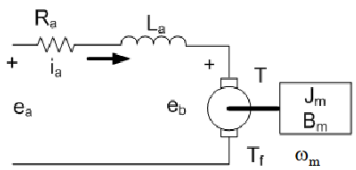
[redacted]

**PROBLEM 1(60pts)** Throughout the semester a recurrent example of a dynamical system has been the DC motor. To this point we have assumed its TF is: $\dfrac{\Omega(s)}{V(s)} = G(s) = \dfrac{g_s}{\tau s + 1} \quad \left[\dfrac{rad/\sec}{volt}\right]$. On p.57 of the book [1], as well as at many other sources (e.g. https://www.engr.siu.edu/staff/spezia/Web438A/Lecture%20Notes/lesson14et438a.pdf [2]) the governing equations include:

(1): $L\dfrac{di}{dt} + Ri = v - K_E\omega$ $[volts]$ ; (2): $J\dot{\omega} + B\omega = K_T i$ $[N-m]$.




**(a)(7pts)** Show that $\dfrac{\Omega(s)}{V(s)} = G(s) = \dfrac{K_T}{(Js+B)(Ls+R)+K_T K_E} \quad \left[\dfrac{rad/\sec}{volt}\right]$

<u>*Solution*</u>:

**(b)(6pts)** In [1] on p.58 the authors state "In many cases the relative effect of the inductance, *L*, is negligible compared to the mechanical motion and can be neglected." In [2] the author states on p.9 that "The electrical time constant $\tau_e = L/R$ is much smaller than the mechanical time constant $\tau_m = J/B$, and is usually neglected. In both [1] and [2] the effect is to set $L = 0$. Even so, they are not saying the same thing. To say that *L* is negligible compared to mechanical motion is like comparing bananas and rock music. And even though it is true that $\tau_e << \tau_m$, neither of these time constants is a time constant associated with $G(s)$. Hence, this argument is weak. Show that the poles of $G(s)$ can be expressed as

$$s_{1,2} = \frac{-(1+r) \pm \sqrt{1+r^2+2(1-2K)r}}{2\tau_e} \quad \text{where } r \overset{\Delta}{=} \tau_e/\tau_m \text{ and } K \overset{\Delta}{=} 1 + (K_T K_E)/(RB). \qquad (1b)$$

<u>*Solution*</u>:

**(c)(10pts)** From (a), for $L$=0 , we have $\hat{G}_c(s) = \dfrac{K_T}{JRs + BR + K_T K_E}$ with a single pole $\hat{s}_1^c = -(BR + K_T K_E) / JR = -[1 + (K_T K_E) / (BR)] / \tau_m$, and associated time constant $\hat{\tau} = \tau_m / [1 + (K_T K_E) / (BR)]$.

In (b) it is clear that we <u>cannot</u> assume $\tau_e = 0$. For $r << 1$, (1b) gives $\hat{G}_b(s) = \dfrac{K_T / JL}{(s - \hat{s}_1^b)((s - \hat{s}_2^b)}$. Consider the numerical information in the Appendix code at 1(a). It was taken from p.12 of [2]. Compute the numerical values of (i) the exact poles and time constants, (ii) the pole and time constant for $L = 0$, and (iii) the poles and time constants for $\tau_e << \tau_m$.
*Solution*: [See code @ 1(c).]

**(d)(6pts)** Regardless of your answers in (c) assume here that: $s_{1,2} = [-5.49\, , -54.67]$, $\hat{s}_1^c = -5$, and $\hat{s}_{1,2}^b = [-5.41\, , -54.59]$. Clearly, the roots closest to the imaginary axis are all approximately $-5$. Since the poles $\hat{s}_{1,2}^b$ are close to the exact poles $s_{1,2}$, we will restrict our attention to $s_{1,2}$. (i) Overlay the step responses associated with $G(s)$ and $\hat{G}_c(s)$. (ii) Explain the physical significance of the difference in behavior at $t = 0$.
*Solution*: [See code @ 1(d).]
(ii)

**Figure 1(d)** Step responses for $G(s)$ and $\hat{G}_c(s)$.

**(e)(6pts)** The similarity of the step responses you should have found in (c) provides support for the assumption $L = 0$ in relation to the speed/voltage transfer function. In this part , (i) overlay the Bode plots for the **<u>position</u>**/voltage TFs. (ii) For each TF obtain the phase information for the frequency at which the magnitude is $-10\,\text{dB}$.
*Solution*: [See code @ 1(e).]

**Figure 1(e)** Bode for $G_p(s)$ and $\hat{G}_p(s)$.

**(f)(11pts)** Using a proportional controller with a gain of $10\,\text{dB}$ in a unity feedback configuration, (i) overlay the closed loop responses for a $10^o$ step. (ii) Explain how their difference relates to CL stability information available in (e). (iii) Compute the maximum time delay for each system before it becomes unstable.
*Solution*: [See code @ 1(f).]
(ii):

(iii):

**Figure 1(f)** CL responses for a $10^o$ step.

**(g)(8pts)** (i) Overlay the CL Bode plots. Use the data cursor to estimate (ii) the $\pm 3\,\text{dB}$ bandwidth for each system, and (iii) the amplification (in dB) at resonance for each system.
*Solution*: [See code @ 1(g).]

(ii)

(iii):

**Figure 1(g)** CL Bode plots with data cursor information.

**(h)(6pts)** From (a), for a controller gain K, the exact OL TF is $KG(s) = \dfrac{KK_T}{s[(Js+B)(Ls+R)+K_T K_E]}$. (i): Use the Routh array to obtain the general inequality that is needed for CL stability. (ii) Using the numerical values for all parameters (including K=10 dB) but *L*, find the minimum value of *L* such that the CL will remain stable, and (iii) compare it to the given $L = 0.02$.
*Solution*:
(i):

(ii):

**PROBLEM 2(40pts)** The unity feedback position control block diagram for the DC motor at right includes a speed changer block: $\dot{\theta} = \gamma\theta_m$. With an external load connected to the output shaft of the speed changer, the effective total mechanical inertia and damping are [from EM345]:

$$J_T = J + J_{load} / \gamma^2 \text{ and } B_T = B + B_{load} / \gamma^2 .$$

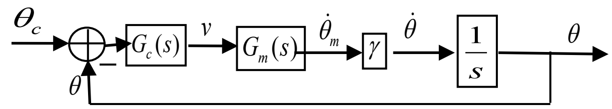



**Figure 2** Position control block diagram.

The motor + speed changer TF is now: $\dfrac{\Omega(s)}{V(s)} = \breve{G}(s) = \dfrac{K_T\gamma}{J_T L s^2 + (B_T L + J_T R)s + (K_T K_E + B_T R)}$.

Hence, for $G_c(s) = K_P$ the CL TF is: $W(s) = \dfrac{K_T K_P \gamma}{J_T L s^3 + (B_T L + J_T R)s^2 + (K_T K_E + B_T R)s + K_T K_P \gamma}$.

**(a)(8pts)** For a small load we can assume that $J_{load} = 0$ and $B_{load} = 0$. For the numbers given in 1(c) and $K_P = 10dB$, (i) obtain a plot of the CL root locus for varying $\gamma$. (ii) Use the data cursor to determine the range of $\gamma$ for CL stability.

<u>Solution</u>: [See code @ 2(a).]

**Figure 2(a)** CL root locus for varying $\gamma$.

**(b)(10pts)** Throughout the remainder of this problem we will make the assumptions in (b) and let $\gamma = 1$. (i) From the OL Bode plot obtain a proportional controller that will yield a CL $PM = 70^o$. (ii) Plot the CL response to a $10^o$ step. (iii) Use the data cursor to estimate the angle position at $\theta(2\,\text{sec})$.

<u>Solution</u>: [See code @ 2(b).]

(i):

(iii):

**Figure 2(b)** OL Bode plot (left). CL response to a $10^o$ step (right).

**(c)(11pts)** In (b) you should have found that the CL response time is approximately 1.5 to 2 seconds. (i) Design a PD controller that will result in a CL $PM = 70^o$ at an OL gain crossover frequency $\omega_{gc} = 10\ r/s$. To this end, begin with an OL Bode plot with pertinent information. (ii) Verify your design using the 'margin' command (no plot!) (iii) Plot the CL response to a $10^o$ step, and (iv) give $\theta(t=1)$.

_Solution_: [See code @ 2(c), but show key work HERE.]

(i):

(iii):

**Figure 2(c)** OL Bode plot (left). CL response to a $10^o$ step (right).

(iv):

**(d)(11pts)** You should have observed a slight oscillation in the beginning of the CL step response in (c). For $G_c(s) = 0.5(1+s/3.5)/(1+s/29)$ it is easy to show that the CL has an underdamped pole $s_1 \cong -11.5 + i15$ with $\zeta \cong 0.6$. In this part consider a controller $G_c(s) = K(1+s/4)/(1+s/\omega_2)$. (i) Use the root locus method to arrive at the values for *K* and $\omega_2$ that will place a CL pole at $s_1 = -15 + i11$ to achieve $\zeta \cong 0.8$. (ii) Use the 'margin' command to give the resulting CL PM. (iii) Overlay the CL response to a $10^o$ step with that given in (c). (iv) Briefly comment on the plots.

_Solution_: [See code @ 2(d).]

(i):

(ii):

(iv):

**Figure 2(d)**. Space for a helpful root locus plot (left). Overlaid CL responses to a $10^o$ step (right).

**Final Remark**[redacted]

## Appendix Matlab Code

```
%exam2.m  (3/20/22)
% https://www.engr.siu.edu/staff/spezia/Web438A/Lecture%20Notes/lesson14et438a.pdf
KE=.06; KT=.06; R=1.2; L=.02;
J=6.2*10^-4; B=10^-4;
%----------------------
%PROBLEM 1
%(c):









%(d):






%(e):




%(f):






%(g):




%==================================
%PROBLEM 2
%(a):






%(b):
```

```
%(c):
```

```
%(d):
```