INHN0013 Information Theory & Theory of Computation
Exercise Sheet 1

Assigned: Wednesday October 15 2025
Due: Wednesday October 22 2025

You should know how to solve these problems and ideally you will solve them in the allotted one week. Some of the specific problems below might appear on the optional or actual final exam. You will see the solutions for instances that appear on the optional exams. Many of the problems (although not necessarily every single subproblem of every problem) will be discussed in the tutorials.

1. Prove the following
   (a) $2 + 5 + 8 + ... + (3n - 1) = (n/2)(3n + 1)$, for $n \geq 1$.
   (b) $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$, for $n \geq 0$.
   (c) $n(n^2 + 5)$ is a multiple of 6 for $n \geq 0$.
2. Let $S_1 = \{A, C, T, G\}$ and $S_2 = \{A, G\}$
   (a) Is $S_1 \subset S_2$? Is $S_1 \subseteq S_2$? Is $S_2 \subset S_1$? Is $S_2 \subseteq S_1$?
   (b) What is $S_1 \cup S_2$?
   (c) What is $S_1 \cap S_2$?
   (d) What is $S_1 \times S_2$?
3. Let X and Y be two sets with n and m elements, respectively.
   (a) How many elements are there in $X \times Y$ (the cross product of X and Y)? Why?
   (b) How many elements are there in 2^X (also written as $P(X)$, or the power set of X)? Why?
4. Give state diagrams of DFAs recognizing the languages below. In all cases $\Sigma = \{0, 1\}$.
   (a) $L_1 = \{w | w$ begins with a 1 and ends with a 0$\}$.
   (b) $L_2 = \{w | w$ contains the substring 101110$\}$.
   (c) $L_3 = \{w | w$ has even length and its third symbol is 1$\}$.
   (d) $L_4 = \{w | w$ does not contain the substring 101$\}$.
5. Each of the following languages is the intersection of two simpler languages. Construct DFAs for the simpler languages and combine them to obtain a state diagram of a DFA for the language. In all cases $\Sigma = \{0, 1\}$. (Hint: This means you should have 3 DFAs for each part.)
   (a) $L_1 = \{w | w$ starts with a 1 and has at most one 0$\}$.
   (b) $L_2 = \{w | w$ has an odd number of 1s and an even number of 0s$\}$.
   (c) $L_3 = \{w | w$ has exactly three 1s and any number other than two of 0s$\}$.
   (d) $L_4 = \{w | w$ has a multiple of three of 1s and even length$\}$.
6. Each of the following languages is the union of two simpler languages. Construct DFAs for the simpler languages and combine them to obtain a state diagram of a DFA for the language. In all cases $\Sigma = \{0, 1\}$. (Hint: This means you should have 3 DFAs for each part.)
   (a) $L_1 = \{w | w$ every odd position is a 1 or ends in 010$\}$.

(b) $L_2 = \{w | w$ has an even number of 1s or at most two 0s$\}$.

(c) $L_3 = \{w | w$ starts with 0 and has even length or starts with 1 and has odd length$\}$.

(d) $L_4 = \{w | w$ is ϵ or 0$\}$.

7. Show that each of the following languages is regular and prove the underlying mathematical property:

(a) $L_1 = \{w | w$ is a binary number equal to $3n$, for some integer $n \geq 0\}$.

(b) $L_2 = \{w | w$ is a binary number equal to $5n$, for some integer $n \geq 0\}$.

(c) $L_3 = \{w | w$ is a binary number equal to $7n$, for some integer $n \geq 0\}$.

8. (10 pts) Convert the NFA below to a DFA and show intermediate steps: