Ch3_Four_Vector

Every 4-vector $\mathbf{A}$ have a corresponding displacement vector $d\mathbf{s}$ that is parallel to it. $\mathbf{A}$ is parallel to ds in all frames.

Some **frame independent** quantities:

- The dot product between two 4-vector is defined:

$$\mathbf{A} \cdot \mathbf{B} \equiv -A^t B^t + A^x B^x + A^y B^y + A^z B^z$$

- The square magnitude of a 4-vector is defined:

$$A^2 \equiv \mathbf{A} \cdot \mathbf{A} = -(A^t)^2 + (A^x)^2 + (A^y)^2 + (A^z)^2$$

- The differential squared gives the usual invariance of interval:

$$d\mathbf{s} \cdot d\mathbf{s} = ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

- A scalar remains invariance across reference frames (invariance under Lorentz transform)

4-velocity

- **4-velocity** is defined as the rate of change of 4-position of a particle with respect to the proper time. denoted by u:

$$\mathbf{u} = \begin{pmatrix} \frac{dt}{d\tau} \\ \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{pmatrix}, \text{where } d\tau = \sqrt{1 - v^2}\ dt$$

  - In contrast, the 3-velocity of the particle in its proper frame is called the **ordinary velocity** denoted by $\vec{v}$, with magnitude v.
- 4-momentum