

Quaternion EKF

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文章导航

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1 Quaternion Fusion

1. Initializes the Quaternion:

$$q0 = 1, q1 = 0, q2 = 0, q3 = 0 \quad (1)$$

2. Calculate direction of gravity indicated by algorithm:

$$Gravityx = 2.f * (q1 * q3 - q0 * q2) \quad (2)$$

$$Gravityy = 2.f * (q0 * q1 + q2 * q3) \quad (3)$$

$$Gravityz = q0 * q0 - q1 * q1 - q2 * q2 + q3 * q3 \quad (4)$$

3. Calculate direction of gravity indicated by measurement:

$$axNorm = ax / InverseSqrt(ax * ax + ay * ay + az * az) \quad (5)$$

$$ayNorm = ay / InverseSqrt(ax * ax + ay * ay + az * az) \quad (6)$$

$$azNorm = az / InverseSqrt(ax * ax + ay * ay + az * az) \quad (7)$$

4. Calculate accelerometer feedback scaled by 0.5:

$$deviategx = 0.5f * (ayNorm * Gravityz - azNorm * Gravityy) \quad (8)$$

$$deviategy = 0.5f * (azNorm * Gravityx - axNorm * Gravityz) \quad (9)$$

$$deviategz = 0.5f * (axNorm * Gravityy - ayNorm * Gravityx) \quad (10)$$

5. Apply accelerometer feedback to gyroscope, Convert gyroscope to radians per second scaled by 0.5:

$$halfgxdt = 0.5f * (gx - deviategx) * dt \quad (11)$$

$$halfgydt = 0.5f * (gy - deviategy) * dt \quad (12)$$

$$halfgzdt = 0.5f * (gz - deviategz) * dt \quad (13)$$

6. Integrate rate of change of quaternion:

$$q0 = q0 - halfgxdt * q1 - halfgydt * q2 - halfgzdt * q3 \quad (14)$$

$$q1 = q1 + halfgxdt * q0 + halfgzdt * q2 - halfgydt * q3 \quad (15)$$

$$q2 = q2 + halfgydt * q0 - halfgzdt * q1 + halfgxdt * q3 \quad (16)$$

$$q3 = q3 + halfgzdt * q0 + halfgydt * q1 - halfgxdt * q2 \quad (17)$$

$$(18)$$

7. Normalise quaternion:

$$q0* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (19)$$

$$q1* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (20)$$

$$q2* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (21)$$

$$q3* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (22)$$

8. Converts a quaternion to Euler angles in radians:

$$Euler[0] = atan2f(2.f * (q0 * q3 + q1 * q2), 2.f * (q0 * q0 + q1 * q1) - 1.f); \quad (23)$$

$$Euler[1] = asinf(-2.f * (q1 * q3 - q0 * q2)); \quad (24)$$

$$Euler[2] = atan2f(2.f * (q0 * q1 + q2 * q3), 2.f * (q0 * q0 + q3 * q3) - 1.f); \quad (25)$$

2 Extended Kalman Filter

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \quad (26)$$

$$z_k = h(x_k, v_k) \quad (27)$$

$$x = \begin{bmatrix} q0 \\ q1 \\ q2 \\ q3 \\ deviategx \\ deviategy \end{bmatrix} \quad z = \begin{bmatrix} axNorm \\ ayNorm \\ azNorm \end{bmatrix}$$

1. $A = \frac{\partial f}{\partial x}$:

$$A = \begin{bmatrix} 1, & -halfgxdt, & -halfgydt, & -halfgzdt, & 0.5f * q1 * dt, & 0.5f * q2 * dt \\ halfgxdt, & 1, & halfgzdt, & -halfgydt, & -0.5f * q0 * dt, & 0.5f * q3 * dt \\ halfgydt, & -halfgzdt, & 1, & halfgxdt, & -0.5f * q3 * dt, & -0.5f * q0 * dt \\ halfgzdt, & halfgydt, & -halfgxdt, & 1, & 0.5f * q2 * dt, & -0.5f * q1 * dt \\ 0, & 0, & 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 1 \end{bmatrix}$$

2. $H = \frac{\partial h}{\partial x}$:

$$H = \begin{bmatrix} -2.f * q2, & 2.f * q3, & -2.f * q0, & 2.f * q1, & 0, & 0 \\ 2.f * q1, & 2.f * q0, & 2.f * q3, & 2.f * q2, & 0, & 0 \\ 2.f * q0, & -2.f * q1, & -2.f * q2, & 2.f * q3, & 0, & 0 \end{bmatrix}$$

3. Extended kalman filter

$$\hat{x}_k^- = f(\hat{x}_{k-1}^-, u_{k-1}, 0) \quad (28)$$

$$P_k^- = AP_{k-1}A^\top + WQW^\top \quad (29)$$

$$K_k = \frac{P_k^- H^\top}{HP_k^- H^\top + VRV^\top} \quad (30)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0)) \quad (31)$$

$$P_k = (I - K_k H)P_k^- \quad (32)$$