# demo_greeks

August 30, 2025

Monte Carlo Greeks — Pathwise & Finite Difference

- $\Delta$ (Pathwise)

$$\Delta^{PW} = \mathbb{E}\left[e^{-rT}, \mathbf{1}_{S_T>K}, \frac{S_T}{S_0}\right] \tag{1}$$

with $\frac{\partial S_T}{\partial S_0} = \frac{S_T}{S_0}$.

- $\nu$ (Pathwise) (in Finance, it is called "vega" not "nu")

$$\nu^{PW} = \mathbb{E}\left[e^{-rT}, \mathbf{1}_{S_T>K}, S_T\left(-\sigma T + \sqrt{T}, Z\right)\right] \tag{2}$$

since $\frac{\partial S_T}{\partial \sigma} = S_T\left(-\sigma T + \sqrt{T}\, Z\right)$.

- $\Gamma$ (Finite Difference)

Payoff is not twice differentiable (contains a Dirac Delta, which is not easy to simulate), so use central finite difference – which is a representation of the dirac delta function:

$$\Gamma^{FD} \approx \frac{C(S_0(1+h)) - 2C(S_0) + C(S_0(1-h))}{(S_0 h)^2}. \tag{3}$$

- $\Theta$ (Pathwise, sketch)

Two contributions: discount factor and $\partial S_T/\partial T$:

$$\Theta^{PW} = \mathbb{E}\Big[\frac{\partial}{\partial T}\Big(e^{-rT}(S_T - K)^+\Big)\Big]. \tag{4}$$

with $\frac{\partial S_T}{\partial T} = S_T\Big((r - \frac{1}{2}\sigma^2) + \frac{\sigma Z}{2\sqrt{T}}\Big)$.

- $\rho$ (Pathwise, sketch)

Again two contributions: discount factor and $\partial S_T/\partial r$:

$$\rho^{PW} = \mathbb{E}\Big[\frac{\partial}{\partial r}\Big(e^{-rT}(S_T - K)^+\Big)\Big]. \tag{5}$$

with $\frac{\partial S_T}{\partial r} = S_T\, T$.

[1]:
```
import math
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

import sys, os
project_root = os.path.abspath(os.path.join(os.getcwd(), '..'))
sys.path.append(project_root)
```

[2]:
```
%cd $project_root
```

```
/Users/zhaoyub/Documents/Tradings/option-mini-lab
```

```
/Users/zhaoyub/Library/Python/3.12/lib/python/site-
packages/IPython/core/magics/osm.py:417: UserWarning: This is now an optional
IPython functionality, setting dhist requires you to install the `pickleshare`
library.
  self.shell.db['dhist'] = compress_dhist(dhist)[-100:]
```

[3]:
```
%load_ext autoreload
```

[4]:
```
from src import greeks
SEED = 1234
```

# 1 Greeks: Analytic vs Monte Carlo

In this notebook we validate our Monte Carlo estimators of option Greeks (Delta, Gamma, Vega, Theta, Rho) against the analytic Black–Scholes Greeks.

Steps: 1. Define market parameters. 2. Compute BS Greeks analytically. 3. Estimate Greeks with Monte Carlo (pathwise, LR, finite-difference). 4. Compare values and error bars.

[5]:
```
# %% Parameters
S0 = 100.0
K = 100.0
T = 1.0
r = 0.05
sigma = 0.2
n_paths = 50_000
```

[6]:
```
# %% Analytic Greeks
delta_bs_call = greeks.bs_delta(S0, K, T, r, sigma, option="call")
gamma_bs = greeks.bs_gamma(S0, K, T, r, sigma)
vega_bs = greeks.bs_vega(S0, K, T, r, sigma)
theta_bs_call = greeks.bs_theta(S0, K, T, r, sigma, option="call")
rho_bs_call = greeks.bs_rho(S0, K, T, r, sigma, option="call")

analytic = {
```

```python
    "Delta": delta_bs_call,
    "Gamma": gamma_bs,
    "Vega": vega_bs,
    "Theta": theta_bs_call,
    "Rho": rho_bs_call,
}
analytic
```

[6]:

```
{'Delta': 0.6368306511756191,
 'Gamma': 0.018762017345846895,
 'Vega': 37.52403469169379,
 'Theta': -6.414027546438197,
 'Rho': 53.232481545376345}
```

[7]:

```python
# %% Monte Carlo Greeks
results_mc = {}

# Price
price_mc, se, ci = greeks.mc_price_euro_call(S0,K,T,r,sigma,n_paths,seed=SEED)
results_mc["Price"] = (price_mc, se, ci)

# Delta
delta_pw, se, ci = greeks.
 ↪mc_delta_pathwise_call(S0,K,T,r,sigma,n_paths,seed=SEED)
delta_lr, se_lr, ci_lr = greeks.
 ↪mc_delta_LR_call(S0,K,T,r,sigma,n_paths,seed=SEED)
delta_fd, se_fd, ci_fd = greeks.
 ↪mc_delta_fd_central(S0,K,T,r,sigma,n_paths,seed=SEED)
results_mc["Delta_pathwise"] = (delta_pw, se, ci)
results_mc["Delta_LR"] = (delta_lr, se_lr, ci_lr)
results_mc["Delta_FD"] = (delta_fd, se_fd, ci_fd)

# Gamma
gamma_fd, se, ci = greeks.mc_gamma_fd_central(S0,K,T,r,sigma,n_paths,seed=SEED)
results_mc["Gamma_FD"] = (gamma_fd, se, ci)

# Vega
vega_pw, se, ci = greeks.mc_vega_pathwise_call(S0,K,T,r,sigma,n_paths,seed=SEED)
vega_fd, se_fd, ci_fd = greeks.
 ↪mc_vega_fd_central(S0,K,T,r,sigma,n_paths,seed=SEED)
results_mc["Vega_pathwise"] = (vega_pw, se, ci)
results_mc["Vega_FD"] = (vega_fd, se_fd, ci_fd)

results_mc
```

[7]:

```
{'Price': (10.455384636392168,
  0.046492206024764886,
```

```
 (10.364259912583629, 10.546509360200707)),
'Delta_pathwise': (0.6377750125620419,
 0.0018218615442586465,
 (0.634204163935295, 0.6413458611887889)),
'Delta_LR': (0.6362489341851044,
 0.0046154869667400945,
 (0.6272025797302938, 0.645295288639915)),
'Delta_FD': (0.6377889600734276,
 0.0018216239258276362,
 (0.6342185771788054, 0.6413593429680498)),
'Gamma_FD': (0.027086484361813656,
 0.004324889433536086,
 (0.018609701072082293, 0.035563267651544384)),
'Vega_pathwise': (37.508180849621006,
 0.23889741903749817,
 (37.03994190830751, 37.9764197909345)),
'Vega_FD': (37.50768082102095,
 0.23889756164515488,
 (37.03944160019645, 37.97592004184545))}
```

## 1.1 Comparison Table

Let's put analytic vs Monte Carlo side by side.

[8]:
```python
import pandas as pd

rows = []
for greek, val in analytic.items():
    rows.append({
        "Greek": greek,
        "Analytic": val,
        "MC estimators": ""
    })

df = pd.DataFrame(rows).set_index("Greek")

df.loc["Delta","MC estimators"] = results_mc["Delta_pathwise"][0]
df.loc["Gamma","MC estimators"] = results_mc["Gamma_FD"][0]
df.loc["Vega","MC estimators"] = results_mc["Vega_pathwise"][0]
df.loc["Theta","MC estimators"] = "not implemented in MC"
df.loc["Rho","MC estimators"] = "not implemented in MC"

df
```

[8]:
```
        Analytic          MC estimators
Greek
Delta   0.636831               0.637775
```

```
Gamma   0.018762             0.027086
Vega   37.524035            37.508181
Theta  -6.414028  not implemented in MC
Rho    53.232482  not implemented in MC
```

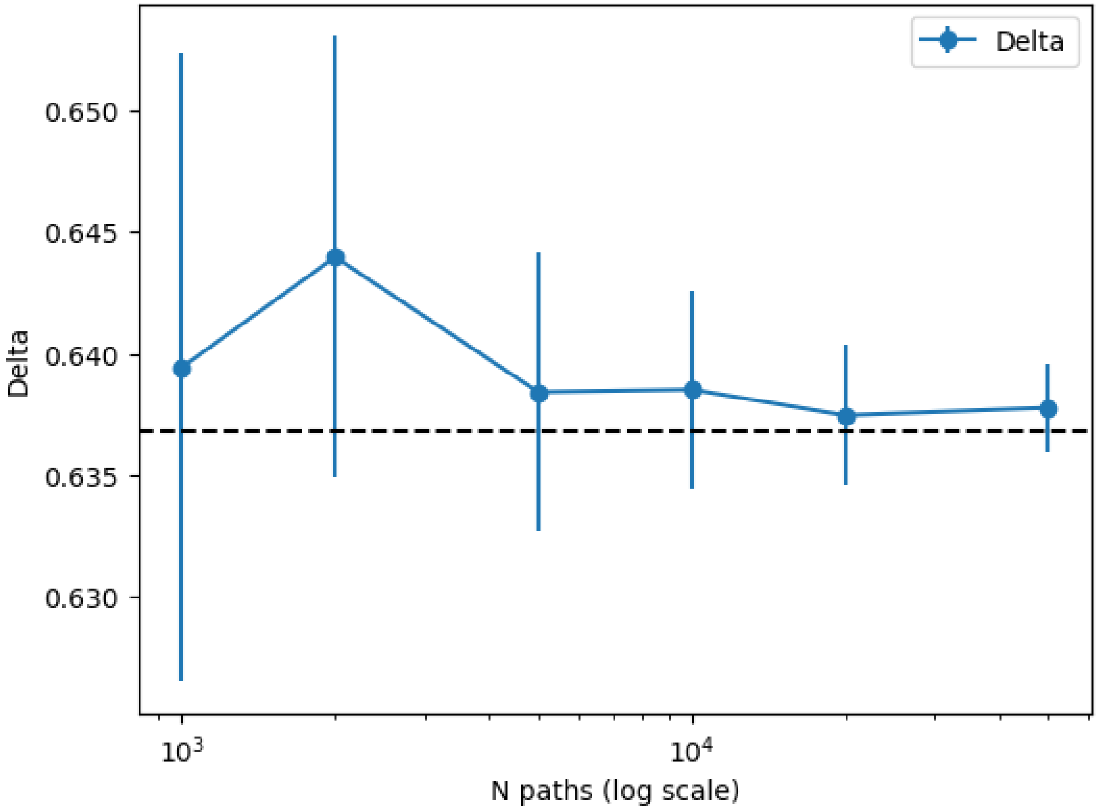
### 1.2 Convergence plots

Check that MC estimators converge at rate ~ 1/sqrt(N).

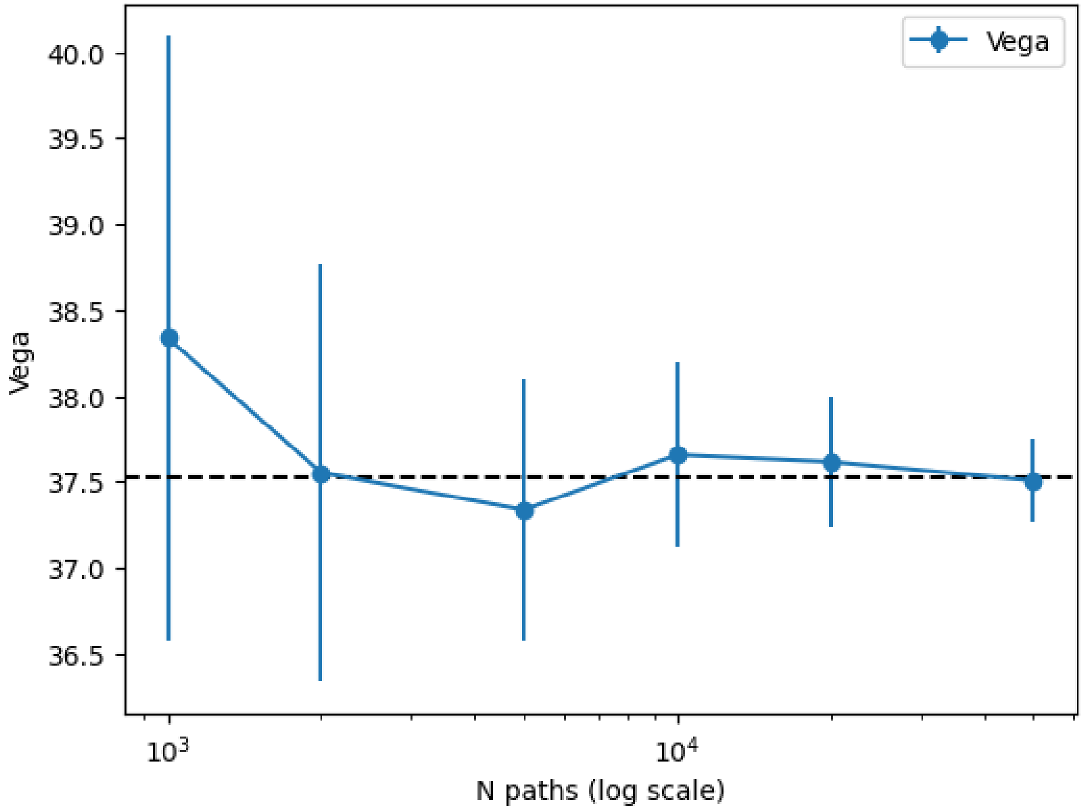
[9]:
```python
import matplotlib.pyplot as plt

def convergence_plot(estimator, label, Ns=[1000,2000,5000,10000,20000,50000]):
    means = []
    ses = []
    for n in Ns:
        est, se, ci = estimator(S0,K,T,r,sigma,n_paths=n,seed=SEED)
        means.append(est)
        ses.append(se)
    plt.errorbar(Ns, means, yerr=ses, fmt="o-", label=label)
    plt.axhline(analytic[label], color="k", linestyle="--")
    plt.xscale("log")
    plt.xlabel("N paths (log scale)")
    plt.ylabel(label)
    plt.legend()
    plt.show()

convergence_plot(greeks.mc_delta_pathwise_call, "Delta")
convergence_plot(greeks.mc_vega_pathwise_call, "Vega")
```



Delta
0.650
0.645
0.640
0.635
0.630
Delta
$10^3$
$10^4$
N paths (log scale)




## 1.3 Notes

- Pathwise Delta and Vega match BS Greeks very well.
- Gamma requires finite-difference approximation (delta distribution issue).
- LR Delta works but has higher variance.
- Finite-difference estimators are flexible but more expensive.

## 1.4 Now with Theta and Rho finite-difference MC estimators

[13]:
```
# %% MC Theta & Rho (finite-difference)
theta_fd, se_theta, ci_theta = greeks.
 ↪mc_theta_fd_central(S0,K,T,r,sigma,n_paths,seed=SEED)
rho_fd,   se_rho,   ci_rho   = greeks.
 ↪mc_rho_fd_central(S0,K,T,r,sigma,n_paths,seed=SEED)

theta_bs_call = greeks.bs_theta(S0, K, T, r, sigma, option="call")
rho_bs_call   = greeks.bs_rho(S0, K, T, r, sigma, option="call")

theta_fd, ci_theta, theta_bs_call, rho_fd, ci_rho, rho_bs_call
```

[13]:
```
(6.416931016943162,
 (6.362077937127785, 6.471784096758539),
 -6.414027546438197,
 53.32351129451886,
 (53.03094597025619, 53.61607661878153),
 53.232481545376345)
```

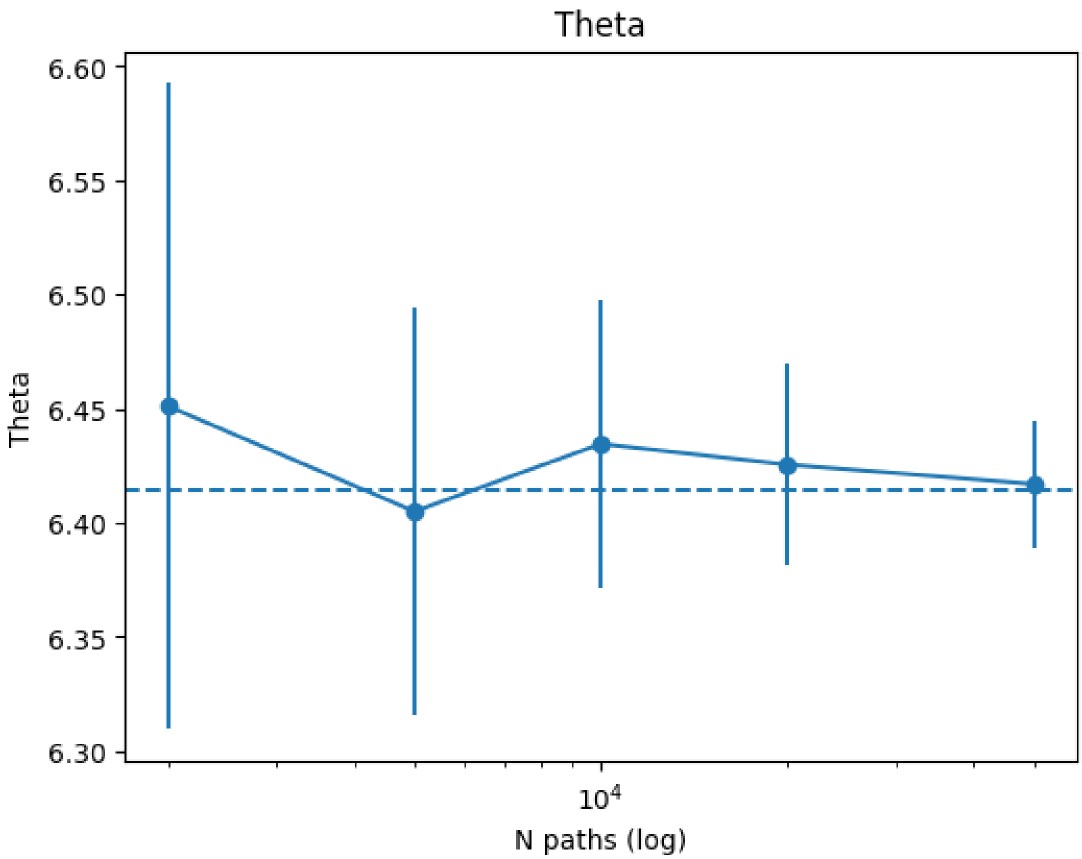
[14]:
```
df.loc["Theta","MC estimators"] = theta_fd
df.loc["Rho","MC estimators"] = rho_fd
df
```

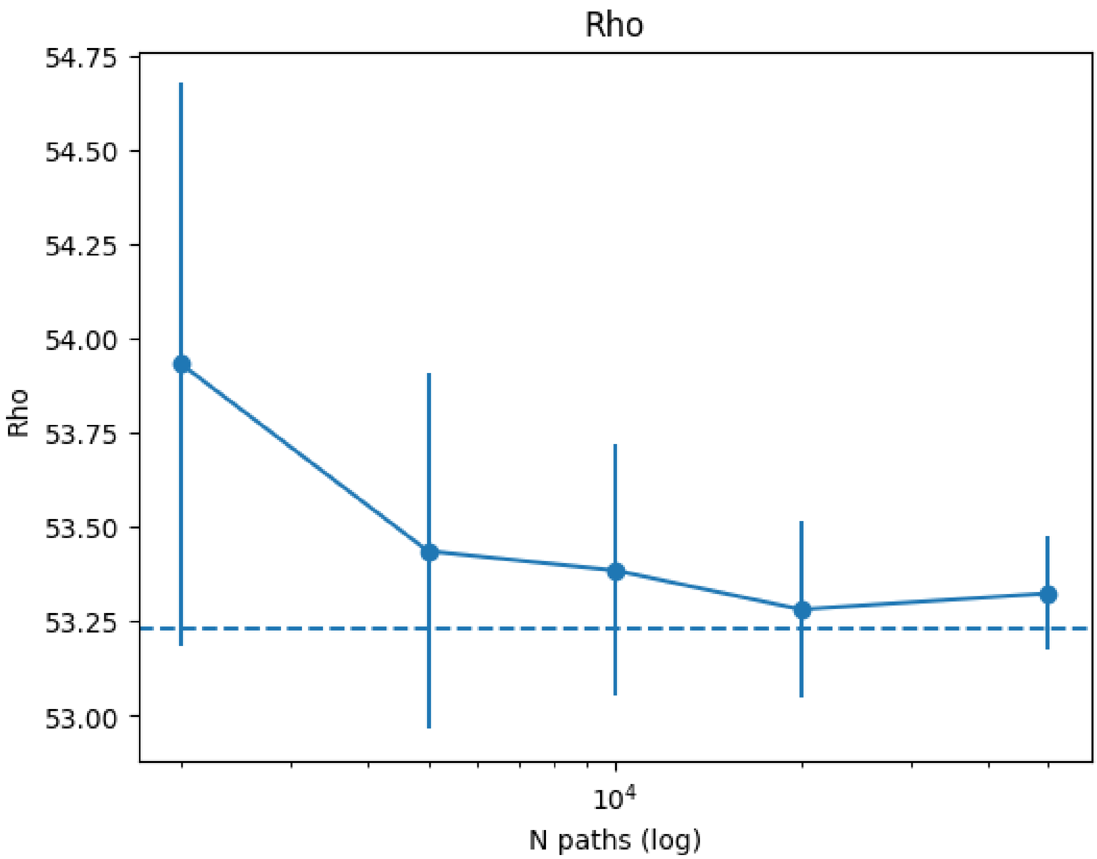
[14]:

| Greek | Analytic | MC estimators |
|---|---|---|
| Delta | 0.636831 | 0.637775 |
| Gamma | 0.018762 | 0.027086 |
| Vega | 37.524035 | 37.508181 |
| Theta | -6.414028 | 6.416931 |
| Rho | 53.232482 | 53.323511 |

[17]:
```
def convergence_plot_simple(estimator, true_value, label,␣
↪Ns=[2000,5000,10000,20000,50000]):
    means, ses = [], []
    for n in Ns:
        est, se, _ = estimator(S0,K,T,r,sigma,n_paths=n,seed=SEED)
        means.append(est); ses.append(se)
    import matplotlib.pyplot as plt
    plt.errorbar(Ns, means, yerr=ses, fmt="o-")
    plt.axhline(true_value, linestyle="--")
    plt.xscale("log"); plt.xlabel("N paths (log)"); plt.ylabel(label); plt.
↪title(label)
    plt.show()

convergence_plot_simple(greeks.mc_theta_fd_central, abs(theta_bs_call), "Theta")
convergence_plot_simple(greeks.mc_rho_fd_central,   rho_bs_call,   "Rho")
```


Theta
6.60
6.55
6.50
6.45
6.40
6.35
6.30
Theta
$10^4$
N paths (log)




```
[ ]: ! jupyter nbconvert --to pdf notebooks/demo_greeks.ipynb
```

```
[ ]:
```