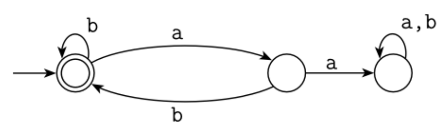
INHN0013 Information Theory & Theory of Computation
Exercise Sheet 4

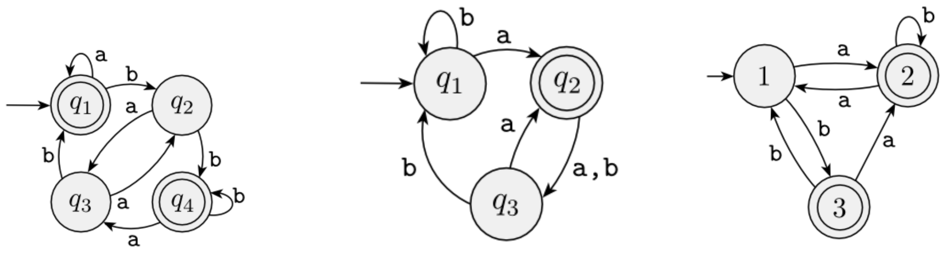
Assigned: Wednesday November 5 2025
Due: Wednesday November 12 2025

You should know how to solve these problems and ideally you will solve them in the allotted one week. Some of the specific problems below might appear on the optional or actual final exam. You will see the solutions for instances that appear on the optional exams. Many of the problems (although not necessarily every single subproblem of every problem) will be discussed in the tutorials.

1. Prove or disprove the following claims:
   (a) The class of context free languages is closed under union.
   (b) The class of context free languages is closed under intersection.
   (c) The class of context free languages is closed under concatenation.
2. For each of the languages below indicate whether or not it is Context Free. In each case, prove your answer: if you believe L_i is a CFL, give a CFG or a PDA for it and explain why the construction works; if you believe L_j is not a CFL, then prove this using the Pumping Lemma.
   (a) $L_1 = \{a^n b^n a^n b^n | n \geq 0\}$
   (b) $L_2 = \{w | w \in \{a,b\}^*$ and twice the number of a's is equal to three times the number of b's$\}$
   (c) $L_3 = \{a^n b a^{2n} b a^{3n} | n \geq 0\}$
   (d) $L_4 = \{w\#u |$ where $u, w \in \{a,b\}^*$, and w is a substring of $u\}$
   (e) $L_5 = \{www | w \in \{a,b\}^*\}$
   (f) $L_6 = \{a^i b^j a^i b^j | i,j \geq 1\}$
   (g) $L_7 = \{a^i b^j | i = kj$ for some positive integer k$\}$ (Hint: Use PL. Let m,n be the number of a's and b's in vy from uvxyz. Choose a prime q > p and show $q - 1 < \frac{q^4+m}{q^3+n} < q + 1$ and derive a contradiction)
3. Design the simplest (highest numbered) grammar for $L = \{a^n b^n c^n | n \geq 1\}$
4. Consider the grammar below and (i) describe the language L that it generates, (ii) specify what type of grammar this is and what type of language L is:
   $S \to EAE$
   $E \to ER$
   $RA \to AAR$
   $RE \to E$
   $EA \to A$
   $AE \to A$
   $A \to a$



5. Convert the following FAs into right-linear Type3 grammars:



6. Convert the following right-linear Type3 grammars into NFAs and briefly describe the corresponding regular languages:

   - $S \to aA|a$
     $A \to aA|bB$
     $B \to bB|b$
   - $S \to aX|a$
     $X \to aY|aZ$
     $Y \to a$
     $Z \to a$
   - $S \to bA|aB|\epsilon$
     $A \to bA|b$
     $B \to aB|a$

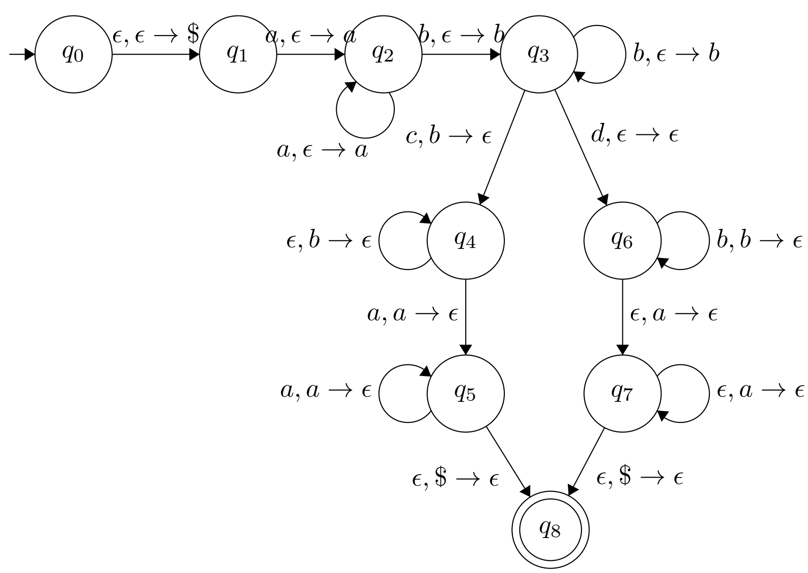
7. Convert the CFG below to an equivalent PDA.
   $S \to XSX|R$
   $R \to aTb|bTa$
   $T \to XTX|X|\epsilon$
   $X \to a|b$

8. (a) Let C be a context-free language and R be a regular language. Prove that the language C $\cap$ R is context free.

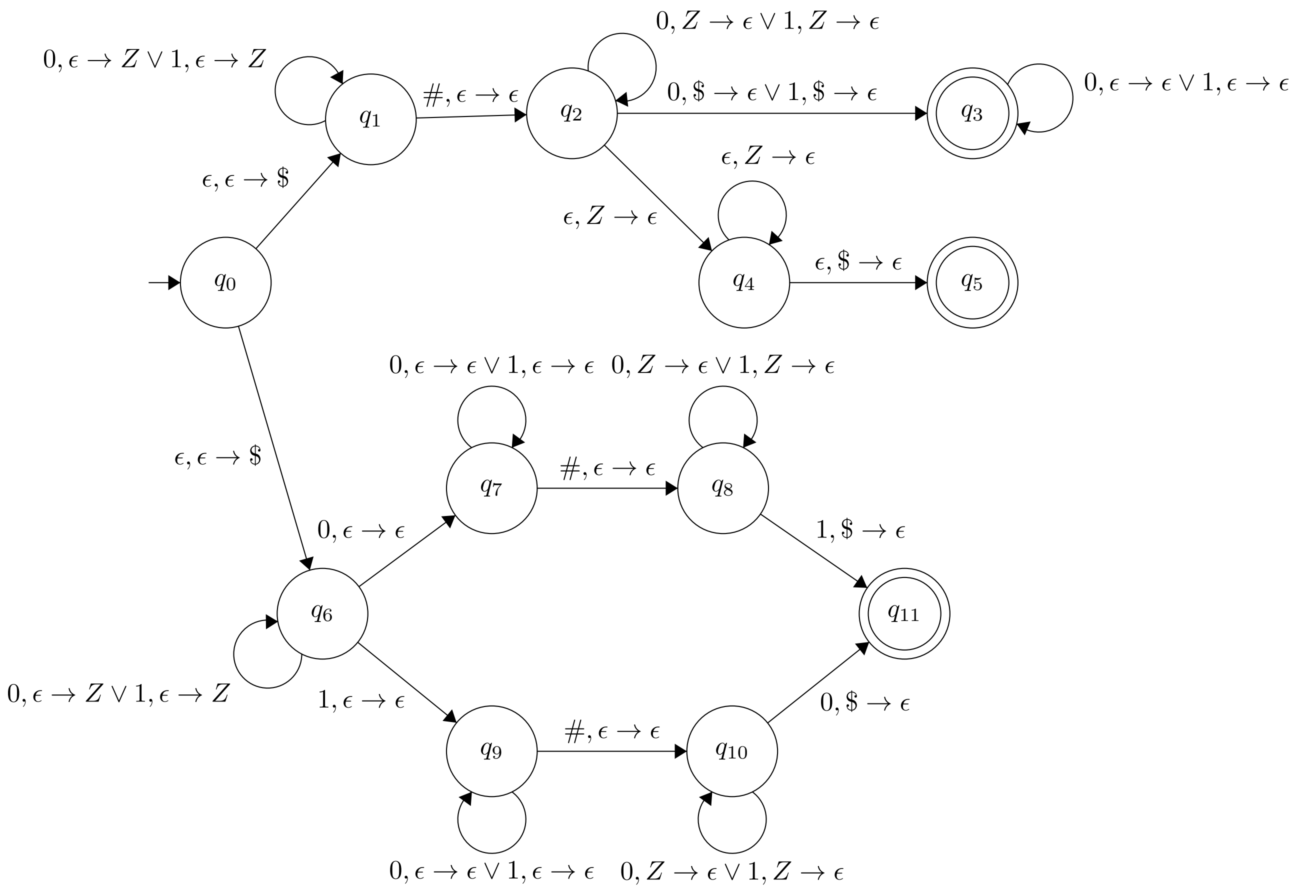
   (b) Let $A = \{w|w \in \{a,b,c\}^*$ and w contains equal numbers of a's, b's, and c's$\}$. Use part (a) to show that A is not a CFL.

9. Describe the language recognized by each of the following PDAs and give brief explanations why.

   (a)



(b)


$0, \epsilon \to Z \vee 1, \epsilon \to Z$
$\#, \epsilon \to \epsilon$
$0, Z \to \epsilon \vee 1, Z \to \epsilon$
$0, \$ \to \epsilon \vee 1, \$ \to \epsilon$
$0, \epsilon \to \epsilon \vee 1, \epsilon \to \epsilon$
$\epsilon, \epsilon \to \$$
$\epsilon, Z \to \epsilon$
$\epsilon, Z \to \epsilon$
$\epsilon, \$ \to \epsilon$
q_0
q_1
q_2
q_3
q_4
q_5
$0, \epsilon \to \epsilon \vee 1, \epsilon \to \epsilon$
$0, Z \to \epsilon \vee 1, Z \to \epsilon$
$\epsilon, \epsilon \to \$$
$\#, \epsilon \to \epsilon$
q_7
q_8
$0, \epsilon \to \epsilon$
$1, \$ \to \epsilon$
q_6
q_{11}
$0, \epsilon \to Z \vee 1, \epsilon \to Z$
$1, \epsilon \to \epsilon$
$0, \$ \to \epsilon$
$\#, \epsilon \to \epsilon$
q_9
q_{10}
$0, \epsilon \to \epsilon \vee 1, \epsilon \to \epsilon$
$0, Z \to \epsilon \vee 1, Z \to \epsilon$