Ch3_Four_Vector

Every 4-vector $\mathbf{A}$ have a corresponding displacement vector $d\mathbf{s}$ that is parallel to it. $\mathbf{A}$ is parallel to ds in all frames.

Some **frame independent** quantities:

- The dot product between two 4-vector is defined:

$$\mathbf{A}\cdot\mathbf{B} \equiv -A^tB^t + A^xB^x + A^yB^y + A^zB^z$$

- The square magnitude of a 4-vector is defined:

$$A^2 \equiv \mathbf{A}\cdot\mathbf{A} = -(A^t)^2 + (A^x)^2 + (A^y)^2 + (A^z)^2$$

- The differential squared gives the usual invariance of interval:

$$d\mathbf{s}\cdot d\mathbf{s} = ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

- A scalar remains invariance across reference frames (invariance under Lorentz transform)

4-vectors

- **4-velocity** is defined as the rate of change of 4-position of a particle with respect to the proper time. denoted by u:

$$\mathbf{u} = \begin{pmatrix} \frac{dt}{d\tau} \\ \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{pmatrix}, \text{where } d\tau = \sqrt{1-v^2}\,dt$$

  - In contrast, the 3-velocity of the particle in its proper frame is called the **ordinary velocity** denoted by $\vec{v}$, with magnitude v.

law of conservation of 4 momentum

- Logic: 'Conservation of p' + 'Linearity of LT' = 'Conservation of p across frames' Imagine a system going through a collision. The initial and final momentum can be calculated. The conservation of 4 momentum in a frame can be stated as

  $$\sum_j \mathbf{p}_{j,init} - \sum_j \mathbf{p}_{j,final} = 0$$

  Assume the momentum in some frame S, $[\mathbf{p}]_S$, is conserved, question is does does it conserve in some other frame S'. Each individual momentum vector in the sum can be Lorentz transformed to the other frame, but that is equivalent to Λ on the sum of all 4-momentum:

  $$\begin{aligned}[\mathbf{p}'_{1i}] + [\mathbf{p}'_{2i}] + \cdots - [\mathbf{p}'_{1f}] - [\mathbf{p}'_{2f}] - \cdots \\ = [LT]\left([\mathbf{p}_{1i}] + [\mathbf{p}_{2i}] + \cdots - [\mathbf{p}_{1f}] - [\mathbf{p}_{2f}] - \cdots\right)\end{aligned}$$

  But the RHS of this equation is just 0! (because lorentz transform of 0 is just 0). The conservation of momentum in the other frame (LHS) is thus shown.
- ⚖ If the law of Conservation of 4-vectors are valid in one frame, it is valid across frames.
- ⚖ The actual conservation of 4-vectors are experimentally justified by particle physicists.