***Integral sliding mode control design for system***

Let us consider a class of nonlinear dynamics of system as

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|  | (1)  (2) |

In order to design the nonlinear integral sliding mode control (ISMC) for (1) and (2), the dynamics described in (1) and (2) can be written as

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| --- | --- |
|  | (3) |

The ISMC method is presented to achieve the control objective . Before the discussion, the tracking error is defined as . Instead of using a classical linear sliding surface, we introduce nonlinear sliding manifold- sing integral sliding mode as

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| --- | --- |
|  | (4) |
|  | (5) |

where **is a positive constant, and  is the integration of and has initial valueIf  is always kept at zero such that , then the sign integration (5) yields

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| --- | --- |
|  |  |

Thus,  will converge to zero in finite time

|  |  |
| --- | --- |
|  | (6) |

Furthermore, the convergence of the tracking error  is accomplished in the same finite time (6) due to the fact  on the sliding surface .

**Remark 1**. For smoother control signals, the sign integrator (5) of the integral sliding manifold can be modified into

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|  | (7) |

where  is a positive constant. On the sliding surface ****, the tracking error  and integration  are also assured to be finite time convergence stable from argument. The time  is spend for error convergence.

Next, to keep the system on the integral terminal sliding surface , we take the time derivative of  as

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|  | (8) |

**Theorem 1.** For the dynamic model of system, described by (3), by designing the control input as

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| --- | --- |
|  | (9) |

and choosing control gain  as

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|  | (10) |

with  and , then the control errors  and  are guaranteed with finite time convergence stability.

**Proof.** Take a Lyapunov function candidate as

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|  | (11) |

Taking the time derivative of  gives:

|  |  |
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|  | (12) |

by substituting (9) into (12), one can obtain

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|  | (13) |

Applying the equalities

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|  |  |

into (13), yields

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|  | (21) |

We have chosen . according to ,  is assured once the control gain  is set to (10). since  and , the system always kept on the sliding manifould . As a result, the tracking errore  and  converge to zero in finite time (6).