
_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

UNIVERSITAT POLITÈCNICA DE VALÈNCIA


# Advanced methods of artificial vision

## Chapter 1: Hand crafted feature extraction

# Index

1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

# Index

1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

# What is a feature?

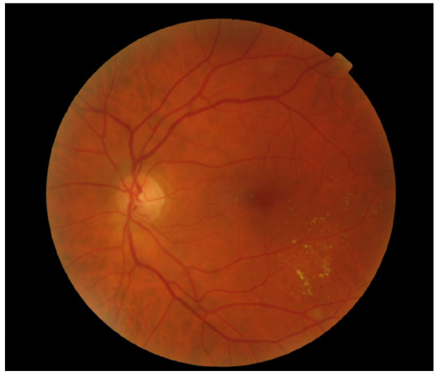


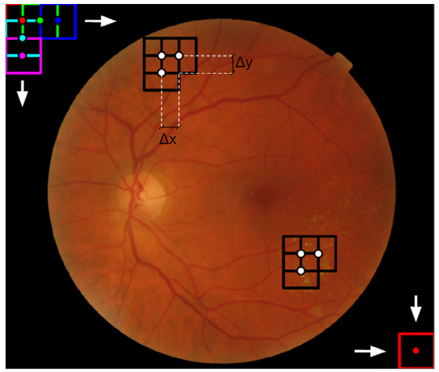
- A feature (also called a **descriptor**) is a usually numerical value that is related to some relevant aspect of an entire image, part of an image, or a shape resulting from a segmentation process.
- Supposedly, a feature should have close values when the relevant aspect whose essence it is trying to capture (colour, texture, area, etc.) is similar from a human point of view.
- Another highly desirable property of a good feature is its **robustness**, understood as immunity to noise (the values of the feature should not change too much if the image or shape on which it is calculated is affected by a moderate level of noise).
- Generally, not a single feature is associated with the object of interest but several or many of them grouped together in what is known as a **feature vector**.

# Feature extraction methodology



- How do we analyse the image?
  - Global descriptors: descriptors for the complete image
  - Local descriptors:
    - Rectangular blocks or patches
    - Landmark extraction: Harris, SIFT,…
- Descriptors
  - Statistical descriptors
  - Cooccurrence Matrix
  - Histograms of Oriented Gradients
  - Local Binary Patterns
  - Gabbor Filters
  - SIFT, SURF
  - ….





# Statistical descriptors



- Analyse the statistical distribution of some property for each of the pixels in the image.

- Classified into: first order methods (those based on the histogram), second order methods (those based on co-occurrence matrices), and higher order methods.

- First-order statistics:
  - The normalised histogram of the image is calculated.
  - Properties that are obtained from this histogram.

| Mean | Variance | Skewness | Kurtosis | Entropy |
|---|---|---|---|---|
| $\mu = \sum_{i=1}^{n} i h(i)$ | $\sigma^2 = \sum_{i=1}^{n} (i-\mu)^2 h(i)$ | $\mu_3 = \frac{1}{\sigma^3} \sum_{i=1}^{n} (i-\mu)^3 h(i)$ | $\mu_4 = \frac{1}{\sigma^4} \sum_{i=1}^{n} (i-\mu)^4 h(i)$ | $-\sum_{i=1}^{n} h(i) \log h(i)$ |

# Co-occurrence matrix



- Histogram-based statistics have the disadvantage of losing spatial information.
- The same information would be obtained for an image of a chessboard with the black and white squares swapped.
- To capture the spatial dependencies of grey level values, which contribute to the perception of textures present in an image, a 2D structure called co-occurrence matrix is defined to analyse textures.
- The co-occurrence matrix $P(i, j)$ is defined by specifying a shift direction $d = d(i, j)$ and counting all pairs of pixels separated by d and having grey values $i$ and $j$.

https://scikit-image.org/docs/stable/user_guide/index.html

# Co-occurrence matrix

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 5 | 6 | 8 |
| 2 | 3 | 5 | 7 | 1 |
| 4 | 5 | 7 | 1 | 2 |
| 8 | 5 | 1 | 2 | 5 |

GLCM

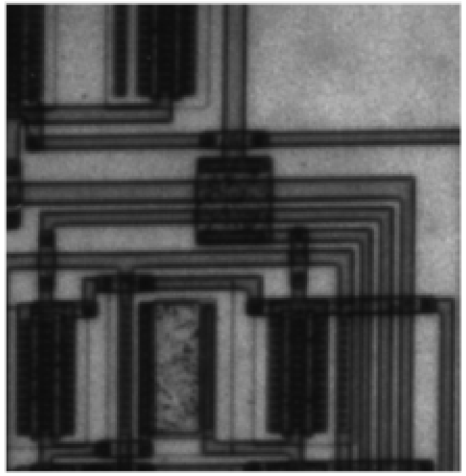
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 1 | 2 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 7 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

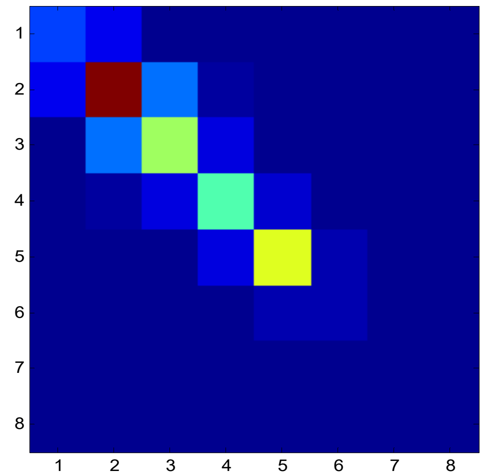
Contrast: 0.3307 $\sum_{i,j} |i-j|^2 p(i,j)$

Correlation: 0.9032 $\sum_{i,j} \frac{(i-\mu i)(j-\mu j)p(i,j)}{\sigma_i \sigma_j}$

Energy: 0.1323 $\sum_{i,j} p(i,j)^2$

Homogeneity: 0.8534 $\sum_{i,j} \frac{p(i,j)}{1+|i-j|}$





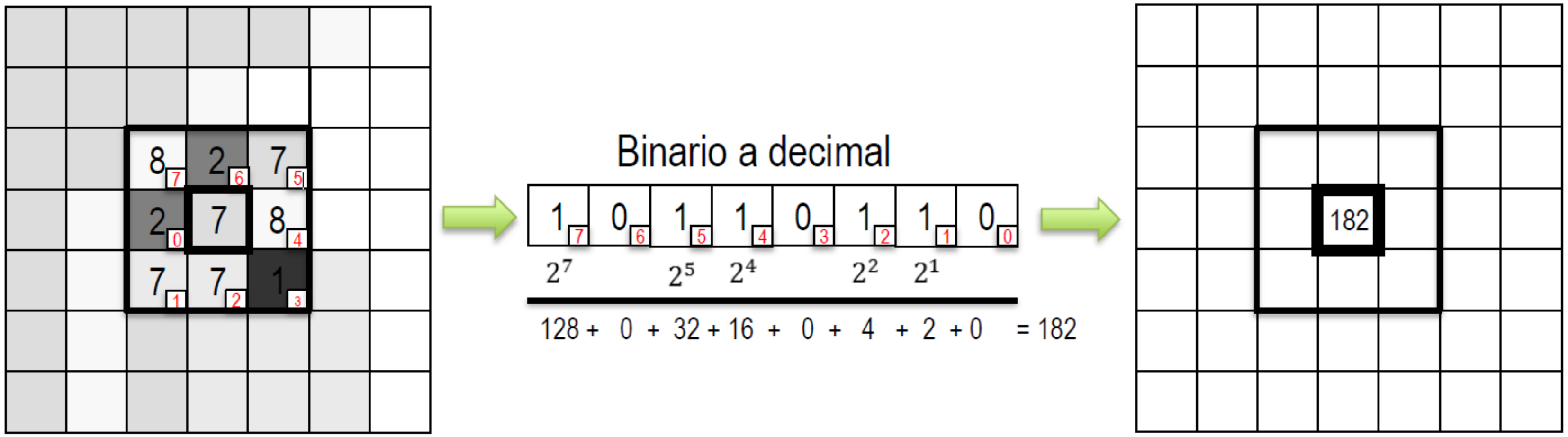
# Index



1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

# Local Binary Patterns(LBP)

$$LBP_{P,R}(i,j) = \sum_{p=0}^{P-1} s(g_p - g_c) \cdot 2^p, \;\; s(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$



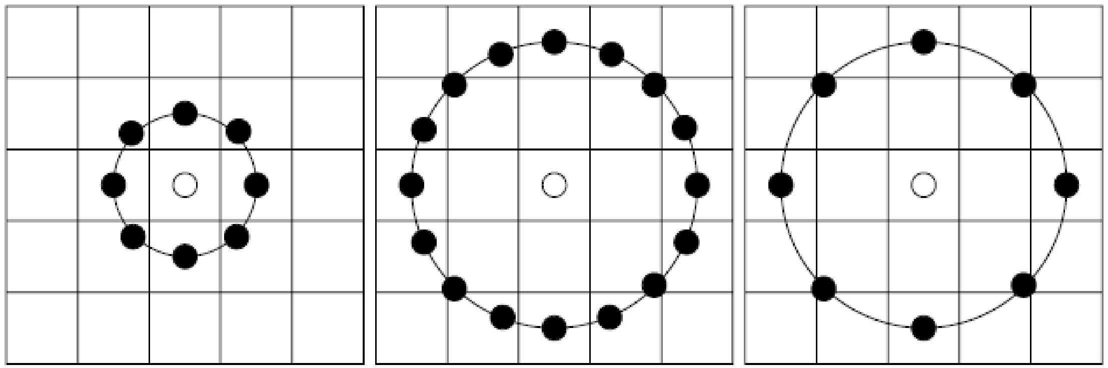

Ojala T, Pietikäinen M & Mäenpää T (2002) Multiresolution gray-scale and rotation invariant texture classification with Local Binary Patterns. IEEE Transactions on Pattern Analysis and Machine Intelligence 24(7):971-987.

# Local Binary Patterns(LBP)



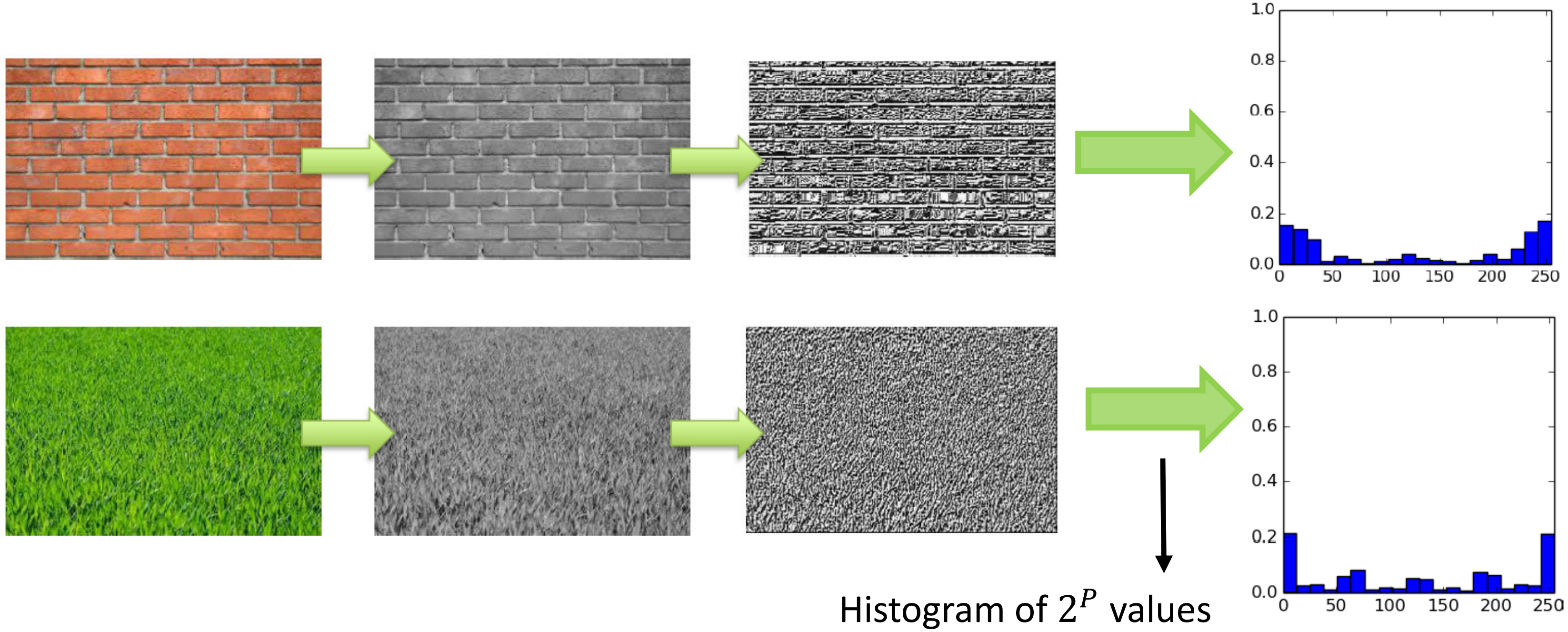
$$LBP_{P,R}(i,j) = \sum_{p=0}^{P-1} s(g_p - g_c) \cdot 2^p, \;\; s(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$P$ P is the number of neighbours and $\mathrm{R}$ is the neighbourhood radius.



$P = 8$ y $\mathrm{R} = 1$ $\quad$ $P = 16$ y $\mathrm{R} = 2$ $\quad$ $P = 8$ y $\mathrm{R} = 2$

# Local Binary Patterns(LBP)




Histogram of $2^P$ values

# Local Binary Patterns(LBP)

**LBPRi (LBP Rotational Invariant)**

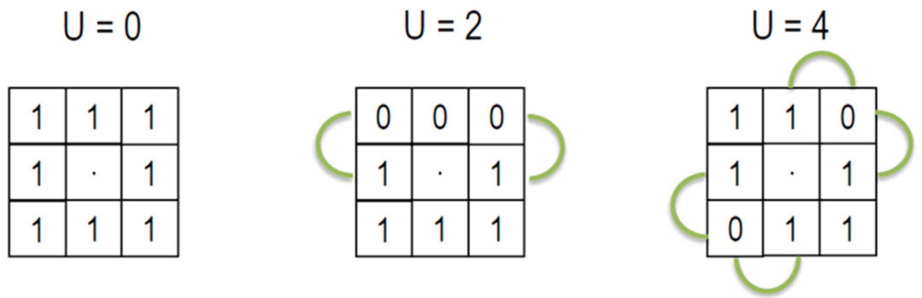
$$LBP^{ri}_{P,R} = min\{ROR(LBP_{P,R}\ ,\ t)\ \ |\ \ t = 0, 1, ..., P-1\}$$

$ROR(x, t)$ performs $t$ different rotations of the $x$ bits

- e.g. 10000010, 00101000, and 00000101 are equivalent to the minimum code 00000101.
- $P = 8$ 256 different and 36 rotationally invariant LBP patterns

## Uniform LBP

- Objetive: to reduce the number of possible patterns to the really discriminating ones.
- How? By analysing the uniformity (U) of the patterns : number of transitions from 0 to 1 and 1 to 0.



_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

# Local Binary Patterns(LBP)

MUIT Master Universitario en Ingeniería de Telecomunicación

**Uniform rotationally invariant LBP: LBPriu2**

- Patterns with U=0 or U=2 are reassigned an individual pattern code.
- All other patterns are reassigned the same code (they become indistinguishable).
- LBP: 256 patterns, LBP Uniform: (58 + 1) patterns, (9+1) rotationally invariant . In general (P+2)

| LBP patterns | Uniformity |
|---|---|
| 00000000 | 0 |
| 01111111 | 2 |
| 00000011 | 2 |
| 01000111 | 4 |
| 01010101 | 8 |

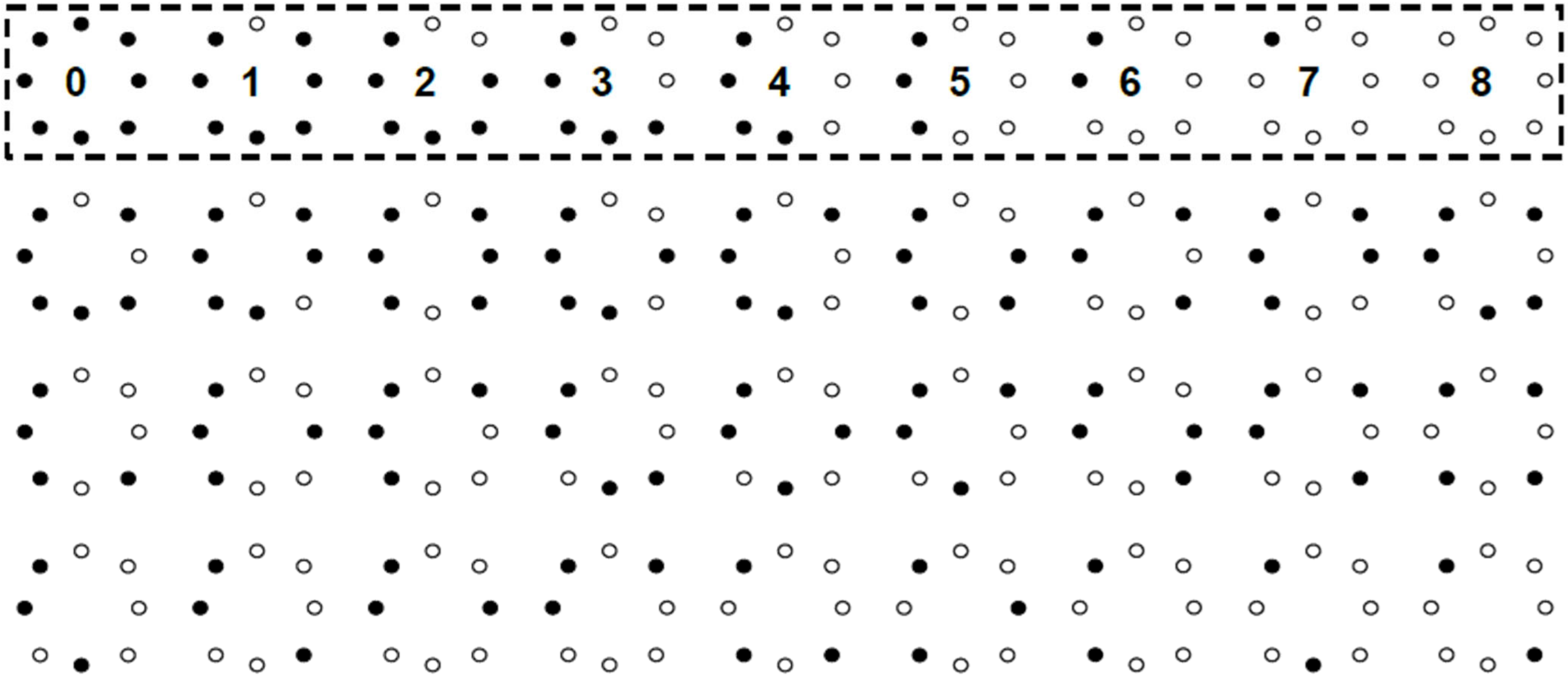
$$LBP_{P,R}^{riu2} = \begin{cases} \sum_{p=0}^{P-1} s(g_p - g_c) & \text{if } U(LBP_{P,R}) \leq 2 \\ P+1 & \text{otherwise} \end{cases}$$

$$U(LBP_{P,R}) = |s(g_{P-1} - g_c) - s(g_0 - g_c)| + \sum_{p=1}^{P-1} |s(g_p - g_c) - s(g_{P-1} - g_c)|$$

# Local Binary Patterns(LBP)

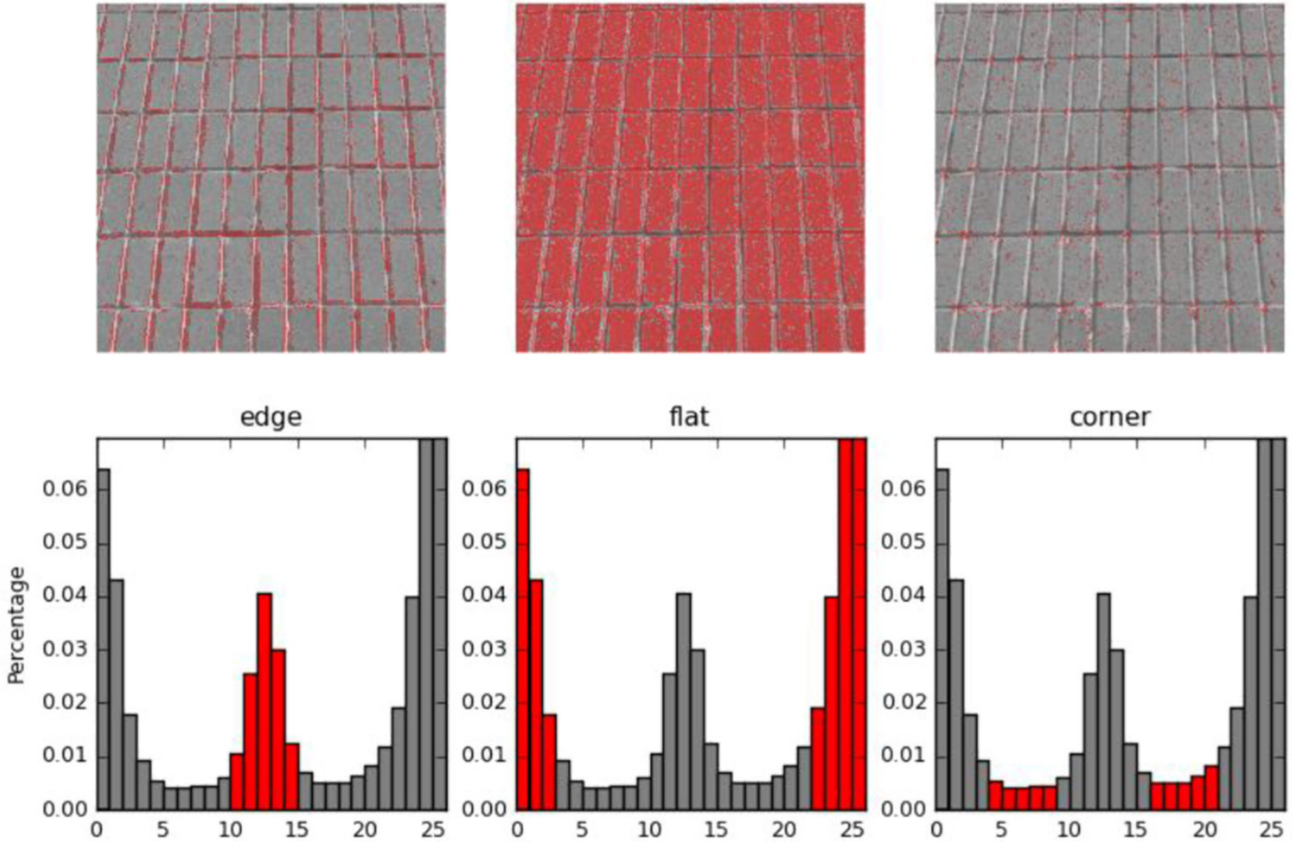


**Uniform rotationally invariant LBP: LBPriu2**



# Local Binary Patterns(LBP)



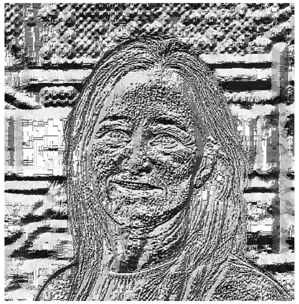



http://scikit-image.org/docs/dev/auto_examples/plot_local_binary_pattern.html

TELECOM ESCUELA
TÉCNICA VLC SUPERIOR
DE UPV INGENIEROS
DE TELECOMUNICACIÓN

# Local Binary Patterns(LBP)

MU
IT
Master
Universitario en
Ingeniería de
Telecomunicación

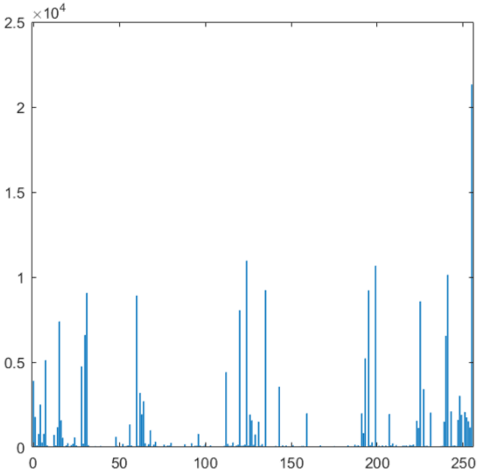


LBP

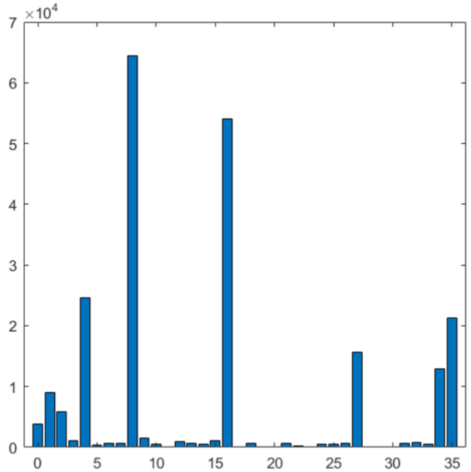
P=8 R=1

LBPri

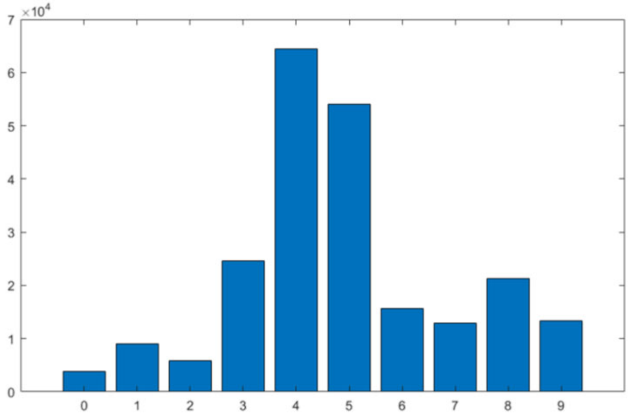
LBPriu2




LBP

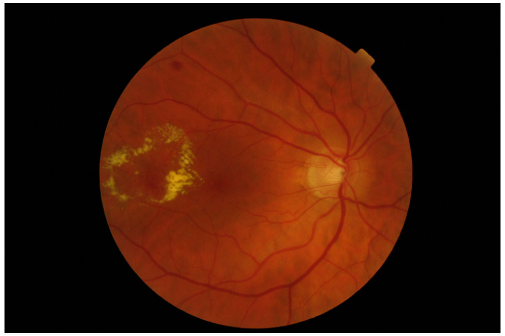
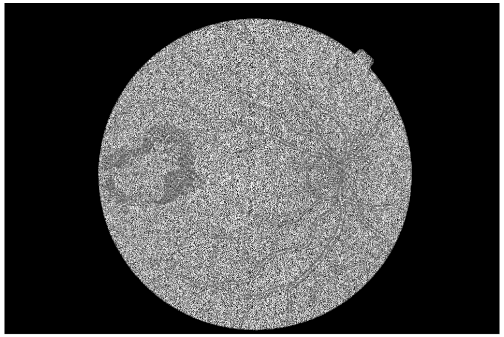
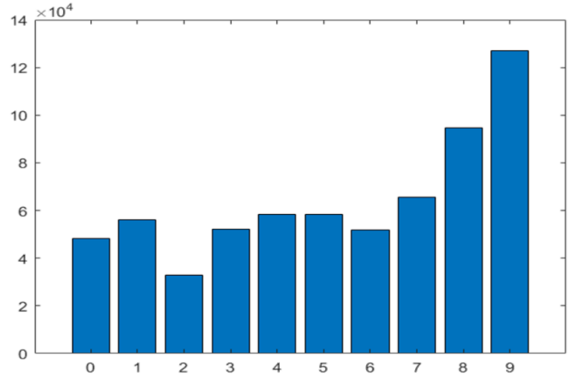



LBPri

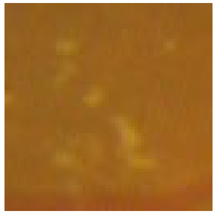


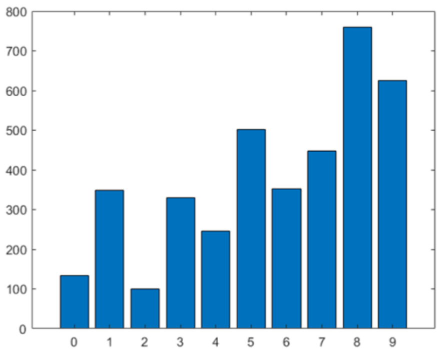

LBPriu2

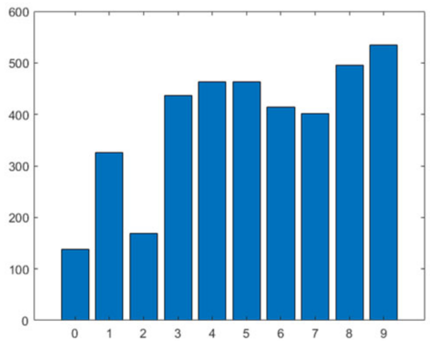
# Local Binary Patterns(LBP)



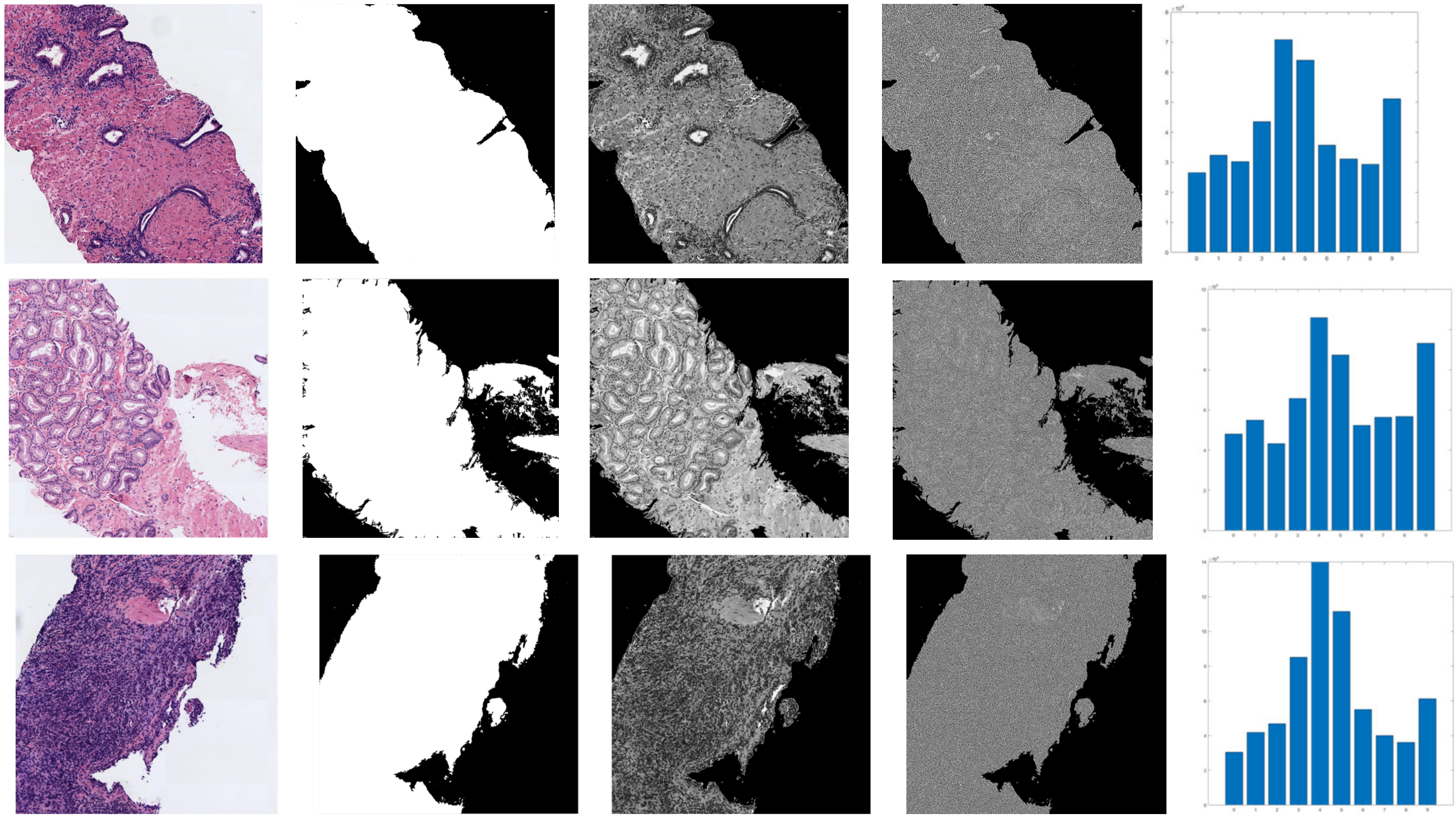
# Local Binary Patterns(LBP)

Original: i | Máscara: mask | Luminancia: ig | LBP | Histograma LBP

sana

Cáncer grado 3

Cáncer grado 5



# Index

1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

# Histograms of Oriented Gradients (HOG)

- The histogram of oriented gradients (HOG ) is a widely used descriptor in image processing for the purpose of object detection. The technique counts the occurrences of gradient orientation in localised portions of an image.

- Algorithm:
  - Gradient calculation
  - Grouping the orientations
  - Block descriptor
  - Block normalisation

# Convolution

$$y[m,n] = x[m,n] * h[m,n] = \sum_k \sum_l x[k,l]h[m-k,n-l]$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 74 | 66 | 62 | 59 | 63 | 71 | 75 | 72 |
| 72 | 65 | 61 | 58 | 62 | 70 | 75 | 72 |
| 71 | 68 | 63 | 59 | 63 | 69 | 73 | 69 |
| 70 | 67 | 62 | 59 | 62 | 69 | 73 | 69 |
| 70 | 64 | 59 | 57 | 61 | 70 | 74 | 72 |
| 68 | 66 | 64 | 63 | 65 | 68 | 69 | 68 |
| 70 | 69 | 67 | 66 | 67 | 69 | 69 | 67 |
| 69 | 68 | 66 | 64 | 65 | 67 | 66 | 63 |

Imagen de salida

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | 3 | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

Kernel

$h[k,l]$

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

$h[-k,-l]$

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

$$(-1)*62+0*59+1*62+(-2)*59+0*57+2*61+(-1)*64+0*63+1*65=3$$

# Gradient Computation

$G_x$

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

$G_y$

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

$$Magnitude = \sqrt{G_x^2 + G_y^2}$$

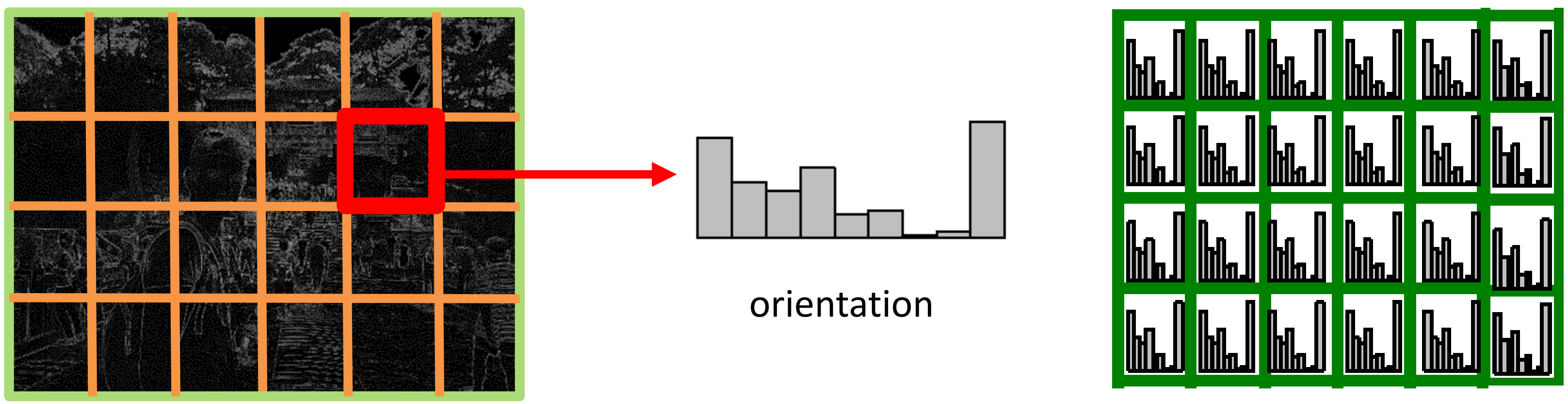
$$Orientation = \tan^{-1}\frac{G_y}{G_x}$$

_ **TELECOM** ESCUELA TÉCNICA **VLC** SUPERIOR DE **UPV** INGENIEROS DE TELECOMUNICACIÓN

# Histograms of Oriented Gradients (HOG)

**M**aster **U**niversitario en **I**ngeniería de **T**elecomunicación

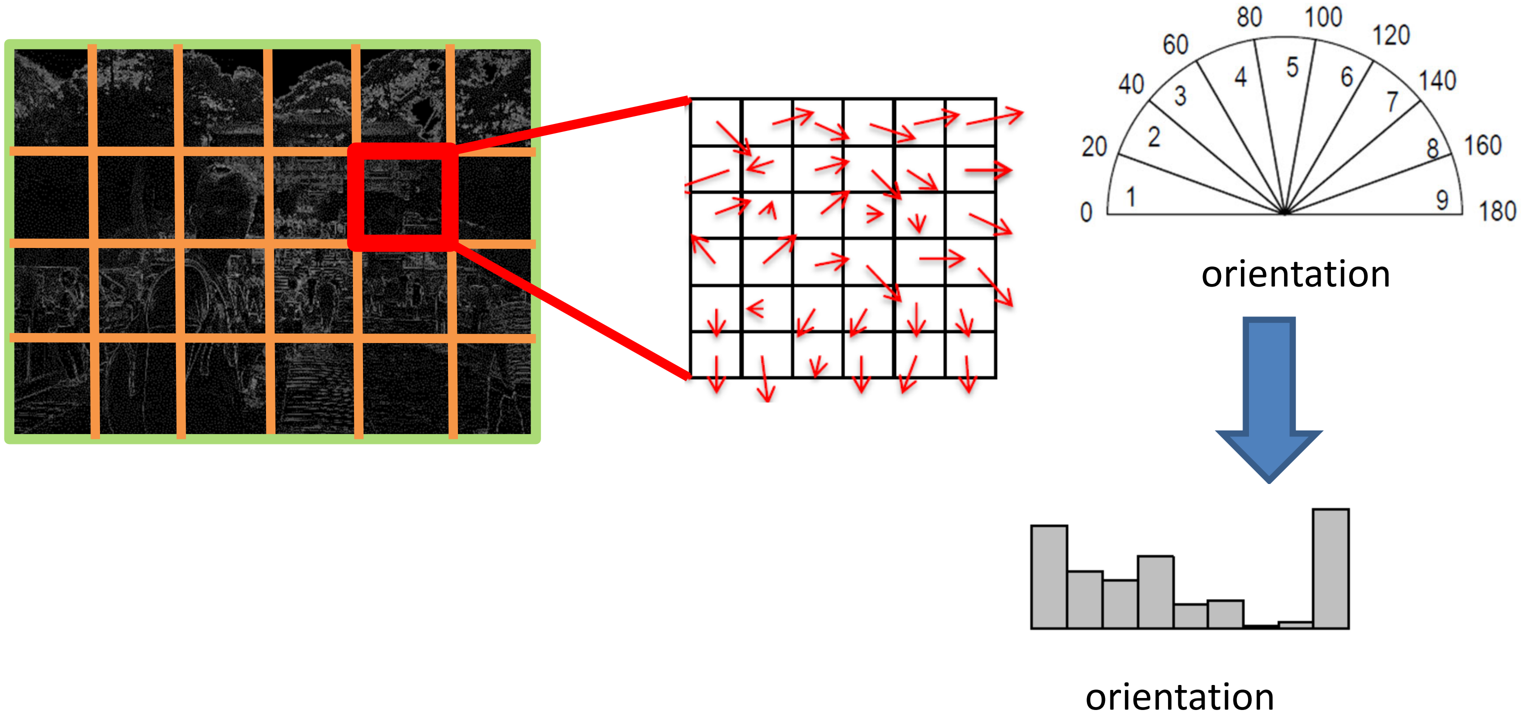
- Division of the image into cells of fixed size.
- Calculation of a histogram of the orientations in each cell.
- Global descriptor combines histograms of all cells.



# Histograms of Oriented Gradients (HOG)



- Division of the range of orientations into a fixed number of intervals.
- Assign each pixel in the cell to an interval based on the gradient orientation.
- Accumulate the gradient magnitude of all pixels assigned to an interval.

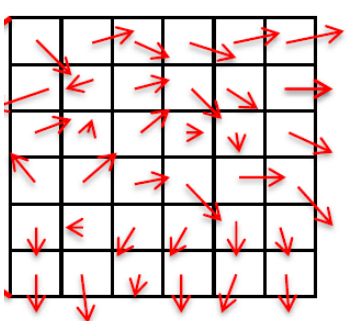


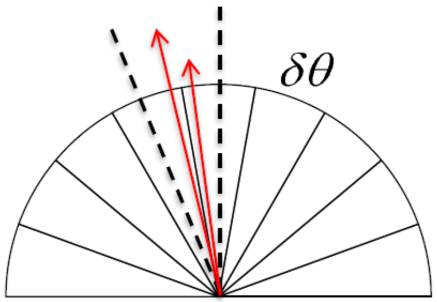
# Histograms of Oriented Gradients (HOG)



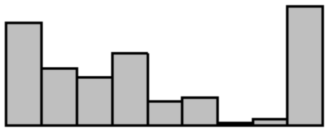
## Orientation histogram calculation: interpolation in orientation

- Problems
  - Gradients with very similar orientations can be assigned to different intervals.
  - Sensitive to small gradient variations.
- Solution:
  - Assign each gradient to the two closest intervals with a weight proportional to the distance from the orientation to the centre of each interval.






orientation



orientation

$$\omega_k(x,y) = \max\left(0, 1 - \frac{\theta(x,y) - \theta_k}{\delta\theta}\right)$$

$$h(k) = \sum_{(x,y)\in C} \omega_k(x,y) g(x,y)$$

# Histograms of Oriented Gradients (HOG)



Assign each gradient to the two closest intervals with a weight proportional to the distance from the orientation to the centre of each interval.

$$g(x,y) = 100$$
$$\theta(x,y) = 45º$$

Assuming that for the histogram calculation the orientation is divided into 9 intervals (without considering the sign), what would be the contribution of this pixel to each of the intervals of the histogram?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

# Histograms of Oriented Gradients (HOG)



Assign each gradient to the two closest intervals with a weight proportional to the distance from the orientation to the centre of each interval.

$$g(x,y) = 100$$
$$\theta(x,y) = 45º$$

Assuming that for the histogram calculation the orientation is divided into 9 intervals (without considering the sign), what would be the contribution of this pixel to each of the intervals of the histogram?

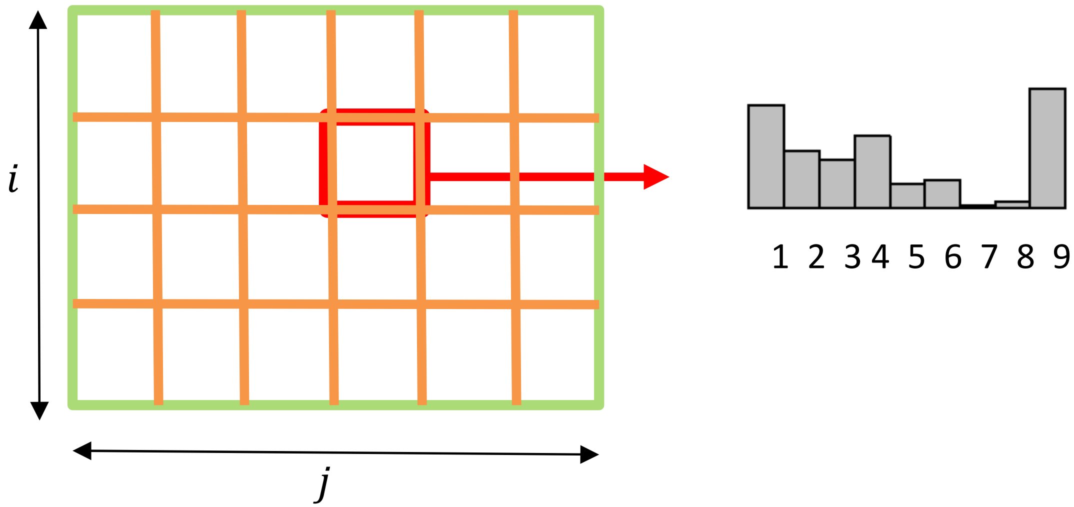
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 0 | 25 | 75 | 0 | 0 | 0 | 0 | 0 | 0 |

# Histograms of Oriented Gradients (HOG)



## Orientation histogram calculation: spatial integration

- A histogram is calculated for each of the cells.
- Each pixel contributes to the histogram of its corresponding cell.



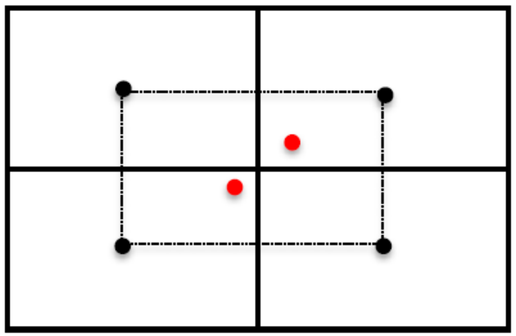

$$h_{ij}(k) = \sum_{(x,y)\in C_{ij}} \omega_k(x,y) g(x,y)$$

# Histograms of Oriented Gradients (HOG)



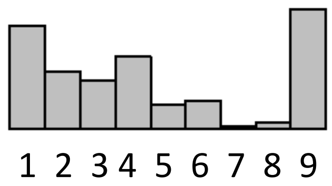
## Orientation histogram calculation: spatial integration

- Problems
  - Pixels in close proximity can be assigned to different cells.
  - Sensitive to small variations in object shape.
- Solution:
  - Assign each pixel to the four nearest cells with a weight proportional to the distance of the pixel from the centre of each cell.



$$\omega_{ij}^{x}(x,y) = \max\left(0, 1 - \frac{d_{ij}^{x}}{\delta x}\right) \qquad \omega_{ij}^{y}(x,y) = \max\left(0, 1 - \frac{d_{ij}^{y}}{\delta x}\right)$$

$$h_{ij}(k) = \sum_{(x,y)} \omega_{ij}^{x}(x,y)\,\omega_{ij}^{y}(x,y)\,\omega_{k}(x,y)\,g(x,y)$$

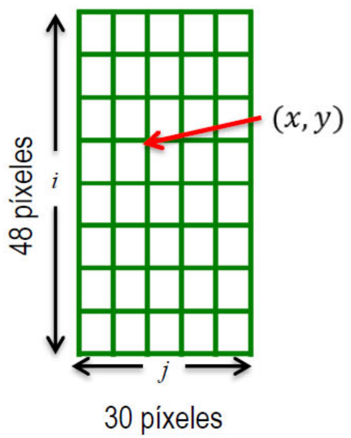


_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

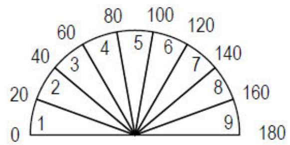
# Histograms of Oriented Gradients (HOG)

MUIT Master Universitario en Ingeniería de Telecomunicación

Given an image and the division into cells as shown and assuming that the orientation is divided into 9 intervals without considering the sign (0º-180º), in which histograms $(i,j)$ of the final representation and in which intervals $k$ and with which weight will the pixel ($x$=12,$y$=18) with orientation $\theta = 60º$ contribute?




| Histograma (i,j) | Intervalo orientación k (1-9) | Peso del gradiente $\omega^x_{ij}(x,y)\omega^y_{ij}(x,y)\omega_k(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

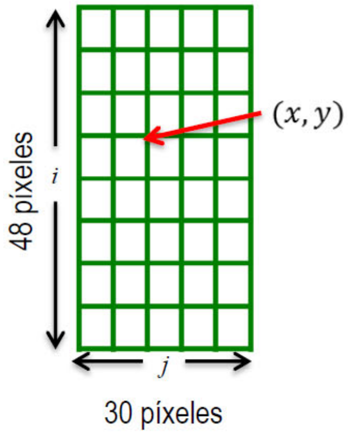


_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

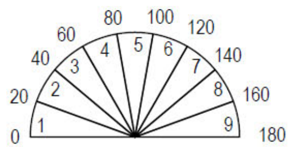
# Histograms of Oriented Gradients (HOG)

MUIT Master Universitario en Ingeniería de Telecomunicación

Given an image and the division into cells as shown and assuming that the orientation is divided into 9 intervals without considering the sign (0º-180º), in which histograms $(i,j)$ of the final representation and in which intervals $k$ and with which weight will the pixel ($x$=12,$y$=18) with orientation $\theta = 60º$ contribute?




| Histograma (i,j) | Intervalo orientación k (1-9) | Peso del gradiente $\omega_{ij}^{x}(x,y)\omega_{ij}^{y}(x,y)\omega_{k}(x,y)$ |
|---|---|---|
| (3,2) | 3 | 0,125 |
| (3,3) | 3 | 0,125 |
| (4,2) | 3 | 0,125 |
| (4,3) | 3 | 0,125 |
| (3,2) | 4 | 0,125 |
| (3,3) | 4 | 0,125 |
| (4,2) | 4 | 0,125 |
| (4,3) | 4 | 0,125 |

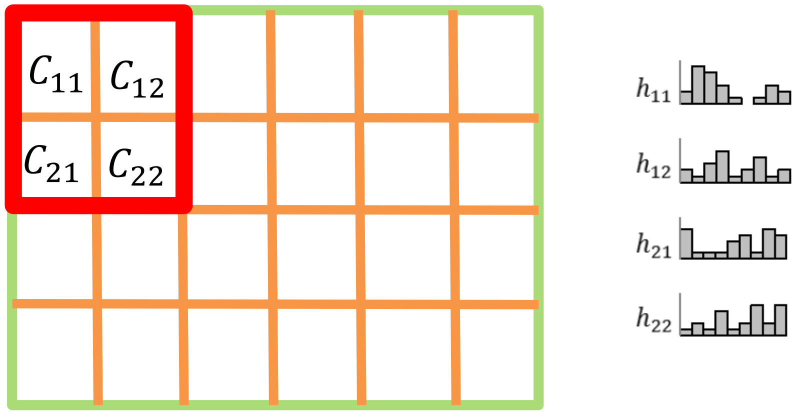


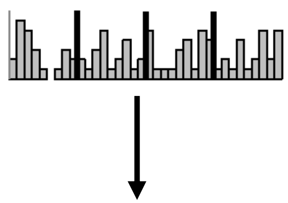
# Histograms of Oriented Gradients (HOG)



## Block normalisation

- Grouping of cells in blocks of b x b cells.
- Single vector per block concatenating the histograms of all cells.
- Vector normalisation using the L2 norm.



$$v = (x_1, x_2, \dots, x_N)$$

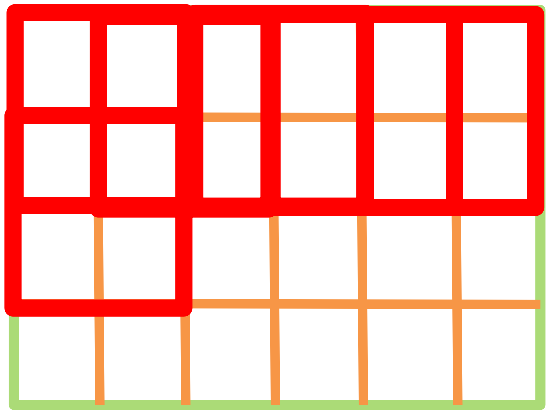
$$v' = \frac{v}{\sqrt{\|v\|_2^2 + \varepsilon}} = \frac{v}{\sqrt{x_1^2 + x_2^2 + \cdots + x_N^2 + \varepsilon}}$$

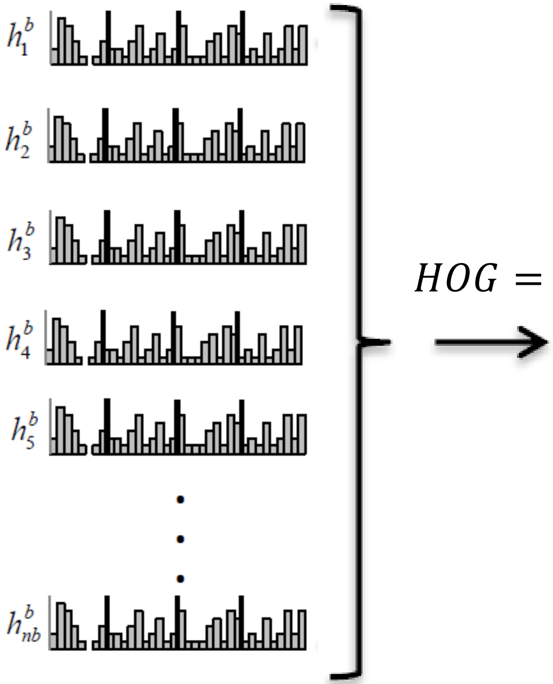
# Histograms of Oriented Gradients (HOG)



**Final descriptor**

- Block overlap.
- For each block, normalised histogram of cells.
- Final descriptor: concatenation of all block histograms






$$HOG = (x_1, x_2 \ldots x_N)$$

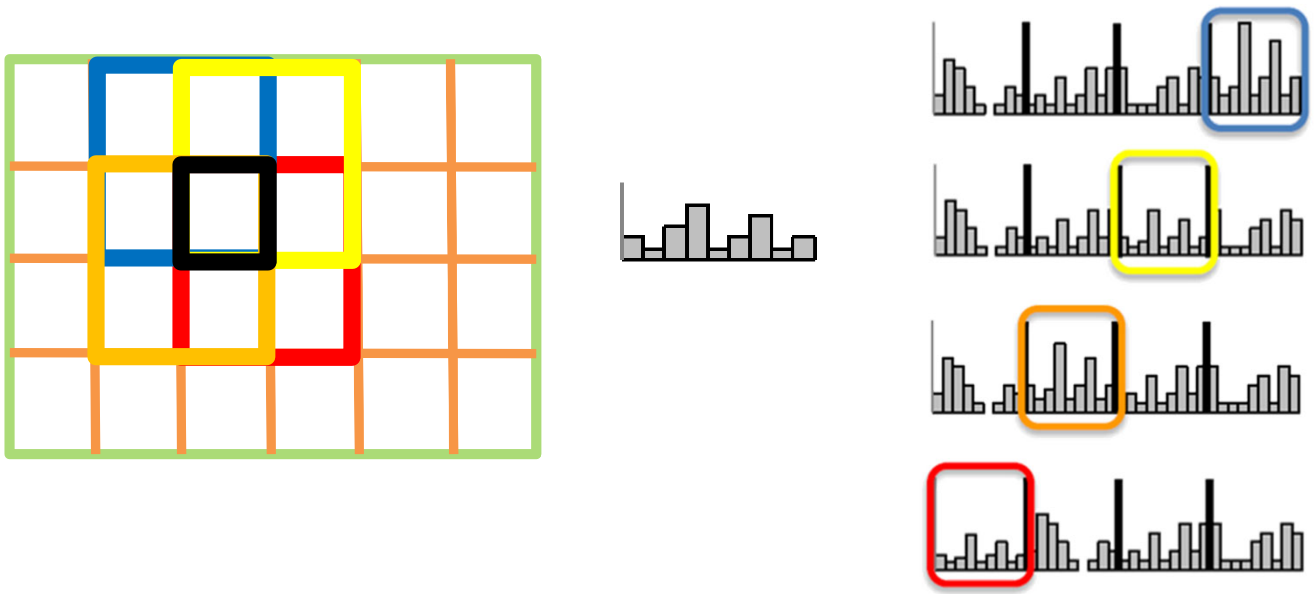
_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

# Histograms of Oriented Gradients (HOG)

MUIT Master Universitario en Ingeniería de Telecomunicación

**Final descriptor**

- Each cell contributes to the description of several blocks.
- In each block with a different normalization.



_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

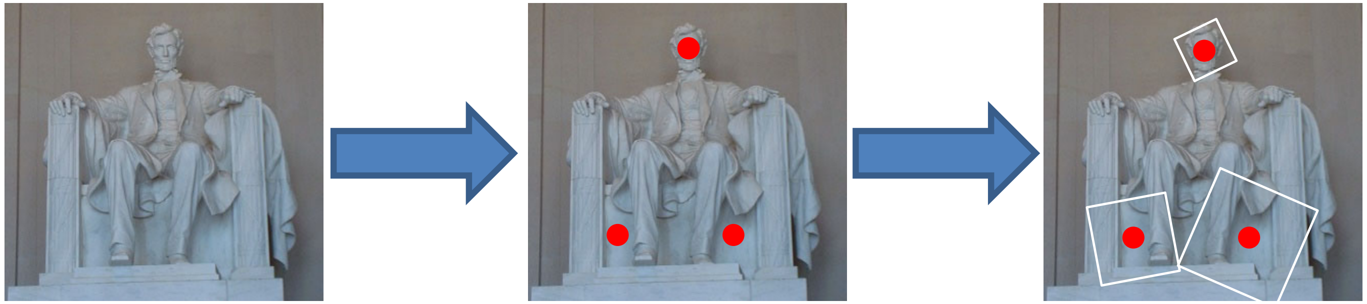
# Index

Master Universitario en Ingeniería de Telecomunicación

1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

# Keypoint detector





Test image

**Detector**: locates single-scale or multi-scale points of interest.

**Descriptor**: Content invariant representation (HOG, SIFT, etc.)

# Harris' detector



For each pixel is calculated: $C(\mathbf{x}) = \begin{bmatrix} \sum_{\mathbf{x}\in N_{\mathbf{x}}} I_{\mathbf{x}}^2(\mathbf{x}) & \sum_{\mathbf{x}\in N_{\mathbf{x}}} I_{\mathbf{x}}(\mathbf{x}) I_{\mathbf{y}}(\mathbf{x}) \\ \sum_{\mathbf{x}\in N_{\mathbf{x}}} I_{\mathbf{y}}(\mathbf{x}) I_{\mathbf{x}}(\mathbf{x}) & \sum_{\mathbf{x}\in N_{\mathbf{x}}} I_{\mathbf{y}}^2(\mathbf{x}) \end{bmatrix}$

Example $C(\mathbf{x}) = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ $\Rightarrow$

- $\lambda_1$ and $\lambda_2$ high: corner
- $\lambda_1$ high and $\lambda_2$ small: edge
- $\lambda_1$ small and $\lambda_2$ high: edge
- $\lambda_1$ and $\lambda_2$ small: flat area

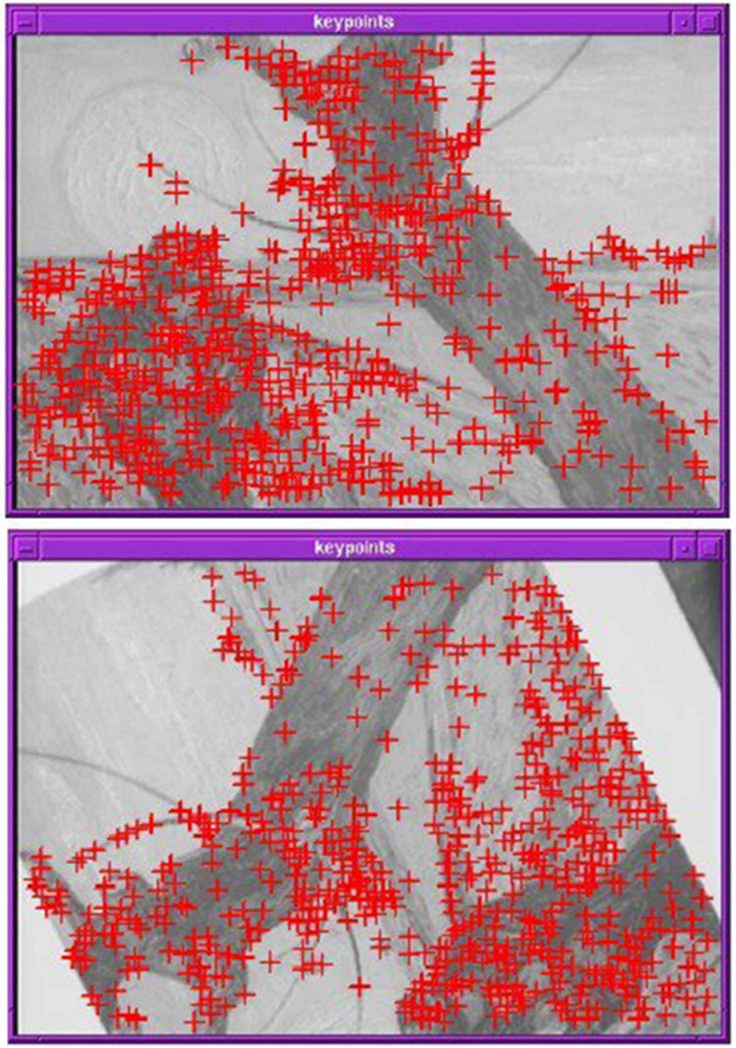
General case $C(\mathbf{x}) = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

# Harris' detector



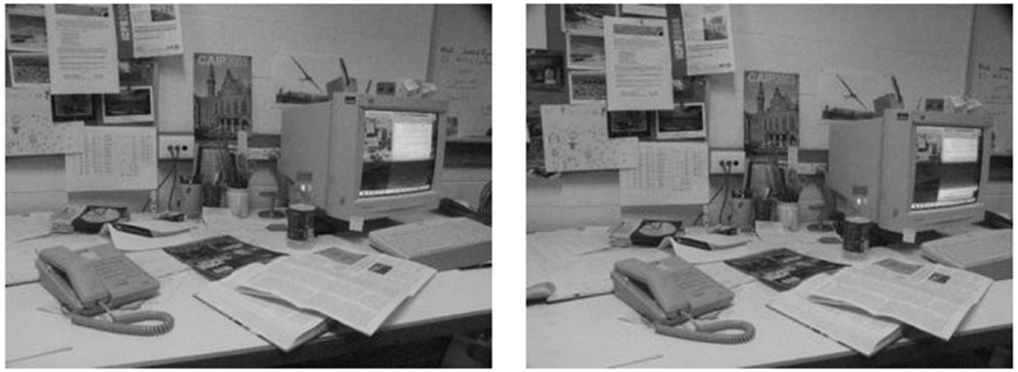
- Algorithm designed for motion tracking.
- Reduces calculation time compared to tracking all points.
- Invariant to translation and rotation.
- Not invariant to scale changes.

- Algorithm
  - Reduce image noise (Gaussian filter for example).
  - Image gradients calculation.
  - Construct the matrix C for each pixel.
  - Obtain and analyse determinant ($\lambda_1 \cdot \lambda_2$) and trace ($\lambda_1 + \lambda_2$) of the matrix C.

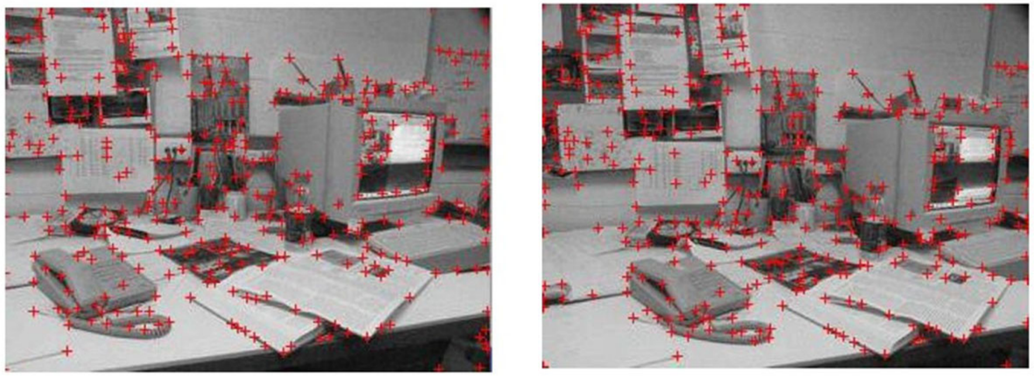


# Matching of features





Original images



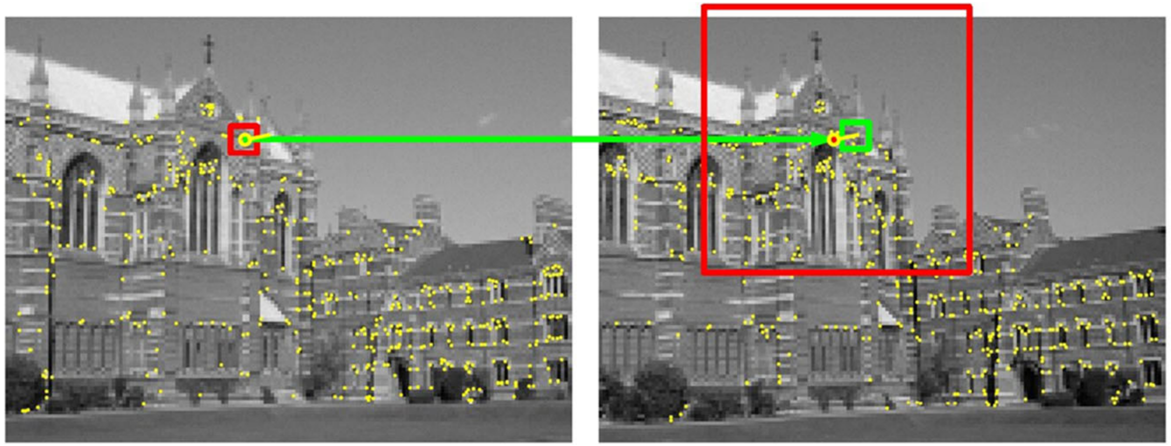
Keypoints (corners) detected with Harris' detector

# Matching of features



**CORRELATION**

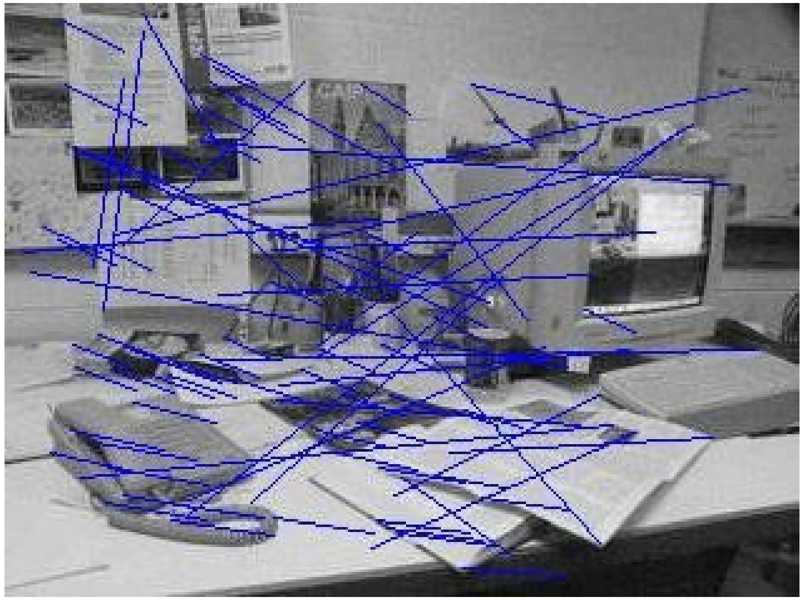
$$R(\mathbf{x}_1, \mathbf{x}_2) = \frac{\sum_{\mathbf{x}_1 \in N_{\mathbf{x}_1}, \mathbf{x}_2 \in N_{\mathbf{x}_2}} s(\mathbf{x}_1) s(\mathbf{x}_2)}{\sqrt{\sum_{\mathbf{x}_1 \in N_{\mathbf{x}_1}} s_1(\mathbf{x})^2} \sqrt{\sum_{\mathbf{x}_2 \in N_{\mathbf{x}_2}} s_2(\mathbf{x})^2}}$$



# Matching of features



**Search of matchings: Putative matchings**



Need for a robust algorithm

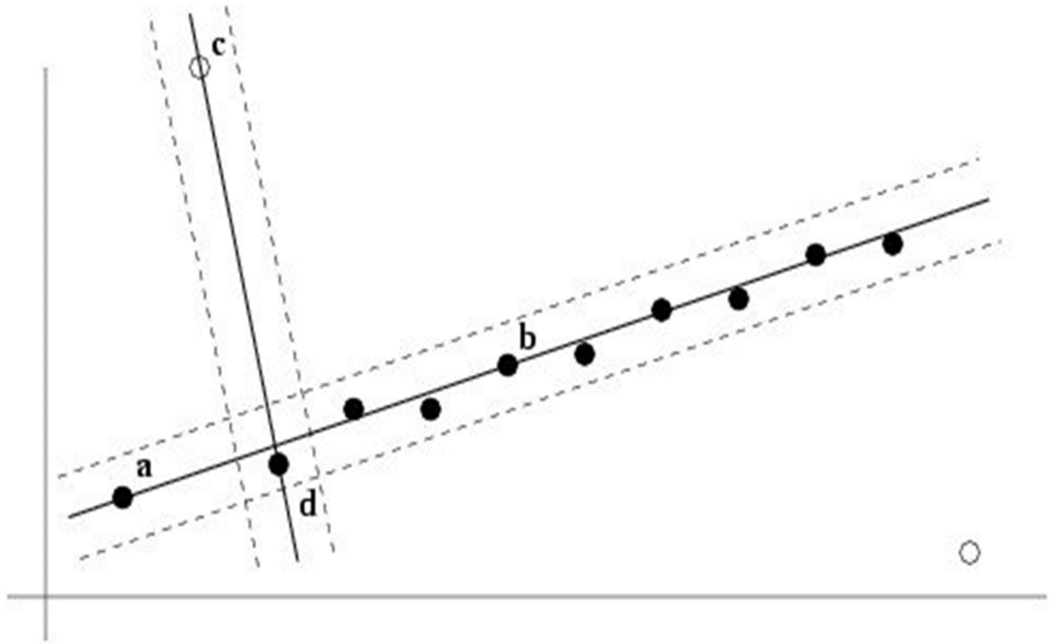
# Robust estimation



## RANSAC (RANdom SAmple Consensus)

### Algorithm

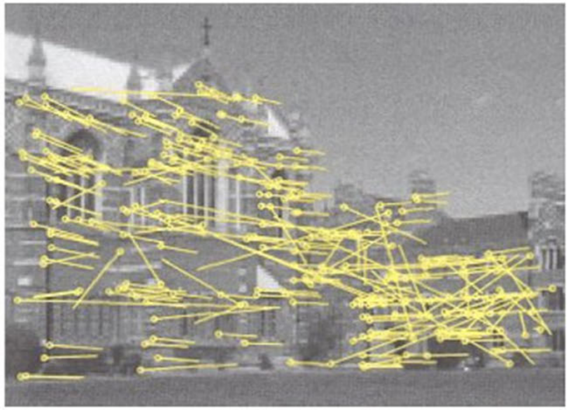
1. Take a sample: minimum number of points to estimate the model.
2. Look at the support of that model: number of points below a distance t (consensusset).
3. Repeat the process for N samples and select the model with the highest support.
4. Points with a distance less than t are inliers.Reestimate the model with all inliers.



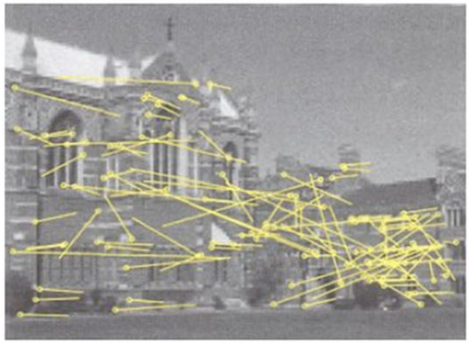
# Robust estimation



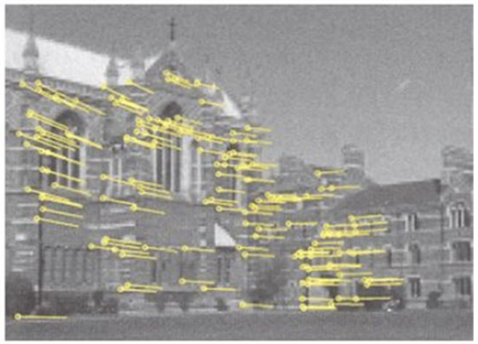
**Example RANSAC**



Initial matches



117 outliers

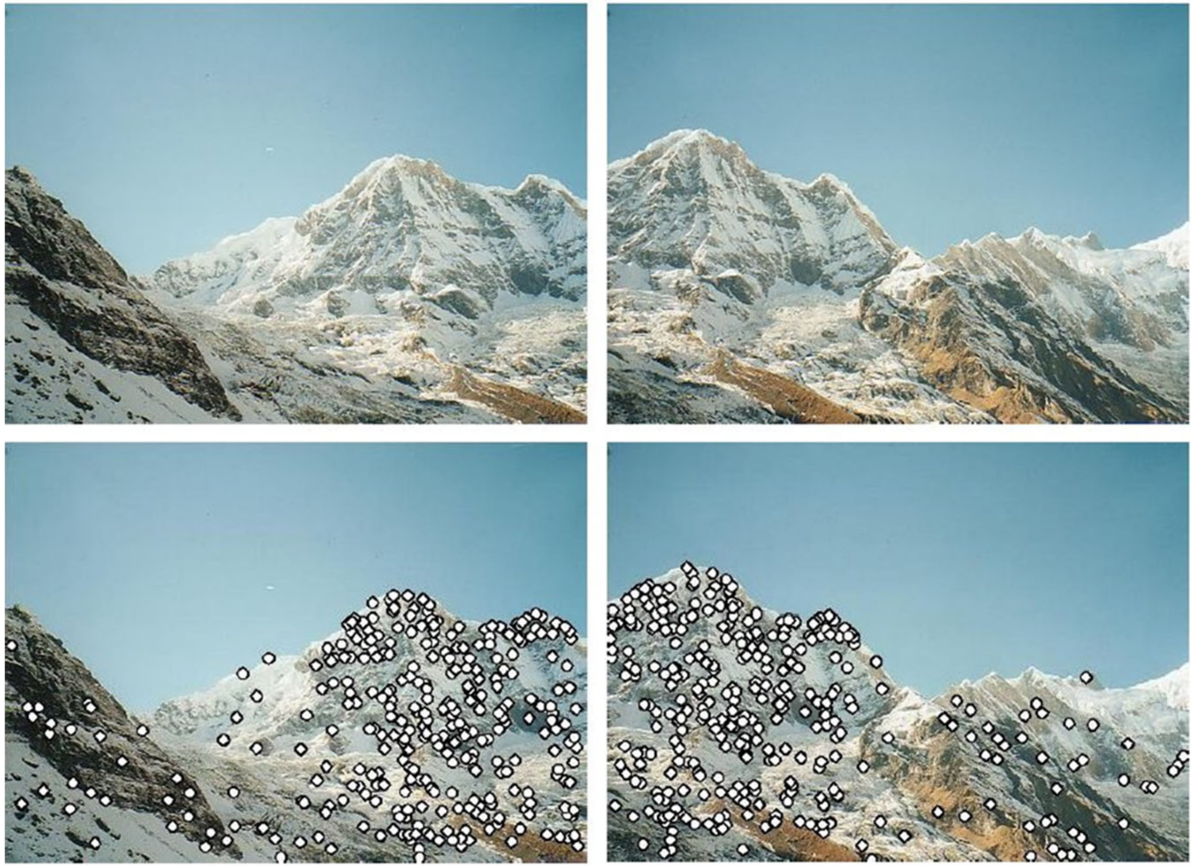


151 inliers

# Robust estimation



**RANSAC estimation: Panoramic image construction**

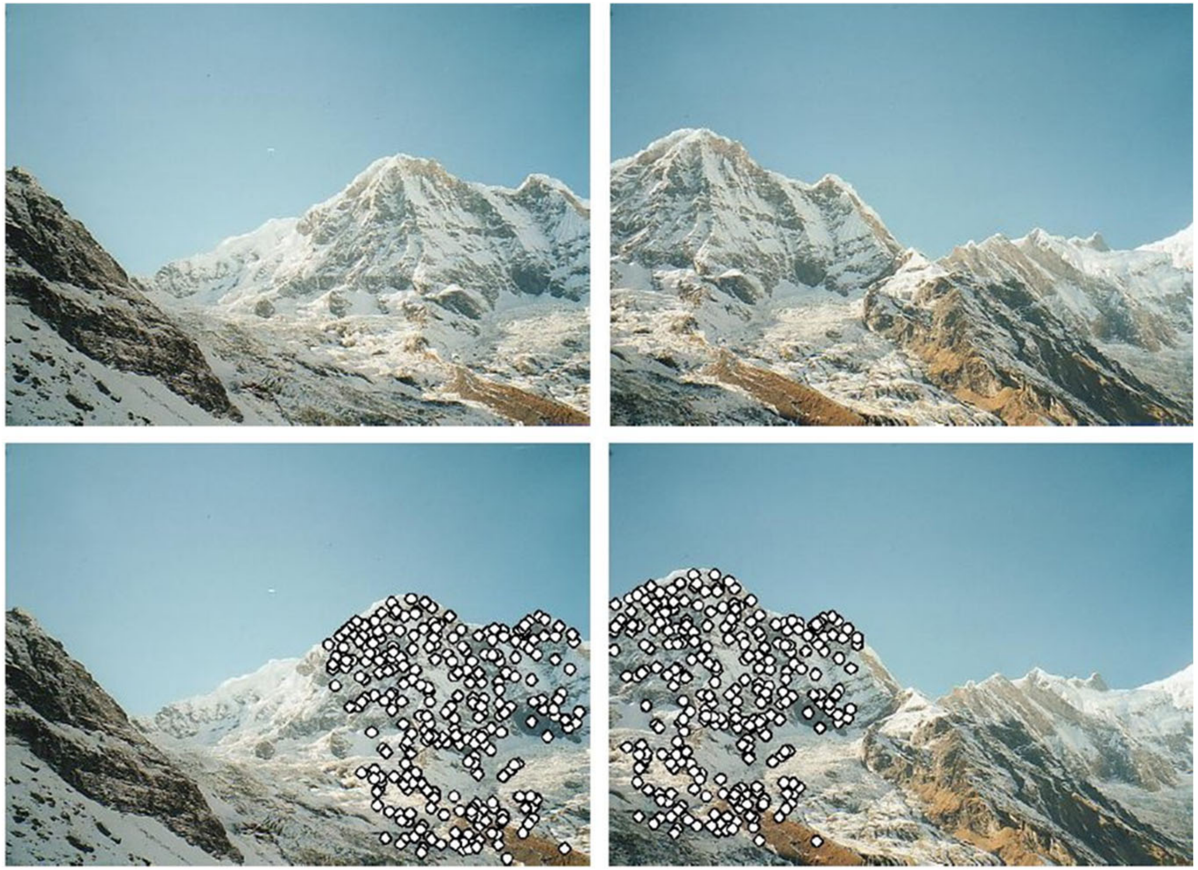


_ TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

# Robust estimation

Master Universitario en Ingeniería de Telecomunicación

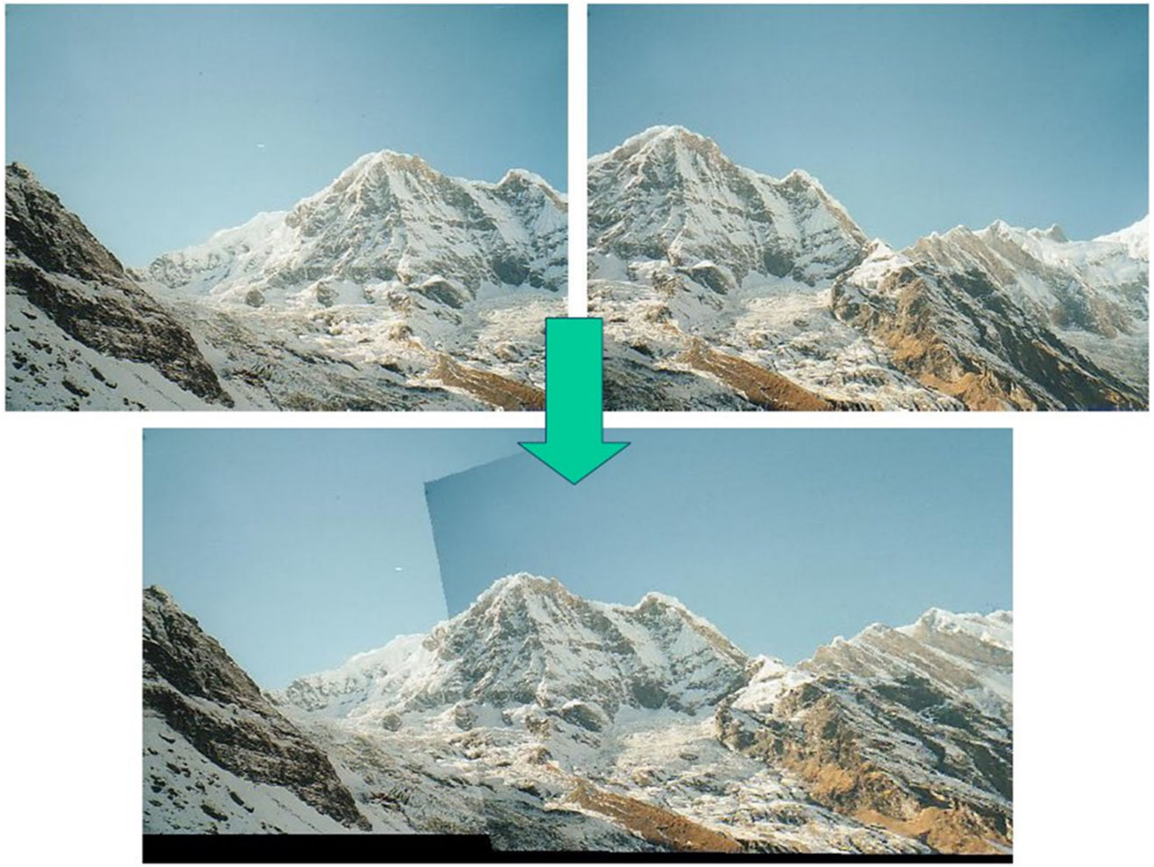
**RANSAC estimation: Panoramic image construction**



# Robust estimation



**RANSAC estimation: Panoramic image construction**



# SIFT Detector



**SIFT (S**cale **I**nvariant **F**eature **T**ransform**)**

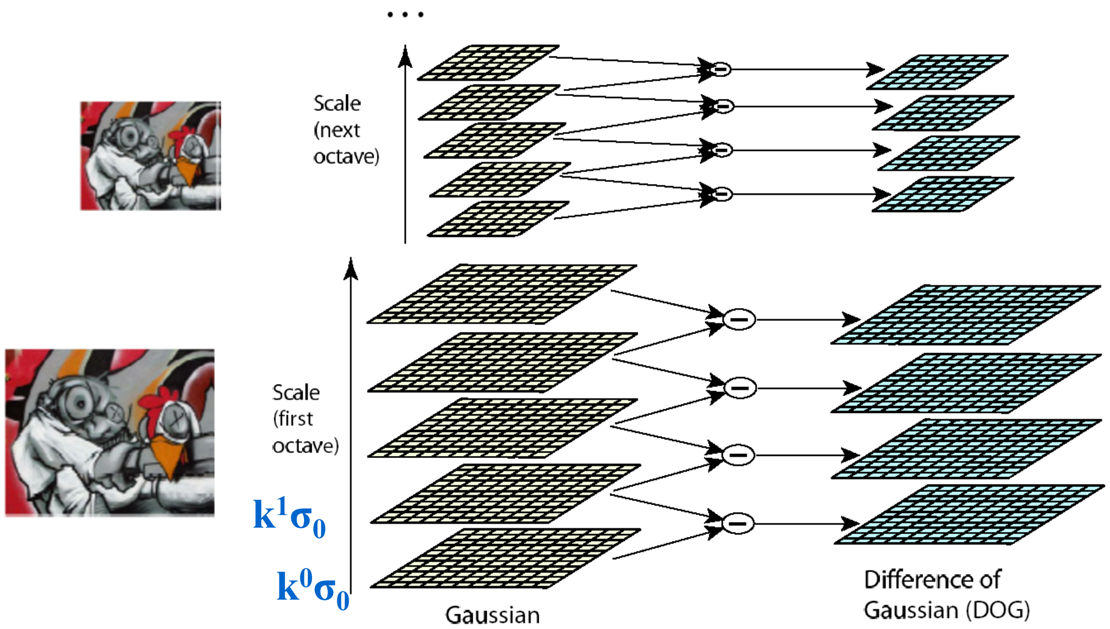
- Invariant descriptor to position, scale, rotation, illumination, contrast and point of view.

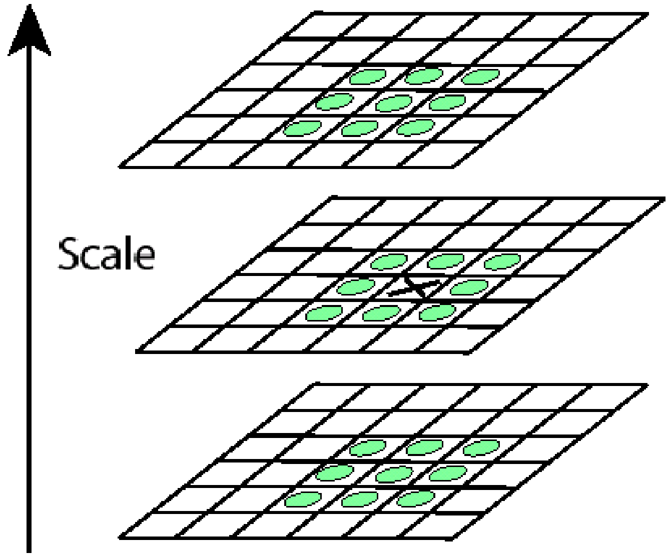
- **Algorithm**

  1. Detection of extremes in scale and space: Extract rotation and scale invariant points of interest (keypoints).
  2. Eliminate 'weak' keypoints.
  3. Orientation assignment: Assign one or more orientations to each point of interest.
  4. Keypoint descriptor: Use local gradients at the selected scale.

***D. Lowe, "Distinctive Image Features from Scale-Invariant Keypoints", International Journal of Computer Vision, 60(2):91-110, 2004.***

# SIFT: Extrema detection




$$k^S = 2^S$$

$$D(x, y, \sigma) = L(x, y, k\sigma) - L(x, y, \sigma)$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y,)$$

TELECOM ESCUELA TÉCNICA VLC SUPERIOR DE UPV INGENIEROS DE TELECOMUNICACIÓN

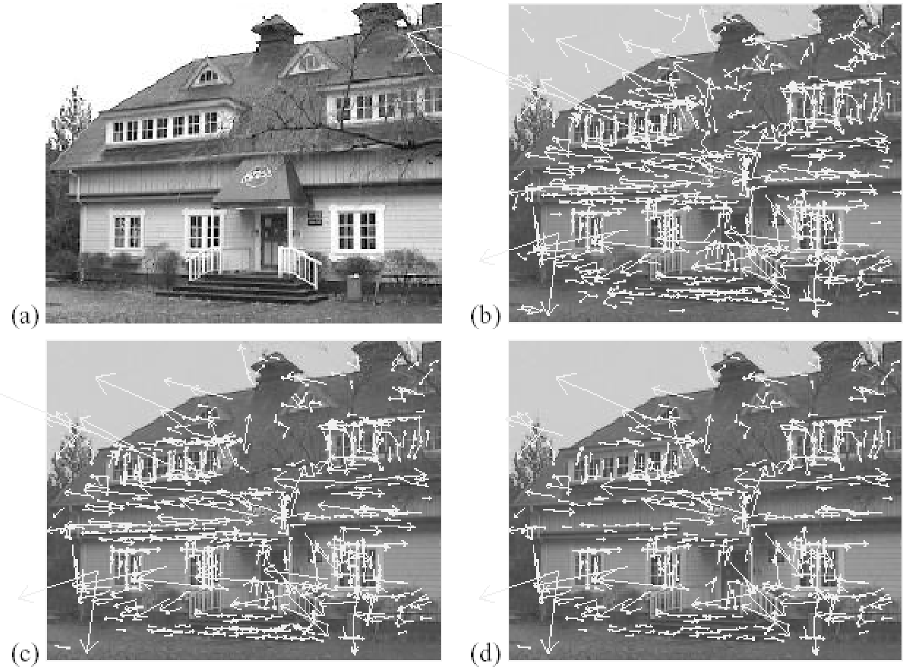
# SIFT: Weak keypoints elimination

MUIT Master Universitario en Ingeniería de Telecomunicación

**Weak keypoints**

- keypoints with low contrast (<0.03).
- Bad edge

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r} \qquad \mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$



(a)Image 233x189
(b) 832 extreme DoG
(c) 729 left after low contrast thresholding
(d) 536 after Hessian ratio

# SIFT Descriptor

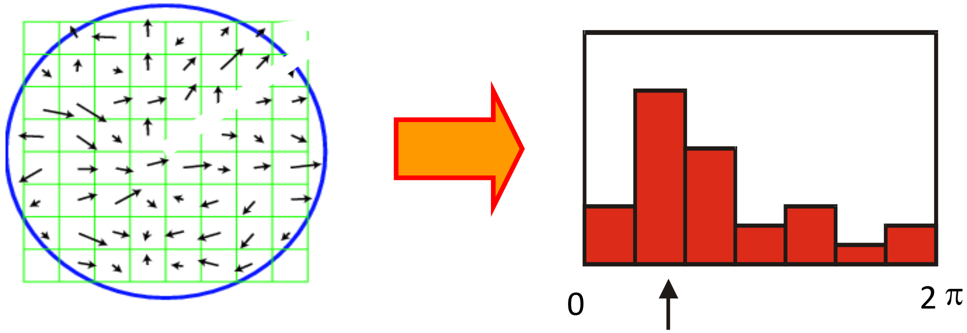


## Orientation assignment

- Histogram of gradient directions for each keypoint.

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y, )$$

$$m(x,y) = \sqrt{(L(x+1,y)-L(x-1,y))^2 + (L(x,y+1)-L(x,y-1))^2}$$
$$\theta(x,y) = a\tan 2((L(x,y+1)-L(x,y-1))/(L(x+1,y)-L(x-1,y)))$$



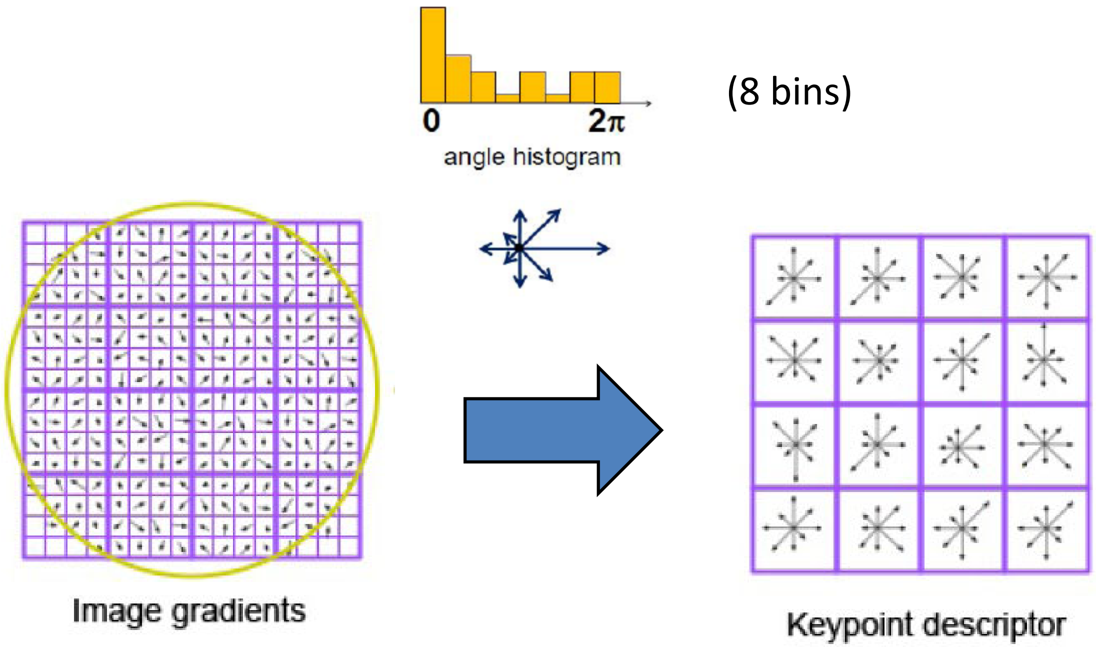

- Histogram weighted by the magnitude of the gradient and by a Gaussian function with σ=1.5 s.

# SIFT descriptor



**Orientation assignment**

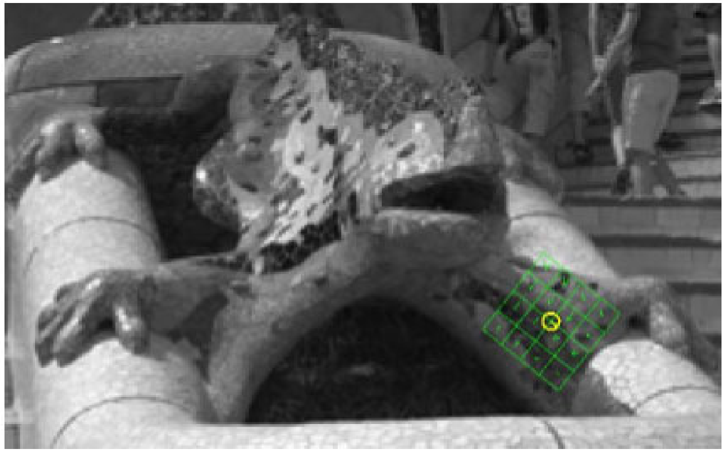
1. 16 x16 window around each keypoint.
2. Divide into 4x4 cells.
3. Calculate the histogram in each cell (partial vote).

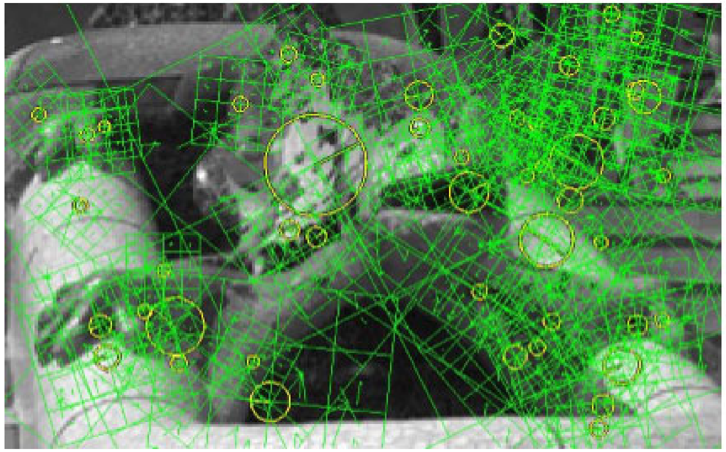



(8 bins)

16 histograms x 8 orientations
= 128 features

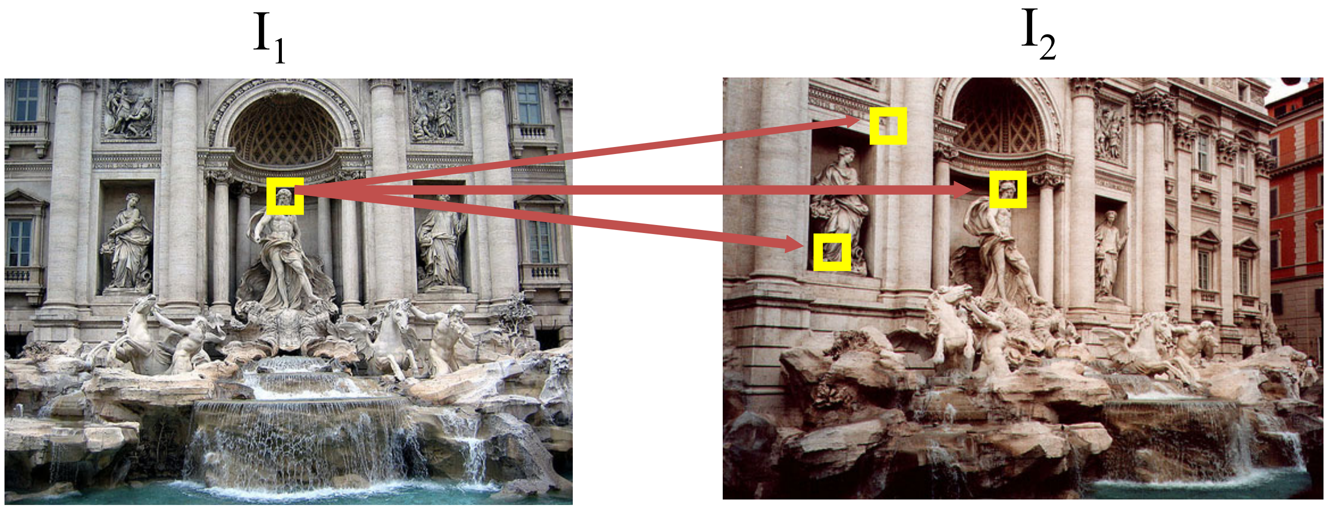
# SIFT Detector





# SIFT: Matching of features

Descriptor with mínimum distance



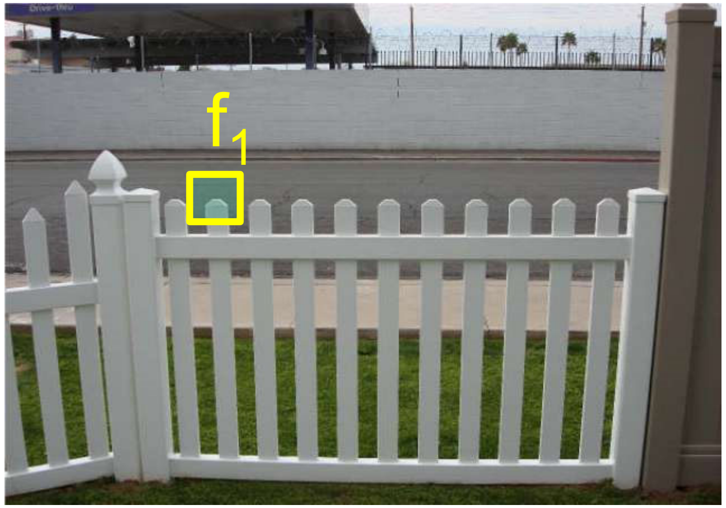

$$SSD(f_1, f_2) = \sum_{i=1}^{N} (f_{1i} - f_{2i})^2$$

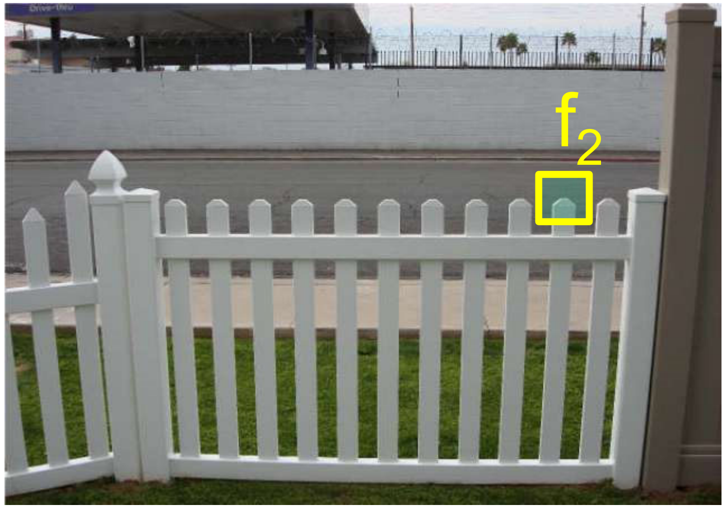
Problems with ambiguities

# SIFT: Matching of features

Problems with ambiguities

$I_1$ $I_2$



# Index

1. Introduction
2. Statistical descriptors
3. Local Binary Patterns
4. Histogram of Oriented Gradients
5. SIFT
6. Gabor Filters

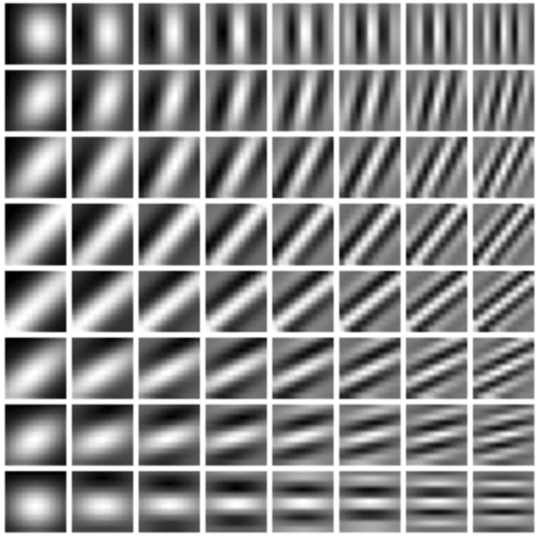
# Gabor filters

- Bank of linear filters.
- Each filter analyzes whether there is any specific frequency content in the image in specific directions.
- Image analysis with Gabor filters is thought by some to be similar to perception in the human visual system.

$$G(x,y) = A\exp\left(-\alpha\widetilde{x}^2 - \beta\widetilde{y}^2\right)\sin\left(\omega\widetilde{x} + \phi\right)$$

where

$$\widetilde{x} = (x - x_0)\cos(\theta) + (y - y_0)\sin(\theta)$$
$$\widetilde{y} = -(x - x_0)\sin(\theta) + (y - y_0)\cos(\theta).$$



Examples of Gabor filters defined by (10.6). The orientation angle $\theta$ varies from $0$ in the top row to $\pi/2$ in the bottom row, whereas the frequency varies from $\omega = 1$ in the left column to $\omega = 10$ in the right column.

**Source: Deep Learning: Foundations and Concepts. Christopher M. Bishop and Hugh Bishop**