Gabe_On vectors

Manifold and Euclidean space? :"Sphere and tangent plane"

Image the surface of a sphere. Take a point p on the surface and gradually zooming in onto p, we see that the surface around p gets flatter and looks like a plane. At this scale, the scope of camera can be effectively approximated by a plane, with euclidean geometry. The plane has vectors that lives on it and If we expand the size of the plane along its basis vectors, by doing so we constructed a tangent plane. This plane we constructed at p is the euclidean spacetime. And the sphere that euclidean spacetime approximates is the spacetime manifold.

The tangent space locally approximates the manifold. On a point p of the spacetime manifold M lives a tangent space T_p. On a differentially small patch around p, the tangent space T_p approximates the manifold.

- The manifold (sphere) It is locally $\mathbb{R}^2$ (plane)
- For every patch on the manifold, you can find a transformation that transform the tangent plane of it to the tangent plane of a neighbor patch.

The tangent space is a vector space. The manifold is not. Draw an arrow tangent to the manifold. Take a non-differential step along the direction of the arrow, you would step out of the domain of the manifold.

Defining Vectors on manifold

The problem of defining a vector on a manifold arises because of two reasons:

1. **There is no global coordinate system on a manifold**. A well-defined coordinate system allowed us to define vector the same way everywhere. For manifold, there lack a natural common definition of vector at different points.

2. **There is no inherent meaning of length and direction of vector**. There is no meaningful way to compare vectors at different points. That means you cannot compare vectors across euclidean tangent space.

So the task is setup to address these problems and try to define a vector that is legit across the manifold.