Data Structures and Algorithms
(CS221)

Computational Complexity of Algorithms

Scenario!!

Approach 1 (Slow Way)

Which approach do you think is faster?

How can we measure "fast" in a systematic way?

"Imagine you have a huge list of names (e.g., student records). If I ask you to find a specific name, how would you do it?

Approach 2 (Faster Way): If the list is sorted, start from the middle and eliminate half the list each time.

Scenario!!

"Computers can solve problems in different ways, but not all methods are efficient. As input size grows, some algorithms become painfully slow. Our goal is to understand how an algorithm's execution time grows with input size."

"Would you rather use a method that takes 1 second or 1 hour for the same problem?"

Computational Complexity

Computational complexity indicates how much effort is needed to apply an algorithm or how costly it is.

Computational complexity refers to the study of the efficiency of algorithms in terms of the resources they consume.

Effort/efficiency criteria: **time and space**

Computational Complexity

Asymptotic Notation

Asymptotic notation is used in computer science to describe the efficiency of algorithms, specifically their time and space complexity

It helps to understand how an algorithm performs as the input size grows.

expressing the *main component* of the cost of an algorithm, using idealized units of computational work

Computational Complexity

Asymptotic Notation

Simplify analysis

Compare algorithms

Why is Asymptotic Notation Important?

Focus on growth rates

Asymptotic Notation

Big O Notation (O)

Big O tells us the worst-case scenario (how fast the algorithm will run in the worst case)

the *upper bound* of an algorithm's time or space complexity

It provides a worst-case scenario for the growth rate of the algorithm

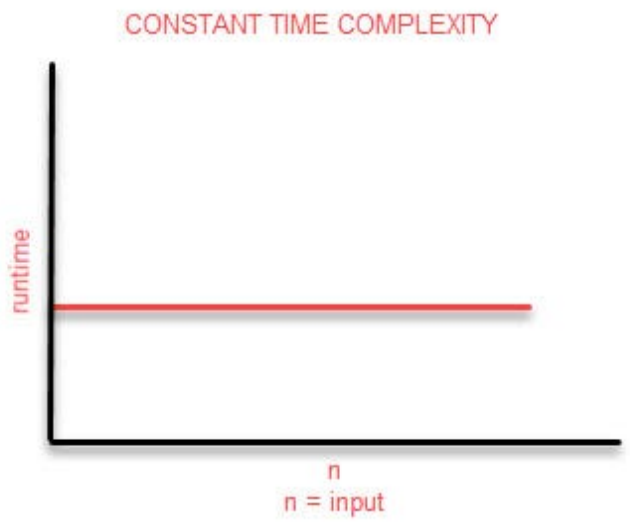
Time Complexity

Constant time complexity- O(1)

An algorithm has **O(1) (constant time complexity)** if its execution time does not depend on the input size n. It always takes a fixed amount of time to complete, regardless of how large the input is.

Looking up a student's name in a class register by roll number

"What is the name of the student with roll number 23?



If students' names are stored in a **list (array) by roll number**, finding a student name is an **O(1) operation**

Since the roll number directly maps to the index in an array, we retrieve the name instantly, no matter how many students are in the register

Time Complexity

Constant time complexity

The function **getFirstElement()** accesses the first element of the array in **O(1) time**.

No matter how large the array is, accessing arr[0] always takes **constant time**

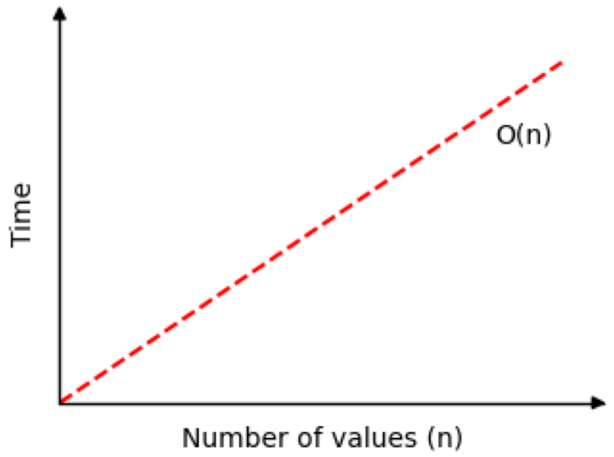
```
int getFirstElement(int arr[], int size) {
    return arr[0];
}

int main() {
    int numbers[] = {10, 20, 30, 40, 50};
    int size = sizeof(numbers) /
sizeof(numbers[0]);
    cout << "First element: " <<
getFirstElement(numbers, size) << endl;
    return 0;
}
```

Time Complexity

Linear Time Complexity – O(n)

An algorithm has **O(n) (linear time complexity)** if the execution time grows **proportionally** with the input size n

double the input size, the execution time will also **double**.



Time Complexity

Linear Time Complexity – O(n)

Checking Student Attendance in a Class

Suppose you have a class list with **100 students**.

You need to **call each student name one by one** to check attendance.

if the class size grows to **200 students**, the time taken will also **double**.

Time Complexity

Linear Time Complexity – O(n)

The function **printArray()** loops through **each element** in the array

.If size = 5, the loop runs **5 times**.

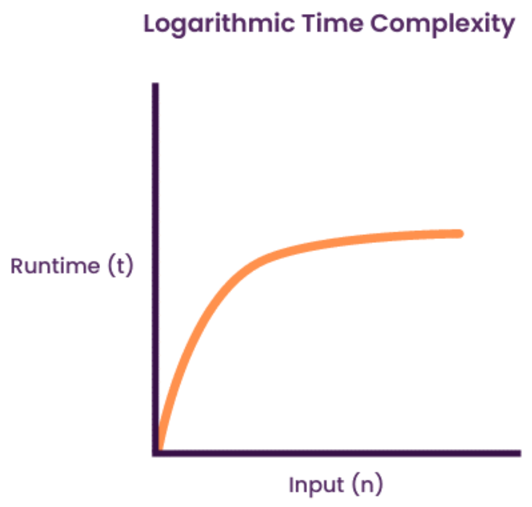
If size = 1000, the loop runs **1000 times**

```
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++) {
        cout << arr[i] << " ";
    }
    cout << endl;
}

int main() {
    int numbers[] = {10, 20, 30, 40, 50};
    int size = sizeof(numbers) /
sizeof(numbers[0]); // Calculate array
size

    printArray(numbers, size);
    return 0;
}
```

Time Complexity

Logarithmic Time Complexity – Binary Search

if the number of operations **reduces exponentially** as the input size increases



Time Complexity

Logarithmic Time Complexity – Binary Search

Finding a Video on YouTube's Search Algorithm

When you search for a video on **YouTube**, the platform doesnt check every video (**O(n)** search). Instead

It first **narrows down** videos based on your query

It ranks them based on **relevance, popularity, and personalization**

The search space is **continuously reduced**, making it **logarithmic**

Time Complexity

Logarithmic Time Complexity – Binary Search

```
int binarySearch(int arr[], int size, int
target) {
    int left = 0, right = size - 1;

    while (left <= right) {
        int mid = left + (right - left) / 2;
        if (arr[mid] == target)  // Found the
target
            return mid;
        else if (arr[mid] < target)
            left = mid + 1;  // Search the right
half
        else
            right = mid - 1; // Search the left
half
    }

    return -1; // Not found
}
```

```
int main() {
    int numbers[] = {2, 5, 8, 12, 16, 23,
38, 45, 56, 72, 91};
    int size = sizeof(numbers) /
sizeof(numbers[0]);

    int target = 23;
    int result = binarySearch(numbers,
size, target);

    if (result != -1)
        cout << "Element found at index: "
<< result << endl;
    else
        cout << "Element not found!" <<
endl;

    return 0;
}
```

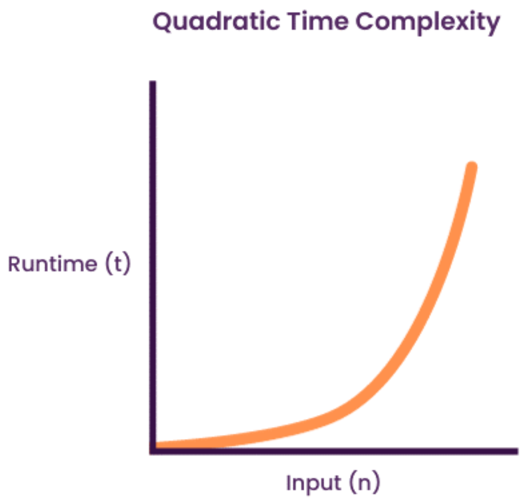
Time Complexity

Quadratic Time Complexity – O(n²)

If the number of operations **grows quadratically** with the input size

If the input size **doubles**, the number of operations **quadruples**

If the input size **triples**, the number of operations **becomes nine times larger**



Time Complexity

Quadratic Time Complexity – O(n^2)

Example: Imagine a classroom where every student has to **shake hands** with every other student.

If there are **5 students**, there are **5 × 5 = 25** possible interactions

If there are **10 students**, there are **10 × 10 = 100** interactions.

Thats quadratic growth!

Time Complexity

Quadratic Time Complexity – O(n²)

```
void bubbleSort(int arr[], int n) {
    for (int i = 0; i < n - 1; i++) {   //
Outer loop (n)
        for (int j = 0; j < n - i - 1;
j++) {  // Inner loop (n)
            if (arr[j] > arr[j + 1]) {  //
Swap if out of order
                swap(arr[j], arr[j + 1]);
            }
        }
    }
}
```

```
int main() {
    int arr[] = {5, 2, 9, 1, 5, 6};
    int n = sizeof(arr) / sizeof(arr[0]);

    bubbleSort(arr, n);

    cout << "Sorted Array: ";
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";

    return 0;
}
```

Time Complexity

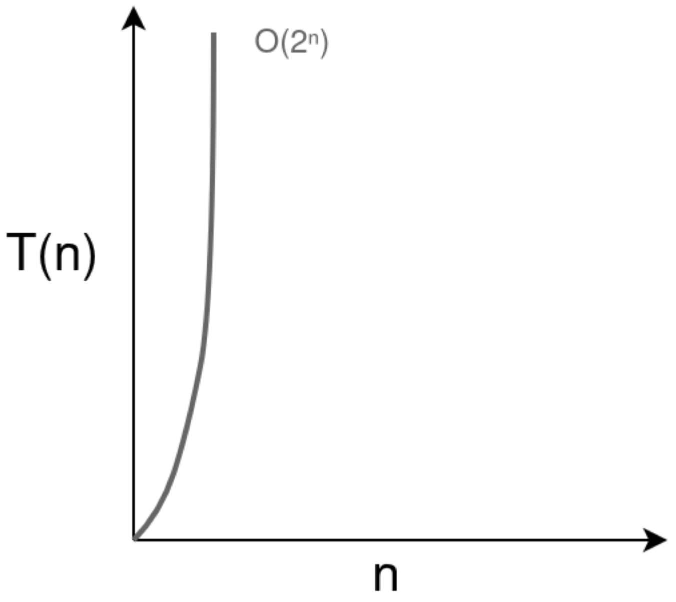
Exponential Time Complexity – $O(2^n)$

if the number of operations **doubles** with each additional input element

If the input size **increases by 1**, the time taken **doubles**

If the input **doubles**, the time taken **grows exponentially** (much faster than quadratic $O(n^2)$)

Example:
For **n = 10**, operations = 2^{10} = **1,024**.
For **n = 20**, operations = 2^{20} = **1,048,576** 😱.



Exponential algorithms quickly become impractical for large inputs!

Time Complexity

Exponential Time Complexity

Password Cracking (Brute Force Attack) 🔓

A hacker trying **every possible password** needs to check **2^n** combinations.

Thats why long passwords (e.g., 12+ characters) are much harder to crack

Time Complexity

Factorial Time Complexity – O(n!)

if the number of operations **grows factorially** as the input size increases

Factorial Growth Example:
If **n = 5**, operations = **5! = 5 × 4 × 3 × 2 × 1 = 120**

If **n = 10**, operations = **10! = 3,628,800** 😱

If **n = 20**, operations = **20! = 2,432,902,008,176,640,000** (Massive explosion!)

Factorial time complexity is the slowest-growing complexity and quickly becomes infeasible for large inputs.

Time Complexity

Factorial Time Complexity – O(n!)

Assigning Jobs to Workers (Job Scheduling)

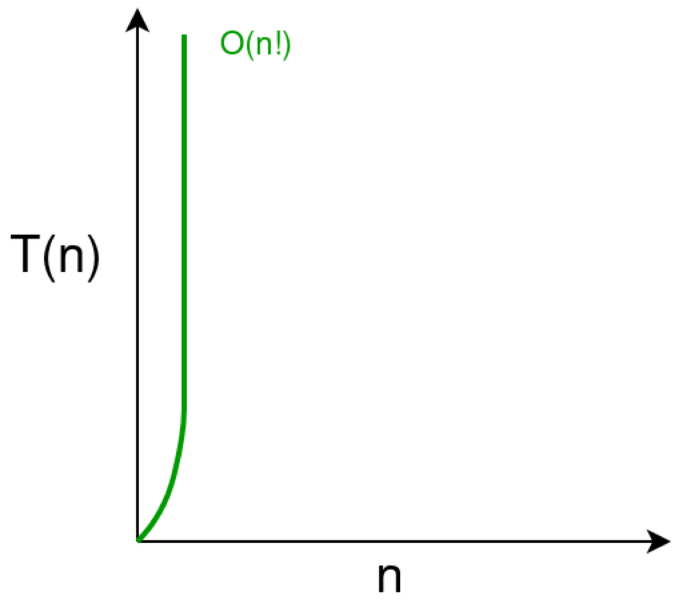
If you have **n workers** and **n tasks**, and eac worker can do any task,

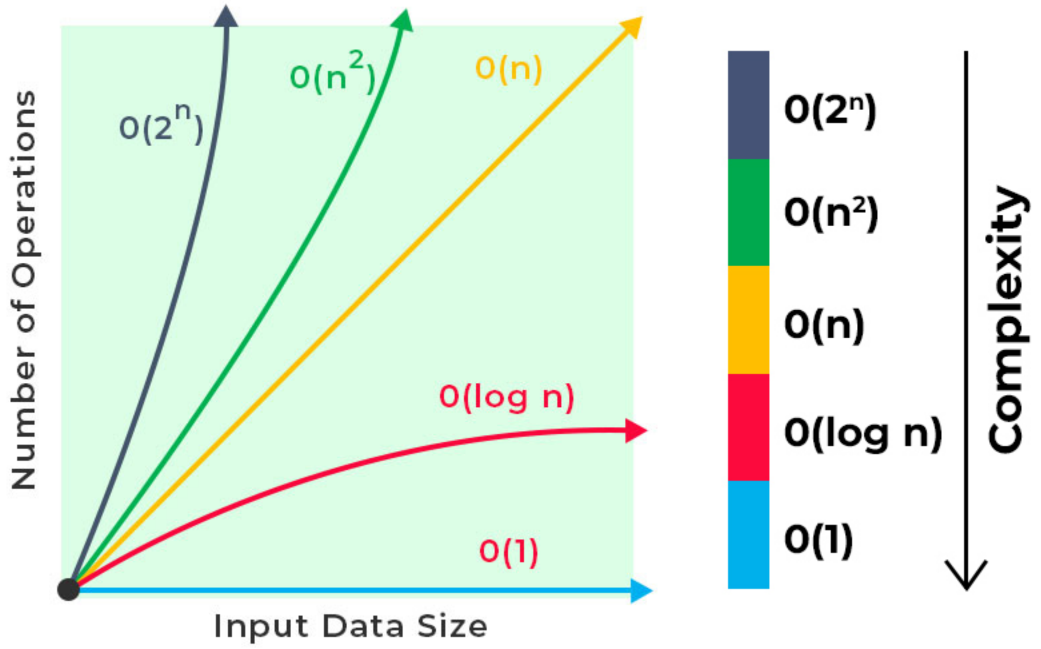
The number of ways to assign jobs = **n!** (factorial complexity)

Organizing People in a Photo Shoot

If you have **n people** and must arrange them in a line,

The number of different ways to arrange them is **O(n!)**.




O(2^n)
O(n^2)
O(n)
O(log n)
O(1)
Number of Operations
Input Data Size
O(2ⁿ)
O(n²)
O(n)
O(log n)
O(1)
Complexity

Questions?

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