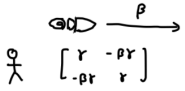
Ch4_Notation

Essence of the chapter:

$$\begin{bmatrix} A'^t \\ A'^x \\ A'^y \\ A'^z \end{bmatrix} = \begin{bmatrix} \Lambda^t_t & \Lambda^t_x & \Lambda^t_y & \Lambda^t_z \\ \Lambda^x_t & \Lambda^x_x & \Lambda^x_y & \Lambda^x_z \\ \Lambda^y_t & \Lambda^y_x & \Lambda^y_y & \Lambda^y_z \\ \Lambda^z_t & \Lambda^z_x & \Lambda^z_y & \Lambda^z_z \end{bmatrix} \begin{bmatrix} A^t \\ A^x \\ A^y \\ A^z \end{bmatrix}$$



Say prime frame S' moves at β wrt S

$$\Lambda^t_t = \Lambda^x_x = \gamma, \quad \Lambda^t_x = \Lambda^x_t = -\gamma\beta$$

Notation simplification:

① $A^t, A^x, \cdots \rightarrow A^\mu$ = μ'th component ($\vec{A}$)

② $\Lambda^t_t, \Lambda^t_x, \cdots \rightarrow \Lambda^\mu_\nu$

Rewrite Lorentz

$$A'^\mu = \sum_\nu \Lambda^\mu_\nu A^\nu$$

Einstein summation convention:

> If the same Greek-letter index appears exactly once as a superscript *and* exactly once as a subscript in any single *term* of an equation, we will *assume* that term is to be summed over all four possible values of that index.

$$A'^\mu = \Lambda^\mu_\nu A^\nu$$

$$A^\mu = (\Lambda^{-1})^\mu_\nu A'^\nu$$

The Metric Tensor

$$\begin{bmatrix} \eta_{tt} & \eta_{tx} & \eta_{ty} & \eta_{tz} \\ \eta_{xt} & \eta_{xx} & \eta_{xy} & \eta_{xz} \\ \eta_{yt} & \eta_{yx} & \eta_{yy} & \eta_{yz} \\ \eta_{zt} & \eta_{zx} & \eta_{zy} & \eta_{zz} \end{bmatrix} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$
- dot product $\vec{A} \cdot \vec{B} = \eta_{\mu\nu} A^\mu B^\nu$

  $\vec{A}^2 = \eta_{\mu\nu} A^\mu A^\nu$

EM field Tensor

(skipped)

Free and Bound Indices

$$p'^{\mu}_{tot} = p'^{\mu}_{1} + p'^{\mu}_{2} = \Lambda^{\mu}_{\nu}\, p^{\nu}_{1} + \Lambda^{\mu}_{\alpha}\, p^{\alpha}_{2}$$
$$= \Lambda^{\mu}_{\nu} \left(p^{\nu}_{1} + p^{\nu}_{2} \right)$$

- μ is called free indecies (free to assign)
- ν is called bound indecies "dummy"

Useful Identities

- $\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta}$
- $\frac{d}{d\tau} (\vec{A} \cdot \vec{A}) = 2\, \eta_{\mu\nu} A^\mu \frac{dA^\nu}{d\tau}$ [chain rule, so there's a 2]

- Also, the superscript in denominator should be viewed as a subscript