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**AUTOMATIC DOCUMENT SEGMENTATION AND  
SUMMARIZATION USING NLP-BASED METHODS**

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# **AUTOMATIC DOCUMENT SEGMENTATION AND SUMMARIZATION USING NLP-BASED METHOD**

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## **ABSTRACT**

In today's exceedingly information-rich world, the efficiency in which humans can absorb, consume, and integrate information into their own repertoire of knowledge is dependent upon one's reading speed and their own understanding of said information. As a result, one solution to said problem is for text to be summarized and information to be condensed into forms that can be quickly absorbed briefly. For the solution to be viable, the conversion of a PDF file of a research paper must be successfully converted into a document containing the summary of each segment of the paper. The main structure of the project, therefore, can be separated into 2 different sections: PDF Segmentation and text summarization of each segment.

Although information from a PDF can easily be extracted using tools such as PyPDF2, the structure of each paper remains an issue due to the wide variety in which research papers are formatted. As such, traditional methods cannot be relied upon but require the usage of machine learning models.

The process of text summarization is more varied as many approaches are available. In general, text summarization can be achieved either through extractive or abstractive methods. An extractive method can only generate a summary using the provided text. On the other hand, abstractive method's output is a summary that is made using text that is not present in the document itself.

This thesis's aim is to test multiple different approaches to text summary, attempt to improve said methods and determine the most suitable approach and a final product that can demonstrate said result.

# **Chapter 1 INTRODUCTION**

## **1.1 Background**

As the speed at which information is generated in the age of digitalization, the speed of information consumption and digestion must also be improved through the usage of tools and techniques. As such, the development of a tool that can help with that issue is the focus of the thesis.

## **1.2 Problem Statement**

Due to the amount of information that each research paper holds are tremendous, the potential and possibilities to which a research paper can make an impact on the world can be lost. With topics ranging from Physics, Medicine, Psychology, etc., potential loss of valuable insights that could be gleaned from research papers or papers that are overlooked is immense.

Although methods to improve reading speed, such as Speed reading, Skimming, Active reading, etc., exist, all these methods aim to improve the person that is acquiring information. Even though this method is viable and can improve efficiency, much time is required to be spent for one person to acquire said skill. Human beings, innately, improve their existence and society through the development of tools. As such, developing a tool that can be used to assist said endeavor is this project's main purpose.

## **1.3 Scope and Objectives**

For the thesis to be considered completed, the development of a tool that can be used to parse a PDF file, turn it into a document of different segments, then automatically summarize the information must be completed and presented as the result of the thesis.

## **1.4 Structure of thesis**

The main structure of the paper can be divided into 6 different chapters, with each chapter having its own purpose in providing full coverage of the topic. Chapter 1 serves as the paper's introduction as the problem and aim of the thesis is introduced. Chapter 2 gives an overview of the information and knowledge used in the paper. Moving onto chapter 3, the methodologies and techniques used for the implementation are discussed in detail. The results of the project are shown and displayed in chapter 4. Said results are then evaluated and compared based on

predefined measurements to determine the effectiveness of the model itself. Lastly, chapter 6 is the summary of the project, including the strengths, weaknesses and improvements that could be made to the thesis.

## Chapter 2 LITERATURE REVIEW

### 2.1 Overview of Automatic Document Segmentation

The general goal of the Automatic Documentation Segmentation (ADS) is to scan through a document (in the form of a PDF file), identify each segment of the document, then output the result into a well-defined form for further use cases.

As most scientific texts in online databases, which are well-defined and documented, are in the form of a PDF file, this has become the priority in terms of input format. This choice is further enhanced due to an estimate of around 50 million research papers [1] that are stored as a PDF file in 2008. Since then, the number of research papers has only grown ceaselessly, further enhancing the decision of using PDFs as inputs.

Segments of a document referred to each part of a paper that is frequently required to be a part of any research paper, such as Abstract, Introduction, Methodologies, etc. However, the number of segments that are presented in the paper should not have any severe effects on the functionalities of the tool itself. As the aim of the application is not to adhere to a certain principle or guideline regarding the writing style, but to effectively summarize a paper, all segments of a paper are treated as equal, and can equally influence the reader.

The act of identifying each segment of the document referred to the process of acquiring the texts that are available in the paper, classifying it accordingly based on its segment, and formatting and storing it into a predefined form of storage. The process of text acquisition can be achieved through using multiple different methods. It can be as simple as extracting the text from the PDF as is without any regards for the structure of the paper, by making use of tools such as the PyPDF2 package in Python. The structure of a paper can be difficult to acquire due to the variety of methods and standards in which a paper is written. For example, whether the content is divided into two columns or one, whether the spacing on each line is 1.5 or Double, all aspects of a PDF can influence the way that the text is perceived by the machine. Currently, two methods in which the structure can be parsed are using Optical Character Recognition (OCR) and classification of content into predefined categories using Machine learning. While the first method is viable, its output is highly dependent on how the document is rendered. It is, therefore, the second method that will be the focus of the paper. To achieve this, **GeneRation Of Bibliographic Data** (or GROBID) is the tool of choice according to the criteria that has been

set. Since GROBID integrates with a third-party file conversion tool called **pdftoxml** to convert file from PDF to XML, XML is chosen as the form of storage for the output of the extraction process.

### 2.1.1 GROBID

GROBID [2] makes use of a cascade of sequence labelling models to parse its documents. This means that a document must go through multiple different models, each specializing in a specific task, to achieve the result. In the case of extracting the segments of the entire document, the model must first go through each layer and get the result, then pass said result to the next layer. Each of these layers are then combined and formatted accordingly. The GROBID cascade of sequencing model can be seen from Figure 2-1.

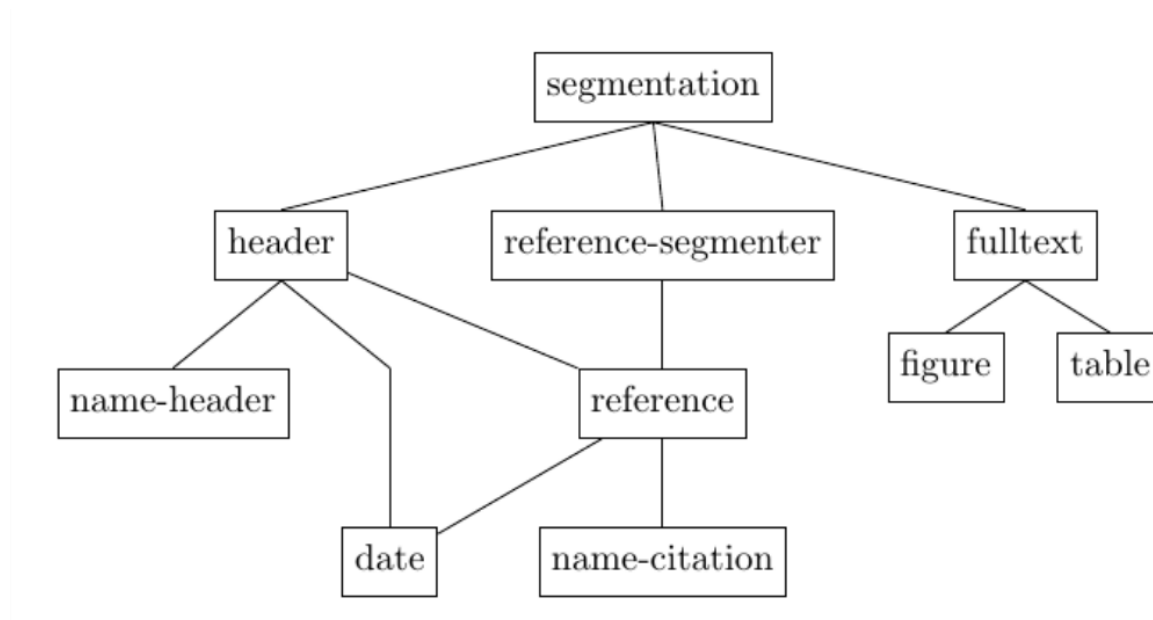


Figure 2-1 - GROBID cascade of sequencing model [3]

Although each model can create output from the original text, some models' purpose is only to provide the lower layer with input, creating a *cascade* of information. As a result, the accuracy of the model is highly dependent on the combination of multiple models. However, any errors that result from the higher layer will have a negative effect on the final output.

Although GROBID's implementation support the use of both a Deep Learning model and a *Conditional Random Field (CRF)* model, for the purpose of segmenting a scientific PDF paper, only the CRF model can be utilized as the number of parameters that are needed for processing the entire document exceeds the limit of what the deep learning model is currently capable of.

Although the CRF model is lacking in terms of accuracy in comparison to the Deep Learning model, it makes up for it through the speed with which it processes each PDF.

## 2.2 Overview of Natural Language Processing

Natural Language Processing (NLP) is an interdisciplinary field that focuses on the process of comprehending and manipulating human language. The general goal is to achieve better understanding of natural language using simple and durable techniques for fast processing of text.

Although differences exist in the function and usage of NLP between different use cases, the common Natural Language process typically includes seven steps, which are Sentence segmentation, Word tokenization, Stemming, Lemmatization, Stop word analysis, Dependency parsing, and Part-of-speech tagging.

The first step of the process is to retrieve the list of sentences from the provided text through the process known as *sentence segmentation*. In most cases, a sentence ends with a dot, which can make processing this step simple as that is the most common separator of a sentence. Only sentences that end with an “e.g.”, “?”, and “etc.” should be taken note of as these can happen occasionally.

After sentence segmentation, *word tokenization* is the next step. This step’s main purpose is to segment the sentences into separate words. These words are separated by white space, comma, dash, dot, etc. This process can be achieved simply through storing each word of a sentence into a data structure, then proceeding to filter out unwanted whitespace words.

*Stemming* is the process of stripping the words of its prefixes and suffixes, allowing the algorithm to obtain the basic form of a word. As an example, through stemming the word “Improvement”, “Improv” is the result of this step. Multiple different words can result in the same result through the stemming process as can be seen when applying this process to both “Improvement” and “Improvise”. An implementation of this process can be implemented through the Porter Stemmer algorithm, the Lancaster Stemmer algorithm, etc. The output of the process, however, does not bear resemblance to an English word.

Input from the stemming function is then piped into the *Lemmatization* function. This process made improvements upon the stemming process by turning it into a *lemma*, which is the canonical form of a word. Instead of “Improv”, which is not a proper word, “Improve” is returned instead.

After the word list has been refined and reduced to contain a list of lemmas, words that are not considered important to the linguistic analysis process are removed from the list through the process of *stop-word filtering*. Words such as “a”, “the”, “an”, etc., appear frequently throughout the content of a document, and are considered as *stop words* since they do not contribute any additional meaning or context to the document. Filtering these values can improve the accuracy of the overall process.

Through filtering stop words, the word list can now be considered clean, as it contains only lemmas that influence the meaning of the document and can be used for the *dependency parsing process*. During the process, the structure of a sentence can be broken down into its basic form, from Sentences to Noun phrase and Verb phrase, to Verb and Determiner, etc. The sentence is parsed, creating a tree like structure with each lower layer being more specific than the one above it.

*Part-of-speech tagging* improves upon the dependency parsing functionality by assigning a part-of-speech label to each sequence of words [4]. Tags can be divided into *closed class* words, which are frequent and ambiguous, and *open-class* words, which are divided differently depending on the set of tags that is being used. Typically, a tag set can include tags such as Nouns, Verbs, adjectives, etc. The result from both dependency parsing and part-of-speech tagging can be seen from the Figure 2-2.

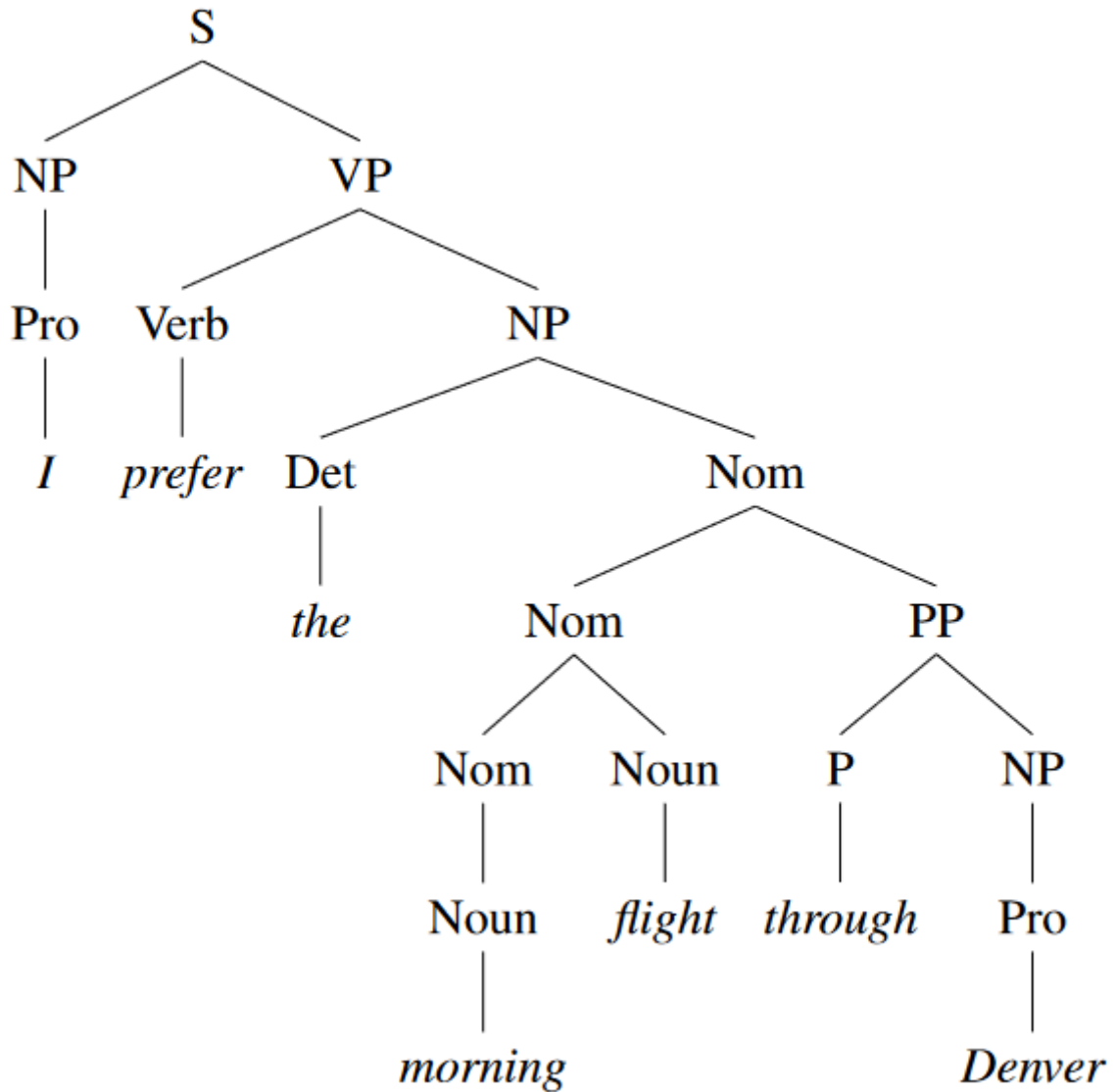


Figure 2-2 - Dependency style parse tree [4]

## 2.3 Overview of Text Summarization

Text summarization, as the name suggests, is the process of parsing a single or multiple documents and constructing a summary for each respective document. Applications of this system are widely available, from summarization of news, opinion, sentiment, story and novels to scientific paper summaries in all fields.

The general framework of the system involves a pre-processing step, a processing step and a post-processing step with the final step being optional in some cases. Text summarization system can be classified based on many of its aspects including input size (single-document or multi-document), approach (extractive, subtractive, or hybrid), nature of output summary (generic-based or query-based), language (monolingual, multilingual, or cross-lingual), type



(headline, sentence-level, highlights, or full summary), domain (generic or domain specific)[5]. The framework's visualization can be seen in Figure 2-3.

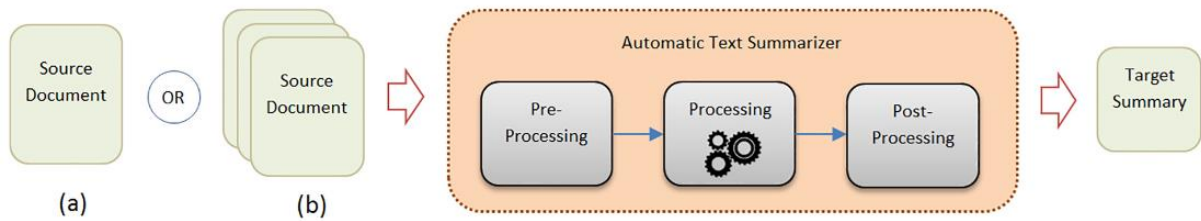


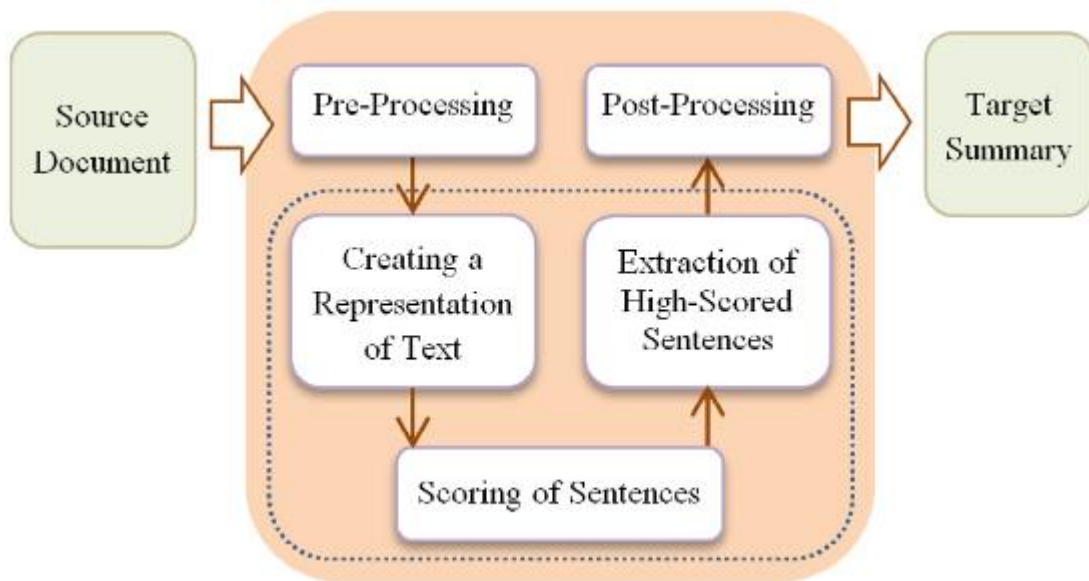
Figure 2-3 - Text summarization general framework [5]

### 2.3.1 Extractive text summarization

The *extractive text summarization* approach selects the most important sentence using one approach amongst many to create the output. Among the available methods, approaches that have proven to produce positive results include statistical-based methods, concept-based methods, graph-based methods, deep-learning methods, etc.

Due to only reusing words from the original text, some advantages that this approach may provide include having faster response time in comparison with abstractive text summarization techniques and having higher accuracy in its output. However, for the same reason, some disadvantages of the techniques were created. The resulting text can contain redundancy due to having two closely-related-in-meaning sentences included. Chosen sentences may also be too lengthy to serve as a summary or the chosen sentence may lack the required context that is necessary for the reader to fully comprehend the summary. Additionally, sentences with different temporal status may conflict with each other when included in the same paragraph. Some notable extractive methods include TextRank (which made use of a graph-based system to determine the importance of words) [6], and Latent Semantic Analysis (LSA) (which made use of Single Value Decomposition technique to analyze the relationship between documents) [7]

The general framework of a text summarization system in the previous section can then be updated to include general steps in an extractive system. These changes can be seen in Figure 2-4.



*Figure 2-4 - Extractive text summarization general framework [5]*

### **2.3.2 Abstractive text summarization**

Abstractive text summarization differs from the extractive method in that before the input is processed, it is turned into one of the intermediate forms that will then be the input of the summarization process. Similar to the extractive text summarization, multiple different approaches can be used to the summary generation process, ranging from graph-based, tree-based, template-based, machine-learning-based, etc.

An abstractive text summarization system can generate greater summary with words that do not belong to the original text. For a summary generation system, the ability to create new sentences defines what an abstractive text summary is. The output of this type of system is generally more similar to summary created by humans in comparison to the extractive type. However, due to the highly advanced and complex nature of this system, creating one is typically difficult and can require the use of natural language generation or machine learning, which is still a growing field.

A new version of the general framework containing the architecture of an abstractive text summarization can then be visualized, resulting in Figure 2-5.

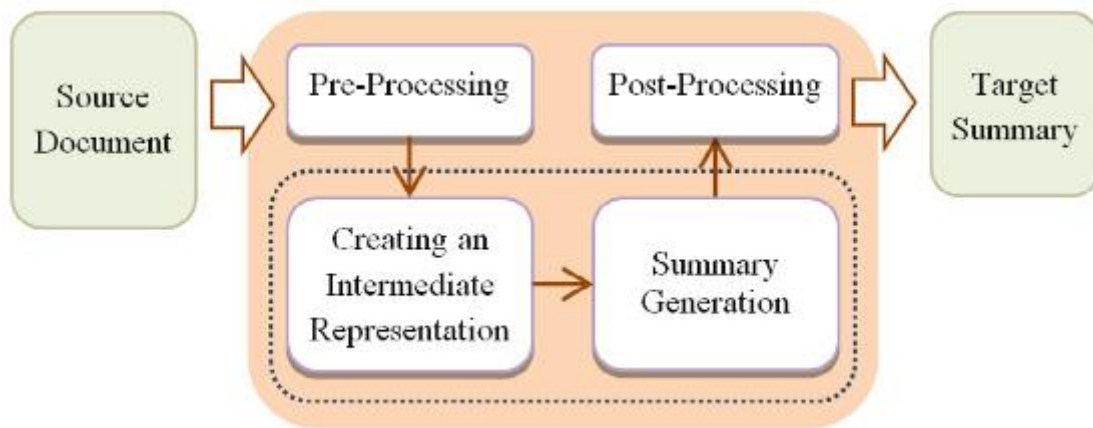


Figure 2-5 - Abstractive text summarization general framework [5]

### 2.3.2.1 Machine-learning-based Summary Generation

Due to the recent advancements in the progress of improving Artificial Intelligence (AI), techniques that make use of AI models have increasingly become both more effective and efficient at a rapid rate. Models such as **Bidirectional and Auto-Regressive Transformers (BART)** [8], or **Text-to-Text Transfer Transformer (T5)** [9] have continuously improve the process of text summarizations.

Following the development of pretrained models, utilizing trained and optimized models has become trivial due to the reduction in training time required for these models. This is especially relevant in cases where resources are limited such as laptops or lightweight servers. The primary method in which this is achieved is through a method known as transferred learning [10]. Inspired by how humans acquire new knowledge by applying and adapting old one, AI model are trained in a two-phase framework [11], with the first being pre-training, allowing for the model to obtain knowledge from large labeled datasets, and the second being finetuning which allow a model to adjust to a specific task with less data required.

Amongst the models, three, which are possible to be run in a local environment efficiently, were chosen for this purpose. This includes the Falcon AI's text-summarization model [12] (a T5-based model), the Facebook's bart-large-cnn model [13] (a BART-based model). Even though another model which is not suitable for this use case, a relatively large model was chosen to for the benchmark, which was the Stability AI's stablelm-2-1\_6b model [14].

As these models are pretrained, the dataset that was used also differs between each models. While some models were trained mostly on one dataset, others used multiple simultaneously. For instance, Facebook’s bart-large-cnn model’s base BART model was pretrained on a large corpus using a denoising autoencoder method. BART is then fine-tuned using the CNN Dailymail Dataset ([15], [16]), which is a . Falcon AI’s text-summarization model, on the other hand, is finetuned using a variety of documents and their generated summaries. Since text-summarization is fine-tuned based on Google’s t5-small model [9], the model is pre-trained many datasets, some important dataset includes:

- **C4** [9]: A cleaned version of Common Crawls’ web crawl corpus.
- **Wiki-DPR** [17]: contain 21 million passages from Wikipedia.

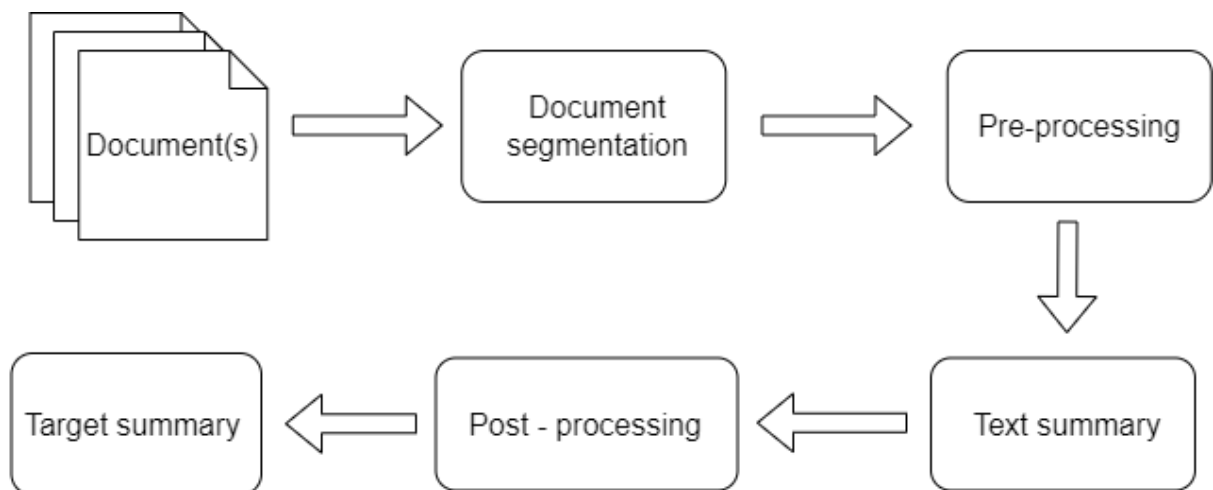
As for stablelm-2-1\_6b, 3 main datasets were use for the pre-training process, including:

- **UltraChat 200k** [18], [19]: Consisting of 1.4 million dialogues generated by ChatGPT.
- **MetaMathQA** [20]: Containing questions and answers related to Math.
- **Orca DPO Pairs**: A modified version of the OpenOrca dataset ([21], [22], [23], [24], [25])

## Chapter 3 METHODOLOGY

### 3.1 Overview

Although this project does have exposed APIs and a minimal web user interface, the application only lacks refinement that can effectively serve its purpose. Currently, most core sections of the project are well defined, resulting in a fully featured tool that can be utilized, albeit, not very user-friendly. The general structure of the desired application can be surmised to be the combination of the document segmentation system and the text summarization system, with the output of the first system being the input of the next, with pre-processing being applied to the segmented text between the two system. The original input of the system should take into consideration both the single document and the multiple documents situations. As such, the general structure of the application can be described through Figure 3-1.



*Figure 3-1 - General structure of the application*

Since many summarizations techniques can be utilized in this situation, five were chosen to be used as the summarization methods. These methods would then be compared using specific benchmarks to determine the most suitable for this specific use case. Amongst them, aside from LSA and TextRank which is an extractive method, two previously mentioned pretrained machine learning models are utilized to achieve abstractive results, including text-summarization and bart-large-cnn.

## 3.2 System design

Although Figure 3-1 depicts a multitude of steps involved in the process, the whole system can be condensed into 2 different parts, which are the segmentation subsystem, and the text summarization subsystem which includes all steps after segmenting the documents. The first subsystem contains the data path and logic behind the segmenting of a document using GROBID. Following, the output of the first subsystem is then preprocessed, summarized and postprocessed in the second subsystem. The subsystem would then deliver the response to the corresponding request, completing all tasks required. The overall design of the system can be expressed through the component diagram in *Figure 3-2*.

The system is heavily dependent upon the usage of Docker as a method to ensure system stability and interoperability. The usage of containers can decrease unforeseen errors caused by operating systems or package dependencies by having Dockerfiles predefining steps that are required to complete before each containers start. Through these steps, both the version of the operating system and the project's dependencies can be defined, ensuring that the system would work for as long as these packages are supported and available.

The entire system is defined through the use of Docker Compose since each container on its own would not be able to communicate with each other. While each subsystem can be initialized easily through Docker, the act of connecting these containers repeatedly for either debugging purposes or deployment is both inefficient and tedious, making the whole process error prone. As such, through utilizing Docker Compose, the connectivity and prerequisite between all related containers can be defined, allowing for Docker to construct each container orderly through predefined steps.

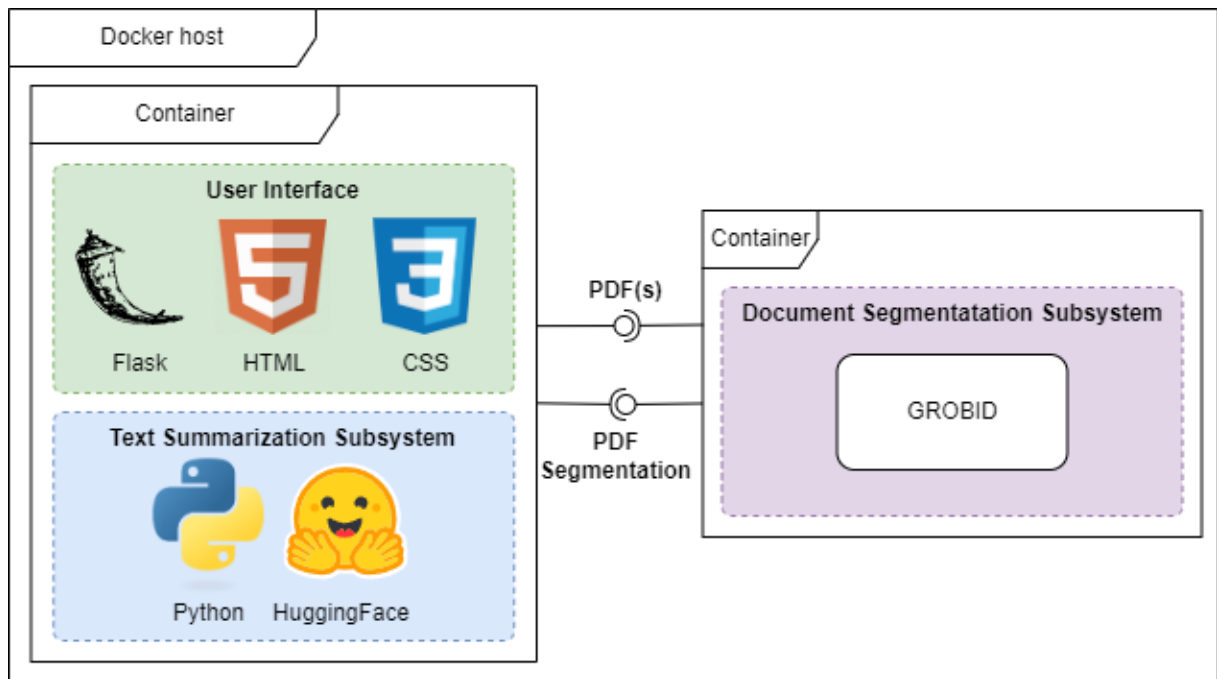


Figure 3-2 – The component diagram of the system

### 3.2.1 Data format

The usage of Docker as the main method for cross-platform compatibility is inevitable as the application needs to be flexible and adaptable to new system requirements. In addition, as each subsystem is designated as a different container, communication between each container is limited to HTTP request. Due to which, the application must, therefore, be data-driven, controlling its logic through changes made to the data. As a result, JSON file format was chosen as the common interface between each of the subsystem and processes.

Although the original output of the GROBID subsystem is an XML file, which can act as the interface for the overall system, a large amount of unnecessary data is included due to its highly generalized nature. Amongst the included data, tags containing header information and reference information, for instance, are of no use for the current application. Therefore, the truncation of said file is necessary to declutter and isolate the important information for further use. The resulting truncation is then reformatted into a JSON file due to the necessity of moving data between containers.

For the purposes of this application, a research paper can be divided into many sections, with each section being the header (i.e. Header 1. Introduction) of the paper. Each sub-header’s content (i.e. Header 1.1’s content) will be consolidated and appended into the parent header, creating a section. A section’s name, however, is defined only by the parent header. This process can be visualized through Figure 3-3.

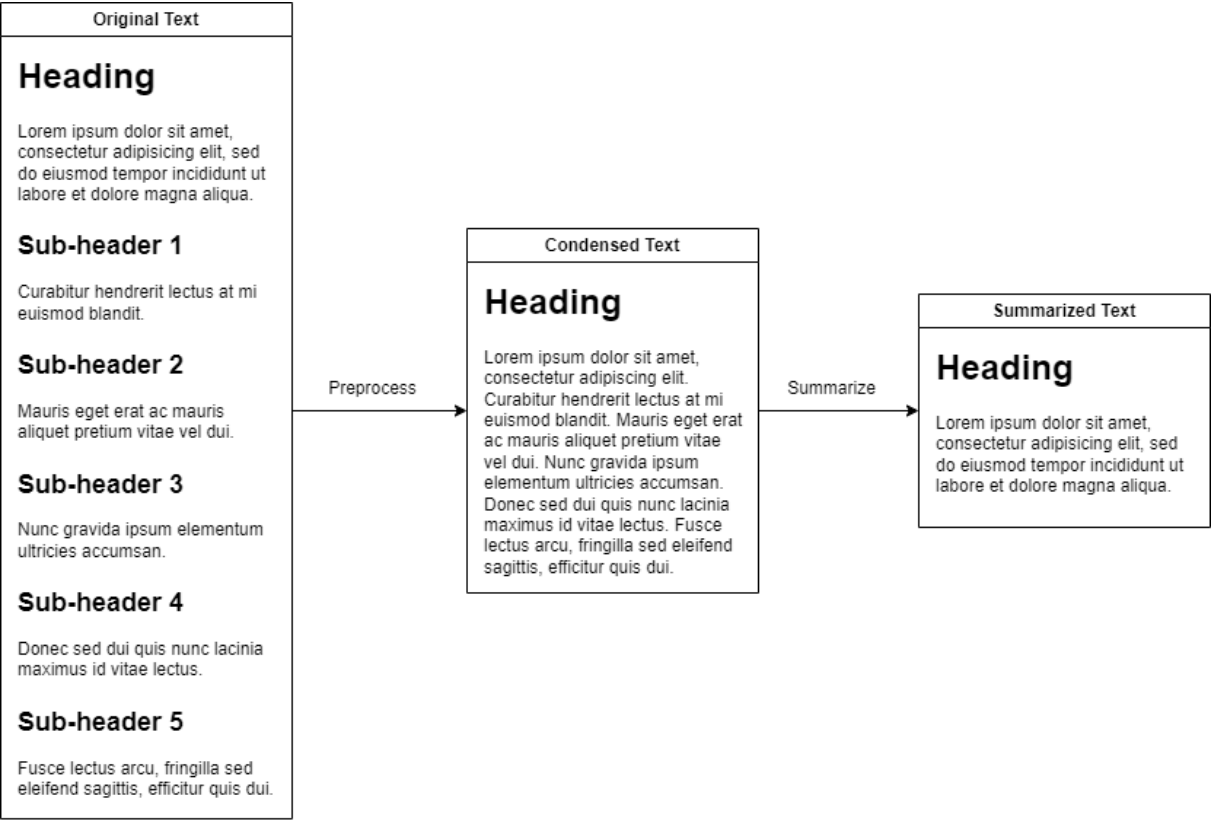


Figure 3-3 – Data condensing process

Since each paper can be divided into its sections and the section’s content, necessary information of each paper includes the name of the header and the content of each header. The name of the file is also included in the final summary JSON file due to the need for differentiation between different files when multiple PDFs are uploaded. However, the abstract section is included in the JSON file due to its usefulness for testing purposes. As such, the JSON file may contain the abstract section depending on the current setup of the system, the section list which includes multiple header and content.



### **3.2.2 Document Segmentation Subsystem**

As explained in the Literature Review chapter, GROBID has been chosen due to its high compatibility with the needs of the project. As a result, most classes and functionalities of this subsystem are for integrating the GROBID's functionalities with the main application. One of its main purposes is to initialize and configure the basic GROBID settings such as the GROBID home (the GROBID source location), changing the input PDF file into an appropriate format, and getting the result after parsing. Said result, which is in the form of an XML file, then became the input for the XMLParser class and YAMLParser class, creating the final output of the subsystem, which is a YAML file containing the segmentation of the provided PDF.

GROBID is set up as a separate Docker container in which a PDF file or a whole PDF can be used as an input to the system. Since GROBID provide prebuilt Docker containers which can help users build and use them on both Windows and Linux, building the entire system from source is not necessary as you would be forced to use Linux to develop the entire system. Communicating with said container is simply through the use of the provided APIs. Since wrappers libraries are provided to user for quick integration of their system with GROBID, the act of utilizing these APIs are trivialized.

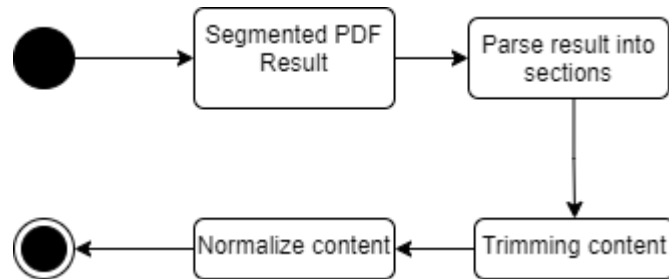
### **3.2.3 Text Summarization Subsystem**

As the text summarization subsystem is not only responsible for the text summarizer, but also both the pre-processing and postprocessing of the input and output, this subsystem is further divided into three main modules, with each being pre-processing, summary, and postprocessing correspondingly. Additionally, a wrapper module is used to encapsulate the whole subsystem, wiring everything together, and exposing the appropriate APIs and web interface.

#### **3.2.3.1 Pre-processing module**

In the pre-processing module, each section's content from the segmented paper is first trimmed and normalized. During the trimming process, excess white space from both between, before, and after words are removed, cleaning the text syntactically. After said process, all capitalized characters are lowercased in the word normalization process, allowing for the model to read the text more effectively. Each section and its name are then stored in a custom data object called Paper. Each paper contains the name of the currently processing PDFs and a list containing the multiple data objects called Section. A Section is responsible for containing the section's name

and content. Utilizing the custom data object can help move data more effectively between each module.



*Figure 3-4 – Pre-processing module diagram*

### 3.2.3.2 Text summarization module

Moving to the text summarization module, although many different pre-processing and summarization options are available, only a few options are included in the final product due to the speed at which these methods process the provided data. Especially of note are the text summarization methods involving the use of pretrained machine learning models, as these methods not only consume a significant amount of resource while running, but also during its start up. Amongst the aforementioned models, only Facebook’s bart-large-cnn and Falcon AI’s text-summarization are suitable for running in production, while Stability AI’s stablelm-2-1\_6b can only be used for testing purposes. This can be observed more clearly through the execution time in the evaluation section. Therefore, a list of available options for summarizing text includes: TextRank, LSA, text-summarization model, and bart-large-cnn. These methods are then connected to the overall system to produce the corresponding summaries.

The input text is simply passed into the system in the case of extractive summarization methods (LSA and TextRank). However, since all chosen subjective summarization methods use machine learning methods, a corresponding prompt needs to be constructed, instructing the model on what to do. The input would also need to be tokenized and calculated for the total number of tokens. This is due to some models having a limit on the number of tokens it can accept with each prompt, requiring the input to be truncated before being passed into the model. With bart-large-cnn, 1024 tokens are the limit, while for stablelm-2-1\_6b, the model can handle up to 2048 tokens. As for text-summarization, it boasts a 2048 token limit despite being lightweight.

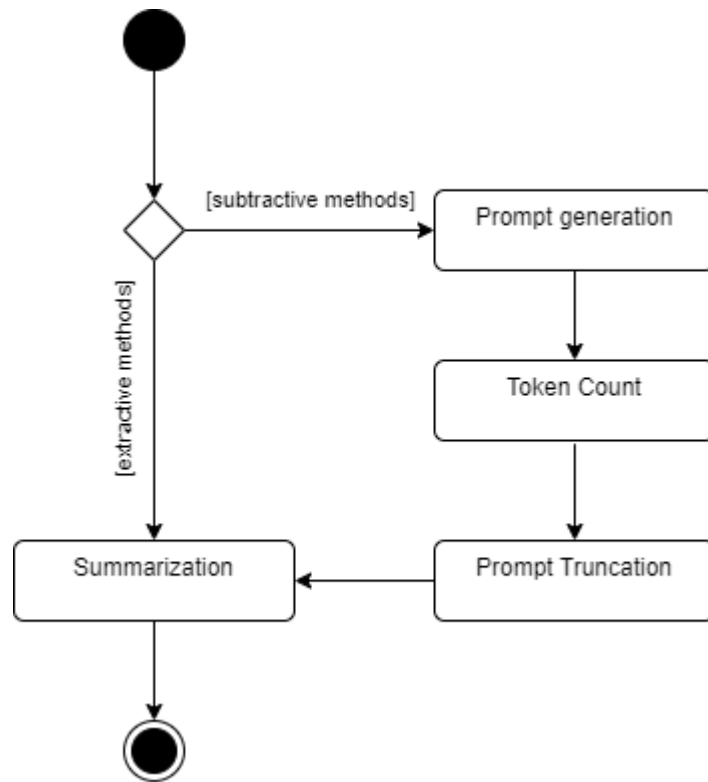


Figure 3-5 – Text summarization module diagram

### 3.2.3.3 Post-processing module

Since the resulting summaries are formatted in different ways depending on the method in which the summary was produced. For example, stablelm-2-1\_6b’s output will start with “<|im\_end>\n<|endoftext>”, which needs to be removed. As such it is the post-processing module’s responsibility to reformat the resulting text according to the corresponding summarization method. Furthermore, since each section of a paper is passed through the summarization method individually, the module must recompile all these sections and add them under the correct section header for the result, which is in the form of a JSON file. In order to display the results to the Web UI, the JSON result is required to be translated into HTML, with the correct format and CSS style class, which would then be used to display the result through web interface.

Amongst the results from each subtractive summarization model, only bart-large-cnn does not requires any additional processing, the remaining 2 methods would require cleaning as text-summarization model would have “Summarize the following segment into 1 sentence:” (which

was part of the prompt) at the start of the result, and stablelm-2-1\_6b would start with “<|im\_end>\n<|endoftext|>”.

### 3.2.3.4 Wrapper module

As for the final module which is the application wrapper, Flask is used as the main library to construct and host the website due to its simplicity and ease of use. The APIs are constructed as the main method for communication with the system. Of note are two APIs, including a method for uploading a file and a method for summarizing the provided files. Port 5000 is also available by default as a method of choice for the system to be more accessible through a simple Web UI. Overall, an overview of the system can be observed through Figure 3-6.

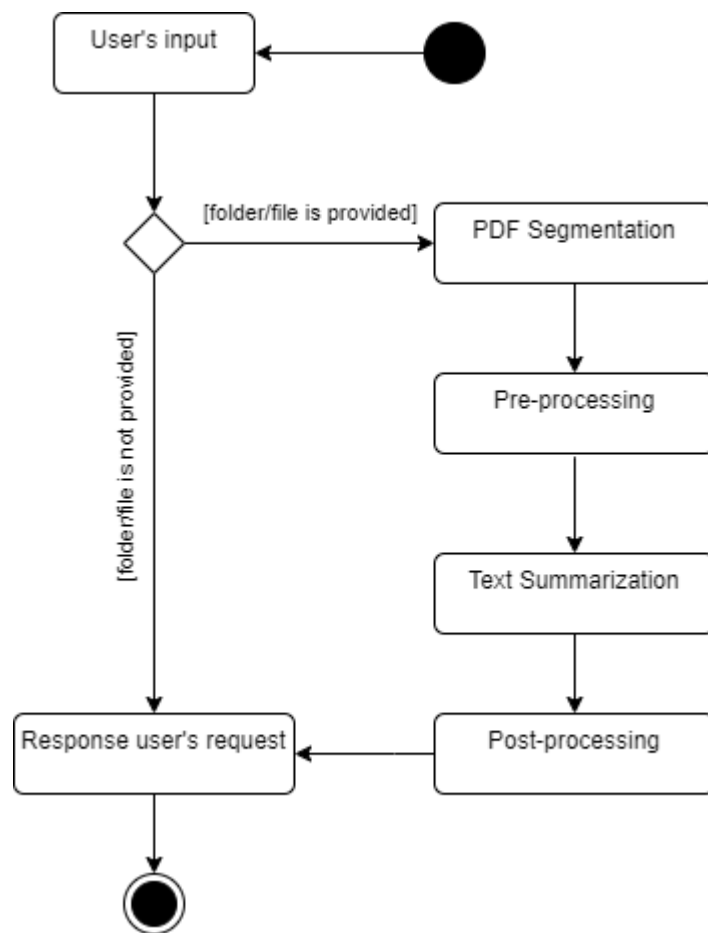


Figure 3-6 – The application's activity diagram

### 3.3 Utilizing Extractive summarization methods in Pre-processing

#### 3.3.1 Overview

During the normal pre-processing procedure for text, methods such as text normalization, whitespace trimming, etc. are frequently used. Although these methods can improve the input to the summarization system and clean up the input text, the remaining paragraphs can be exceedingly lengthy, spanning more than 1000 words each, which can include unnecessary information, as well as, possibly exceeding the token limit of the model. As such, filtering important sentences from the others can potentially improve the output of the whole system.

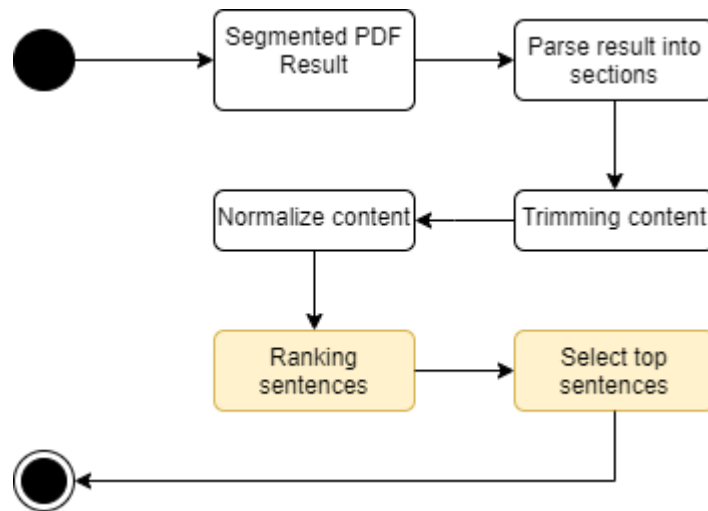
#### 3.3.2 Detailed Description

Extractive summarization methods such as LSA and TextRank have been decided to be used as methods for extracting the specified number of sentences to be passed into another summarization model. For this approach, the top five sentences from each section's content are chosen for further summarization through abstractive method.

Since both LSA and TextRank use different approaches to rank sentences, the result can change drastically. Although with the addition of this step, only subtractive summarization methods can utilize this method, as applying the same method on itself does not make sense and would not yield meaningful result. When comparing the process of Figure 3-8 to Figure 3-3, changes made to the original text can be seen clearly. In this specific case, the phrase "*Donec sed dui quis nunc lacinia maximus id vitae lectus.*" is determined to be the lowest ranking sentences amongst the condensed sentences as an example, causing the sentence to be filtered after being passed through the process. Through **Error! Reference source not found.**, the flow of the data in the pre-processing section can further be dissected, allowing for a clearer view of the data pipeline.

In the first step, which is to rank each sentence, each summarization has its own way to do so, either by calculating distance between each sentences using Levensthein distance in the case of TextRank or through constructing a matrix of term frequency in the case of LSA. As such, the biggest difference between each method would be in this step since each method can contain multiple steps within themselves, causing each one to be different from one another. In theory,

a ranking can be made using other methods beside the provided extractive methods, as the only requirement being providing a method to compare the sentences with each other, allowing for automatic ranking between each sentence. Some examples of this can be seen through utilizing embedded AI model to vectorize each sentence, then compare them. Moving on to the second step, the list provided in the first step is simply sorted, permitting the top sentences to be picked.



*Figure 3-7- Proposed pre-processing module diagram*

Due to changes made to the data pipeline, it follows that changes must also be made to the overall design of the Pre-processing subsystem. These changes to the layout can be viewed in Figure 3-7, in comparison with Figure 3-4. Even though a new step is added into the process, no significant changes were made as the results from the segmentation system still require cleaning and compilation before becoming usable. Since the system is only utilized when user choose abstractive method, the proposed pre-processing method will not be used if extractive methods are chosen. This would mean that in cases where TextRank and LSA are chosen as the main summarization method, the ranking sentence step and top sentence selection step are ignored, stopping only at normalizing the text content.

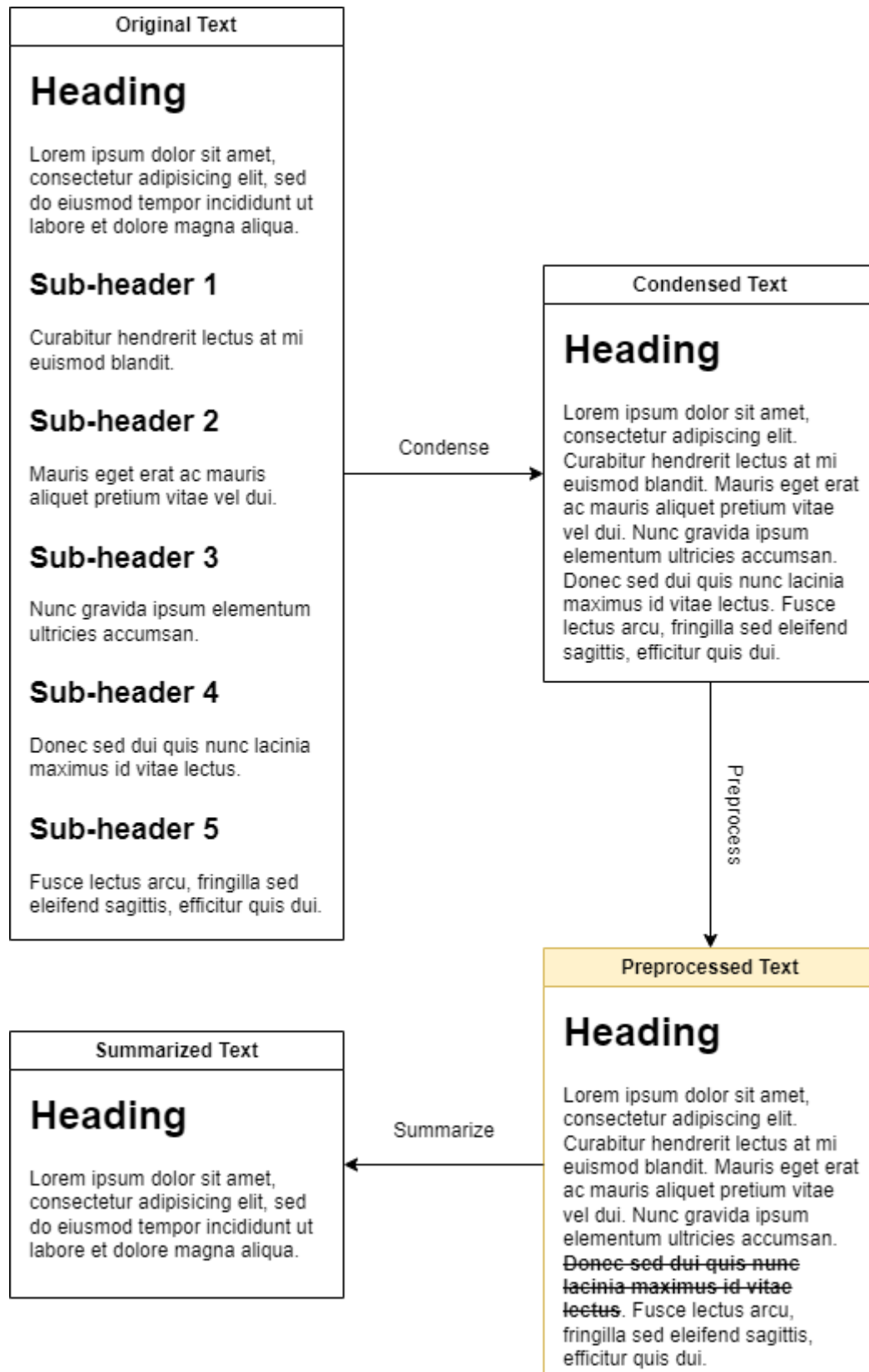


Figure 3-8 – Pre-processing data processing

### 3.3.3 Rationale

When implementing and executing the system while utilizing the abstractive summarization method, none of the available model would be practical enough to be used as an application practically due to its long inference time. This is especially true in cases where user have no access to powerful GPU but can only make use of the computer CPU. A possible solution to this issue is to set the system up as a cloud service, allowing for ease of access to users, and horizontal scaling of the entire system to make use of powerful GPUs through cloud providers. Although this method is technically viable, the cost involved with running an AI model through cloud services can be tremendous, especially when powerful GPUs are involved.

As such, the second method, which is to allow users to host their own application node, is more preferable. This requires the system to somehow be able to optimize the time of inference, while maintaining a good enough result that low-end machine can effectively utilize the application locally. Therefore, a hybrid approach in which extractive summarization methods are utilized to reduce the provided input, which in turn would reduce the amount of time required to process the summaries.

Although the method makes sense in theory, it is unknown whether these changes to the data pipeline can improve the results in terms of their effectiveness. This is due to the possibility of preprocessed text losing important information from the process instead of filtering unnecessary one.

Since each method executes both methods in different ways, the combination of each method with an abstractive summarization method must be examined and evaluated. Effectively, after implementing this change, the entire system would execute the summarization process twice, with the first pass going through an extractive summarization process while producing 5 sentences. These 5 sentences would then be passed through the extractive summarization system, finalizing the result of the system. Presumably, through reducing the input to the abstractive system, the provided prompt would become shorter, which will ultimately reduce the time needed for the model to infer the result from the input text. Additionally, it can potentially increase the overall quality of the resulting prompt may improve in comparison to its non-pre-processed counterpart.



### 3.4 User interface

Due to the system's structure being constructed in a composition of docker modules, the project lent itself well to using a Web User Interface as its default method of presenting information to users. Since cross-compatibility and flexibility are some of the project's main concern since its conception, utilizing web browser as the method of choice for communicating with the UI. The design and implementation of the UI can be seen in Figure 3-9.

By default, the home directory of the website would direct the user to the Web UI interface of the application, The UI includes simple but important settings that can help with configuring the whole summarization process. Said configurations are separated into 2 distinct parts, including the pre-processing settings and the summarization settings. The user is allowed to choose one of three available pre-processing options, including **"None"** (which skip the use of extractive summarization in the pre-processing step), **"TextRank"**, and **"Latent Semantic Analysis"**. By default, **"None"** is chosen in case user do not choose any method. As for the summarization method, as explained, only text-summarization and bart-large-cnn model are available as choices due to stablelm-2-1\_6b being unsuitable for usage in production. Along with the subtractive model, both TextRank and LSA are available as choices for the users.

The user would then be required to choose a file that would be processed. This action can either be done through dragging and dropping the file into the designated zone or just by simply clicking on the zone, allowing the user to choose a file from the file explorer. In order to prevent unintended consequences from users' actions, some simple fail saves are implemented including checking the file extension for any combination of the phrase ".pdf", filtering the file's name from including inappropriate keyword for certain operating systems through using the provided *secure\_filename* function from Flask, and providing extensive error messages in various other cases.

Report Generation

Pre-processing:

NoneTextRankLatent Semantic Analysis

Processing:

TextRankLatent Semantic AnalysisFalcon AI Text SummarizerBart Large CNN

Drag & drop your PDF file here, or click to select a file

File: Deienno\_et\_al.\_-2024\_-\_Accretion\_and\_Uneven\_Depletion\_of\_the\_Main\_Asteroi.pdf

Generate

INTRODUCTION

we show that, unless the primordial mass of the mab was indeed very low ( $\lesssim 2.14 \times 10^{-3} M_{\oplus}$ ) before accretion started, the number of s-complex objects with  $d > 500$  km that would form and survive solar system evolution largely exceed the number currently observed (i.e.,  $n_{\text{s-complex}}(d > 500 \text{ km}) = 1$ ; (4) vesta; we present our work in the following structure: section 2 describes our modeling.

MODEL

nonetheless, in term of primordial mab mass, pebble accretion models should likely lead to similar conclusions. for the total primordial mass of the mab we assumed that the initial distribution of planetesimals, uniformly distributed between 1.8 and 3.6 au, follows a density profile  $\sigma = \sigma_0 r^{-\gamma}$ , which is analogous to the minimum-mass solar nebula (mmsn; hayashi 1981) radial density profile, and for simplicity we refer to it as such.

MAB, DATASET, AND COMPARISON WITH MPC

labels all and  $\text{amd}_{\text{jsps/pj}}$  are for different cuts in the data by.  $\bullet$  c19 all refers to results taken from while excluding runs 4, 7, and 7a 7, as well as those with embryos (runs 1b and 2b) 8.  $\bullet$  c19  $\text{amd}_{\text{jsps/pj}}$  is also for results from but while only considering cases where  $\text{amd}_{\text{js}} < 0.19$  and  $2.3 \lesssim \text{ps}/\text{pj} \lesssim 2.5 \times 10$  at the end of the giant planet instability phase (runs 1, 2a, and 5a). the work by reported on mab depletion within a few 100s myr after the end of the giant planet instability, and they did not account for the 50% additional depletion during subsequent 4.5 gyr of solar system evolution.

RESULTS

yet, the fall-off (steepening) of our depleted sfd for  $d \gtrsim 200\text{-}300$  km (figure; e.g. yellow curve) suggest that objects larger than those sizes, including the differentiated asteroid (4) vesta, were likely terrestrial planetesimals implanted into the mab (during terrestrial planet accretion, rather than asteroids formed in situ at 0.5 myr (case of 4 vesta), or at 2-3 myr (case of undifferentiated large objects) after cais. primordial mab masses larger than about  $\approx 2.14 \times 10^{-3} M_{\oplus}$  formed at 0.5 myr after cais would generate sfd incompatible with our predicted s-complex sfd, unless depletion occurred much earlier than accretion could have taken place.

CONCLUSIONS

the overall mab depletion factor reported in the last column of table should only be used as a guide for estimating overall mass depletion, but not size-dependent population depletion, as the latter is also semimajor-axis dependent. the second main result of the present work is that we found the maximum total mass that could have formed in the primordial mab at around 0.5 myr after cais is likely to be smaller than  $\approx 2.14 \times 10^{-3} M_{\oplus}$ .

Figure 3-9– Web user interface



## Chapter 4    IMPLEMENT AND RESULTS

### 4.1 Overview

This chapter provides explanations and descriptions of the work that has been done. Amongst them, most of the work can be divided into 2 distinct parts, which are implementing the document segmentation using GROBID and the development of the text summarization capability along with the multitude of summarization methods. Since the whole system is encapsulated in a cluster of Docker nodes, configured through the use of Docker Compose, the whole system can be reproduced easily.

### 4.2 Implementation

Some code snippets explaining what has been discussed in Chapter 3 from each subsystem can be found in the following part of this chapter. By providing these code sections, the method in which the application is implemented can be better understood.

#### 4.2.1 Document Segmentation

As GROBID is a machine learning approach to the segmentation problem, its initialization requires the initialization of the GROBID container. The code snippet in Figure 4-1 is the function through which the initialization is made.

Since this method would find all PDF files available in the specified UPLOAD FOLDER, no additional set up is required except for having to upload the corresponding files to the correct folder. Toward that end, user can directly upload the PDF file by submitting it from the Web UI, or by utilizing the folder-uploading request, an example of which can be found in Figure 4-2.

After the file content as well as its name is checked sufficiently, including the file extension, the content of the file, the filename. Additionally, the *secure\_filename* function also provides a method to adjust the file name so that it does not violate any rules regarding the naming scheme of files from any operating system. The file is then saved to the corresponding folder, awaiting further usage.

```

async def parse_pdf() -> bool:
    """
    Connect to the GROBID and summarize the PDF in the folder.
    """
    with app.app_context():
        with concurrent.futures.ThreadPoolExecutor(max_workers=1):
            try:
                client = GrobidClient(config_path=GROBID_CONFIG_PATH)
                client.process(
                    "processFulltextDocument",
                    UPLOAD_FOLDER,
                    output=SEGMENT_FOLDER,
                    consolidate_citations=False,
                    tei_coordinates=False,
                    verbose=True
                )
            except:
                return False
    return True

```

*Figure 4-1 – GROBID initialization code snippet*

### 4.2.2 Text summarization

As both LSA and TextRank are used during the development process, implementation of both systems is available. Even though TextRank was implemented through the usage of a library, its implementation is worth mentioning as some parameters and its choice have effects on the result of the summary. LSA, however, does not have a library that supports it from the start, so it needs to be implemented step by step. As for the pretrained models, HuggingFace [26] was utilized as the library of choice for implementation due to the large number of tools, support, and models available.

```

@app.route("/upload", methods=['POST'])
def upload():
    if (request.method != 'POST'):
        return "No GET method", 404

    if 'files' not in request.files:
        return "No 'files' header in payload.", 404

    pdf_file = request.files['files']

    if pdf_file.filename == '':
        return "File has no name.", 404

    if not pdf_file:
        return "File does not exist.", 404

    if not allowed_file(pdf_file.filename):
        return "File does not end with .pdf", 404

    filename = secure_filename(pdf_file.filename)

    pdf_file.save(os.path.join(app.config['UPLOAD_FOLDER'], filename))

    return 'Successfully send files', 200

```

*Figure 4-2 – PDF-uploading method*

#### 4.2.2.1 LSA

The LSA summarization approach goes through 7 steps in total, including Tokenizing, NGram generation, stop words filtering, word frequency sorting, sentence detection, word searching, and finally summarization. While the first 3 steps are a part of the `_create_dictionary` function, the remaining steps are distributed among the remaining function calls due to their complexity. The summary is then generated by using the output from the steps above. This can be seen in Figure 4-3.

```

def lsa (text:str, sent_num:int=SENT_NUM) -> str:

    if (len(text) == 0):

        return ""

    dictionary = _create_dictionary(text)

    sentences = sent_tokenize(text)

    matrix = _create_matrix(text, dictionary)

    matrix = _compute_term_frequency(matrix)

    u, sigma, v = singular_value_decomposition(matrix, full_matrices=False)

    ranks = iter(_compute_ranks(sigma, v))

    result = _get_best_sentences(

        sentences,

        sent_num,

        lambda s: next(ranks)

    )

    result_str = ''

    for sentence in result:

        result_str += sentence

    return result_str

```

*Figure 4-3 – LSA Implementation*

The summarization process above works by looping through each summary sentence, conducting word searching on it, and storing the results in a list. The summary is then concatenated and constructed from said list.

#### **4.2.2.2 TextRank**

As mentioned, the TextRank implementation is made using a library called *summa*, this can be seen through Figure 4-4, where the summarization function is defined.

```
def summarize_text(text:str, sent_num:int=SENT_NUM) -> str:

    return summarize(

        text = text,

        words = sent_num * (_get_max_length_sentence(text) + 5),

        additional_stopwords= summary.get_stop_word_list()

    )
```

*Figure 4-4 – TextRank summarization function*

In said function, “text” refers to each PDF segment that the user wants to summarize. “words” is the upper word limit of the resulting summary and “additional\_stopwords” refers to the list of words that need to be filtered out before processing the summary. In order to control the number of sentences the resulting summary will have, the length of the longest sentence in the provided text must be calculated, which is the responsibility of the `_get_max_length_sentence` function. Utilizing which, only the text that needs to be summarized and the number of sentences needs to be passed in as parameter.

#### 4.2.2.3 Pretrained models

Even though small differences exist when implementing different models with regards to how each one is initialized and utilized, a familiar structure can be distilled. Due to loading a model requiring an extended amount of time, it is preloaded and cached, allowing for subsequent usage requiring less memory and time to complete a task. An example of preloading the model can be found in Figure 4-5.

```
async def preload_stable_lm_chat_1_6b():

    global tokenizer

    global model

    tokenizer = AutoTokenizer.from_pretrained(

        'stabilityai/stablelm-2-1_6b-chat'

    )

    model = AutoModelForCausalLM.from_pretrained(

        'stabilityai/stablelm-2-1_6b-chat',

        device_map="auto",

    )
```

*Figure 4-5 – Model preloading*



The text is then passed into the chosen model and further processed to be suitable for the required text generation. The input is first passed into a prompt constructor, where it will instruct the model for the required text generation. The prompt would then be tokenized in some models such as stablelm-2-1\_6b-chat as per the requirement of each model. Finally, the text is then passed into the model where it would be processed, and the result can be obtained. The process can be seen in Figure 4-6 in the case of stablelm-2-1\_6b-chat.

```

async def stable_lm_chat_1_6b(content:str) -> str:

    prompt = [{'role': 'user', 'content': prompt_constructor(content)}]

    inputs = tokenizer.apply_chat_template(

        prompt,

        add_generation_prompt=True,

        return_tensors='pt'

    )

    tokens = model.generate(

        inputs.to(model.device),

        max_new_tokens=get_max_length(content),

        temperature=0.01,

        do_sample=True

    )

    output = tokenizer.decode(

        tokens[:, inputs.shape[-1]:][0], skip_special_tokens=False)

    return output.rstrip("<|im_end|>\n<|endoftext|>")

```

*Figure 4-6 – Model text generation*

### 4.3 Results

The results from the process of the entire system can be condensed into 2 different JSON files with the first being the document segmentation result and the second being the summary. As an example, one research paper named has been chosen. As the result of the document segmentation being the content of the entire paper, only the first header will be showcased in Figure 4-7. Since many different methods of summarizations were used, only LSA and

TextRank are shown as an example of the data format, which can be seen in Figure 4-8 and Figure 4-9.

```
{
  "name": "Hansen_et_al._-2024_-_Productivity_and_quality-
adjusted_life_years_QALY.grobid.tei.xml",
  "segments": [
    {
      "header": "Introduction",
      "content": "Few aspects concern human beings more than health. But resources are scarce
and, as we face demographic changes, with increased demand from retirees and a constrained
labour market due to shrinking working age share of the population, there is a pressing need
to protect the health and productivity of the economically active population. Therefore,
critical decisions on health care interventions, as well as occupational health and safety
policies, have to be made constantly. The evidence from clinical trials and observational
studies, in addition to assessments about the productivity consequences, are crucial to make
those decisions. The purpose of this paper is to develop a unified framework for the
measurement and valuation of outcomes of such programmes and policies. It is widely accepted
that the health benefit a patient derives from a particular health care intervention can be
defined according to two natural dimensions: quality of life and quantity of life. An
alternative to QALYs is the so-called productivity-adjusted life years (in short, PALYs),
which are calculated by multiplying a productivity index by years lived. The productivity
index ranges from 0 (completely unproductive) to 1 (completely productive), and may take into
consideration factors such as absence from work due to ill health (absenteeism), reduced
productivity while at work (presenteeism) and premature exit from the workforce (e.g.,
Economic evaluation of policies to improve occupational health and safety is one field of
research where productivity outcome measures following the broader PALY idea are applied
extensively (e.g., Our approach builds upon the framework introduced in Hougaard et al. We
conclude this introduction stressing that our model treats health and productivity as
different individual attributes. In doing so, we obviously depart from the literature that
considers only one of them, but also from the simplistic assumption that both concepts are
perfectly correlated (which would allow to use a reduced model). The precise relationship
between health and productivity is complex and the anticipated correlation might actually be
positive or negative, depending on the viewpoint. For instance, The rest of the paper is
organized as follows. In Section 2, we introduce the framework and the basic common axioms
that all our evaluation functions will satisfy. In Section 3, we characterize the focal (and
somewhat polar) evaluation functions QALYs and PALYs. In Section 4, we characterize classes of
evaluation functions which compromise among the previous two. In Section 5, we characterize
more general functional forms, that evolve around the notion of healthy productive years
equivalent. In Section 6, we discuss our contribution with a special emphasis on the choice
among the different evaluation functions we characterize. Finally, in Section 7, we provide
some concluding remarks providing further connections to related literature and pointing out
possible extensions of our work. For a smooth passage, we defer all proofs to the Appendix."
    }, ...
  ]
}
```

*Figure 4-7 – Segmentation result*

```
{
  "name": "Hansen_et_al._-2024_-_Productivity_and_quality-
adjusted_life_years_QALY.grobid.tei.xml",
  "segments": [
    {
      "header": "Introduction",
      "content": "The productivity index ranges from 0 (completely unproductive) to 1
(completely productive), and may take into consideration factors such as absence from work due
to ill health (absenteeism), reduced productivity while at work (presenteeism) and premature
exit from the workforce (e.g., Economic evaluation of policies to improve occupational health
and safety is one field of research where productivity outcome measures following the broader
PALY idea are applied extensively (e.g., Our approach builds upon the framework introduced in
Hougaard et al."
    }, ...
  ]
}
```

*Figure 4-8 – TextRank summarization result*

```
{
  "id": "Hansen_e",
  "name": "Hansen_et_al._-2024_-_Productivity_and_quality-
adjusted_life_years_QALY.grobid.tei.xml",
  "segments": [
    {
      "header": "Introduction",
      "content": "The precise relationship between health and productivity is complex and the
anticipated correlation might actually be positive or negative, depending on the viewpoint."
    }, ...
  ]
}
```

*Figure 4-9 – LSA summarization result*

## Chapter 5 DISCUSSION AND EVALUATION

### 5.1 Evaluation

As text segmentation utilized GROBID, which is a tool that has been used and benchmark through multiple sources ([27], [28], [29], [30]). As such the main focus of the evaluation section would be on the summarization method.

Evaluating and comparing the chosen summarization methods are an important task so as to choose the appropriate sentences for the product to be adaptable for a range of topics that can be covered for by a research paper. Additionally, the provided summarization methods must be efficient enough to be able to be executed by an average user. As such, additional information such as the evaluation method, the dataset, the specification of the test machine, and the evaluation results will be included in the following sections.

#### 5.1.1 Evaluation methods

Due text summarization being highly dependent upon the perspective and knowledge of the reader. As such, having testers with the appropriate knowledge to evaluate the result is the most desirable outcome as this can realistically reflect the opinion of normal readers. Otherwise, a dataset containing a reference summary result can also be used to compare to the generated text of the system being tested.

However, since the current summarization use case is highly specific, no dataset containing the required referenced summary was found. Since most open-sourced datasets focus on summarizing the entire paper's content to a small paragraph, this could not be applied for this current use case since the PDF is segmented and each segment is further summarized. Additionally, many datasets were rejected due to them being based on sources that are not research papers, including newspapers and books, this can be seen amongst the list of frequently used dataset for this purpose [5] as most of them are from the news and books domain.

As such, the only available option is to utilize a statistical calculation in order to determine the value of each summary. The calculation of choice was ROUGE-1, ROUGE-2, and ROUGE-L [31]. Although some traditional measures such as Recall (Equation 5-1), Precision (Equation 5-2), and F-Measure (Equation 5-3) can be used to calculate quantitatively how much of the

summary has recapture the context of the original text, and how much of that context is relevant, ROUGE measure provides more granularity as it has the benefits of the simpler measures, but also applies it to n-gram and longest matching sequence. Using these measures, the process of evaluating text summarization can be done automatically, allowing for faster feedback and improvement to the system.

$$P = \frac{S_{ref} \cap S_{cand}}{S_{cand}}$$

*Equation 5-1 – Precision equation*

$$R = \frac{S_{ref} \cap S_{cand}}{S_{ref}}$$

*Equation 5-2 – Recall equation*

$$F - Measure = \frac{2(Precision)(Recall)}{Precision + Recall}$$

*Equation 5-3 – F-Measure equation*

ROUGE measures build upon the existing methods of Recall, Precision, and F-Measure. In the case of ROUGE-1 and ROUGE-2, Recall and Precision for the unigram and bigram are calculated respectively. Afterwards, F-Measure is calculated, resulting in the final product. As for ROUGE-L, instead of N-Gram, the longest common subsequence is used to calculate Recall and Precision. From which, the corresponding F-Measure is the result of ROUGE-L measure. In essence, ROUGE measures make use of F-Measure and add small additional improvements to help account for many situations.

### 5.1.2 Dataset

In order to evaluate the system, a random set including 10 research papers PDF relating multiple topics (medicine, astrophysics, machine learning, etc.). Amongst these files are:

- Accretion and Uneven Depletion of the Main Asteroid Belt – Deienno et al. [32]
- Karma: An Experimental Study – Elokda et al. [33]
- Productivity and quality-adjusted life years: QALYs, PALYs and beyond – Hansen et al.[34]
- A Data-driven Latent Semantic Analysis for Automatic Text Summarization using LDA Topic Modelling – Onah et al.[35]
- AI and the Problem of Knowledge Collapse – Peterson. [36]
- The rules of multiplayer cooperation in networks of communities – Pires and Broom [37]
- Mixture-of-Depths: Dynamically allocating compute in transformer-based language models – Raposo et al. [38]
- The economic burden of antimicrobial resistance: Why it is more serious than current studies suggest – Smith and Coast. [39]
- Sequential Monte Carlo Bandits – Urteaga and Wiggins. [40]
- History Trees and Their Applications – Viglietta. [41]
- GreedLlama: Performance of Financial Value-Aligned Large Language Models in Moral Reasoning – Yu et al. [42]

These papers were all randomly chosen by browsing through Arxiv, finding different topics, and choosing a paper at random. By having a more diversified list of PDFs, the dataset can be less biased overall. These papers are first segmented through GROBID, then condensed by removing sub-headers. The condensed content would then be summarized individually, resulting in a reference version of the summary of each paper.

### 5.1.3 Evaluation results and Environment Settings

The machine being used to evaluate the system is currently an AMD RYZEN 5 5600 with 16 of RAM. After running ROUGE-1, ROUGE-2, and ROUGE-L measures, the results are tabulated in Table 5-1.

Pre-processing method	Summarization method	ROUGE-1	ROUGE-2	ROUGE-L
None	<b>TextRank</b>	0.446608	0.134224	0.446608
None	<b>LSA</b>	0.502153	0.129708	0.502153
None	<b>Falcon AI's text-summarization</b>	0.475585	0.132557	0.475585
None	<b>Facebook's bart-large-cnn</b>	0.500143	0.154753	0.500143
TextRank	<b>Falcon AI's text-summarization</b>	0.521697	0.181454	0.521697
TextRank	<b>Facebook's bart-large-cnn</b>	<b>0.565338</b>	<b>0.204300</b>	<b>0.565338</b>
LSA	<b>Falcon AI's text-summarization</b>	0.484316	0.136793	0.484316
LSA	<b>Facebook's bart-large-cnn</b>	0.532834	0.151566	0.532834
None	<b>Stability AI's stablelm-2-1_6b</b>	0.530595	0.221119	0.530595
TextRank	<b>Stability AI's stablelm-2-1_6b</b>	<b>0.585241</b>	<b>0.269883</b>	<b>0.585241</b>
LSA	<b>Stability AI's stablelm-2-1_6b</b>	0.524413	0.208317	0.524413

Table 5-1 - ROUGE-1, ROUGE-2, and ROUGE-L evaluation n results.

Since **stablelm-2-1\_6b** are selected as an example of some of the latest models, it is not surprising to see that its performance being ranked amongst the top of the benchmark result. Interestingly, while using TextRank as a pre-processing method increase the performance of the model on all measures, using LSA for the same purpose would decrease the performance on all testing measurements.

Amongst the evaluation result of the remaining methods, the combination of having TextRank as the pre-processing method and using the resulting sentences to be summarized by the Facebook's bart-large-cnn model obtained the most positive result, being the top ROUGE-1, ROUGE-2, and ROUGE-L measure. The combination between TextRank and Falcon AI's text-summarization model also yields positive results since it claims the 2<sup>nd</sup> place amongst the ROUGE-2 and ROUGE-L measure. Noticeably, all summarization methods being pre-processed by the use of an extractive summarization model would increase the evaluation results over the executing just the base summarization method.

Additionally, in order to measure the efficiency of each method, the total time required to generate the summaries for ten chosen PDFs were documented in Table 5-2.

Summarization method	Pre-processing method		
	None	TextRank	LSA
<b>TextRank</b>	0.885442	-	-
<b>LSA</b>	8.959388	-	-
<b>Falcon AI's text-summarization</b>	571.973681	318.815027	<b>288.749598</b>
<b>Facebook's bart-large-cnn</b>	828.024041	719.341524	<b>681.081772</b>
<b>Stability AI's stablelm-2-1_6b</b>	11332.362937	<b>6656.221096</b>	7058.812489

*Table 5-2 – Summarization time (in second) of each combination*

Two observations can be obtained through the data above, with the first being passing the provided text through a layer of pre-processing method can significantly reduce the total text generation time across all summarization methods, aside from the two extractive summarization methods which serves as an example. In the case of Falcon AI's text summarization, a 43% decrease in execution time was observed for the TextRank pre-processing and a 49% decrease when using LSA. However, time saved does not appear to be consistent between methods, as only a peak of 17% in the case of Facebook's bart-large-cnn model and 41% in the case of stablelm-2-1\_6b. Another observation is that while LSA on its own requires more time than TextRank to complete the task, using it as the pre-processing method will save more time in comparison to TextRank with the exception of stablelm-2-1\_6b. The reason for the reduction in execution time is possibly due to the input being truncated to only maintain 5 sentences. This cut down on time needed to process the input to each model, allowing them to finish significantly quicker.

Conclusively, using an extractive summarization method for pre-processing resulted in a decrease in summarization time. Amongst these, aside from the extractive models, text-summarization model finished executing the quickest, followed closely by bart-large-cnn. As stablelm-2-1\_6b takes hours to complete on average, it is not suitable for any application that requires low processing time. When considering the performance of each model, bart-large-cnn remains the summarization method of choice for this task with text-summarization being in the second place, with both model's input being processed by TextRank. Overall, the system of choice for the current application would be having GROBID segmenting the PDF file, TextRank choosing the important sentences, bart-large-cnn summarizing the sentences.



## **Chapter 6      CONCLUSION AND FUTURE WORK**

### **6.1 Conclusion**

Although the project can be considered as functional, much is left to be desired as the project does not consider much of the user experience. Additionally, even though pre-trained models were used, finetuning the model to fit with the current use case was not possible due to limited time and resources. Overall, the project can be considered as the steppingstone for future improvements.

### **6.2 Future work**

Ideas for extending the current work that can be done in the future:

1. Finetuning the model may allow for better summarization results, which can further improve the evaluation metrics
2. More PDFs can be added to the pool of dataset, allowing for more accurate results.
3. Human-evaluated reference PDF can be considered moving forward to improve the quality of the current dataset.
4. A self-developed model can be considered as having a simple model that is tailored to the current task can improve both the application's efficiency and output if done correctly.

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## APPENDIX

### The full result from the Segmentation process

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resources are scarce and, as we face demographic changes, with increased
demand from retirees and a constrained labour market due to shrinking
working age share of the population, there is a pressing need to protect
the health and productivity of the economically active population.
Therefore, critical decisions on health care interventions, as well as
occupational health and safety policies, have to be made constantly. The
evidence from clinical trials and observational studies, in addition to
assessments about the productivity consequences, are crucial to make those
decisions. The purpose of this paper is to develop a unified framework for
the measurement and valuation of outcomes of such programmes and
policies.It is widely accepted that the health benefit a patient derives
from a particular health care intervention can be defined according to two
natural dimensions: quality of life and quantity of life. An alternative to
QALYs is the so-called productivity-adjusted life years (in short, PALYs),
which are calculated by multiplying a productivity index by years lived.
The productivity index ranges from 0 (completely unproductive) to 1
(completely productive), and may take into consideration factors such as
absence from work due to ill health (absenteeism), reduced productivity
while at work (presenteeism) and premature exit from the workforce (e.g.,
Economic evaluation of policies to improve occupational health and safety
is one field of research where productivity outcome measures following the
broader PALY idea are applied extensively (e.g., Our approach builds upon
the framework introduced in Hougaard et al. We conclude this introduction
stressing that our model treats health and productivity as different
individual attributes. In doing so, we obviously depart from the literature
that considers only one of them, but also from the simplistic assumption
that both concepts are perfectly correlated (which would allow to use a
reduced model). The precise relationship between health and productivity is
complex and the anticipated correlation might actually be positive or
negative, depending on the viewpoint. For instance, The rest of the paper
is organized as follows. In Section 2, we introduce the framework and the
basic common axioms that all our evaluation functions will satisfy. In
Section 3, we characterize the focal (and somewhat polar) evaluation
functions QALYs and PALYs. In Section 4, we characterize classes of
evaluation functions which compromise among the previous two. In Section 5,
we characterize more general functional forms, that evolve around the
notion of healthy productive years equivalent. In Section 6, we discuss our
contribution with a special emphasis on the choice among the different
evaluation functions we characterize. Finally, in Section 7, we provide
some concluding remarks providing further connections to related literature
and pointing out possible extensions of our work. For a smooth passage, we
defer all proofs to the Appendix."
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identified with the set  $N = \{1, \dots, n\}$ . Each individual  $i \in N$  is
described by a profile, formalized by a triple  $d_i = (a_i, p_i, t_i)$ ,
where  $a_i \in A$  is a health state,  $p_i \in [0, 1]$  is the productivity
level, and  $t_i$  is a time. We can think of the health state  $a_i$  as a chronic or
representative health state over time. The productivity  $p_i$  is measured by
any chosen indicator. For instance, it can be an indicator of absence from
work (e.g., number of sick days per year for a person). Note that such an
indicator may reflect productivity and contributions to society in a broad
sense. For example, work may include both labour market activities and
domestic work. Alternatively, a measure could be chosen that reflects the
value of the work contributed by the individuals, as would (very roughly)
be the case if measured by e.g., (relative) earnings (e.g., Finally, there
are two plausible interpretations of time in our model. On the one hand, it
could be identifying the individual total lifetime. On the other hand, it
could be identifying incremental individual lifetime from a given status
quo up to the end of life or retirement. Let  $d = (d_1, \dots, d_n)$  denote a
distribution of individual profiles, as described above, and let  $D$  denote
the set of possible distributions. We now give an example in which two
hypothetical distributions are presented. We shall return to this example
later in the text several times to illustrate how these two hypothetical
distributions can be (relatively) evaluated, by means of various evaluation
functions we consider. Example 1 Consider the following two distributions,
involving five individuals each (that could be interpreted as
representative agents of five different groups). In the first distribution
( $d^a$ ), all individuals are experiencing full health. The first one is
also experiencing maximum productivity as well as forty years of lifetime
(until retirement, or the end of life), i.e.,  $d^a_1 = (a^*, 1, 40)$ .
The second individual is experiencing 50% of maximum productivity and forty
years of lifetime, i.e.,  $d^a_2 = (a^*, 0.5, 40)$ . The third individual
is experiencing zero productivity and forty years of lifetime, i.e.,  $d^a_3 = (a^*, 0, 40)$ .
The fourth individual is experiencing 50% of
maximum productivity and ten years of lifetime, i.e.,  $d^a_4 = (a^*, 0.5, 10)$ .
The last individual is experiencing zero productivity and
lifetime, i.e.,  $d^a_5 = (a^*, 0, 0)$ . In the second distribution ( $d^b$ ), the first individual is also experiencing full health and maximum
productivity as well as forty years of lifetime, i.e., Note that if  $E$ 
represents then any strictly increasing transformation of  $E$  would also do
so. An evaluation function  $E$  may be interpreted as an effect measure if it
is used for the economic evaluation of health care or working environment
interventions. In what follows, we present some basic axioms for social
preferences in the current context, that will be common to all the
evaluation functions we consider in this paper. In this section, we present
a set of seven axioms that forms the necessary conditions for the theorems
presented in the remaining sections of this paper. These are termed the
COMMON axioms. The axioms reflect basic principles adapted to our framework
that are widely accepted in economics. In the following sections,
additional axioms are presented, which together with the COMMON axioms
close the characterizations of the evaluation functions we highlight. The
first three COMMON axioms apply to all three attributes in the same way;
whereas the latter four COMMON axioms introduce some conditions on time
which distinguish it from the other two attributes. The first axiom,
anonymity, reflects the principle of impartiality, with a long tradition in

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the theory of justice (e.g., The third axiom, continuity, is the adaptation of a standard operational condition to our context. It says that small changes in productivity or life years should only produce small changes in the evaluation of the distribution. We then move to the second group of COMMON axioms. First, the social zero condition, which is reminiscent of a well-known condition for individual utility functions on health (e.g., LMFHP: For each  $d \in D$  and each  $i \in N$ , such that  $(a_i, p_i) = (a^*, 1)$  and each where  $q : A \rightarrow [0, 1]$  is a function satisfying  $0 \leq q(a_i) \leq q(a^*) = 1$ , for each  $a_i \in A$ . The unweighted aggregation of individual QALYs, as specified in This evaluation function ignores productivity. More precisely, it satisfies the following axiom, productivity independence, which states that for a fixed health state and lifetime, the productivity is irrelevant for the evaluation of the distribution. It also satisfies the time invariance at common health and full productivity axiom, which states that for two individuals at common health and maximum productivity, extra life years are interchangeable. That is, it does not matter to the social planner which individual (among those with common health and maximum productivity) gets extra life years. TICHFP: For each  $d \in D$ , each pair  $i, j \in N$  with  $a_i = a_j = a$  and  $p_i = p_j = 1$ , and each Our first result states the QALY evaluation function is characterized by the combination of the previous two axioms and the COMMON axioms. 10 Theorem 1 The following statements are equivalent: 1. is represented by a QALY evaluation function (1). A possible interpretation of Theorem 1 could be a situation where a social planner decides to evaluate and prioritise among a set of interventions using the QALY evaluation function A counterpart axiom of productivity independence is health independence, which states that, for fixed productivity and lifetime, the health state of an individual is irrelevant for the evaluation. And, likewise, a counterpart of time invariance at common health and full productivity is time invariance at full health and common productivity, which states that for two individuals at full health and common productivity, it does not matter to the social planner who receives the extra life years. 11 As all theorems in this paper require the COMMON axioms, these will not be mentioned again in the interpretations of the remaining theorems. TIFHCP: For each  $d \in D$ , each pair  $i, j \in N$  with  $a_i = a_j = a^*$  and  $p_i = p_j = p$ , and each As the next result states, if the previous two axioms replace their counterparts at Theorem 1, we characterize the following generalized PALY evaluation function, which evaluates distributions by means of the aggregation of individual PALYs in society, when submitted first to a continuous function ( $v$ ). Formally, where The following statements are equivalent: 1. is represented by a generalized PALY evaluation function (2). The generalized PALY evaluation function is a counterpart of the QALY evaluation function. As such, neither the  $v$  function nor the  $q$  function (in their respective functional forms) has a monotonic structure. This makes sense in the latter case because the domain of health states  $A$  does not have a mathematical structure. But the domain of productivity levels is naturally ordered and, therefore, it would make sense to impose  $v$  a non-decreasing structure. The following axiom will guarantee such a feature as a byproduct. The axiom productivity invariance at full health and common time states that, for any two individuals with common lifetime and full health, it makes no difference who gains in productivity. PIFHCT: For each  $d \in D$ , each pair  $i, j \in N$  with  $a_i = a_j = a^*$  and  $t_i = t_j = t$ , and each As the next result states, adding this axiom to those in Theorem 2 we characterize the affine PALY evaluation



function, which evaluates distributions by means of the aggregation of individual PALYs in society, when submitted first to an affine and non-decreasing function. Formally, where  $\alpha \in [0, 1]$ . The following statements are equivalent: 1.  $\alpha$  is represented by an affine PALY evaluation function (3). 2.  $\alpha$  satisfies COMMON, HI, TIFHCP, and PIFHCT. The previous families are obvious generalizations of the focal linear PALY evaluation function, which evaluates distributions by means of the unweighted aggregation of individual PALYs in society, or, in other words, by the weighted (through productivity levels) aggregate time span the distribution yields. Formally, The unweighted aggregation of individual PALYs as specified in (TIUP: For each  $d \in D$  and each  $i \in N$  such that  $p_i = 0$ , and each  $t_i > 0$ , the following statements are equivalent: 1.  $\alpha$  is represented by a PALY evaluation function (4). 2.  $\alpha$  satisfies COMMON, HI, TIFHCP, PIFHCT, and TIUP. According to Theorem 4, a social planner wishing to conduct an economic evaluation of working environment interventions using the PALY evaluation function (4) will also hold a number of specific values in the process of priority setting. These value choices include, for example, that changes in health among individuals following an intervention (axiom HI) and that increases in lifetime among unproductive individuals (axiom TIUP) both have no influence on the choice of intervention. We conclude this section applying the evaluation functions characterized in this section to the distributions from Example 1. More precisely, we summarize the computations in the next table. We infer from there that the first distribution is preferred from the viewpoint of QALYs, whereas the second distribution is preferred from the viewpoint of the PALYs-based evaluation functions."

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      "content": "As mentioned above, the evaluation functions
characterized in the previous section ignore one dimension of our model. In
other words, they all rely on a very demanding axiom of (productivity or
health) independence. The purpose of this section is to dismiss those
axioms, while obtaining characterizations of natural compromises between
those focal (albeit polar) evaluation functions. For instance, the
productivity-and-quality-adjusted life years (PQALY) evaluation function
evaluates distributions by means of the weighted (through productivity and
health levels) aggregate time span the distribution yields, so that health,
productivity and lifespan of individuals enter the evaluation function
multiplicatively. Formally, where  $q : A \rightarrow [0, 1]$  is a health state
quality weight satisfying  $0 \leq q(a_i) \leq q(a^*) = 1$  for each  $a_i$ . As
the next result states, this evaluation function is characterized when we
dismiss health independence in the previous result (characterizing PALYs),
and strengthen the other two independence axioms to consider the following
ones. First, time invariance at common health and productivity, which states
that for two individuals at common health and productivity, it does not
matter to the social planner who receives the extra life years. TICH: For
each  $d \in D$ , each pair  $i, j \in N$  with  $a_i = a_j = a$  and  $p_i = p_j = p$ ,
and each  $c > 0$ , Second, productivity invariance at common health and
time, which says that for two individuals at common health and time, it
does not matter who gains in productivity. PICT: For each  $d \in D$ , each
pair  $i, j \in N$  with  $a_i = a_j = a$  and  $t_i = t_j = t$ , and each  $c > 0$ 
Theorem 5 The following statements are equivalent: 1.  $\alpha$  is represented by a
PQALY evaluation function (5)."
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  "header": "satisfies COMMON, TICHP, PICT, and TIUP.",
  "content": "A social planner may view both health effects and
productivity effects as important outcomes of interventions and may
therefore choose an evaluation function like the PQALY where health status
and productivity of individuals enter the evaluation function
multiplicatively. This implies according to Theorem 5 that one of the
values applied by the social planner in this situation is that increases in
lifetime among unproductive individuals following an intervention (axiom
TIUP) will not increase the desirability of that intervention. The social
planner subscribes to two further values regarding invariance between
different effect components where the first states that if an intervention
leads to extra life years, it does not matter to the social planner which
particular individual (among individuals with the same level of health and
productivity) receives these extra life years (axiom TICHP). The second
value specifies that if an intervention leads to improved productivity, it
does not matter which particular individual (among individuals with the
same level of health and lifespan) is able to perform better in the
workplace (axiom PICT). Our next result states that dismissing the TIUP
axiom in the previous statement we obtain the following alternative
intriguing compromise (dubbed QALY-PQALY) which evaluates distributions by
means of a convex combination of the QALYs and PQALYs the distribution
yields. Formally, where  $q, r : A \rightarrow [0, 1]$  are health state quality
weight functions satisfying  $0 \leq q(a_i) \leq q(a^*) = 1$  and  $0 \leq r(a_i) \leq r(a^*) = 1$  for each  $a_i \in A$ , and  $\beta \in [0, 1]$ . The QALY-PQALY evaluation function (The parameter  $\beta$  measures
the relative importance that the social planner puts on pure health effects
and productivity-and-quality adjusted life years resulting from an
intervention. The following statements are equivalent: 1. is represented by a
QALY-PQALY evaluation function (6). Similar to the previous evaluation
function, a social planner choosing the QALY-PQALY evaluation function (6)
considers both health effects and productivity effects as important
outcomes of interventions. In contrast to the previous theorem, Theorem 6
implies a rejection of axiom TIUP according to which improvements in
lifetime among unproductive individuals following an intervention do not
increase the desirability of that intervention. Apart from that, the social
planner subscribes to the same two axioms regarding invariance between
different effect components as above (TICHP and PICT). The previous family
of evaluation functions (6) includes a natural sub-family of evaluation
functions (QALY-PALY) that evaluate distributions by means of the convex
combinations of the QALYs and PALYs that the distribution yields.
Formally, where  $q : A \rightarrow [0, 1]$  is a health state quality weight
function satisfying  $0 \leq q(a_i) \leq q(a^*) = 1$ , for each  $a_i \in A$ , and  $\gamma \in [0, 1]$ . Health effects and productivity effects
enter the QALY-PALY evaluation function This family of evaluation functions
(PICT: For each  $d \in D$ , each pair  $i, j \in N$  with  $t_i = t_j = t$ ,
and each  $c > 0$  such that The following statements are equivalent: 1. is
represented by a QALY-PALY evaluation function all members of the family if
and only if  $\beta < 1/3$ . As for the family (where But the family of
evaluation functions (8) can also accommodate other evaluation functions
that have not been introduced above. For instance, suppose that  $w$  is a
semimultiplicative function in which productivity enters via a power
function, whereas health enters via QALYs, i.e.,  $w(a_i, p_i) = q(a_i)p$ 

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$\{i\}$ , for each  $(a_i, p_i) \in A \times [0, 1]$ , where  $q : A \times [0, 1] \rightarrow [0, 1]$  is a health state quality weight function satisfying  $q(a_i, p_i) = 1$ , for each  $(a_i, p_i) \in A \times [0, 1]$ . Note that the previous evaluation function is formalizing a concern for the dispersion of productivity levels (as it is a concave function of those levels). The above is somewhat reminiscent of the focal welfare function within the literature on life-cycle preferences over consumption and health status. Therein, multiplicative separability from consumption and health is typically assumed. It turns out that the general family of evaluation functions (The following statements are equivalent: 1. is represented by an evaluation function (8). Theorem 8 implies that if the social planner endorses the view that if an intervention leads to extra life years, it does not matter which particular individual (among individuals with the same level of health and productivity) receives these extra life years (axiom TICHF), then the evaluation will be via a weighted aggregation of the lifetimes the intervention yields. And the weight for each individual lifetime will be obtained via a general function of the health and productivity levels they face. A weaker axiom than time invariance at common health and productivity, is time invariance at full health and productivity, which states that extra years can be interchangeable among individuals with full health and maximum productivity. TIFHP: For each  $d \in D$ , each pair  $i, j \in N$  with  $a_i = a_j = a^*$  and  $p_i = p_j = 1$ , and each  $c > 0$ , If we replace time invariance at common health and productivity by time invariance at full health and productivity we characterize a more general family of evaluation functions, which extend to this context the notion of healthy years equivalent (e.g., More precisely, the healthy productive years equivalent (HPYE) evaluation function evaluates distributions by the unweighted aggregation of HPYEs the distribution yields. Formally, where  $f : A \times [0, 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is continuous with respect to its second and third variables and where, for each  $(a_i, p_i, t)$  The evaluation function (Note that this family (9) includes the previous one As the next result states, the general family of evaluation functions (The following statements are equivalent: 1. is represented by a HPYE evaluation function (9). Theorem 9 implies that if the social planner endorses the view that if an intervention leads to extra life years for individuals with full health and maximal productivity, it does not matter which particular individual receives these extra life years (axiom TIFHP), then the evaluation will be via the unweighted aggregation of the HPYEs the intervention yields (which are well defined due to COMMON). Finally, we can define the so-called generalized HPYE evaluation function by the unweighted aggregation of the image of HPYEs the distribution yields to a certain function. Formally, where  $g : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is a strictly increasing and continuous function, and  $f$  is continuous with respect to its second and third variables and for each  $d$  where, for each  $(a_i, p_i, t)$  Our last result states that the generalized HPYE evaluation function is precisely characterized by the set of COMMON axioms. The following statements are equivalent: 1. is represented by a generalized HPYE evaluation function Theorem 10 implies that if the social planner dismisses the TIFHP axiom, while still endorsing COMMON, then the evaluation will be via a general (but strictly increasing and continuous) function of the HPYEs the intervention yields (which are well defined due to COMMON). Thus, the evaluation will not necessarily be via a simple unweighted aggregation of HPYEs, which was the consequence of the TIFHP axiom, as formalized by the evaluation function "

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  "header": "Discussion",
  "content": "The results presented above in the form of evaluation
functions and their required axioms may be utilised in empirical
applications, for example as part of an economic evaluation of a health
care or working environment intervention. The data collection of the
economic evaluation will typically involve capturing information on all
individuals in the intervention and control group regarding their costs,
health status and productivity level during the follow-up period of the
study.As illustrated throughout the text with the two distributions from
Example 1, the choice of evaluation function matters to a large extent when
it comes to rank different distributions. Instead of making that choice
directly based on their functional forms, we rather believe the choice
should be guided by the axioms they satisfy. Hence the interest of our
axiomatic approach.If the analyst (working on behalf of a social planner)
is of the view that health effects and productivity effects are both
important outcomes when assessing the benefit of the working environment
intervention, evaluation functions ( Intervention A: 1000 individuals
obtaining 1 year in full health and having zero productivity.Intervention
B: x individuals obtaining 1 year in health state a and having zero
productivity.In particular, each respondent would be asked to identify the
number of individuals x in Intervention B to be indifferent between both
interventions. The quality weight can then be derived as  $q(a) = 1000x$  . The
parameter  $\alpha$  may be elicited using a different version of the person
trade-off technique.To wit, respondents would now be asked to state which
of the following interventions are most desirable for society: Intervention
C: one individual obtaining 1 year in full health and having zero
productivity.Intervention D: one individual obtaining y years in health
state a and having maximum productivity.Each respondent would then be asked
to identify the duration y in Intervention D to be indifferent between the
two interventions. Once y is identified, it follows from evaluation
function the need for using a more flexible evaluation function (for
example the QALY-PQALY). If people largely agree with this axiom (i.e., a
majority is largely indifferent between adding productivity to one type or
another, or roughly participants are split into those that favor persons of
one type and those that favor persons of the other type) it provides
support for using the QALY-PALY. 15 The reader is referred to "
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  "header": "Concluding remarks",
  "content": "Some of our models, particularly the more general
functional forms ( They can also be seen as prioritarian evaluation
functions (also known as prioritarian social welfare functionals), which
rank well-being vectors according to the sum of a strictly increasing
(dubbed equivalent health-adjusted lifespan) that is sensitive to
inequality in both age-specific health and health-adjusted lifespan. It is
a life years metric that nests health-adjusted life expectancy.In the
present framework considering health states, productivity and life years,
we could also impose a power transformation of the individual components in
each of the structured evaluation functions (1)-( have no effects on
(labour market) productivity. This might render QALYs more appropriate for
the evaluation of these treatments. But using the same evaluation function
for younger patients misses productivity effects. This is a possible

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motivation for age weights (another motivation is the fair innings argument mentioned above). Our hybrid evaluation functions, such as PQALYs, QALYs-PALYs, QALYs-PQALYs or the more general functional form We conclude mentioning that our framework allows for alternative plausible interpretations, as well as for further generalizations. Regarding the former, we focused on chronic health states, for ease of exposition, but our analysis also allows to consider time-varying health. To wit, we made no assumptions regarding the domain of health states  $A$  (except for the existence of a maximal element). In particular, this allows for time trajectories rather than fixed levels of health (with the trajectory determined by  $t_i$ ). That is,  $a_i = a_i(t_i)$ , where  $a_i(s)$  denotes the health status of individual  $i$  at time  $s$ . This would require a reinterpretation of some of the axioms we considered above. 18 Likewise, instead of assuming that  $p_i \in [0, 1]$  captures the productivity of individual  $i$ , we could assume that it captures the probability to succeed in life that individual  $i$  has. Mariotti and Veneziani (2018) refer to this as "chances of success" and characterize a multiplicative form to evaluate social profiles of "chances of success" (also known as "boxes of life"). 19 We could also derive their characterization results in our model upon endorsing first the axiom of health independence and a counterpart axiom of time independence (not considered in this paper). We, 17 A natural way to start would be by modifying the TICHP axiom, so as to prefer to give the extra life years to the shorter-lived individual. 18 Bossert and D'Ambrosio (2013) is a nice example of an axiomatic analysis of time trajectories (streams) of wealth (rather than health). 19 See also nevertheless, acknowledge that their main axiom is one formalizing a non-interference principle that we do not consider in this paper. 20 As for further generalizations, we stress that the scope of our theory can be enlarged to account for more general evaluations. To wit, our theory deals with the evaluation of population distributions where individuals can be characterized by two instantaneous attributes (one qualitative and one quantitative) and a duration. These attributes can indeed be interpreted as health (qualitative) and productivity (quantitative), as we do in this paper. But there are other potential interpretations (such as happiness or well-being, to name a few). Our results could therefore provide interesting lessons for those settings too.

First, we prove that for each  $d \in D$  and each  $i \in N$ , there exists  $t_i = 0$ , then it follows from ZERO that  $t_i = 0$ . Therefore, assume  $t_i > 0$ . We prove that  $t_i$  exists by contradiction. Therefore, assume that  $t_i$  does not exist. Then,  $T = A \cup B$ , where  $B = \{(a_i, p_i, t_i) \mid d \in N \setminus \{i\}\}$ , implying that either  $t_i = 0$  (a contradiction), or  $t_i \in B$ . Assume the latter. Thus,  $B$  is a non-empty set. By PLD and ZERO, it follows that either  $t_i = 0$  (a contradiction), or  $0 \in B$ . Again, assume the latter. Thus,  $A$  is a non-empty set. By CONT,  $A$  and  $B$  are open sets relative to  $T$ . Altogether, it follows that  $T$  is not a connected set, which is a contradiction. Thus,  $t_i$  exists, and due to LMFHP, it is uniquely determined. Finally, by SEP, we can determine each  $t_i$  separately. Therefore, let  $f_i : A \times [0, 1] \times R \rightarrow R$  be such that  $f_i(a_i, p_i, t_i) = t_i$  for each  $i \in N$ . By ANON,  $f_i() = f_j() = f()$  for each  $i, j \in N$ . By CONT,  $f$  is continuous with respect to its second and third variable and, by the above, we know that  $0 \leq f(a_i, p_i, t_i) \leq t_i$ , so the range of  $f$  is a connected subset of  $R$ . Also, by FHPS,  $f(a_i, p_i, t_i) \leq f(a_i, 1, t_i)$ , and which implies that social preferences only depend on the profile of HPYEs, and, by CONT, they

do so continuously. As in the models of where  $g : R + \mathbb{R}$  is strictly increasing. Suppose first that is represented by a PHEF satisfying and Thus, Conversely, assume now that preferences satisfy all the axioms in COMMON as well as TIFHP. Then, by Theorem 10, for each pair  $d, d \in D$ , where  $g : R + \mathbb{R}$  is strictly increasing. Now, for each pair  $t_i, t_j \in R +$ , and for each  $c > 0$ , it follows by TIFHP that  $g(f(a^*, 1, t_i + c)) + g(f(a^*, 1, t_j)) = g(f(a^*, 1, t_i)) + g(f(a^*, 1, t_j + c))$ . Or, equivalently, In particular, for each  $x, y \geq 0$ . As  $g$  is continuous and strictly increasing, it follows from Theorem 1 in Suppose first that is represented by a PHEF satisfying Then, by Theorem 10, for each pair  $d, d \in D$ , where  $= \omega(a, p, t) = \omega(a, p, 1)t$ , in particular,  $\omega(a, p, t) \in \mathbb{R}$  be such that  $w(a, p) = \omega(a, p, 1)$ , for each  $(a, p) \in A \times [0, 1]$ . Thus, it follows that  $1 = w(a^*, 1) \leq w(a, p) \leq 0$ , for each  $(a, p) \in A \times [0, 1]$ . Then, we may write:  $\omega(a, p, t) = w(a, p)t$ , where  $0 \leq w(a, p) \leq w(a^*, p) \leq w(a^*, 1) = 1$ , and  $0 \leq w(a, p) \leq w(a, 1) \leq w(a^*, 1) = 1$  for each  $(a, p) \in A \times [0, 1]$ , as desired. Suppose first that is represented by a PHEF satisfying Then, for each  $c > 0$ , Conversely, assume now that preferences satisfy all the axioms in the statement of Theorem where  $g : R + \mathbb{R}$  is strictly increasing. PI and TICHFP together imply TICHHP. Thus, preferences satisfy all the axioms in the statement of Theorem 8. Thus, for each  $d \in D$ , where  $0 \leq w(a, p) \leq w(a^*, p) \leq w(a^*, 1) = 1$ , and  $0 \leq w(a, p) \leq w(a, 1) \leq w(a^*, 1) = 1$  for each  $(a, p) \in A \times [0, 1]$ . Now, by PI,  $w(a, p) = w(a, 1)$ , for each  $a \in A$ . Let  $q : A \times \mathbb{R}$  be such that  $q(a_i) = w(a_i, 1)$ , for each  $a_i \in A$ . Then, it follows that  $1 = q(a^*) \leq q(a) \leq 0$ , for each  $a \in A$ . And we may write: where  $0 \leq q(a) \leq q(a^*) = 1$ , for each  $a \in A$ , as desired. Suppose first that is represented by a PHEF satisfying Then, for each  $c > 0$ , Thus, as  $v_i$  and each  $c > 0$  such that  $p_i + c, p_j + c \in [0, 1]$ . In particular, Suppose first that is represented by a PHEF satisfying Then, for each  $c > 0$  such that  $p_i + c, p_j + c \leq 1$ ,  $E[(a, p_i + c, t), (a, p_j, t), d \in N \setminus \{i, j\}] = q(a)(p_i + c)t + q(a)p_j t + k \in N \setminus \{i, j\} q(a_k)p_k t_k$ , and  $E[(a, p_i, t), (a, p_j + c, t), d \in N \setminus \{i, j\}] = q(a)p_i t + q(a)p_j (t + c) + k \in N \setminus \{i, j\} q(a_k)p_k t_k$ . Thus,  $[(a, p_i + c, t), (a, p_j, t), d \in N \setminus \{i, j\}] \preceq [(a, p_i, t), (a, p_j + c, t), d \in N \setminus \{i, j\}]$ . Conversely, assume now that preferences satisfy all the axioms in the statement of Theorem 5. Then, they also satisfy the axioms of Theorem 8. Thus, for each pair  $d, d \in D$ , and, therefore, is indeed represented by an evaluation function satisfying (5), as desired. Suppose first that is represented by a PHEF satisfying Conversely, assume now that preferences satisfy all the axioms in the statement of Theorem for each  $a \in A$ , it follows that is represented by an evaluation function satisfying Case 3.  $\omega(a^*) = 0 = \omega(a^*)$ . Let  $q, r : A \times \mathbb{R}$  be such that  $r(a) = \omega(a)$  and  $q(a) = \omega(a)$ , for each  $a \in A$ . Then, by FHPS,  $1 = q(a^*) \leq q(a) \leq 0$ , and  $1 = r(a^*) \leq r(a) \leq 0$ . Furthermore,  $0 < \omega(a^*) < 1$ . Now, if we let  $\omega(a^*) = \omega(a^*) \in (0, 1)$ , we have  $w(a, p) = \omega(a^*)q(a) + (1 - \omega(a^*))r(a)p$ , for each  $p \in [0, 1]$ , and each  $a \in A$ . Thus, for each pair  $d, d \in D$ , where  $q, r : A \times [0, 1]$  are such that  $1 = q(a^*) \leq q(a) \leq 0$ , and  $1 = r(a^*) \leq r(a) \leq 0$  and  $\omega(a^*) \in (0, 1)$ . Consequently, is indeed represented by an evaluation function satisfying Suppose first that is represented by a PHEF satisfying where  $q, r : A$

$[0, 1]$  are such that  $1 = q(a^*) \iff q(a) = 0$ , and  $1 = r(a^*) \iff r(a) = 0$  and By PICT,  $[(a_i, p_i + c, t), (a_j, p_j, t), d_{N \setminus \{i,j\}}] \preceq [(a_i, p_i, t), (a_j, p_j + c, t), d_{N \setminus \{i,j\}}]$ , for each  $d \in D$ , and  $i, j \in N$  with  $t_i = t_j = t$ . Now, as  $E[(a_i, p_i + c, t), (a_j, p_j, t), d_{N \setminus \{i,j\}}] = (q(a_i)t + q(a_j)t) + (1 - q(a_i))(p_i + c)t + r(a_j)p_jt + k_{N \setminus \{i,j\}}(q(a_k)t_k + (1 - q(a_k))r(a_k)p_kt_k)$ , and  $E[(a_i, p_i, t), (a_j, p_j + c, t), d_{N \setminus \{i,j\}}] = (q(a_i)t + q(a_j)t) + (1 - q(a_i))p_it + r(a_j)(p_j + c)t + k_{N \setminus \{i,j\}}(q(a_k)t_k + (1 - q(a_k))r(a_k)p_kt_k)$ . Thus,  $r(a_i) = r(a_j)$ , for each pair  $a_i, a_j \in A$ . As,  $1 = r(a^*)$ , it follows that  $r(a) = 1$ , for each  $a \in A$ . Thus, letting  $\alpha = 1$ , we obtain that is indeed represented by an evaluation function satisfying "

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*Figure 0-1 – Segmentation full result*