

Caroll_Ch2.1_Motivation for GR

- ? Why do we need GR? How is a universal coordinate system not sufficient?
- ? why can't we define a global inertial frame

2.1 Motivation for GR


Our job as physicists is to construct mathematical models of the world, and then test the predictions of such models against observations and experiments. ---- Carroll

The Weak Equivalence principal



The weak equivalence principal states that, the '**gravitational charge**' m_g of a particle is equivalent to the the '**inertial mass**' m_a of a particle. An analogy to electrostatics gives the following:

$$\begin{aligned}\vec{F}_{grav} &= -m_g \nabla \Phi = m_i a, & a &= \frac{m_g}{m_i} (-\nabla \Phi) \\ \vec{F}_{elec} &= -q \nabla V = m_i a, & a &= \frac{q}{m_i} (-\nabla V)\end{aligned}$$

Note that the gravitational equivalence of charge is the just the inertial mass. Thus it is worth stating that mass has this special property:

-  **The weak equivalence principle** states that the gravitational mass and inertial mass is equivalent:

$$m_i = m_a$$

-  The direct result of this property is that any particle in free fall follows the same **trajectory: the path of least resistance**, independent of their inertial mass
-  Another equivalent statement of WEP is that "*The motion of the particle is the same: (under a gravitational field) and (as observed by an uniformly accelerated frame).*"

The Einstein equivalence principal

The stronger statement of the WEP is the following:

- 🚀 In a local enough region of spacetime, the law of physics reduce to that of special relativity. There is no experiment that can distinguish the existence of a gravitational field by the means of local measurement.
 - 📐 **Physics is reduced to special** means to reduce it to that of flat spacetime. The curvature is thus undetectably small.
 - 📐 The 2nd sentence state that: In this case, even an acceleration is the bi-product of gravity (or normal force by the ground), there is no way to know that this is the case, compare to other cases like accelerated by a moving truck or some other force.
- To put in another way, measurable curvature is the only clue for the observer that he is accelerated by (the normal force that is cause by) gravitational field.

An example of a **case that respects WEP but not EEP** can be illustrated using H atom. The inertial mass of an hydrogen m_H is less than the the sum of its constituent particles $m_p + m_n + m_{e^-}$. The difference is binding energy. The WEP states that $m_i = m_g$, this is also the case here. The **coupling** between gravitation and E&M tune m_g such that it exactly equals to m_I .

- 📐 Coupling: gravity couples not only with mass, but with energy and momentum too
- logic: 'grav couple with mass' + 'grav couple with (binding) energy' \rightarrow
 $m_g < m_{g,separate} \rightarrow m_g = m_I$

Why do we need GR? How is a universal coordinate system not sufficient?

In SR we have **inertial frame** and **rods** that extends through spacetime. It works in a spacetime without curvature. The inhomogeneity of

gravitational field make this idea obsolete. The idea of defining an inertial frame simultaneously for two far away particles is not possible, because however you define it, it violates Newton's 1st law. And much of SR is built on the assumption of inertial frame.

- 🧠 recall: laws of physics holds true, and are the same across inertial frames. It is thus not possible to define a global inertial frames.

To summarize, reasons why we need GR:

- Inertial frames and rods in SR won't work, laws of physics (newton's 1st) doesn't hold on large scales (2 particles falling towards earth).

But it remains possible to create **local** inertial frame. And to construct GR from local inertial frames, we need to have a way to bring stuff from one local inertial frame to another. For that, we will need a different mathematical framework for that. This math structure is called differentiable manifold. It is locally flat, but globally not.

Gravitational Redshift

- ⚠ Skipped
- 🧠 The goal is to test the validity of Einstein Equivalence Principle by showing redshift caused by acceleration is also caused by gravitation. (the no experiment can distinguish claim)

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