Ch4_Boxes

4.1.1 - Practice summation

Practice Summation notation:

$$\delta^t{}_\nu A^\nu = \delta^t{}_t A^t + \delta^t{}_x A^x + \cdots$$
$$= \delta^t{}_t A^t$$
$$= A^t$$
$$\delta^x{}_\nu A^\nu = \delta^x{}_t A^t + \delta^x{}_x A^x + \cdots$$
$$= A^x$$
$$\Rightarrow \delta^\mu{}_\nu A^\nu = A^\mu$$

$$\delta^\mu_t \eta_{\mu\alpha} = \delta^t_t \eta_{t\alpha} + \delta^x_t \eta_{x\alpha} + \delta^y_t \eta_{y\alpha} + \delta^z_t \eta_{z\alpha}$$
$$= \eta_{t\alpha}$$
$$\delta^\mu_x \eta_{\mu\alpha} = \delta^t_x \eta_{t\alpha} + \delta^x_x \eta_{x\alpha} + \cdots$$
$$= \eta_{x\alpha}$$
$$\Rightarrow \delta^\mu{}_\nu \eta_{\mu\alpha} = \eta_{\nu\alpha}$$

4.2.x - Unit of Magnetic and Electric field

$[\vec{E}] = N/C$, recall $[p] = kg$, $[N] = [\frac{dp}{dt}] = kg/m$
$= kg/(m \cdot C)$

$[\vec{B}] = N \cdot s/(m \cdot C)$
$= N/C$
$= kg/(m \cdot C)$

4.3.1 - Low Velocity limit of Lorentz Force Using EM Tensor

$$\frac{dp^x}{d\tau} = q F^{x\nu} \eta_{\nu\alpha} u^\alpha \qquad , \quad u = \begin{pmatrix} \sim 1 \\ v_x \\ v_y \\ v_z \end{pmatrix} \to dt/d\tau \sim \text{same.}$$

$$= q F^{xt} \eta_{t\alpha} u^\alpha + q F^{xx} \eta_{x\alpha} u^\alpha + \cdots , \qquad \eta_{t\alpha} = \begin{cases} -1, \alpha = t \\ 0, \alpha \neq t \end{cases} \cdots$$

$$= q F^{xt} (-1) u^t + q F^{xx} u^x + q F^{xy} u^y + q F^{xz} u^z$$

$$= -q (-E_x u^t) + 0 + q B_z u^y + q (-B_y) u^z$$

$$= q (E_x + B_z v^y - B_y v^z)$$