



# Artificial and Computational Intelligence

**DSECLZG557**

**Contributors & Designers of document content : Cluster Course Faculty Team**



**M2 :: Problem Solving Agent using Search**

**BITS Pilani**

Pilani Campus

**Presented by**

**V Indumathi –Guest Faculty –BITS-W  
indumathi.p@wilp.bits-pilani.ac.in**

# Artificial and Computational Intelligence

## Disclaimer and Acknowledgement



- Few content for these slides may have been obtained from prescribed books and various other source on the Internet
- I hereby acknowledge all the contributors for their material and inputs and gratefully acknowledge people others who made their course materials freely available online.
- I have provided source information wherever necessary
- This is not a full fledged reading materials. Students are requested to refer to the textbook w.r.t detailed content of the presentation deck that is expected to be shared over e-learning portal - taxilla.
- I have added and modified the content to suit the requirements of the class dynamics & live session's lecture delivery flow for presentation
- **Slide Source / Preparation / Review:**
- From BITS Pilani WILP: Prof.Raja vadhana, Prof. Indumathi, Prof.Sangeetha
- From BITS Oncampus & External : Mr.Santosh GSK

# Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

# Learning Objective

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At the end of this class , students Should be able to:

1. Design problem solving agents
  2. Create search tree for given problem
  3. **Apply uninformed search algorithms to the given problem**
  4. **Compare performance of given algorithms in terms of completeness, optimality, time and space complexity**
  5. **Differentiate for which scenario appropriate uninformed search technique is suitable and justify.**
  6. **Differentiate between Tree and Graph search**
-

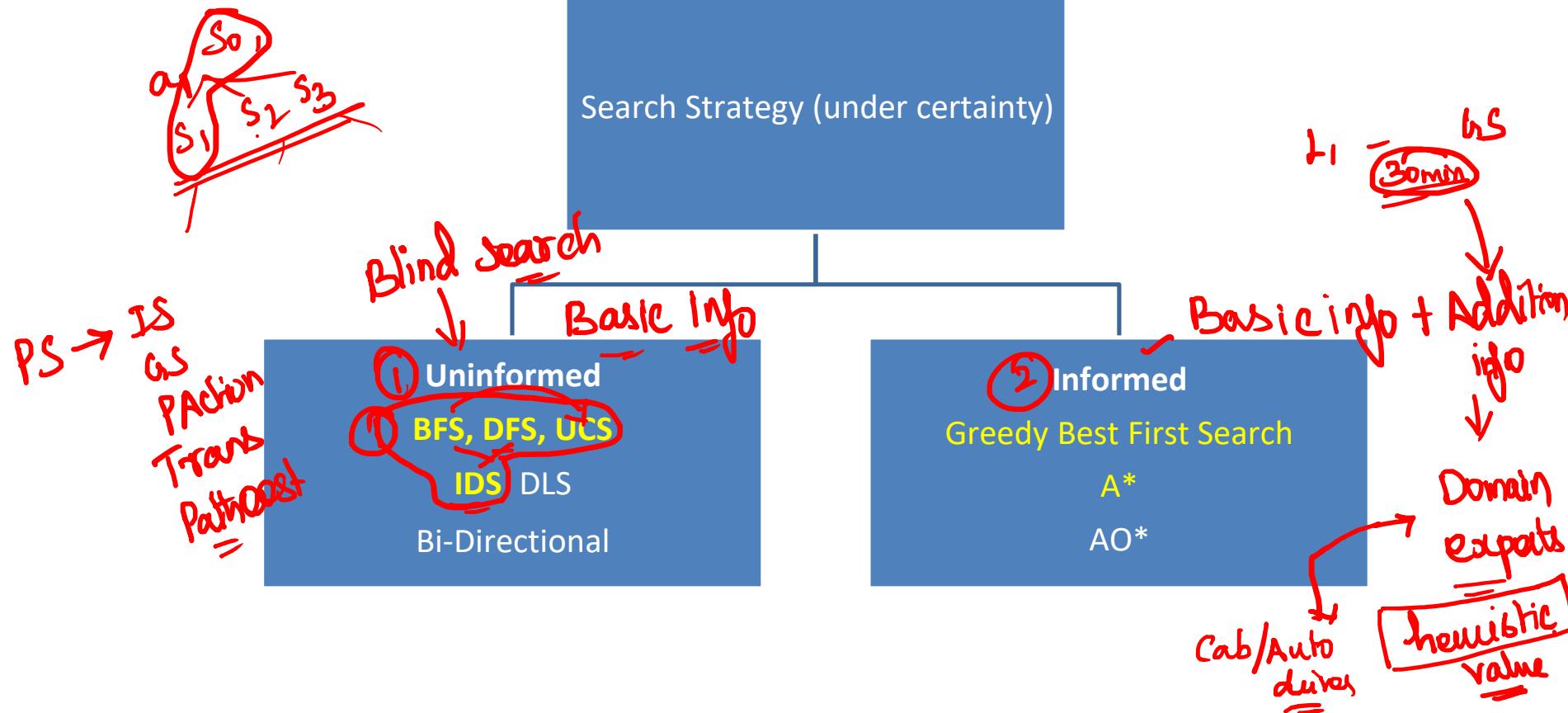
# Problem Formulation

# Searching for Solutions

M2 : Assumptions:

Certain, static, Full observable,  
Discrete,  
Deterministic

Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy



①

## Uninformed Search – BFS & its Variant



TS

Start state  $\rightarrow$  Goal state

Breadth First Search  
(BFS) = Level wise

Level wise

Queue  
FIFO

innovate

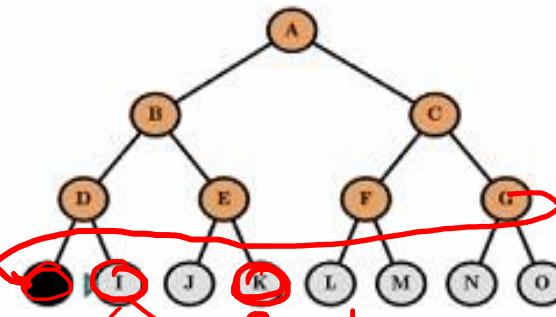
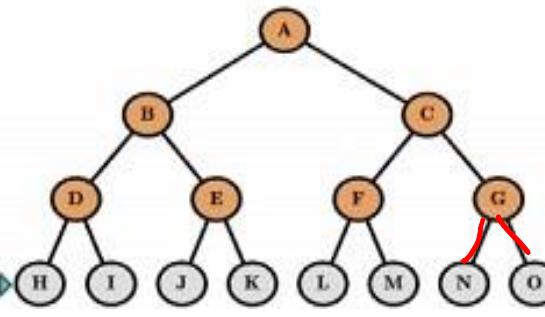
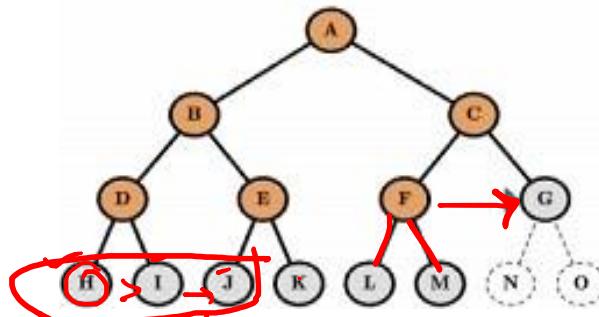
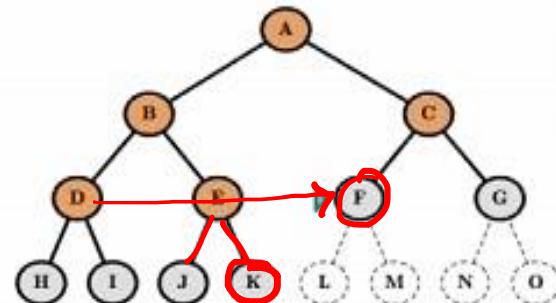
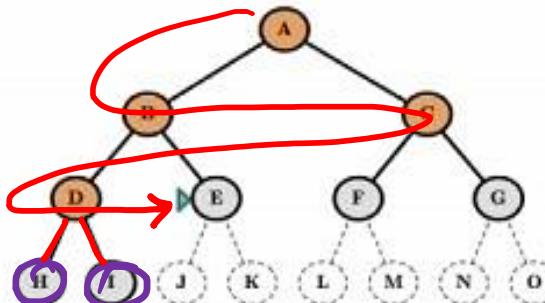
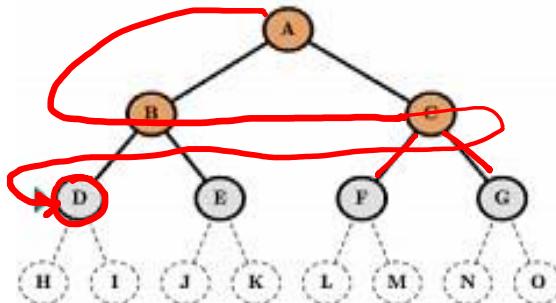
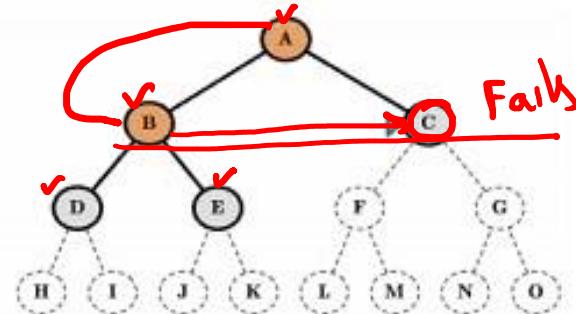
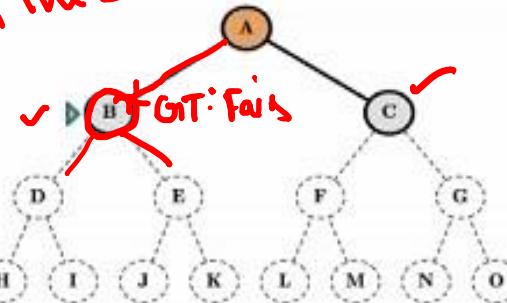
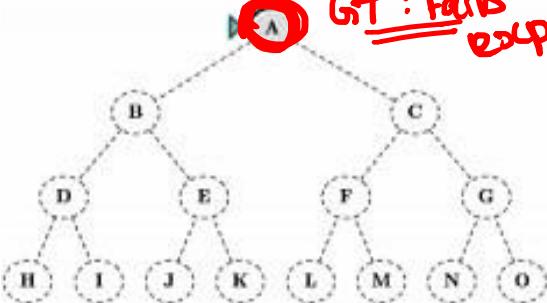
achieve

lead

GS: K  
SS: A

Start node

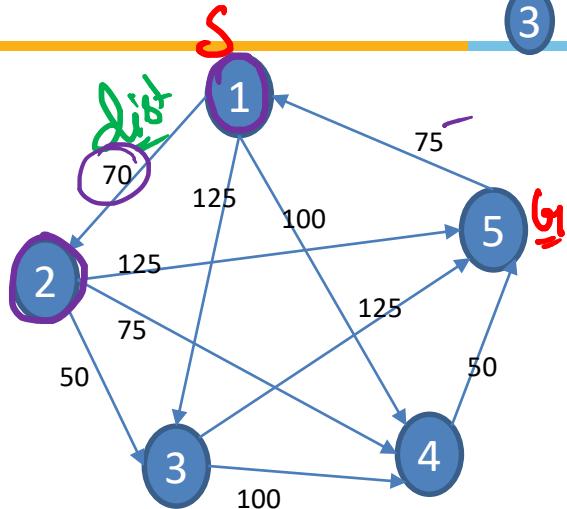
GT : fails  
expand the successor



Passed

FIFO - Queue

BFS - Uninformed

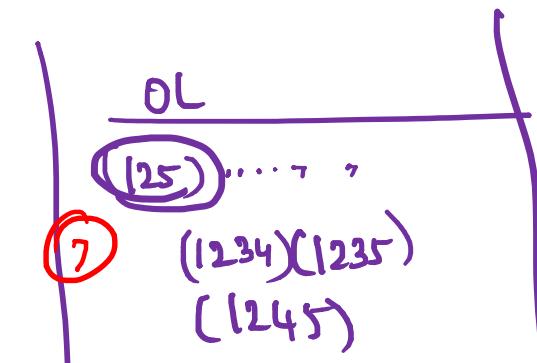
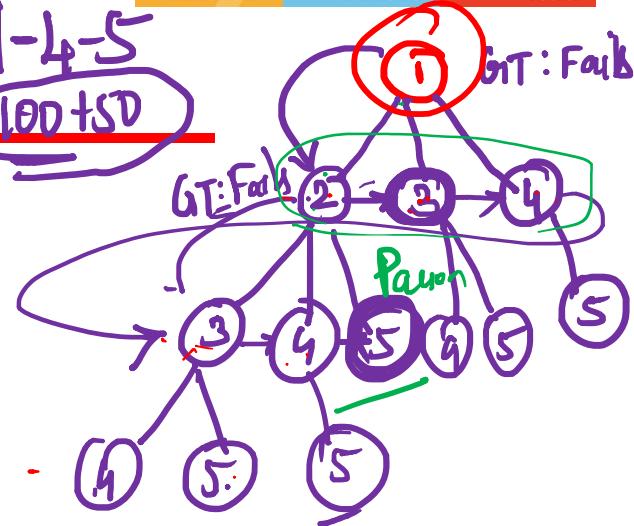


$$\begin{array}{c} \text{1-2-5} \\ \hline 70 + 125 = 195 \end{array}$$

	open list	closed list	Goal test
1	(1)		$F(1)$
2	(12) (13) (14) ←	(1)	$F(12)$
3	(13) (14) (123) (124) (125) (112)		$F(13)$
4	(14) (123) (124) (125) (134) (135) (145) (1) (12) (13) (14)	(1) (12) (13)	$F(14)$
5	(123) (124) (125) (134) (135) (145) (1) (12) (13) (14) (123)	(1) (12) (13) (14)	$F(123)$
6	(124) (125) " " " "	(123)	$F(124)$
7	(124) (125) " " " (1234) (1235)		

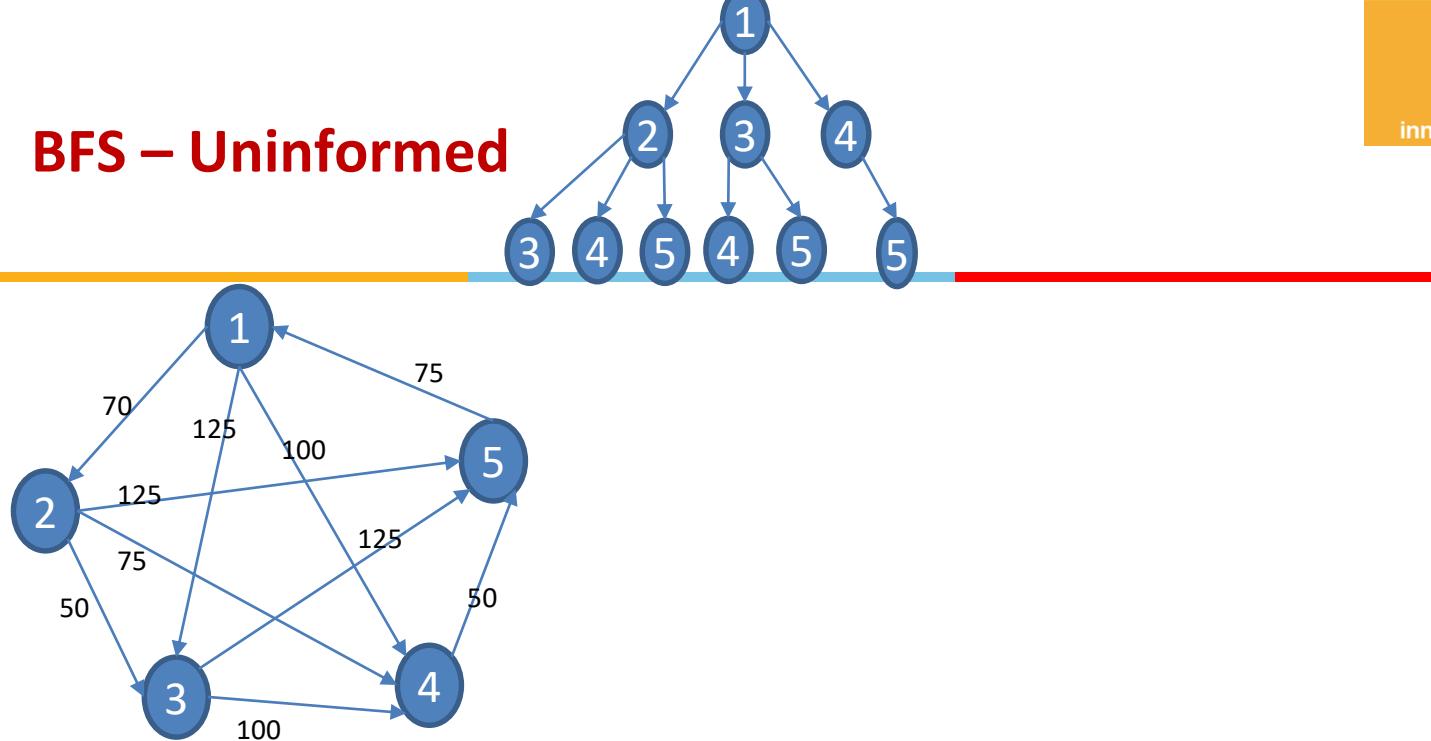
Optimal, cost  
1-4-5  
 $100 + 50$

Y?

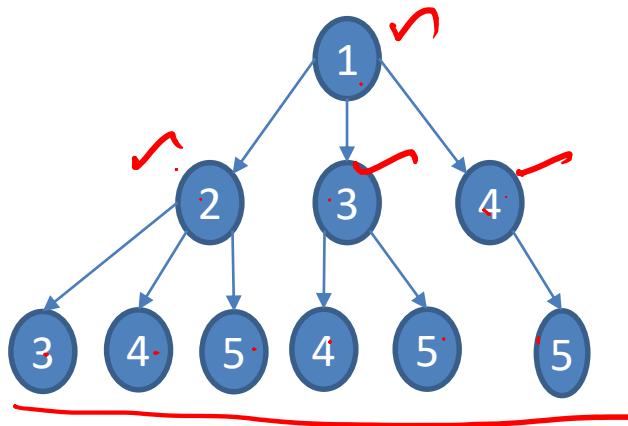
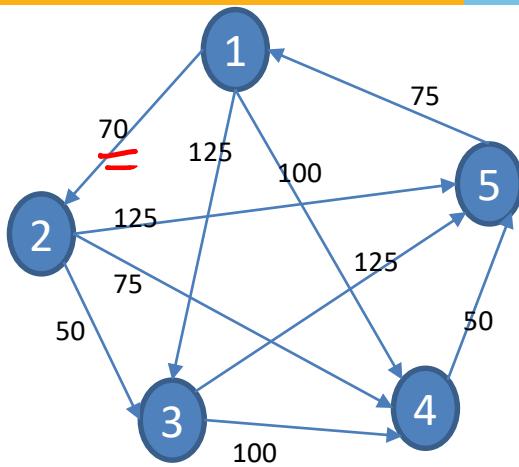


# nodes generated :- 19  
Max Queue length :- 7  
Expanded :- 6 //

## BFS – Uninformed



## BFS – Uninformed



(1)  
(1 2) (1 3) (1 4)

TEST FAILED

:

:

(1 3) (1 4) (1 2 3) (1 2 4) (1 2 5)  
(1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5 ) (1 4 5)

:

:

TEST PASSED

✓  $C(1-2-5) = 70 + 125 = \underline{\underline{195}}$   
✓ Expanded : 4  
✓ Generated : 10  
✓ Max Queue Length : 6

## Breadth First Search – Evaluation

---

**Complete** – If the shallowest goal node is at a depth  $d$ , BFS will eventually find it by generating all shallower nodes

**Optimal** – Not necessarily. Optimal if path cost is non-decreasing function of depth of node. E.g., all actions have same cost

**Time Complexity** –  $\mathcal{O}(b^d)$   $b$  - branching factor,  $d$  – depth

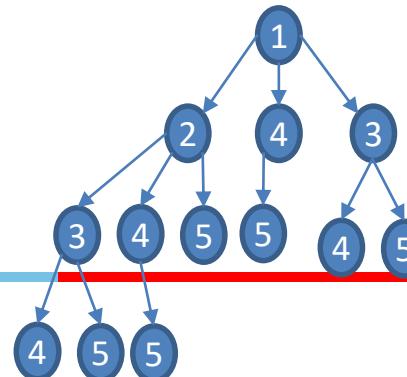
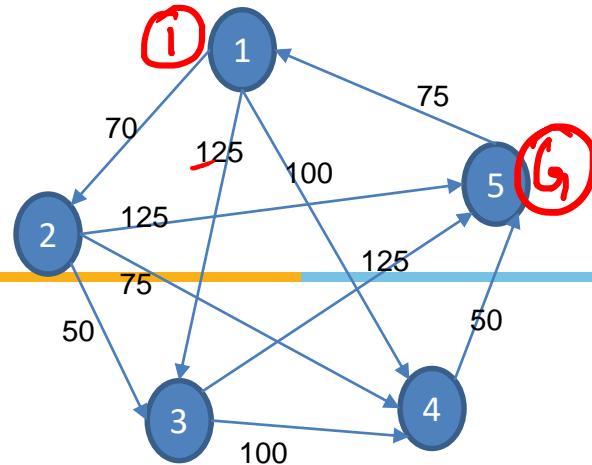
- Nodes expanded at depth 1 =  $b$
- Nodes expanded at depth 2 =  $b^2$
- Nodes expanded at depth  $d$  =  $b^d$
- Goal test is applied during generation, time complexity would be  $\mathcal{O}(b^{d+1})$

**Space Complexity** –  $\mathcal{O}(b^d)$

- $\mathcal{O}(b^{d-1})$  in explored set
- $\mathcal{O}(b^d)$  in frontier set

- Instead of expanding the shallowest node, Uniform-Cost search expands the node  $n$  with the lowest path cost  $g(n)$
- Sorting the Frontier as a priority queue ordered by  $\underline{\underline{g(n)}}$
- Goal test is applied during expansion
  - The goal node if generated may not be on the optimal path
  - Find a better path to a node on the Frontier

UCS



Table

open list

(1)

(12: 70) (14: 100) (13: 125)

(14: 100) (123: 120) (13: 125) (124: 145) (125: 195)

(123: 120) (13: 125) (124: 145) (145: 180) (125: 195)

↓

closed list

(1)

F(1)

(12)

F(12)

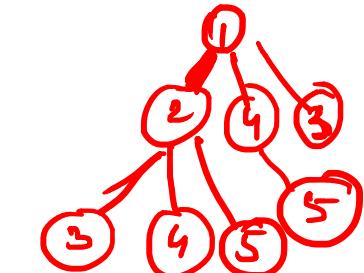
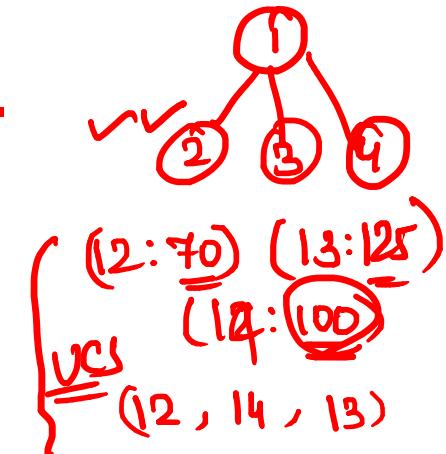
(1)

F(14)

(12)

(14)

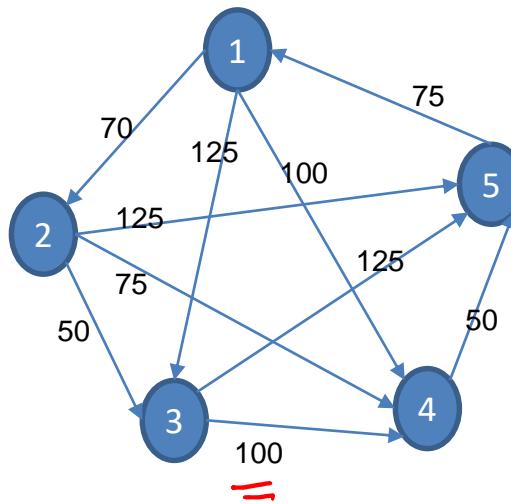
.



(123: 120) (124: 145)

(125: 195)

145: 180



(1)

**(1 2 : 70) (1 4 : 100) (1 3 : 125)**

TEST-F

**(1 4 : 100) (1 2 3 :120)(1 3 : 125) (1 2 4 : 145) (1 2 5 : 195)**

TEST-F

**(1 2 3 :120)(1 3 : 125) (1 2 4 : 145) (1 4 5: 150)(1 2 5 : 195)**

TEST-F

**(1 3 : 125) (1 2 4 : 145) (1 4 5: 150) (1 2 5 : 195) (1 2 3 4:220) (1 2 3 5 :245)**

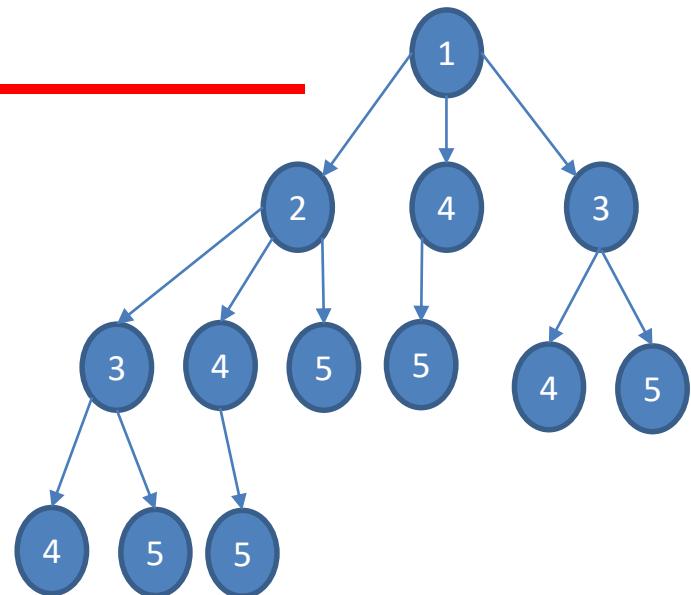
TEST-F

**(1 2 4 : 145) (1 4 5: 150) (1 2 5 : 195) (1 2 3 4:220) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)**

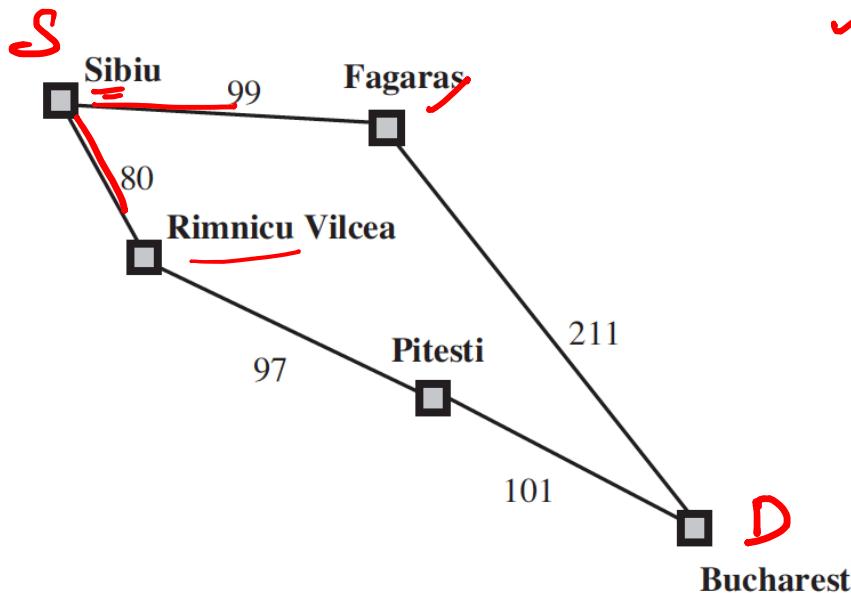
TEST-F

**(1 4 5: 150) (1 2 4 5 : 195) (1 2 5 : 195) (1 2 3 4:220) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)**

**TEST - P**



# Uniform Cost Search

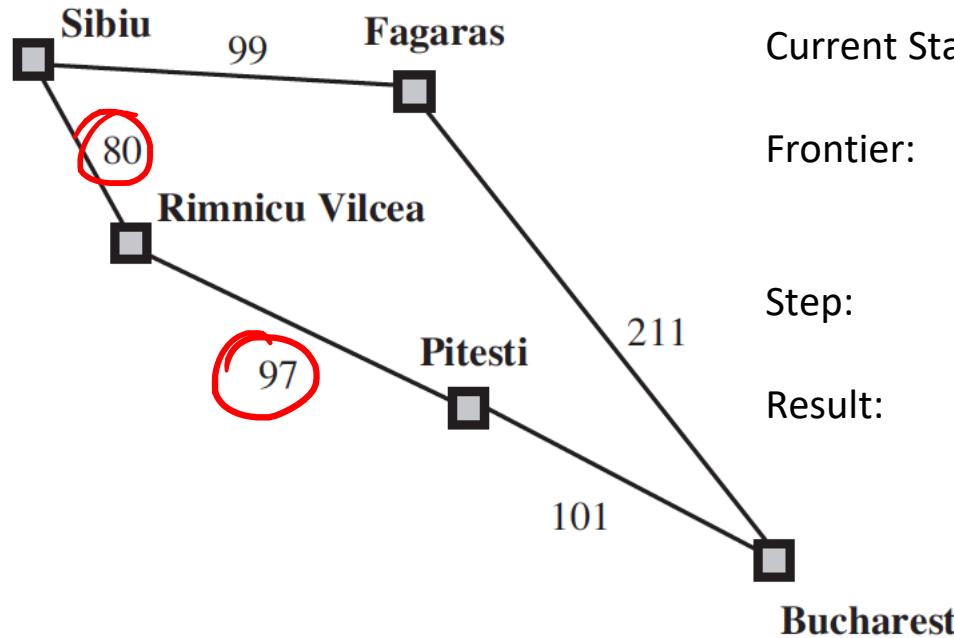


✓ Current State: Sibiu  
 ✓ Frontier: []  
 Step: Expand Sibiu  
 Result: Bucharest  
 Generates ("Rimnicu Vilcea" 80)  
 ("Fagaras", 99)  
 Add to Frontier

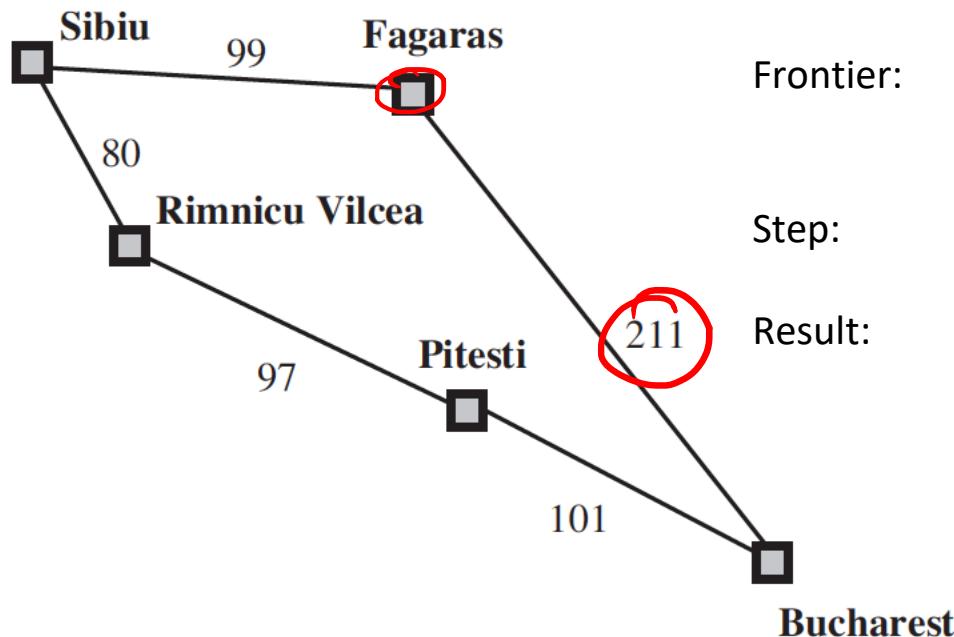
Initial State:  
 Goal State:

Sibiu  
 Bucharest

# Uniform Cost Search



# Uniform Cost Search



Initial State:  
Goal State:

Sibiu  
Bucharest

Current State:

Frontier:

Step:

Result:

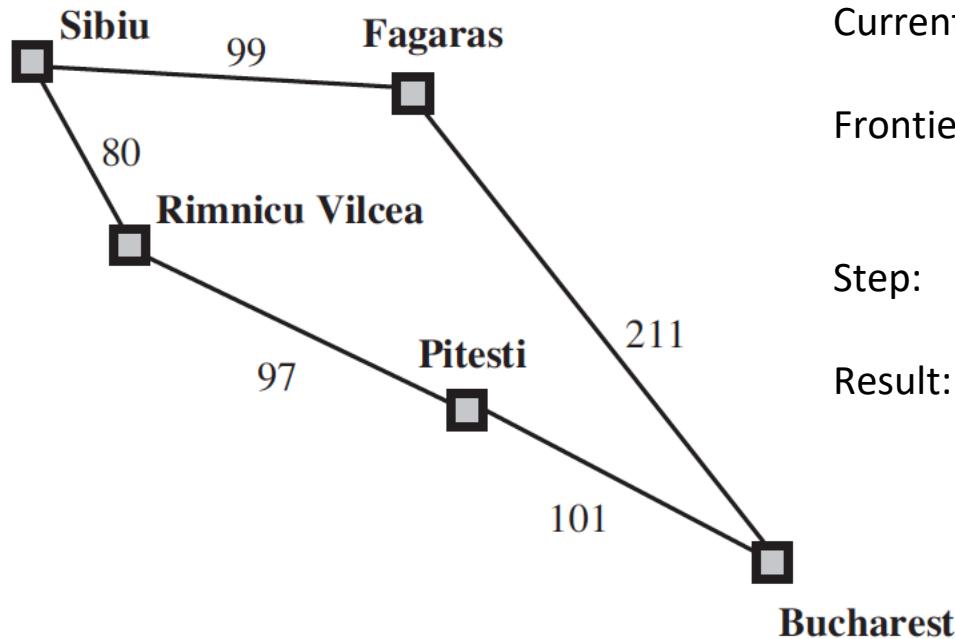
Rimnicu Vilcea (not a Goal state)

[ ("Fagaras", 99),  
("Pitesti", 177)]

Expand "Fagaras" (least cost)

Generates ("Bucharest", 310)  
Add to Frontier  
(It's a Goal State but we won't  
test during generation)

# Uniform Cost Search



Initial State:  
Goal State:

Sibiu  
Bucharest

Current State:

Frontier:

Step:

Result:

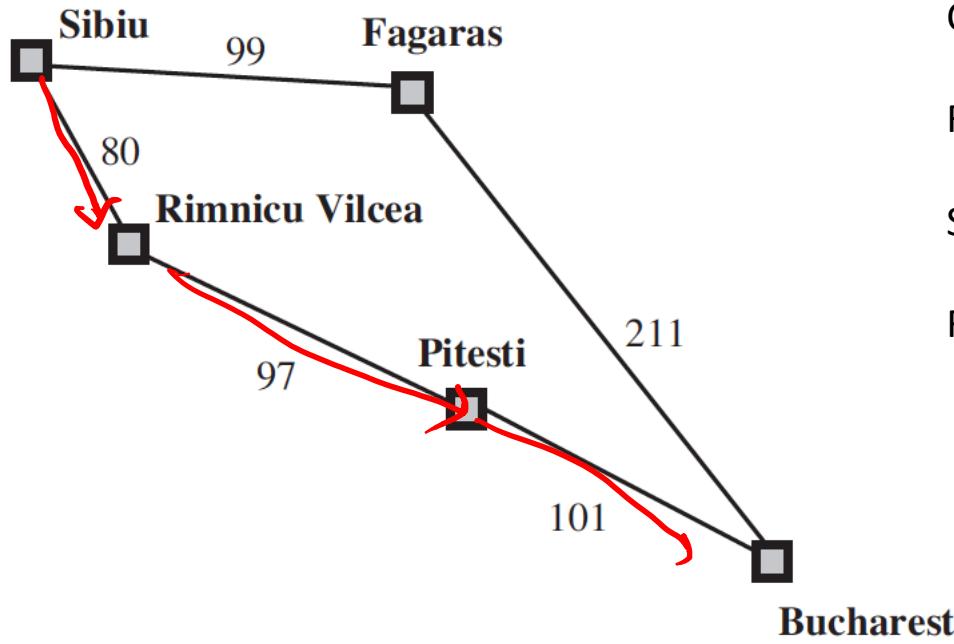
Fagaras (not a goal state)

[ ("Pitesti", 177)  
("Bucharest", 310)]

Expand "Pitesti" (least cost)

Generates ("Bucharest", 278)  
Replace in Frontier  
(It's a Goal State but we won't test during generation)

# Uniform Cost Search



Initial State:  
Goal State:

Sibiu  
Bucharest

Current State: Pitesti (not a goal state)  
 Frontier: [ ("Bucharest", 278)]  
 Step: Expand "Bucharest"  
 Result: No further generation as Goal Test satisfied  
 Return Solution

278

# Uniform Cost Search – Evaluation

**Completeness** – It is complete if the cost of every step > small +ve constant  $\epsilon$

- It will stuck in infinite loop if there is a path with infinite sequence of zero cost actions

**Optimal** – It is Optimal. Whenever it selects a node, it is an optimal path to that node.

**Time and Space complexity** – Uniform cost search is guided by path costs not depth or branching factor.

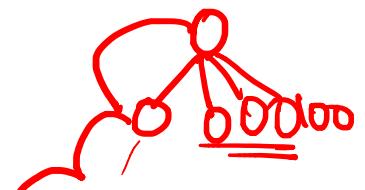
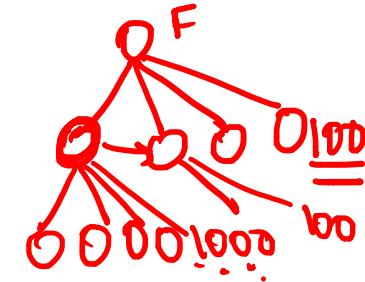
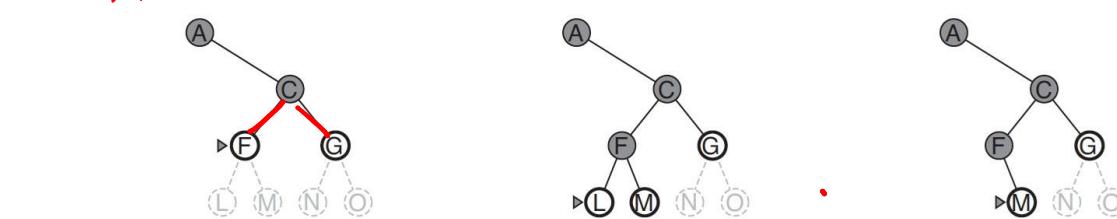
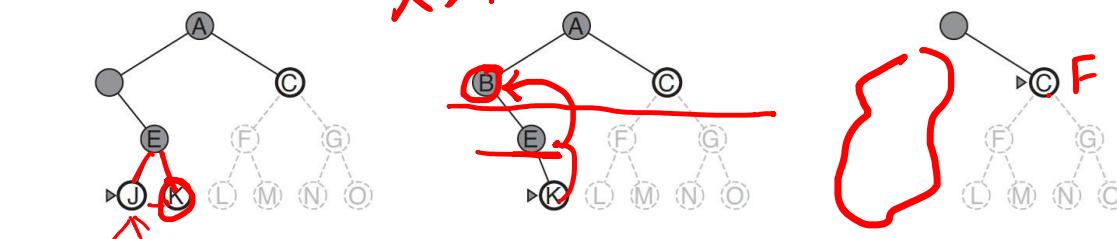
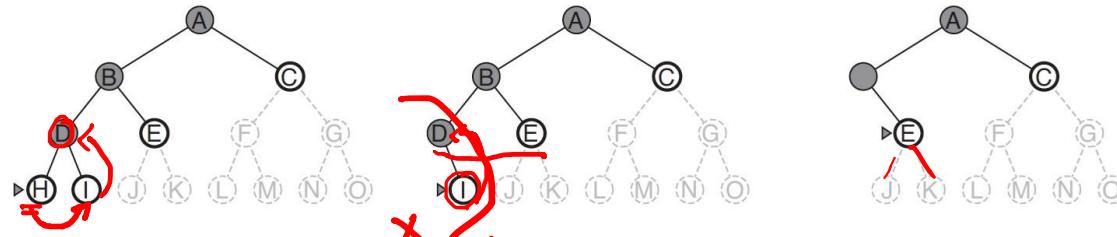
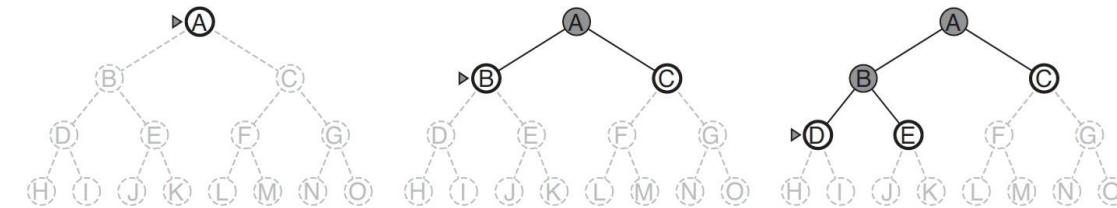
- If  $C^*$  is the cost of optimal solution and  $\epsilon$  is the min. action cost
- Worst case complexity =  $\mathcal{O}(b^{1+\frac{C^*}{\epsilon}})$ ,
- When all action costs are equal  $\rightarrow \mathcal{O}(b^{d+1})$ , the BFS would perform better
  - As Goal test is applied during expansion, Uniform Cost search would do extra



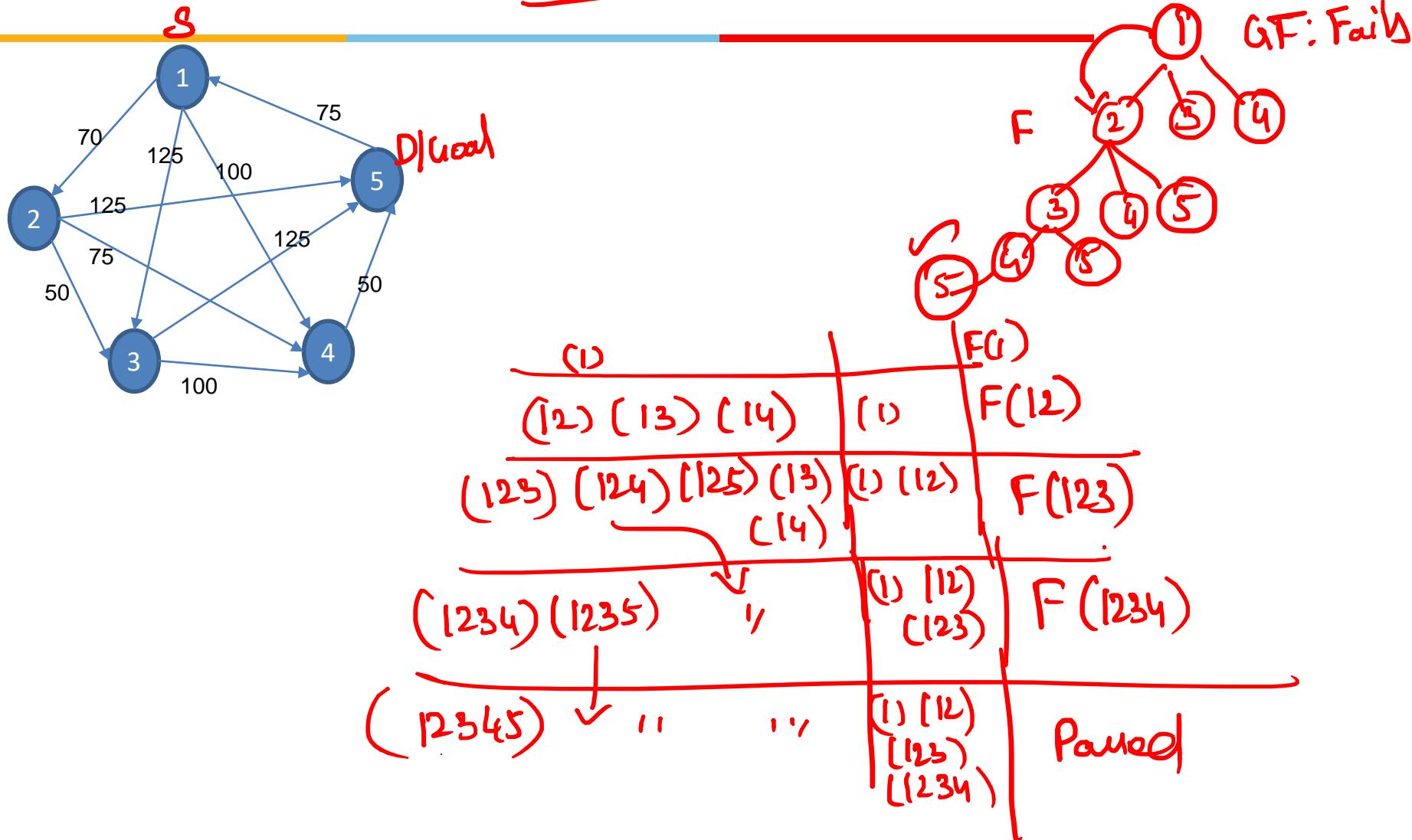
# Uninformed Search – DFS & its Variant

–

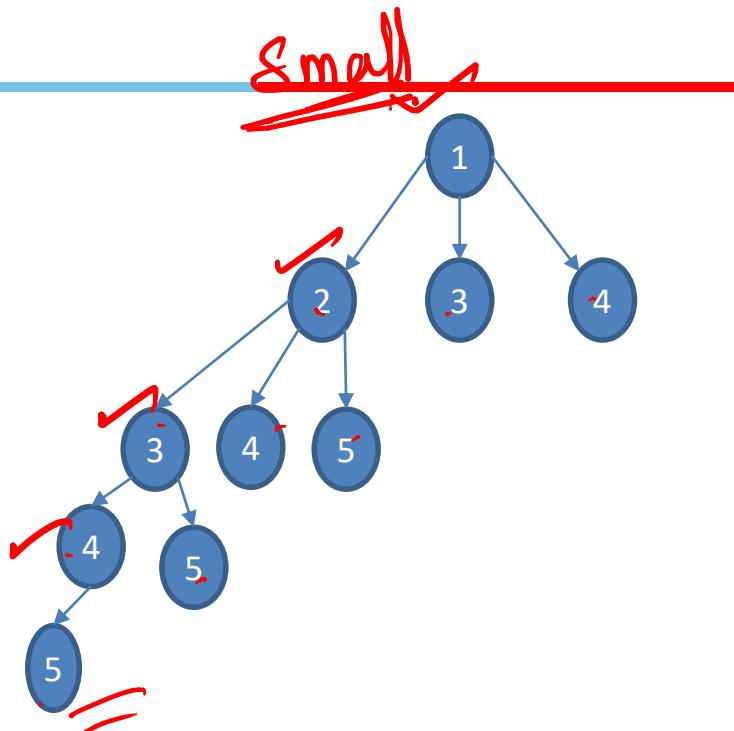
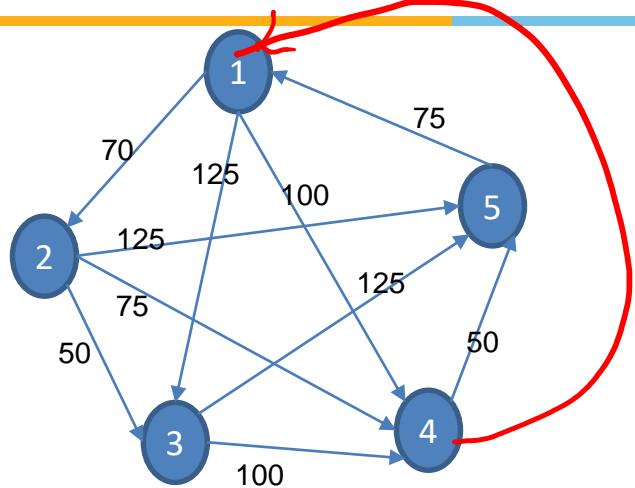
# Depth First Search (DFS)



DFS – Uninformed → LIFO Stack



## DFS – Uninformed



$\{(1)$   
 $(1\ 2)\ (1\ 3)\ (1\ 4)$   
 $(1\ 2\ 3)\ (1\ 2\ 4)\ (1\ 2\ 5)\ (1\ 3)\ (1\ 4)$   
 $(1\ 2\ 3\ 4)\ (1\ 2\ 3\ 5)\ (1\ 2\ 4)\ (1\ 2\ 5)\ (1\ 3)\ (1\ 4)$   
 $\textcolor{red}{(1\ 2\ 3\ 4\ 5)}$   
 $(1\ 2\ 3\ 5)\ (1\ 2\ 4)\ (1\ 2\ 5)\ (1\ 3)\ (1\ 4)$

$C(1-2-3-4-5) = \textcircled{70} + \textcircled{50} + \textcircled{100} + \textcircled{50} = \textcircled{270}$   
 Expanded : 4  
 Generated : 10  
 Max Queue Length : 6

## Depth First Search (DFS)

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**Completeness** – Complete in finite state spaces because it will eventually expand every node

**Optimal** – Not Optimal as it would stop when the goal node is reached without evaluating if there is a better path

**Time Complexity** -  $\mathcal{O}(b^m)$  where m = maximum depth of any node

- Can be much larger than the size of state space
- m can be much larger than d (shallowest goal)

**Space Complexity** – Needs to store only one path and unexpanded siblings.

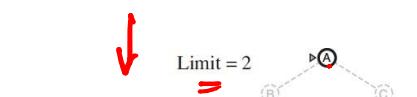
- Any node expanded with all its children can be removed from memory
- Requires storage of only  $\mathcal{O}(bm)$ , b – branching factor, m - max depth

# Iterative Deepening Depth First Search (IDS)

BFST + DFS



STOP  $\rightarrow$  Restart

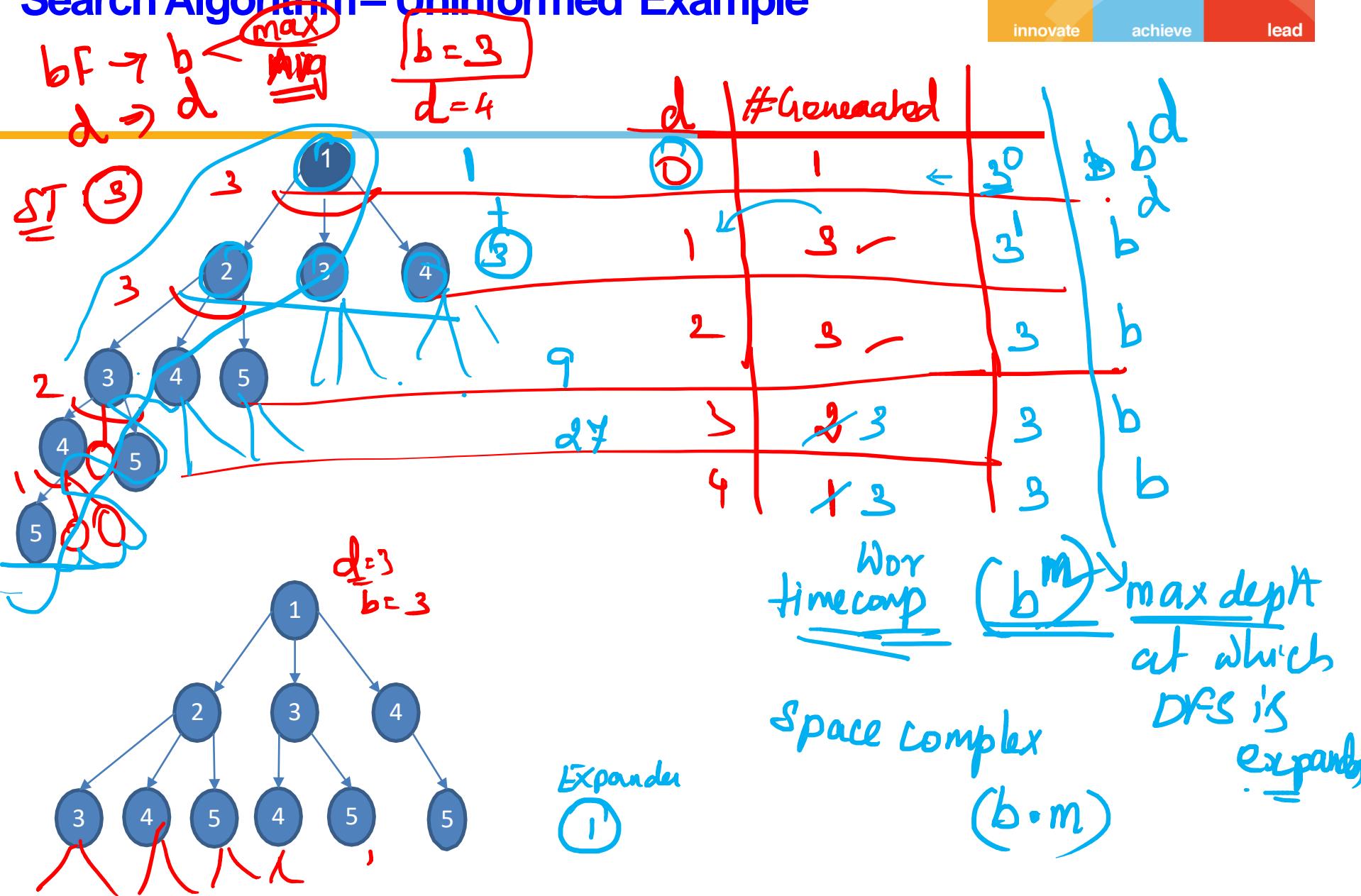


STOP

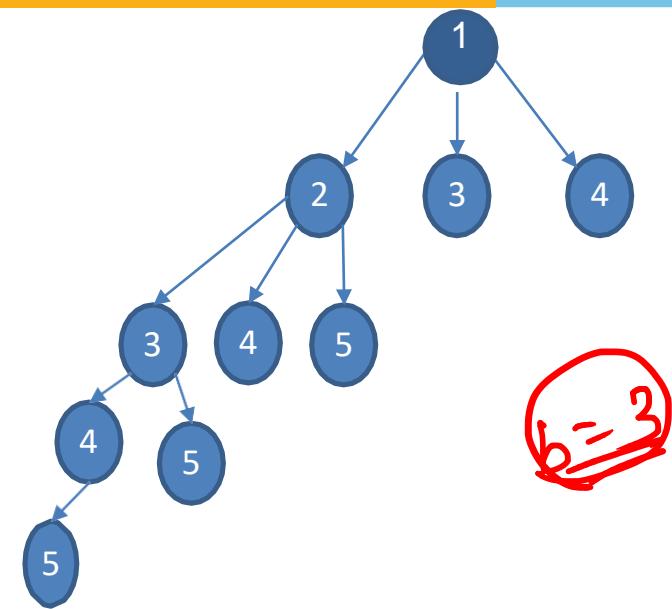




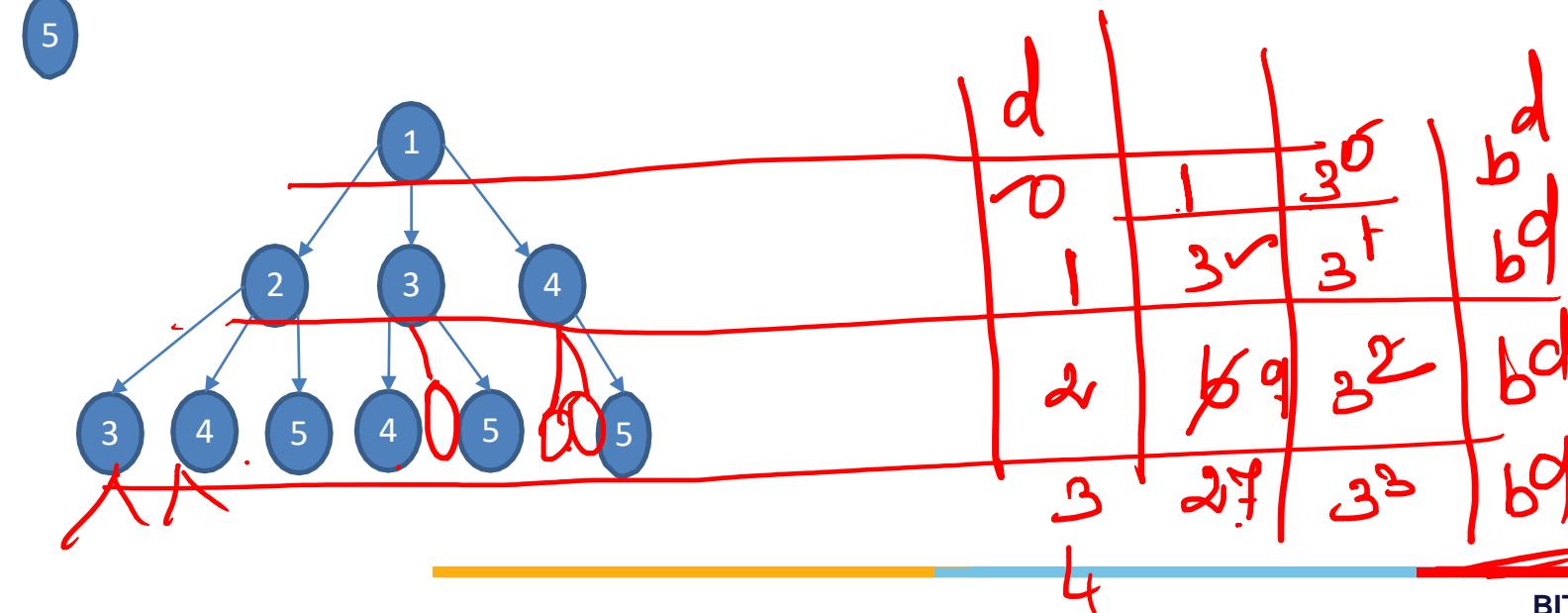
## Search Algorithm – Uninformed Example



# Search Algorithm – Uninformed Example



(b<sup>d</sup>) goal test



# Iterative Deepening Depth First Search (IDS)

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Run Depth Limited Search (DLS) by gradually increasing the limit  $l$

- First with  $l=1$ , then  $l=2$ ,  $l=3$  and so on – until goal is found

It's a combination of Depth First Search + Breadth First Search

Like DFS, memory requirement is a modest  $\mathcal{O}(bd)$  where  $d$  is the depth of shallowest goal

Like BFS

- Complete when branching factor is finite
- Optimal when path cost is non decreasing function of depth

# Breadth First Search – Evaluation

✓ **Complete** – If the shallowest goal node is at a depth  $d$ , BFS will eventually find it by generating all shallower nodes

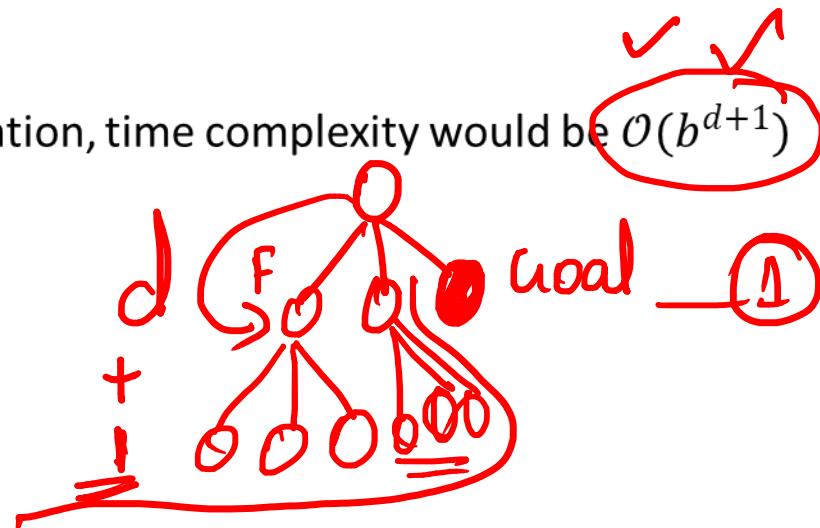
✓ **Optimal** – Not necessarily. Optimal if path cost is non-decreasing function of depth of node. E.g., all actions have same cost – uniform / variant

**Time Complexity** –  $\mathcal{O}(b^d)$  b - branching factor, d – depth

- Nodes expanded at depth 1 = b
- Nodes expanded at depth 2 =  $b^2$
- Nodes expanded at depth d =  $b^d$
- Goal test is applied during generation, time complexity would be  $\mathcal{O}(b^{d+1})$

**Space Complexity** –  $\mathcal{O}(b^d)$

- $\mathcal{O}(b^{d-1})$  in explored set
- $\mathcal{O}(b^d)$  in frontier set



## Depth First Search (DFS)

bf

① **Completeness** – Complete in finite state spaces because it will eventually expand every node

**Optimal** – Not Optimal as it would stop when the goal node is reached without evaluating if there is a better path

**Time Complexity** -  $\mathcal{O}(b^m)$  where m = maximum depth of any node

- Can be much larger than the size of state space
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**Space Complexity** – Needs to store only one path and unexpanded siblings.

- Any node expanded with all its children can be removed from memory
- Requires storage of only  $\mathcal{O}(bm)$ , b – branching factor, m - max depth

3

$$\underline{1+3+9+27}$$

$$1+b+b^2+b^3+b^4-\dots-b^m$$

# Uniform Cost Search – Evaluation

**Completeness** – It is complete if the cost of every step > small +ve constant  $\epsilon$

- It will stuck in infinite loop if there is a path with infinite sequence of zero cost actions

**Optimal** – It is Optimal. Whenever it selects a node, it is an optimal path to that node.

**Time and Space complexity** – Uniform cost search is guided by path costs not depth or branching factor.

- If  $C^*$  is the **cost of optimal solution** and  $\epsilon$  is the min. action cost
- Worst case complexity =  $O(b^{1-\frac{C^*}{\epsilon}})$ ,  $\frac{C^*}{\epsilon} \Rightarrow d$
- When all action costs are equal  $\rightarrow O(b^{d-1})$ , the BFS would perform better
  - As Goal test is applied during expansion, Uniform Cost search would do extra

# Iterative Deepening Depth First Search (IDS)

## Time Complexity:

- Appears that IDS is generating a lot of nodes multiple times
- However, most of nodes are present in the lower levels which are not repeated often
- Generation of nodes
  - At level 1 -  $b$  nodes generated  $d$  times –  $(d)b$
  - At level 2 –  $b^2$  nodes generated  $d-1$  times –  $(d-1)b^2$
  - At level  $d$  –  $b^d$  nodes generated once –  $(1) b^d$
- Time Complexity  $N(\text{IDS}) = \mathcal{O}(b^d)$  same as BFS

IDS is the preferred uninformed search method when search space is large and depth is unknown

# Terminologies – Learnt Today

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- Nodes
- States
- Frontier | Fringes
- Search Strategy : LIFO | FIFO | Priority Queue
- Performance Metrics
  - Completeness
  - Optimality
  - Time Complexity
  - Space Complexity
- Algorithm Terminology
  - d Depth of a node
  - b Branching factor
  - n – nodes
  - l – level of a node
  - m – maximum
  - C\* - Optimal Cost
  - E – least Cost
  - N –total node generated

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**Required Reading:** AIMA - Chapter #3: 3.1, 3.2, 3.3, 3.4

Next Class Plan :  
Informed Search : GBFS & A\*  
Heuristic Design

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials