(HW4)

(7.4)

$$\gamma_{kn}^{(t)} := \mathbb{P}(Z_n = k \mid x_n ; \theta^{(t)})$$

$$= \frac{\mathbb{P}(Z_n=k ; \theta^{(t)})\, \mathbb{P}(x_n \mid Z_n = k ; \theta^{(t)})}{\mathbb{P}(x_n ; \theta^{(t)})}$$

(Bayes formula)

$$= \frac{\mathbb{P}(Z_n=k ; \theta^{(t)})\, \mathbb{P}(x_n \mid Z_n=k ; \theta^{(t)})}{\sum_{i=1}^{K} \mathbb{P}(Z_n=i ; \theta^{(t)})\, \mathbb{P}(x_n \mid Z_n = i ; \theta^{(t)})}$$

(Total prob formula since $\{Z_n = i\}_{1 \leq i \leq K}$ is a partition of the prob space)

$$\Rightarrow \gamma_{kn}^{(t)} = \frac{\pi_k^{(t)} \mathcal{N}(x_n ; \mu_k^{(t)}, \Sigma_k^{(t)})}{\sum_{i=1}^{K} \pi_i^{(t)} \mathcal{N}(x_n ; \mu_i^{(t)}, \Sigma_i^{(t)})}$$

(9.2)

Let us suppose the family $(P\mu)_{P \in \mathcal{P}}$ is independent. Thus, one can write:

$$EPLL(\mu) = \sum_{P \in \mathcal{P}} \log[P(P\mu)]$$

$$= \log\left(\prod_{P \in \mathcal{P}} \mathbb{P}(P\mu)\right)$$

$$= \log\Big(\underbrace{\mathbb{P}\Big(\bigcap_{P \in \mathcal{P}} P\mu\Big)}_{\text{representing the likelihood of the image } \mu \text{ since } (P\mu)_{P \in \mathcal{P}} \text{ forms } \mu.}\Big)$$

Hence, indeed, EPLL can be considered as a log-likelihood of the image μ. Nevertheless, we should pinpoint this is only true for independent patches. This may not be true in practice since patches may overlap.

(9.3)

$$\min_{\varphi_P} E(\varphi_P)$$

$$\Leftrightarrow \min_{\varphi_P} -\log\left(e^{-\beta \frac{\|PU - \varphi_P\|^2}{2\sigma^2}} \mathbb{P}(\varphi_P)\right)$$

$$\Leftrightarrow \max_{\varphi_P} \underbrace{e^{-\beta \frac{\|PU - \varphi_P\|^2}{2\sigma^2}}}_{\propto\, \mathbb{P}(PU \mid \varphi_P)} \mathbb{P}(\varphi_P)$$

constant independent of φ_P

$$\Leftrightarrow \max_{\varphi_P} \mathbb{P}(PU \mid \varphi_P)\, \mathbb{P}(\varphi_P)$$

$$\overset{\text{Bayes}}{\Leftrightarrow} \max_{\varphi_P} \mathbb{P}(\varphi_P \mid PU) \underbrace{\mathbb{P}(PU)}_{\text{independent of } \varphi_P}$$

$$\Leftrightarrow \max_{\varphi_P} \mathbb{P}(\varphi_P \mid PU)$$

Indeed, minimizing $E(\varphi_P)$ is $\Leftrightarrow$ to maximizing $\mathbb{P}(\varphi_P \mid PU)$ which is a MAP assuming φ obeys a GMM.

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