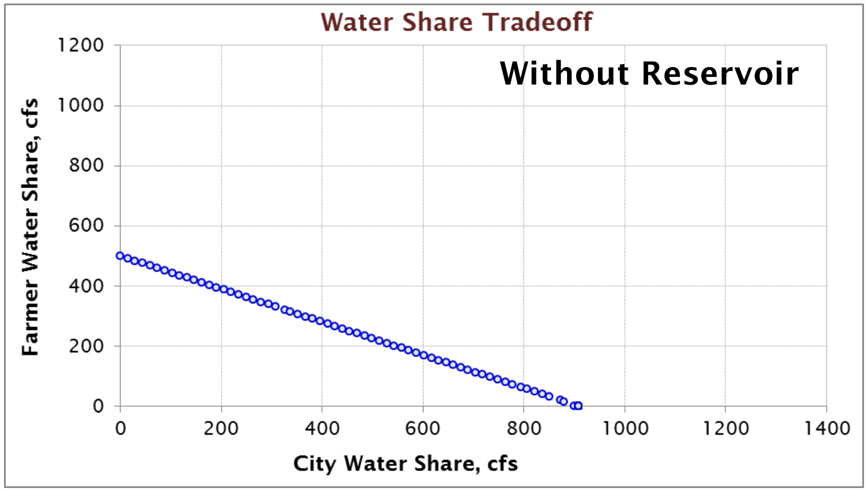

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



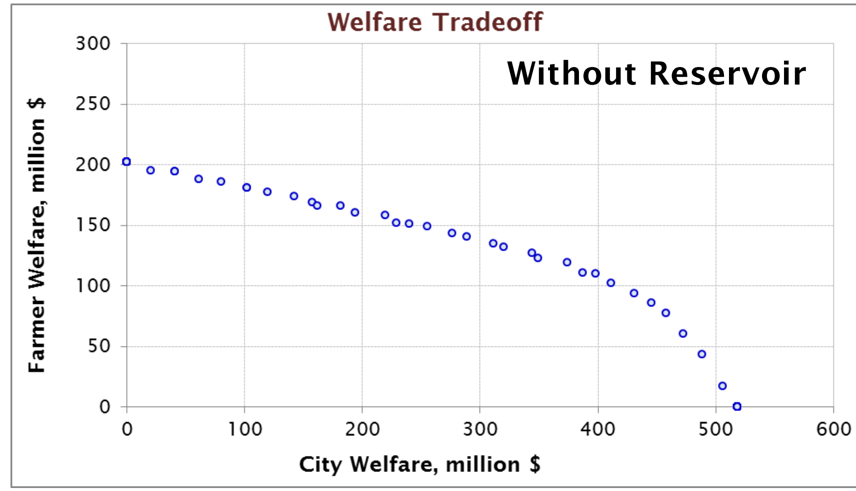
# 3.3.3 Sensitivity to Reservoir-related Interventions[12]

**Results:** Metrics Comparison without Res. [**Top**] vs. with Res. **83.5 bcf** [**Bottom**]:

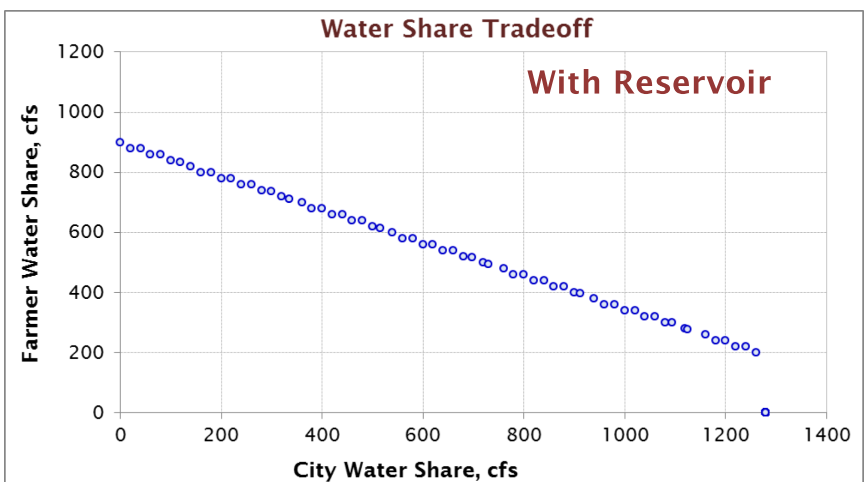
$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$



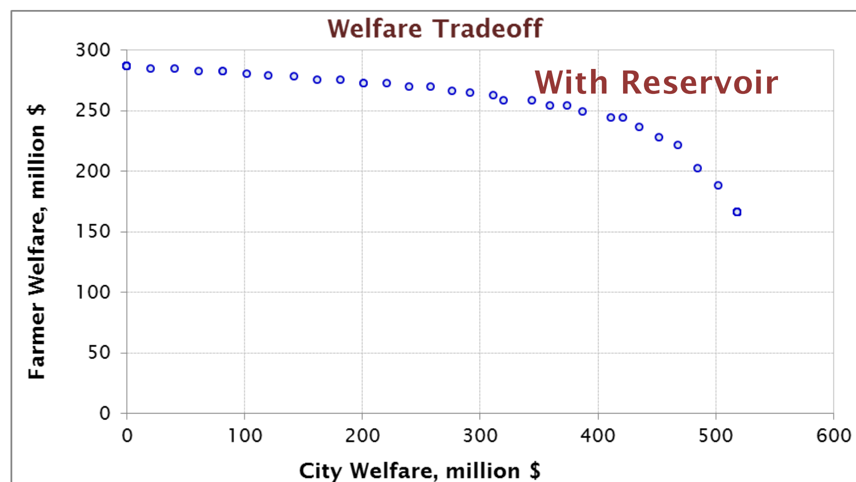

$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\}, i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\}, i=1, 3$



# 3.3.4 Sensitivity to Inflow Uncertainty

For the system described in Example 3.1.1, you are asked to assess the sensitivity of the identified tradeoffs to the uncertainties associated with the inflow process (UIFs). Assume that the reservoir storage capacity is 70 bcf.

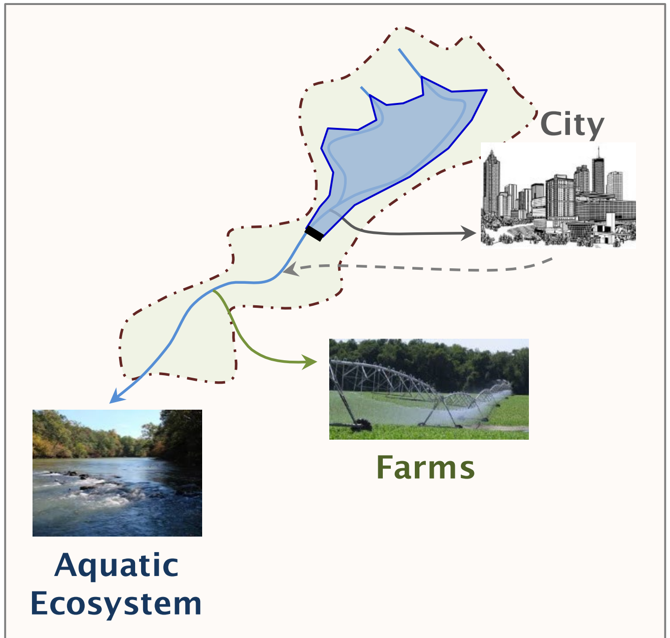



**Figure A:** River and Water Uses Layout

## Approach:

The approach is based on generating many alternative plausible inflow series and repeating the assessment. This procedure provides the means to *(i)* associate reliability measures to any assessment outcome, and *(ii)* characterize the robustness of the assessment results and recommendations. Exhibit 2.9 provides the background for the generation of alternative plausible inflow series.

**Figure E:** Net Evaporation Rate

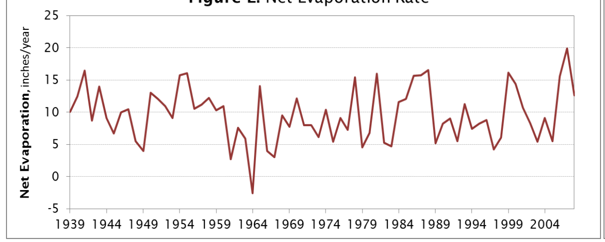



**Figure F:** Reservoir Level vs. Storage Curve

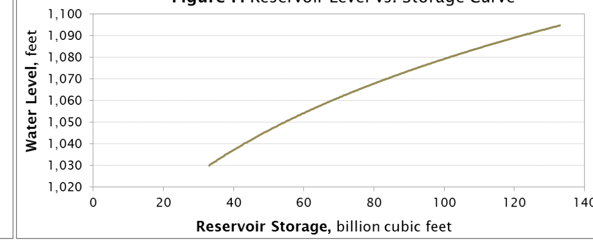



**Figure B:** Natural Inflow Record

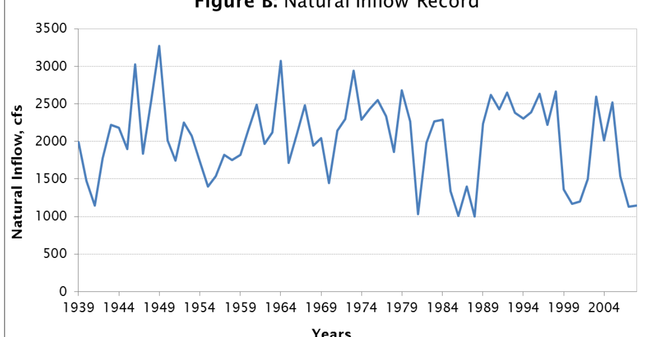



**Figure C:** City Water Demand Function

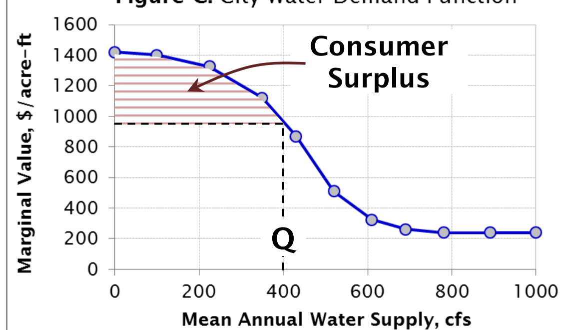
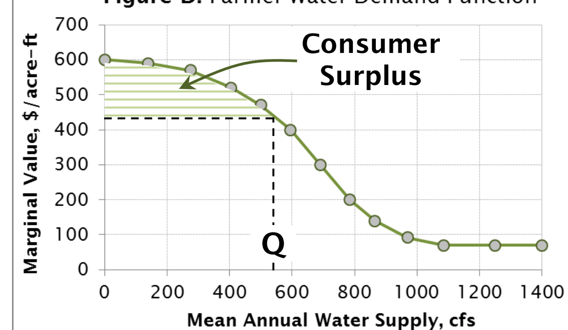



**Figure D:** Farmer Water Demand Function



# Exhibit 3.3.4.1: Alternative Inflow Sequences

In all previous examples, the hydrologic basis for the assessments is the historical inflow series, I(k), k=1,…, N. However, this series is one of many possible realizations of the inflow process, as one can gather if some other historical inflow period were to be selected for the assessments. Thus, a relevant question is, how sensitive are the identified tradeoffs and plan recommendations to this series? This question is especially critical when the available inflow series is relatively short (i.e., less than 30 years). A typical approach to address this question is to generate alternative and equally likely inflow series (typically referred to as *synthetic* series), repeat the assessments, and quantify the sensitivity and uncertainty of the assessment results and plan recommendations. In this exhibit, we outline general procedures to generate alternative but plausible series for inflows or any other inputs.

## (*a*) Series Correlation

A time series with significant serial correlation tends to contain inflow clusters of similar magnitude. This is important for water resources because it implies that low flows (droughts) or high flows (floods) tend to persist. For this reason, synthetic inflows need to be consistent with (i.e., preserve) the correlation, if any, of the historical time series.

The serial correlation of a time series I(k), k=1,…, N, can be estimated by the following expression:

$$r = \frac{\sum_{k=1}^{N-1}\left[I(k)-\left(\frac{1}{N-1}\sum_{k=1}^{N-1}I(k)\right)\right]\left[I(k+1)-\left(\frac{1}{N-1}\sum_{k=2}^{N}I(k)\right)\right]}{\sqrt{\sum_{k=1}^{N-1}\left[I(k)-\left(\frac{1}{N-1}\sum_{k=1}^{N-1}I(k)\right)\right]^2}\sqrt{\sum_{k=2}^{N}\left[I(k)-\left(\frac{1}{N-1}\sum_{k=2}^{N}I(k)\right)\right]^2}}$$

It can be shown that $-1 \le r \le 1$. When r is close to zero, the series exhibits no serial correlation, while when r approaches 1 or -1, subsequent series values exhibit significant positive or negative correlation, respectively.

The above expression provides an *estimate* of the true series correlation $\rho$. However, all estimates are bound to be imprecise because they are derived from finite data samples. This can be understood if we divide the data sample in two parts and compute the sample serial correlation of each. It is highly unlikely that the two estimates will be equal. Thus, even when the sample estimate of the serial correlation is small, we cannot immediately conclude that the true process correlation is *zero*. Reversely, when the sample correlation is 0.2, -0.3, etc., we cannot immediately conclude that the process values are positively or negatively correlated. We need a quantitative way to test the significance or not of $\rho$ given the data sample and the estimated (from the sample) correlation coefficient r.

# Exhibit 3.3.4.1: Alternative Inflow Sequences[2]

## (*a*) Series Correlation cont'd

A powerful such test can be based on the hypothesis that ρ is indeed zero and assessing the likelihood of obtaining a non-zero value of r from a random sample of N values. If this likelihood is very low, then we can infer that the series is uncorrelated; however, if the likelihood is high, our hypothesis (of ρ being zero) is most likely invalid.

## (*b*) Significance Test for Serial Correlation

1. Generate N integer random numbers $u_1(k)$, k = 1,…, N, between 1 and N (inclusive of 1 and N).
2. Construct a new synthetic series $I_1(k)$, k = 1,…, N, such that $I_1(k) = I[u_1(k)]$. (Sampling with replacement.)
3. Compute the sample correlation $r_1$ of the synthetic series $\{I_1(k), k = 1,\ldots, N\}$.
4. Repeat Steps (1), (2), and (3) M times, where M = 1000 (or higher) and obtain M sample correlation values $r_j$, j=1,…, M.
5. Construct the empirical frequency distribution of $r_j$ and obtain the $r_{2.5\%}$ and $r_{97.5\%}$ percentile values.
6. Compare the sample correlation, r, of the original series I(k), k=1,…, N, with the above percentiles, and infer whether the process is uncorrelated or not as follows:

   If $r_{2.5\%} \leq r \leq r_{97.5\%}$, then the process correlation can be assumed to be zero, recognizing that this inference may be in error (i.e., the process correlation is actually non-zero) in 5% of such tests.

   If $r < r_{2.5\%}$ or $r > r_{97.5\%}$, then the process correlation cannot be assumed to be zero, recognizing that this inference may be in error (i.e., the process correlation is indeed zero) in 5% of such tests.

_Note 1:_ The value of M = 1000 should usually suffice, but the test inference sensitivity can also be tested for higher M values.
_Note 2:_ Generally, 2.5% is a typical percentile threshold used in such tests. However, one may examine the sensitivity of the test inference for different error likelihoods such 1%, 2.5%, 5%, etc.

## (*c*) Synthetic Inflow Generation for Uncorrelated Inflows

1. Generate N integer random numbers $u_1(k)$, k = 1,…, N, between 1 and N (inclusive of 1 and N).
2. Construct a new synthetic series $I_1(k)$, k = 1,…, N, such that $I_1(k) = I[u_1(k)]$.
3. Carry out system assessments using the new synthetic series $\{I_1(k), k = 1,\ldots, N\}$.
4. Repeat Steps (1), (2), and (3) M times, where M = 20 (or higher) and assess the sensitivity and uncertainty of all modeling results.

# Exhibit 3.3.4.1: Alternative Inflow Sequences[3]

## (*d*) Synthetic Inflow Generation for Serially Correlated Inflows

Let m and r respectively denote the mean and serial (lag-1) correlation estimates of a series {I(k), k = 1,…, N}. Moreover, denote by z(k) a new process obtained by subtracting m from each I(k) value, z(k) = I(k) – m, k = 1,…, N. Finally, compute the series of residuals:

e(k) = z(k) – r z(k-1), k = 2,…, N.

Then, the generation of alternative synthetic inflow series proceeds as follows:

1. Generate an integer random number $u_1(1)$, between 1 and N (inclusive of 1 and N).
2. Determine the first value, $I_1(1)$, of a new synthetic series such that $I_1(1) = I[u_1(1)]$.
3. Generate N-1 integer random numbers $u_1(k)$, k = 2,…, N, between 2 and N (inclusive of 2 and N).
4. Construct a new synthetic series $I_1(k)$, k = 2,…, N, such that $I_1(k) - m = r\,[I_1(k-1) - m] + e[u_1(k)]$.
5. Carry out system assessments using the new synthetic series $\{I_1(k), k = 1,\ldots, N\}$.
6. Repeat Steps (1) through (5) M times, where M = 20 (or higher) and assess the sensitivity and uncertainty of all modeling results.

*Note 1:* The underlying assumption in the above procedure is that the lag-1 (serial) correlation is the only significant correlation of the I(k) series. This assumption can be tested by applying the significance test of Section 3.3.4.1(b) on the residual series e(k). If this test indicates that the serial correlation of the e(k) is also significant, then the above procedure can first be applied to generate a synthetic residual series, e(k), which can then be used to generate a synthetic inflow series, I(k), as indicated above. Such a procedure can be repeated until no significant serial correlation is detected in the residual series.

## (*e*) Synthetic Generation of Seasonal Inflow Series

The above procedures are pertinent for *annual* inflows which do not exhibit regular periodicities. On the other hand, monthly, weekly, and daily inflows exhibit strong *seasonal* cycles. In these cases, the above procedures are still applicable but must be modified to account for the fact that serial correlations and other statistics may differ for different calendar months, weeks, or days.

Consider, for example, a monthly inflow series $I_{ij}(k)$, where i = 1,…, N, denotes years, and j = 1, …, 12, denotes months. Let also $m_j$ and $r_{j,j-1}$ denote the mean inflow of month j and the serial correlation coefficient between the inflows of months j-1 and j.

# Exhibit 3.3.4.1: Alternative Inflow Sequences[4]

## (*e*) Synthetic Generation of Seasonal Inflow Series Cont'd

Moreover, denote by $z_{ij}(k)$ a new process obtained by subtracting $m_j$ from each $I_{ij}(k)$ value, $z_{ij}(k) = I_{ij}(k) - m_j$, $i = 1,\ldots, N$, $j = 1,\ldots, 12$, and compute the monthly residual series:

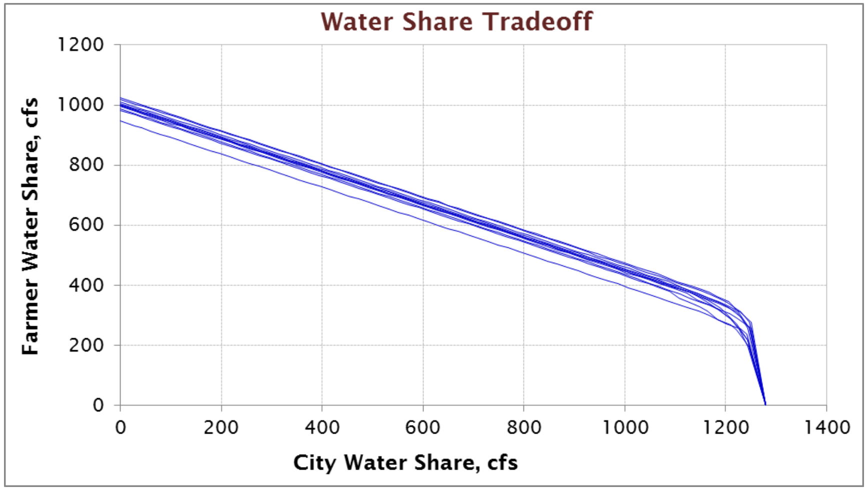
$e_{ij}(k) = z_{ij}(k) - r_{j,j-1}\, z_{ij-1}(k-1)$, $k = 2,\ldots, N$.

Then, the generation of alternative synthetic inflow series proceeds much like the annual series but sequentially using the models for the individual months:

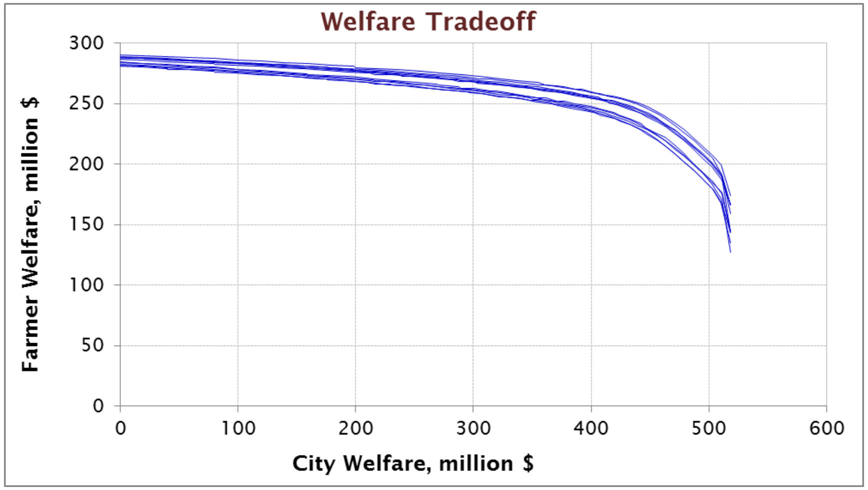
1. Generate an integer random number $u_1(1)$, between 1 and N (inclusive of 1 and N).
2. Determine the first January value, $I_{s11}(1)$ of a new synthetic series such that $I_{s11}(1) = I_{11}[u_1(1)]$.
3. Generate a second integer random number $u_1(2)$ between 2 and N (inclusive of 2 and N).
4. Compute a synthetic inflow value for February, $I_{s12}(2)$, such that $I_{s12}(2) - m_2 = r_{2,1}\,[I_{s11}(1) - m_1] + e_{i2}[u_1(2)]$.
5. Continue in the same sequential fashion for the remaining months of the first year.
6. Repeat for the remaining N-1 years.
7. Carry out system assessments using the new synthetic series $\{I_{sij}(k), i = 1,\ldots, N, j = 1,\ldots, 12\}$.
8. Repeat Steps (1) through (7) M times, where M = 20 (or higher) and assess the sensitivity and uncertainty of all modeling results.

# 3.3.4 Sensitivity to Inflow Uncertainty[2]
## Uncorrelated Synthetic Inflows (10)

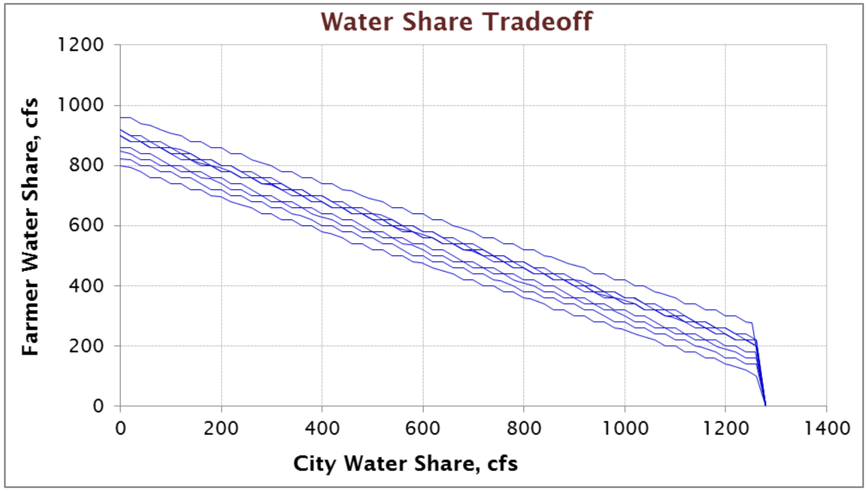
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



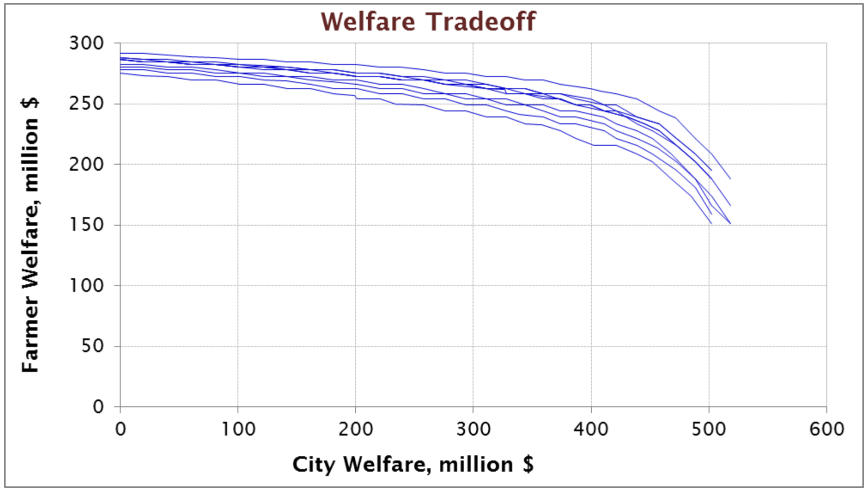

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\},\ i=1, 3$

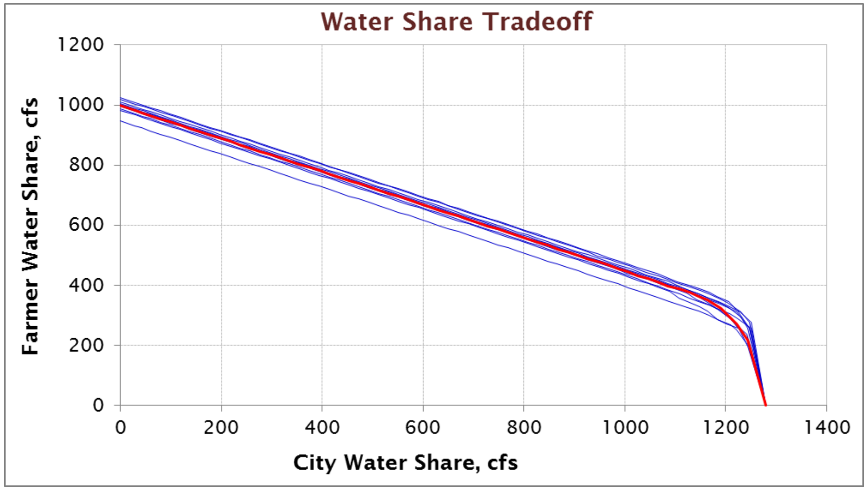



$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\},\ i=1, 3$

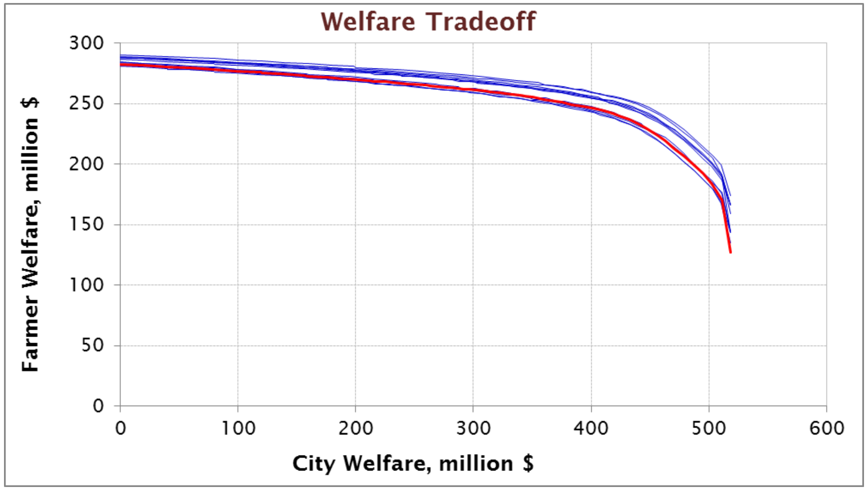


# 3.3.4 Sensitivity to Inflow Uncertainty[3]
## Comparison of Uncorrelated Synthetic, Historical Inflows

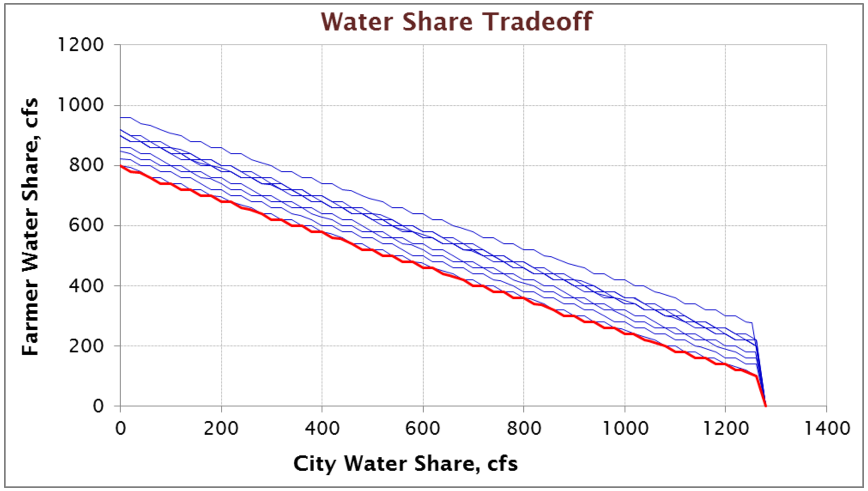
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



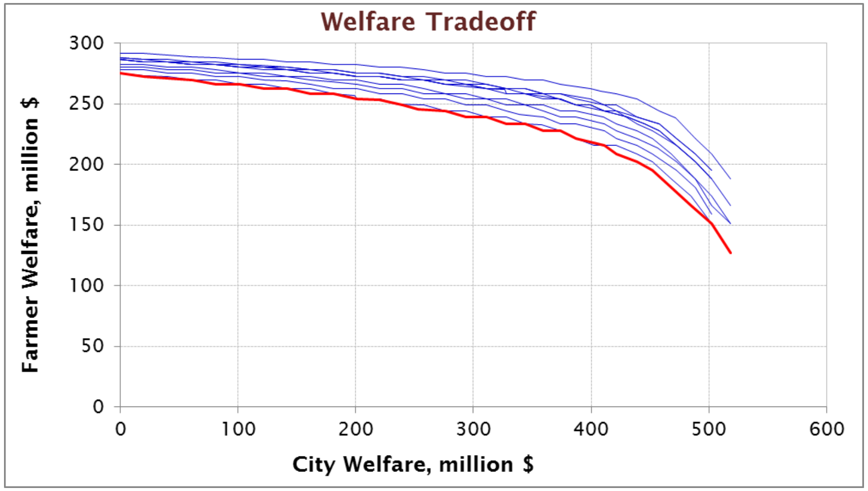

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$



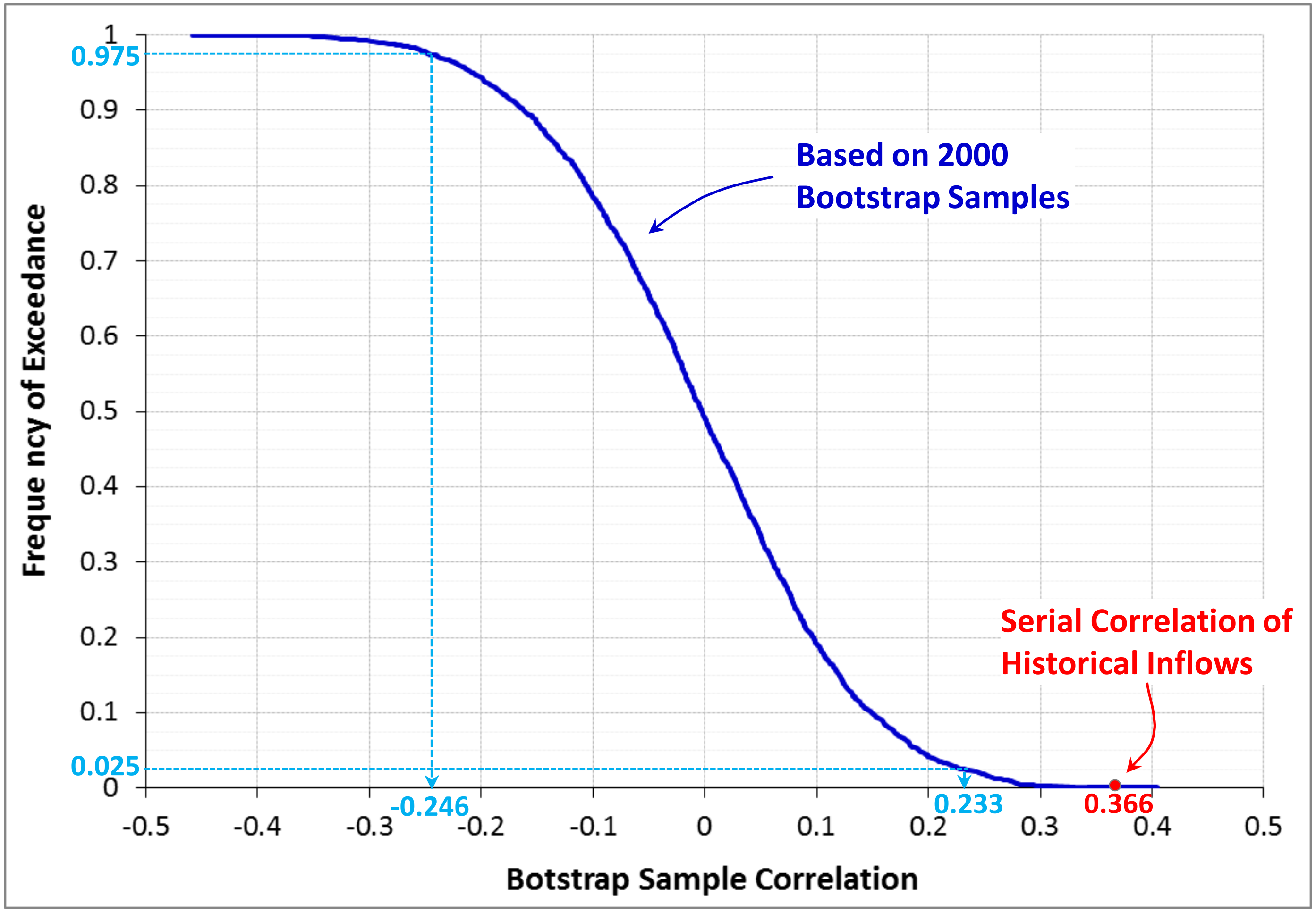

$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\},\ i=1, 3$




$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\},\ i=1, 3$



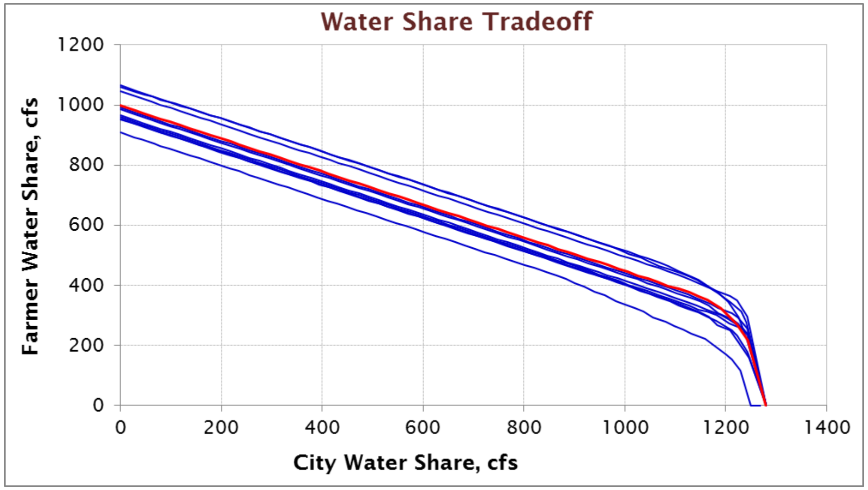
# 3.3.4 Sensitivity to Inflow Uncertainty[4]
## Significant Test (5%) for Serial Correlation of Historical Inflows



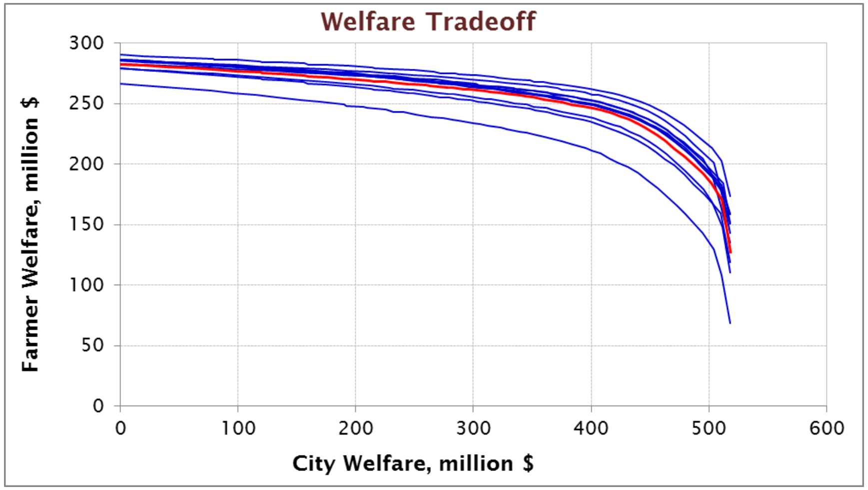

The significance test indicates that the historical inflows *cannot* be assumed to be uncorrelated.

# 3.3.4 Sensitivity to Inflow Uncertainty[5]
## Comparison of Correlated Synthetic, Historical Inflows

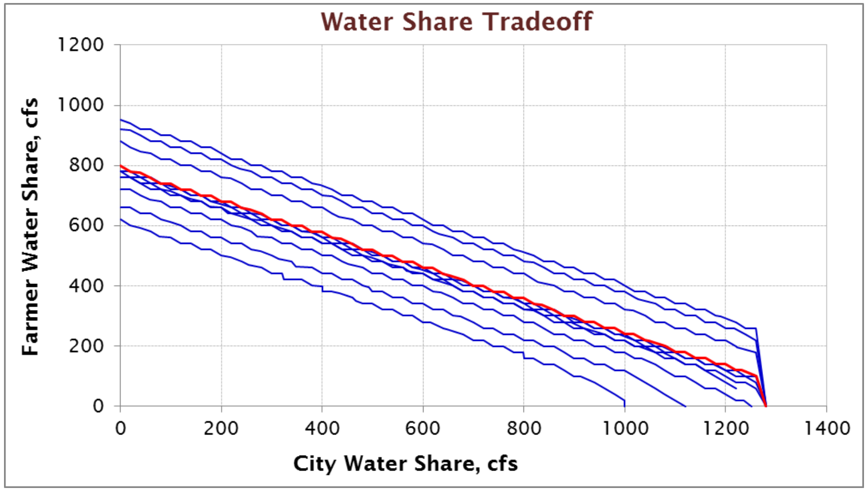
$J_{mean}(U_i) = 1/N\ [u_i(1) + u_i(2) + \ldots + u_i(70)],\ i=1, 3$



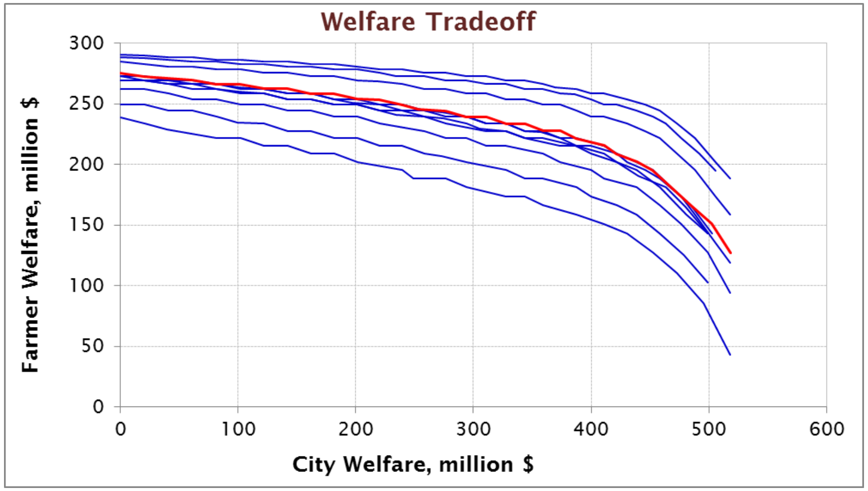

$V_{mean}(U_i) = 1/N\ [EW_i(1) + EW_i(2) + \ldots + EW_i(70)],\ i=1, 3$




$J_{max/min}(U_i) = \max\{\min\{u_i(1), \ldots, u_i(70)\}\},\ i=1, 3$

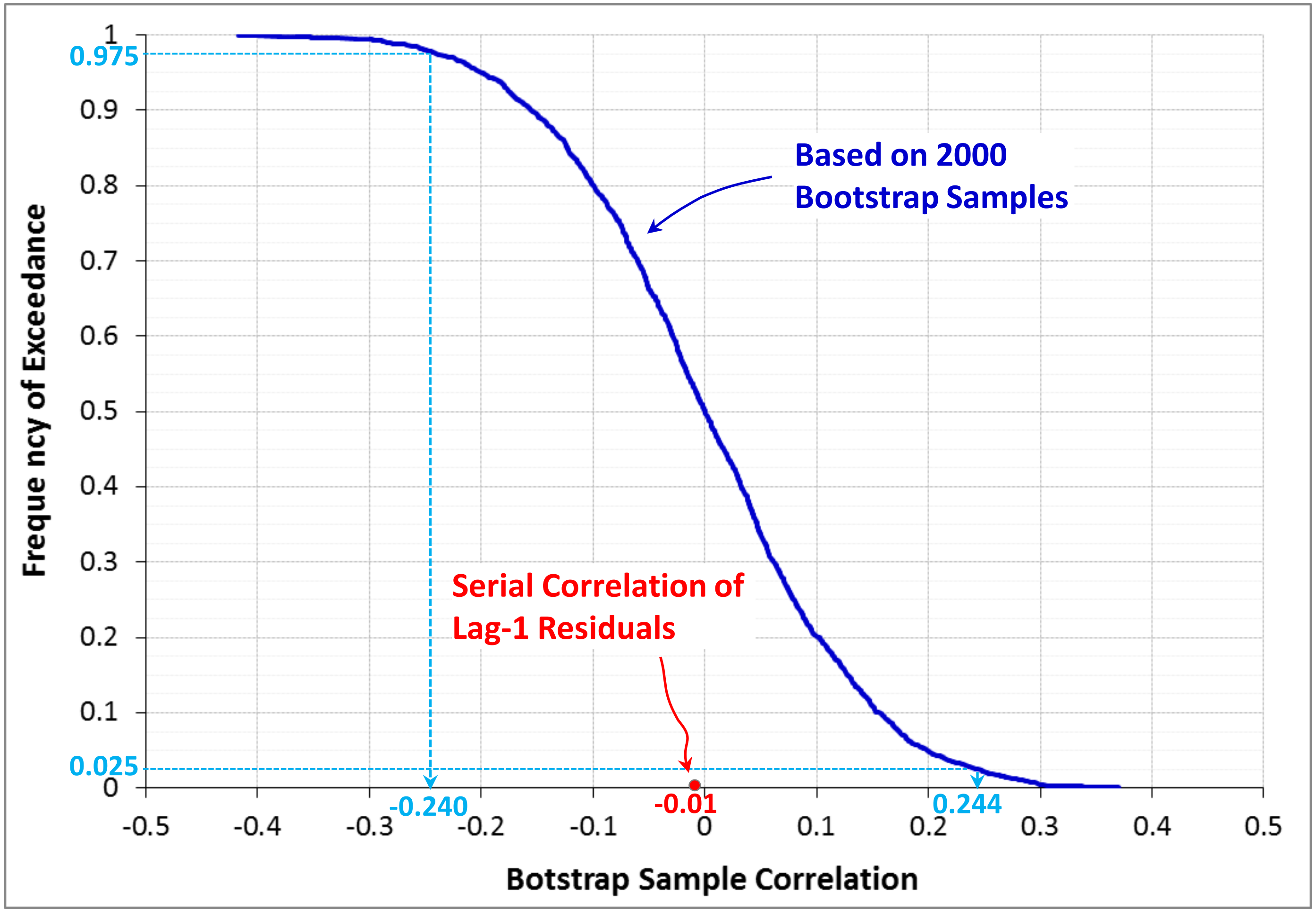



$V_{max/min}(U_i) = \max\{\min\{EW_i(1), \ldots, EW_i(70)\}\},\ i=1, 3$



# 3.3.4 Sensitivity to Inflow Uncertainty[6]

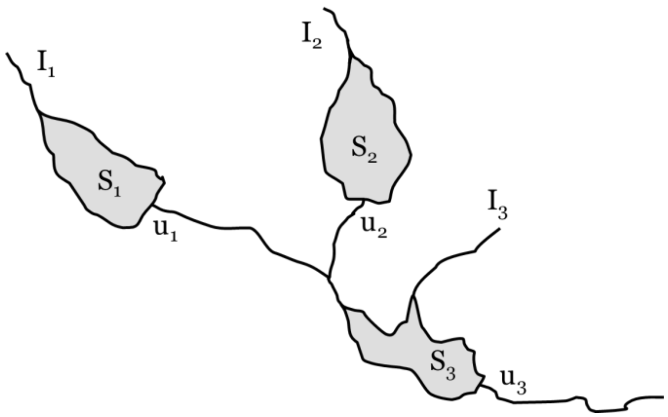
## Significant Test (5%) for Serial Correlation of Lag-1 Residuals




The significance test indicates that the lag-1 residuals *can* be assumed to be uncorrelated, and there is *no need* to consider additional correlation lags.

# Exhibit 3.3.4.2: Alternative Multisite Inflow Sequences

The synthetic inflow generation methods outlined in Exhibit 2.9 can be extended to systems where synthetic inflows are needed *simultaneously* at multiple sites. A typical situation where this arises is depicted in the water resources system below.




This water resources system includes 3 inflow series: $I_1(k)$, $I_2(k)$, and $I_3(k)$. These series may exhibit temporal correlations (e.g., lag-1 autocorrelations) and/or spatial correlations (e.g., correlations between $I_i$ and $I_j$, where $\{i,j\} = 1, 2, 3$, and $i \neq j$).

The significance of all temporal and spatial correlations can be tested using the significance testing procedure outlined in Section 2.9.2 of Exhibit 2.9. Then, depending on the test outcomes, inflow generation can proceed as follows:

**i.** For any series $I_j(k)$ exhibiting no spatial correlations with any other series, synthetic inflow generation can proceed as outlined in Exhibit 2.9.

**ii.** If spatial correlations are present, synthetic inflow generation would have to follow a multivariate approach such as the ones outlined in the next section.

## (*a*) Synthetic Generation of Spatially Correlated, Temporally Uncorrelated Inflows

Denote $I_i(k)$, $i = 1, \ldots, L$ , $k = 1,\ldots, N$, the historical series of interest (comprising a set of L series each with a record length of N values), and $m_i$, i, 1,…, L, their mean values. Moreover, assume that no temporal correlations are significant within and between series. Then, the generation of synthetic inflows can proceed as follows:

1. Generate N integer random numbers $u_1(k)$, $k = 1,\ldots, N$, between 1 and N (inclusive of 1 and N).
2. Construct a new set of synthetic series $I_i(k)$, such that $I_i(k) = I_i[u_1(k)]$, $i = 1,\ldots, L$, $k = 1,\ldots, N$. Essentially, this approach constructs a sequence of new inflows at all sites by random selection (with replacement) of the years from the historical record. However, the year selected at each time step is the same for all series to ensure that spatial correlations are preserved.
3. Carry out system assessments using the new synthetic series $\{I_i(k), i = 1, \ldots, L, k = 1,\ldots, N\}$.
4. Repeat Steps (1), (2), and (3) M times, where M = 20 (or higher) and assess the sensitivity and uncertainty of all modeling results.

# Exhibit 3.3.4.2: Alternative Multisite Inflow Sequences[2]

## (*b*) Synthetic Generation of Spatially *and* Temporally Correlated Inflows

Denote $I_i(k)$, k = 1,…, N, i = 1, …, L, the historical series of interest (comprising a set of L series each with a record length of N values), and $m_i$, i, 1,…, L, their mean values. Moreover, assume that temporal correlations are found to be significant within and between series. Then, the generation of synthetic inflows can proceed as follows:

1. Compute the zero mean historical series: $z_i(k) = I_i(k) - m_i$, k = 1,…, N, i = 1,…, L.
2. Compute all lag-1 spatial and temporal correlations of the $z_i(k)$, k = 1,…, N, i = 1,…, L, and arrange them in two matrices as follows:

$$\mathbf{R}_0 = \begin{bmatrix} r_{0,11} & r_{0,12} & \cdots & r_{0,1L} \\ r_{0,21} & r_{0,22} & \cdots & r_{0,2L} \\ \vdots & \vdots & \vdots & \vdots \\ r_{0,L1} & r_{0,L2} & \cdots & r_{0,LL} \end{bmatrix}, \qquad \mathbf{R}_1 = \begin{bmatrix} r_{1,11} & r_{1,12} & \cdots & r_{1,1L} \\ r_{1,21} & r_{1,22} & \cdots & r_{1,2L} \\ \vdots & \vdots & \vdots & \vdots \\ r_{1,L1} & r_{1,L2} & \cdots & r_{1,LL} \end{bmatrix}$$

where $r_{0,ij}$ = sample correlation between $z_i(k)$ and $z_j(k)$ at lag 0, and $r_{1,ij}$ = sample correlation between $z_i(k)$ and $z_j(k-1)$ at lag 1.

3. Compute the coefficient matrix: $\mathbf{A} = \mathbf{R}_1 \, \mathbf{R}_0^{-1}$ ($\mathbf{R}_1$ times the inverse of $\mathbf{R}_0$, assuming it exists).

4. Compute the L-dimensional vector error series **e**(k), k = 2,…, N: $\mathbf{e}(k) = \mathbf{z}(k) - \mathbf{A}\,\mathbf{z}(k-1)$, where **z**(k) is the L-dimensional vector containing the $z_i(k)$, i = 1,…, L, values computed in Step 1. Each element of the **e**(k) vector is computed as follows:

$$e_i(k) = z_i(k) - \sum_{j=1}^{L} A_{ij}\, z_i(k-1), \quad i = 1,\ldots, L.$$

5. Generate an integer random number $u_1(1)$, between 1 and N (inclusive of 1 and N).
6. Determine the first values, $I_i(1)$, of a new set of synthetic series such that $I_i(1) = I_i[u_i(1)]$, i = 1,…, L.
7. Generate N-1 integer random numbers $u_1(k)$, k = 2,…, N, between 2 and N (inclusive of 2 and N).
8. Construct a new synthetic series **I**(k), k = 2,…, N, such that $\mathbf{I}(k) - \mathbf{m} = \mathbf{A}\,[\mathbf{I}(k-1) - \mathbf{m}] + \mathbf{e}[u_1(k)]$.
9. Carry out system assessments using the new set of synthetic series {**I**(k), k = 1,…, N}.
10. Repeat Steps (5) through (9) M times, where M = 20 (or higher) and assess the sensitivity and uncertainty of all modeling results.

# 3.4 Summary of Key Messages

i. Good system models are very useful in assessing the relative benefits, impacts, and tradeoffs of alternative planning and management decisions.

ii. Models must credibly represent the physical system response to alternative decisions, as well as the impact of these decisions on stakeholder interests characterized by meaningful performance metrics.

iii. Multi-objective tradeoffs provide valuable insights of the interdependencies among stakeholder interests, and of the system capacity to meet their collective expectations.

iv. Planning and management policies and decisions along the Pareto frontier are particularly interesting for stakeholders. The identification of these policies and decisions is often challenging, but it is a key outcome of modeling and assessment efforts. Simulation and optimization methods can facilitate the search for Pareto optimal decisions.

v. Policies and decisions on the Pareto frontier with respect to some stakeholder metrics may be completely undesirable relative to another metrics subset. Thus, policies and decisions must be assessed with respect to *all* stakeholder metrics.

vi. Infrastructure changes can increase the capacity of the system to meet stakeholder expectations, but only if they are combined with Pareto management policies.

vii. Tradeoffs and the desirability of policies and decisions can be rather sensitive to uncertain factors. It is thus important that stakeholders understand and appreciate the sensitivity and robustness of policies and decisions to underlying uncertainties. Uncertainty analysis should be an integral part of IWRM processes, as it can characterize the *risks* associated with alternative policies and decisions. Risk-based decision making is much more likely to lead to *sustainable* long term decisions and plans.