

Section Content:

✚ Asymptotic Analysis

Algorithms Efficiency

Program efficiency is measured by how much of various types of resources it consumes.

The main measures are: -

- Speed or running time
- Space (memory)
- Power consumption

Big O notation

This Big(O) examines the rate of growth of a function by comparing it with some standard functions whose rate of growth is known.

$$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(n^3) < O(n^k) < O(Kn) < O(n!)$$

Finding Big O: -

- Keep the fastest growing term and discard the lower terms and constants
- Ignore coefficients

EX: -

$$\begin{aligned} \text{✚ } f(n) &= 3n^2 + 4n + 7 \\ g(n) &= n^2 = O(n^2) \end{aligned}$$

$$\begin{aligned} \text{✚ } f(n) &= 3n + 5n^2 + 7n^3 + 2n \\ &= O(2n) \end{aligned}$$

$$\begin{aligned} \text{✚ } f(n) &= 4n + 6n + 9n^5 \\ &= O(6n) \end{aligned}$$

$$\begin{aligned} \text{✚ } f(n) &= 7n + 6n + n! \\ &= O(n!) \end{aligned}$$

Big O analysis of Algorithms Calculating Time complexity

Running time is proportional to the number of primitive operations executed during run time.

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1. Int x=2;           constant time c1
   For (int i=0; i<=n; i++) n
   Sum=sum+i;         constant time c2
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$$\begin{aligned} T(n) &= c1 + n + c2 \\ &\text{Is } O(n) \end{aligned}$$

2. For (int i=0; i<=n; i++) n
 For (int j=0; j<=n; j++) n
 Print i+j;

$T(n)=n*n$
 Is $O(n^2)$

3. For (int i=0; i<=n; i++) n
 Print i;
 For (int j=0; j<=n; j++) n
 For (int k=0; k<=n; k++) n
 Print j+k;

$T(n)=n+n*n$
 Is $O(n^2)$

4. For (int i=0; i<=n; i++) n
 Print i;
 For (int j=0; j<=n; j++) n
 For (int k=0; k<=n; k++) n
 For (int l=0; l<=n; l++) n
 Print j+k+l;

$T(n)=n+n*n*n$
 Is $O(n^3)$

5. Int i;
 i=1 constant time c_1
 For (i; i<=n; i*2) $\log_2 n$
 Print i; constant time c_2

$T(n)=c_1 + \log_2 n + c_2$
 Is $O(\log n)$

6. For (int i=n/2; i<=n; i++) $n/2$
 For (int j=0; j<=n; j=j*2) $\log n$
 For (int k=0; k<=n; k=k*2) $\log n$
 Print i+j+k; constant time c_1

$T(n) = n/2 * \log n * \log n + c1$
Is $O(n \log n)^2$