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Definitions *more geometrarum* and Newton's scholium on space and time

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ABSTRACT

Newton's *Principia* begins with eight formal definitions and a scholium, the so-called scholium on space and time. Despite a history of misinterpretation, scholars now largely agree that the purpose of the scholium is to establish and defend the *definitions* of key concepts. There is no consensus, however, on how those definitions differ in kind from the *Principia*'s formal definitions and why they are set-off in a scholium. The purpose of the present essay is to shed light on the scholium by focusing on Newton's notion and use of *definition*. The resulting view is developmental. I argue that when Newton first wrote the *Principia*, he viewed the scholium's definitions as items of "natural philosophy." By the time of the third edition, however, he came to view their methodological status differently; he viewed them as belonging to the more qualified "manner of geometers." I explicate the two methods of natural inquiry and draw out their implications for Newton's account of space.

1. Introduction

Newton's *Principia* begins with eight definitions and a scholium, the so-called scholium on space and time. For much of its history, the scholium was thought to argue for the existence of absolute space by providing empirical evidence that each body had a unique, true state of motion, and assuming that this motion must be motion relative to an immovable, insensible, all-encompassing geometrical structure — absolute space (Mach, 1919, Ch. 2.6; Reichenbach, 1957, p. 34). There were notable exceptions, of course (like Kant and Euler), but this reading of the scholium was largely entrenched as orthodoxy. It is no longer. One of the more influential breaks with orthodoxy was made by Stein (1967). Stein argues that Newton did not frame the scholium in order to provide empirical evidence for the existence of absolute space. Rather, Newton's purpose was to examine the dynamical presupposition of the mechanics his time and show that, in order for those presupposition to have any purchase on the real world, we must adopt the notions of absolute space, time, and motion as he defines them.

Stein's reading has been developed by Laymon (1978), DiSalle (2006), Huggett (2012), and others, albeit with important modifications.[1] Rynasiewicz (1995a; 1995b; 2019) has also reached some corroborating conclusions, although through an independent line of reasoning.[2] There are significant debates here.[3] But there are also points of agreement. Details aside, the scholars just cited mostly agree that Newton's purpose in the scholium was to show that motion, as treated by the mechanics of his time, could not be "adequately *defined* as some distinguished form of motion relative to other bodies, but [had to be] analyzed instead in terms of an absolutely immobile space" (Rynasiewicz, 1995a, p. 135, emphasis added). They also agree that Newton's arguments were grounded in the details of actual mechanical practice, not independently motivated philosophical considerations. According to one version of this view, the mistake of earlier interpretation was to presume that "what [Newton meant] by space, time, and motion, and what [he meant] by claiming that they are 'absolute,' [was] already

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[2] See also Belkind (2007).

[3] One source of disagreement concerns whether the various species of space, time, and motion that Newton identifies result from one underlying distinction/criterion or several, see, e.g., Huggett (2012) and DiSalle (2016). Another source concerns whether all definitions or merely a subset have empirical content; e.g., Brading (2017), Schliesser (2013), and Huggett (2012). A third concerns the robustness of Newton's commitment to the lack of motion of the center of gravity of the solar system given his account of space; see Rynasiewicz (2019) and DiSalle

established on purely philosophical grounds, so that [the scholium merely showed] what physics has to say about these philosophical concepts" (DiSalle, 2016, p. 37). What they overlooked was that "Newton was not taking any such meanings for granted, but defining new theoretical concepts within a framework of physical laws" (*ibid.*). Although the details of this particular analysis are contentious, there is consensus on one claim it captures well: that in the scholium Newton engaged in a highly nuanced conceptual exercise, that of establishing and defending *definitions.* He did so within a framework of already-accepted mechanical principles, but he was establishing and defending definitions nonetheless.

I'll call this shared position "the current interpretation." It is surely correct. However, because it highlights the methodological importance of *defining,* it raises questions about Newton's notion and use of *definition* that were not raised, perhaps not even noticed, by the older, Machian interpretation. Consequently, these questions have been less attended to in the literature. The purpose of this paper is to stress their importance, attempt to answer them, and by so doing shed more light on the scholium. The resulting view of the scholium is developmental. I argue that when Newton first wrote the *Principia*, he viewed the scholium's definitions as items of "natural philosophy." By the time of the third edition, however, he came to view their methodological status differently; he viewed them — or, at least, by his own lights had good reason for viewing them — as belonging to the more qualified "manner of geometers." My discussion focuses on this change insofar as it pertains to Newton's account of space.[4]

The paper runs as follows. In §2, I articulate four questions about "definitions" in the scholium on space and time. The first concerns the methodological difference between formal definitions and definitions in scholia, generally speaking. The second and third questions concern the meaning of a small caveat in the first paragraph of the scholium on space and time, and its later deletion. The fourth concerns the significance of a set of formal definitions Newton drafted for Book III of the third edition of the *Principia* and the notion of definition they embody. In §3, I review some existing answers. In §§4–7, I provide my own, using a developmental narrative. In §4, I distinguish between formal definitions and definitions in scholia by means of Newton's distinction between two methods of physical inquiry: the "manner of geometers" and the natural-philosophical manner. In §5, I argue that when Newton first wrote the *Principia*, he believed that he was providing an account of space in the natural-philosophical manner, one that described spatial ontology as it truly was; no caveats, additions, or deletions. In §6, I show that his belief in the natural-philosophical status of his account of space depended on implicit assumptions about the nature of body. When those assumptions came under attack in the 1710s, he responded by proposing the new definitions for Book III mentioned above, and articulated their methodological status in terms of "the manner of geometers." In §7, I conjecture that because the new definitions were directly connected to Newton's account of space, and because they were methodologically parallel to the scholium's definitions, Newton's engagement with them should have prompted him to reevaluate the methodological status of the scholium's definitions. While there is no direct evidence that he did so, the hypothesis allows us to answer the four questions with which we began, and the circumstantial evidence for it is compelling. Consequently, I argue that by the mid-1710s, Newton came to adopt — or, at least, by his own lights had good reason for adopting — a more epistemically guarded position regarding his account of spatial ontology, one characterized by the "manner of geometers." He no longer believed that his account of space was unqualifiedly true, but that it was only true given assumptions implicit in his own physics.[5]

2. Defining in the *Principia*: four questions

The first and primary question is this: If Newton's strategy in the scholium was to define key concepts, why do so *in a scholium*? Why not present the definitions formally, alongside other "definitions"? The definitions's *locus* is not incidental to their purpose.

To see why this is not a red herring, consider this. As is well known, works written in the geometrical style, like the *Principia*, traditionally consisted of definitions, axioms and postulates, problems, theorems, and lemmas. Each played a specific role: definitions delineated the meaning of terms, axioms introduced commonly agreed-upon suppositions, lemmas provided reusable mathematical results, etc. Scholia sometimes followed any of these. Their role was to provide commentary on the immediately preceding text, usually in the form of experimental justification, methodological reflection, or an expansion of the discussion into tangential topics.[6] As is also well known, the geometrical style was admired for its certainty and the clarity of its foundations. Indeed, the *Elements*'s long history is largely constituted by attempts to render the work's foundations as clearly as possible (De Risi, 2016).

Newton's admiration for the geometrical style is clear. He followed it from the *Principia*'s earliest versions.[7] These *De motu* drafts show him articulating an increasingly rich body of theorems through increasingly refined sets of "Definitions," "Hypotheses," and "Laws" (later "Laws of Motion or Axioms"). Prior to publication, he even modified the first extended version of Book III (the "System of the World") from the discursive style in which it was first written to the geometrical form it now takes. He drew attention to the books's foundations by labeling them "Hypotheses." His efforts did not stop with the work's publication. For example, in preparation for *E2*, Newton divided the heterogeneous "Hypotheses" into two groups: the empirical "Phenomena" and the philosophical "Rules." He took the Rules to be widely accepted principles of causal reasoning and induction, and even called them "Axioms" in early drafts (McGuire, 1968, p. 242).[8] In preparation for *E3*, he considered adding explicit "Definitions" to Book III, ones to which we will return.

[5] This essay extends the argument of (Biener, 2017).

[6] I'm speaking as if scholia are proper parts of the geometrical style, but that is only for ease of exposition. Historically, scholia were conceived to be non-essential, additive elements. In medieval texts, they were simply the commentary that surrounded the graphically central text. Even in 1708, after scholia had been generally graphically incorporated into the main text of various works, Harris (1708) did not include them in his description of the geometrical style. He defined the latter as "proceed[ing] upon Principles in themselves self evident, on *Definitions*, *Postulates*, and *Axioms*, and a previously demonstrated Series of Propositions, step by step, till it give you a clear knowledge of the thing to be demonstrated" (original emphasis). He defined "scholium" as "a remark made leisurely, and as it were by the by, on that Proposition, Subject or Discourse before advanced, treated of, or delivered."

[7] Like many of his contemporaries, Newton held up Huygens's *Horologium Oscillatorium* as an exemplar of the geometrical style (Bos, 1986, xxvi). He repeatedly "recommended … Huygens's style and manner … He thought [Huygens] the most elegant of any mathematical writer of modern times, and the most just imitator of the ancients" (Pemberton, 1728, Preface).

[8] De Risi (2016) notes that although there was debate on the status of "axioms" vs. "postulates," axioms were generally considered more fundamental or at least more commonly received. They were the traditional equivalent of Euclid's "common notions." This equation goes back to Aristotle (Lee, 1935), and fits well with

I rehearse this evidence in order to make a familiar point: Newton attempted to clarify the *Principia*'s foundations over a period of over forty years (e.g., Cohen, 1971, p. ix). However, I want to stress that this attempt consisted, at least partly, in categorizing the work's foundations according to their role within the work, vis-à-vis the geometrical style. That is, it consisted in determining which items were "Definitions," which were "Hypotheses", and which could be taken as widely acceptable and so labeled "Axioms". Of course, the meaning of some terms, particularly "Hypotheses," was itself being negotiated at the time, but it is clear that the task of categorizing foundations was a crucial part of Newton's efforts to secure the certainty, generality, and perspicacity of the *Principia*'s reasoning (Guicciardini, 2009, p. 343ff.).

So what's the issue? According to the current interpretation, the scholium *defines* its key terms. But as the scholium's long history of misinterpretation suggests, situating definitions in a scholium is not the most perspicacious way of presenting them.[9] This is because *defining* isn't usually a scholium's role. Yes, the scholium is the scholium to the definitions. But the scholium to the laws does not introduce additional laws. The scholia to various propositions do not introduce additional propositions. Why does the scholium to the definitions introduce additional definitions, with no explanation as to their nature?[10] In a work whose foundations are categorized according to their function, providing definitions in a scholium (especially with no additional explanation) is *prima facie* odd (especially when the terms defined are both contentious and central to the work's overall purpose). Put differently: Newton thought long and hard about how to present the *Principia*. But according to the current interpretation, he chose to present some definitions as "Definitions" and not to present some definitions as "Definitions." The first issue is simply whether there are different notions of "definition" at play here, and what their implications are. We'll examine possible explanations in §3 and §4.

The second and third questions are more specific and raised by Newton's use of "definition" in the scholium's introductory paragraph. In *E1* and *E2*, the scholium began:

> Thus far it has seemed best to explain the senses in which less familiar words are to be taken in this treatise. Since time, space, place, and motion are very familiar to everyone, *I do not define them* [non definio]. Yet I note that these quantities are popularly conceived solely with reference to the objects of sense perception … (Newton, 1713, p. 5, emphasis added).

Superficially, the passage is in conflict with the current interpretation. It makes a point of claiming that whatever is going on in the scholium in regard to time, space, and motion should not be conceived of as an act of defining. This was no slip of the pen. Newton took the same position in *De gravitatione* (circa 1668–1684; hereafter *DG*), a text that bears pronounced similarities to the scholium.[11] The text begins with a short introduction followed by a list of definitions, but space and time (duration) are excluded from the list:

> The terms 'quantity', 'duration', and 'space' are too familiar to be susceptible of definition by other words (Newton, 2004, pp. 12–13).

Although Newton proceeded almost immediately to articulate the nature of space (in words), he placed his discussion in a "note"; that is, outside the apparatus of formal "Definitions." Later, in the run-up to the *Principia*, he had considered doing otherwise. In *De motu corporum in mediis regulariter cedentibus* (circa winter 1684/5 to spring 1685), Newton treated "motion," "absolute space," "relative space", "absolute time" and "relative time" as bona fide "Definitions," with no caveat concerning indefinability. But subsequently, he eliminated these definitions, placed his treatment in a scholium, and explicitly stated that he was not defining. The choice was clearly considered.

In §3, I will address the idea that Newton only demurred from defining the *genera* time, space, place, and motion, but intended to define their *species,* absolute and relative. For now, I'd like to highlight another bit of complicating evidence. In *E3*, the scholium remained almost unchanged, except that Newton *deleted* the troubling caveat:

> Thus far it has seemed best to explain the senses in which less familiar words are to be taken in this treatise. Time, space, place, and motion are very familiar to everyone, yet it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception … (Newton, 1999, p. 408).[12]

The difference is slight, but important.[13] Without "I do not define them," the scholium can be naturally read as distinguishing between the explanation of "less familiar words" thus far given (in the "Definitions") and the explanation of "very familiar" words to

[9] See also note 14.

[10] William Whiston, one of Newton's earliest expositors, noted the atypicality right away. In *Sir Isaac Newton's Mathematick Philosophy More Easily Demonstrated* (1716), he chose to *not* follow Newton in placing definitions in a scholium, and explained the decision by appealing to the rich early-modern notion of "Order": "Time, Space, Place, and Motion, as being things so well known to all, scarce need to be defined. But how ever, for the taking away some Prejudices out of Mens Minds, it is very expedient, that with the famous Newton we should distinguish these Quantities into Absolute and Relative, True and Apparent, Mathematical and Vul-

[11] It also bears pronounced dissimilarities. Most notably, *DG*'s physical theory is quite immature compared to that of the *Principia*. See (Henry, 2011; Ruffner, 2012). Nevertheless, *DG* remains a methodologically informative text. As DiSalle (2013) stresses, the analytical project of *DG* is refined and distilled in the *Principia*, but it is largely the same analytical project. For methodological differences, see (Biener, 2017).

[12] In *E1* and *E2*: *Nam tempus, spatium, locum et motum ut omnibus notissima, non definio. Dicam quod vulgus tamen* … In *E3*: *Tempus, spatium, locus, & motus, sunt omnibus notissima. Notandum tamen quod vulgus* … Scholars have noted the odd nature of Newton's deletion. For example, Huggett writes: "[a] discussion of how Newton defined these terms may seem paradoxical, because he himself … starts the Scholium [in *E*1 and *E2*] … by declaring that he does 'not define time, space, place and motion' … For reasons I do not understand though plausibly related to the topics of this essay [i.e., Newton's strategy in the scholium] – the phrase is omitted from the third edition" (Huggett, 2012, p. 198 and note). Rynasiewicz too observes that the deletion is "[t]he only difference of any potential note [in *E3*'s version of the scholium]" (1995a, p. 139). But neither author conjectures about its significance. The suspicion that the deletion is "of potential note" and "plausibly related" to Newton's strategy is what I'm trying to cash out.

[13] It is possible that the deletion was just a minor correction to the flow of the sentence. But this seems unlikely. Newton's persnicketiness suggests that there was *some* reason for an edit that spoke to the very strategy of the scholium. This is especially the case if we consider that Newton had once considered supplying explicit definitions instead of the scholium. It is also unlikely that the deletion was

follow in the scholium. That is, it can be read as continuing the task of articulating "the senses in which … words are to be taken in this treatise." However, with the caveat in place (as in *E1* and *E2*), the text seems to recommend a greater distinction between the function of the scholium and the function of the "Definitions" that precede it. It makes it harder to assert, without qualification, that the scholium is concerned with *definitions*. This raises two questions: First, If Newton's innovation in the scholium was to define key concepts in light of accepted mechanical principles, why did he explicitly demure from defining in *E1* and *E2* without explaining the sense of the caveat? Second, why did he stop from demurring in *E3*, given that the contents of the scholium remained almost identical?[14]

The fourth question is akin to the first, but concerns Newton's proposed, yet unpublished, formal definitions to Book III of *E3*. They are of physical terms — "body" and "void" — not mathematical terms like "motive quantity of centripetal force." In them, Newton explicitly reflects on their status as "Definitions." We'll discuss them more in §6. For now, I'd like to suggest that they are quite relevant for understanding the scholium's definitions. This is because the new definitions are methodologically parallel to the scholium's definition, as these are understood by the current interpretation. Take, for example, Newton's definitions of "body":

> Definition I: I call body anything that can be moved and touched, and resists when touched. (UCL MS-Add-3965 504r).[15]

On the current interpretation, the scholium's purpose is to articulate the kinematical concepts that one must adopt in order to make sense of a world in which Newton's dynamics hold true (or, more modestly, a world in which the common dynamical principles of his time hold true). This is what makes the scholium's 'definitions' unique, and distinguishes them from the official "Definitions." However, *E3*'s draft definitions function in much the same way. Namely, the definition of "body" articulates the (minimal) concept of body that we must adopt in order to make sense of the entities for which Newton's dynamics hold true. For this purpose, "body" turns out to be a movable that exerts resistance when impinged upon; other potentially corporeal properties, like extension, need not play a role. As Biener & Smeenk (2012, p. 115) put it, *E3*'s definition of "body" shows that "[t]he nature of body depends [only] upon whatever constraints are implied by satisfaction of the laws [of motion]." Brading (2012, p. 29) makes a similar, but weaker, point: "a necessary condition for the individuation and identity of physical bodies is that they satisfy the laws." On both accounts, Newton's definition is offered from 'within' his physical theory. It is grounded in the dynamical laws just as the scholium's definitions of "absolute time" or "absolute motion" are grounded in the dynamical laws. *Prima facie*, this intra-theoretic definitional strategy is what makes the scholium's definitions different from the *Principia*'s formal "Definitions," which seem to merely stipulate the meaning of words (more on this in §4). Given what we have already said about the the *locus* of definitions in the *Principia*, what are we to make of Newton's decision to present one set of intra-theoretic definitions in a scholium and the other set using the formal apparatus of "Definitions"? Does this square with our answer to the first question?

These are by no means the only questions raised by Newton's use of "definition." But they are sufficient for showing that the topic is worth investigating. They also form the skeleton on which the rest of this essay hangs. I will address them in §4, §5, §7, and §6; respectively. In sum, they are:

A) Why did Newton use *the scholium* in order to provide definitions, as opposed to providing formal definitions? (Or, similarly: Are there different notions of "definition" at play in the "Definitions" and the scholium?)
B) Why did he assert that he was *not* defining space, time, place, and motion in *E1* and *E2*?
C) Why did he remove the caveat in *E3*?
D) *E3*'s draft definitions are methodologically similar to the to the scholium's definitions. Why are they presented as formal definitions? How does this square with our answer to A?

Notice that A+B and C+D set up a contrast between early and later editions of the *Principia*, and thus suggest that we should not assume that Newton's views on these issues were fixed. To my knowledge, there are no existing answers to C and D. We examine answers to A and B in the next section.

3. Some answers

Let's start with a common answer to B, as it is the quickest to dismiss. It is possible that Newton only demurred from defining the *genera* time, space, place, and motion in the scholium. He did, however, intend to define their *species*, absolute and relative. He expected his readers to track the distinction, and so didn't need to clarify further. Rynasiewicz (1995a) entertains this reading, but rejects it. He notes that "in turning to *locus* and *motus* [Newton] does provide what, according to any reasonable standard, would qualify as definitions of the genera" (p. 140; Rynasiewicz, 2019, p. 2). Moreover, Newton is clear that the absolute/relative distinction is not a distinction of species. He writes: "absolute and relative spaces are the same in species and in magnitude …" (p. 409). The answer is also *ad hoc*; it sheds no light on A, C, and D.

Huggett (2012) provides a better answer to B, and a compelling answer to A. He focuses on the relation between definitions and the *Principia*'s mathematical proofs:

> The definitions that [Newton provides] under that designation are all of new or contentious technical terms whose precise

[14] To further motivate the importance of Newton's caveat, allow me to make a tantalizing suggestion. I believe, although cannot fully argue here, that "I do not define" had much to do with how the 'orthodox' interpretation of the scholium became popular to begin with. The caveat provides an error-theory, if you will, for why the scholium was misinterpreted by so many, for so long. Briefly: the popularity of the orthodox interpretation had much to do with the prominence of its most famous expositor, Ernst Mach. However, when Mach quoted from the scholium in *Die mechanik in ihrer entwickelung* (1883), he quoted from the first German translation, by J. P. Wolfers (1872), which was of *E2*. This was the edition in which Newton still stated that "I do not define": *Zeit, Raum, Ort und Bewegung als alles bekannt erklre ich nicht* (Mach, 1883, p. 207; Newton, 1872, p. 25). English translations were guilty of the same omission. Andrew Motte's oft-reprinted translation was also of *E2,* at least as regards the first lines of the scholium: "I do not define space, time, place, and motion, as being known to all" (Newton, 1729, 9). The sentence even remained unchanged in Florian Cajori's 1934 edition, and was thus part of the authoritative English translation until 1999. Consequently, scholarly interpretations of the scholium incorporated, and further propagated, the mistake (e.g., Earman, 1989, p. 20). Perhaps, then, Mach (and Earman, and many others) didn't entertain the idea that the scholium's purpose was to *define* key concepts because the text they used explicitly suggested otherwise. Newton's own words may have thrown his interpreters off the scent.

> not to say that his work does not raise further questions about their meanings, and it is those issues that the Scholium addresses. And thus in the sense that it explicates what the terms mean, he does offer 'definitions' - but since they are not meanings directly appealed to in the proofs, they are not 'definitions' of the formal system of mechanics developed in the *Principia* … [I]n a loose sense — not in the sense of a definition in the formal system of the *Principia*, but in the sense of telling us what he means by the term [s] — Newton *does* define (Huggett, 2012, p. 198, original emphasis).

According to Huggett, the scholium's terms are not necessary for understanding the *Principia*'s mathematical proofs. Thus, they require no formal definition and Newton can justifiably claim that he does not define them. But that is not to say that the scholium's terms are not necessary for understanding the *Principia* as a whole. It is essential for the project of Book III that Book I and II's mathematical results come to bear on observations and so, as Huggett puts it, "there is no formal definition of [the scholium's terms], but there is discussion of [their] meaning in physics" (p. 199). The distinction between "formal definition" and "discussion of meaning in physics" thus accounts for the difference between the "Definitions" and the scholium.

I believe Huggett's analysis is correct, as far as it goes. But it also has shortcomings. First, the distinction between definitions required for mathematical proofs and definitions required for physical application is too restrictive. Definition 1 — quantity of matter — exemplifies the problem. The definition has two parts: first, "quantity of matter is a measure of matter," and second, "that arises from its density and volume jointly" (1999, p. 403). The first part takes the form of a traditional Euclidean definition, following the mold of: "a point is …", "a line is …", etc.[16] But the second part is more curious. As Cohen explains, its Latin construction entails that it is *not* part of the predicate that follows "is" (Cohen, 1999, p. 91). He notes that the second part "is clearly not stated in the primary form in which … all the other definitions of the *Principia* are given … [it] is rather a rule, stating a relation between the new concept of mass (as a measure of matter) and the concepts of volume and the intuitively known [concept of] density" (*ibid*.). Curiously, Cohen also notes that "nowhere in the *Principia* does Newton in fact begin by determining a body's density and volume and then computing the mass" (p. 88). Consequently, it seems that the definition's second part provides the kind of "discussion of meaning in physics" that Huggett expects to find in the scholium, and, moreover, is not necessary for understanding the *Principia*'s proofs. The same might be said of Definitions 3 and 4 ("vis insita" and "vis impressa") which define modes of action, not mathematical quantities (more on Def. 3 shortly). Second, Huggett's account makes it more difficult to answer D. The type of definitions Newton entertained for *E3* are formal, insofar as they are presented as "Definitions," but concern physical meanings that are not essential for understanding proofs. This seems to cut across Huggett's distinction, and thus is in tension with his answer to A. His account is close, but difficult to sustain without exception.

Rynasiewicz offers a third distinction. He holds that while the *Principia*'s formal definitions are nominal and stipulative (definitions of names), the scholium's definitions are real (definitions of things). The difference is this:

> The opening eight definitions labeled as such, which I referred to as *stipulative* definitions, fall under … [the] heading of definitions of names. They are neither true nor false, but set up the way certain words shall be used. In contrast, definitions of things attempt to capture what things really are, their natures or in what they consist. As such they are substantive theses, evaluable as correct or incorrect (Rynasiewicz, 2019, p. 2).

According to Rynasiewicz, the purpose of the scholium is to offer and defend the true definitions of absolute quantities. The purpose of the "Definitions" is to stipulate meanings. The two groups are separated in the text because they have different functions. One advantage of this account is that the real/nominal distinction is historically viable, and would have been familiar to Newton through several sources (Rynasiewicz, 2019, p. 2). A second advantage is that it allows us to claim in a full-throated way that the scholium offers *definitions*, not definitions "in a loose way." And although this makes it difficult to see the sense in Newton's caveat and its deletion (B & C), Rynasiewicz does not set out to address these issues (Rynasiewicz, 1995a, pp. 139–141). A third advantage is that the account does not imply that the division between nominal and real definitions must align with division between definitions required for mathematical proofs and definitions required for physical application, thus avoiding the problems discussed in the previous paragraph.

However, the account does run into another problem; namely, that some of the definitions — Definition 3 most prominently — do seem evaluable as correct or incorrect. Definition 3 states that the *vis insita* of matter is "the power of resisting by which every body, so far as it is able, perseveres in its state either of resting or of moving uniformly straight forward" (p. 404). This is not *merely* a stipulation of meaning. As Cohen (1999, 96) notes, "the definition … implies the first law of motion" and so seems to make existence claims about privileged trajectories in space and the symmetry between rest and motion. Of course, this is a thorny issue and the literature on Definition 3 is vast. But suffice it to say that it is not obvious that the definition embodies no ontological commitments (e.g., McGuire, 1994). Rynasiewicz recognizes a version of this problem. He notes that "it's not always so clear, in general, whether a definition is nominal or real," and that Newton "did not systematically distinguish between [real and nominal definitions] … in the manuscripts leading to the *Principia*" (Rynasiewicz, 2019, note 16). Still, he holds that "this does not count against what can be established about the Scholium on the basis of internal evidence" (*ibid*.).

Definition 3 aside, I agree. However, because my purpose here is to extract from Newton a systematic notion of "definition," I do take more seriously the fact that he did not generally distinguish real and nominal definitions, and that he saw no reason to do so in the scholium whose very subject was the multiple meanings of words. But instead of more negative arguments, I'll offer a competing account and argue for its merits. I also want to save what seems right in Huggett's and Rynasiewicz's accounts; namely, that there is *some* sense in which the scholium's definitions are more closely tied to physical reality than the "Definitions." However, I'll argue that this sense need not entail that the latter have no physical implications or that they are unqualifiedly real.[17]

4. "Definitions," scholia, and the mathematical/physical distinction

Instead of analyzing the scholium's arguments to arrive at its sense of "definition," I'll privilege Newton's explicit statements on definitions and scholia. Newton explicitly reflects on the nature of definitions in mechanics in two notable places. The first is *DG* (examined in this section), the second is the draft definitions to *E3* (examined in §6). As we'll see, although they are separated by at least thirty years, they contain almost identical ideas. In this regard, *DG* can serve as a guide to Newton's later thought.

Let's begin with A. In *DG*, Newton explicitly paired formal definitions and scholia with different sets of methodological commitments:

> It is fitting to treat the science of the gravity and of the equilibrium of fluids … by a twofold method. To the extent that it appertains to the Mathematical sciences, it is right that I abstract it *as much as possible* from Physical considerations. And for this reason I have undertaken to demonstrate its individual propositions strictly, in the manner of Geometers [*more Geometrarum*], from abstract principles sufficiently familiar to the student. And, since this doctrine may be judged to be somewhat akin to natural Philosopy in so far as it is applied to elucidating many of its Phenomena, and moreover, that its use may thereby be made especially clear and the certainty of its principles perhaps confirmed, I shall not be reluctant to illustrate the propositions from abundant experiments as well; but in such a way that this freer kind of discussion, *disposed in scholia*, may not be confused with the former, which is treated in lemmas, propositions and corollaries. The foundations from which this science is to be demonstrated are definitions of certain words, or axioms and postulates no one denies (Newton, 2004, p. 12, emphasis added).

In short, Newton used stylistic elements to segregate philosophical methods. On the one hand is the geometrical style (*mos Geometrarum).* It proceeds rigidly and formally from abstract principles and is only "somewhat akin" to natural philosophy. On the other is the "freer" (*laxius*) kind of method that concerns experimentation and the confirmation of first principles. It is more closely associated with traditional natural philosophy and is located in scholia (*in Scholia dispositum*).

This distinction is often interpreted through the lens of the *Principia*'s more-familiar physical/mathematical distinction (hereafter: PMD). But I believe this is a mistake. Understanding the difference between the two distinctions is crucial for understanding how Newton conceived of formal definitions.

This is how the PMD is introduced in the *Principia*:

> [I consider] forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by words [like attraction and impulse] I am anywhere defining a species or mode of action or a physical cause or reason (Newton, 1999, p. 408).

This and similar statements led the likes of Berkeley to read the *Principia*'s mathematics as mere calculational devices; that is, to hold that we cannot infer any claims regarding causal structure or ontology from the work's mathematical machinery, even when it successfully saves real-world phenomena (e.g., Berkeley, 1721

But this was not Newton's view. The *Principia*'s mathematical treatment of real-world phenomena was meant to be silent about *some* things, but not all. It *was* supposed to capture the actual mathematical properties of an ontology of physical forces and their interactions, but it was also supposed to remain silent about (at the very least) the forces's underlying causes and physical natures.[18] Needless to say, there is disagreement about how to draw the line between what the *Principia* captures and what it remain silent on, and further disagreement about what 'capturing the actual mathematical properties of an ontology of physical forces' might mean.[19] But there is also agreement that: 1) Newton took at least some mathematical claims that the *Principia* made about physical forces to be *true*; i.e., he took them to be representative of reality (even if he didn't spell out what that means); and 2) he didn't take these mathematical claims to entail any further physical claims, and certainly no *false* claims, about matters which the *Principia* did not explicitly address. Without these commitments, Newton could not assert that the gravitational force "really exists and acts according to the laws that we have set forth" *and* that he can remain silent about its underlying causes (Newton, 1999, p. 943).[20]

But the *Principia*'s PMD is *not* the operative distinction in *DG*. *DG*'s PMD is weaker, and commits Newton to an analogue of (1), but not (2). In *DG*, Newton held that his mathematical treatment of hydrostatics is true, but also that it essentially involved him in concomitant physical falsehoods:

> In these definitions … I consider only absolutely hard or fluid bodies, for one cannot reason mathematically concerning bodies that are partially so, on account of the innumerable figures, motions, and connections of the least particles. Thus I suppose that a fluid does not consist of hard particles, but that … it has no small portion or particle which is not likewise fluid. And moreover … I define the parts, not as being in motion …, but only as capable of motion … [Finally,] I suppose that … a hard body is not made up of conglomerate parts, but is a single undivided and uniform body …, whereas a fluid body is uniformly divided at all points (Newton, 2004, pp. 38–39).

[18] Although Newton claimed that Book I and II of the *Principia* are "mathematical" and Book III is "physical", he also clearly intended his statements regarding the "mathematical" treatment of force to qualify the conclusions of Book III. As he wrote in "An Account …:" [Newton] is silent about the cause of gravity, there occurring no experiments or phenomena by which he might prove what was the cause thereof. And this he hath abundantly declared in his *Principles*, near the beginning thereof, in these words: 'I am not now considering the physical causes and sites of forces'. And a little after: 'Moreover, I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity towards a center, considering these forces not from a physical but only from a mathematical point of view'" (Newton, 2004, pp. 123–24). Cf. (Domski, 2020).

[19] For example, Janiak (2008) holds that by treating force mathematically, Newton means treating its measurable (physical) quantities, and by treating it physically, he means treating its non-quantitative features (p. 60). A mathematical treatment is thus a circumscribed treatment; a physical treatment is more comprehensive, perhaps allowing for a "complete physical characterization" of its subject matter (p. 57). Harper (2011) holds that a mathematical treatment ignores deep causal features (even quantitative ones), while a physical treatment does not. But a mathematical treatment *does* capture causal feature at the appropriate level, the level which mathematical description is framed (pp. 94–95). For Ducheyne (2012), a mathematical treatment characterizes genus-level truths about forces, while a physical treatment characterizes the "actual physical [and even causal] conditions or forces in the empirical world" (p. 27–28). Domski (2020) approaches the issue from another angle, focusing on the very applicability of mathematical concepts to the physical world.

Or similarly:

> But in order that you may conceive of this composite body as a uniform one, suppose (*finge*) its parts to be infinitely divided and dispersed everywhere … Certainly such reasoning is suitable for contemplation by mathematicians … but in physics things seem otherwise (Newton, 2004, p. 38).

As regards the *Principia*, Newton suggested that his mathematical treatment of force captures forces as they are in the real world, with no concomitant distortion. He held that "abstract [ing] from physical considerations" introduces no falsehoods (albeit by remaining mum about a good many things), and certainly introduces no falsehoods about fundamental ontology or causal structure. In *DG*, however, Newton suggests that abstract mathematical treatment may *require* positing fictions about fundamental ontology and causal structure, if only for the purpose of enabling that very mathematical treatment. His lifelong position was that bodies and fluids are atomic. But since atomicity introduces intractable complexity, in *DG* bodies and fluids are not treated as they are *in rerum natura.* Rather, they are treated as continua (though they are not) and as absolutely hard or absolutely fluid (though they are not).[21] Thus, *DG*'s PMD is somewhat broader than the *Principia*'s. As regards the theory of force, Newton believed that abstraction from physical considerations introduced no physical falsehoods. But this was not a feature of the PMD itself, it was a feature of his theory of force. The PMD could also be used to justify more "destructive" forms of mathematical abstraction.

So let's return to *DG*'s first paragraph. It suggests that the *mos geometrarum* proceeds formally from abstract principles and is only somewhat akin to natural philosophy, which is treated in Scholia. On the *Principia*'s PMD, the difference between the two boils down to style and scope, but not to verisimilitude. On *DG*'s PMD, the two may also exhibit a difference in verisimilitude: while the natural philosophical method is essentially wedded to truth, the mathematical method may purposely depart from it (not willy-nilly, but in ways that enable mathematical treatment). And not surprisingly, I think that we should understand *DG*'s first paragraph in light of *DG*'s PMD, not the *Principia*'s. The crucial difference between what Newton thought appropriate for scholia and what he thought appropriate for the more formal elements of the geometrical style concerns their relation to physical truth.

Of the various formal elements of the geometrical style, Newton explicitly tied *definitions* to the enabling physical falsehoods that are sometimes necessary for mathematical abstraction:

> [T]hus I have accommodated these *definitions* not to physical things but to mathematical reasoning, *after the manner of geometers* who do not accommodate their definitions of figures to the irregularities of physical bodies. And just as the dimensions of physical bodies are best determined by their geometry — as with the dimension of a field by plane geometry, although a field is not a true plane; and the dimension of the earth by the doctrine of the sphere, even though the earth is not precisely spherical — so the properties of physical fluids and solids are best known from this mathematical doctrine, even though they are not perhaps absolutely nor uniformly fluid or solid (Newton, 2004, p. 39, emphasis added).

Physically false but 'accommodating' assumptions can be introduced into mathematical reasoning by means of *definitions*. Geometers are distinctive because they can make such accommodations licitly. Traditional natural philosophers cannot (or rather, should not). Instead, they should track physical truth and capture their referents as they really are, with no accommodating assumptions. Importantly, this also entails that whether an assumption is 'accommodating' or not is only evaluable from the natural-philosophical perspective. Within the scope of mathematical treatment, there's no sense in asking whether accommodations are true or not; e.g., whether bodies are *really* continuous. They are continuous because they are stipulated to be so — this is the assumption required for "contemplation by mathematicians." We can further ask whether this assumption is true — whether it captures how "in physics things seem" — but by so doing we have crossed into natural philosophy.[22]

I believe this distinction between the *mos geometrarum* and the natural-philosophical manner, *and thus between formal definitions and the contents of scholia*, captures what is appealing in Huggett's and Rynasiewicz's distinctions, without falling into the same pitfalls. Most importantly, it preserves the sense that the contents of scholia are somehow truer to reality than formal definitions, without suggesting that formal definitions are wholly independent of reality. First, the distinction does not suggest that formal definitions are divorced of *all* physical content, as Huggett's distinction suggests. And second it does not suggest that formal definitions are *wholly* stipulative, as Rynasciewicz's distinction suggests. Rather, it only suggests that formal definitions are divorced from physical considerations *as far as possible* and may still be evaluated according to their verisimilitude to reality *from the natural-philosophical perspective*. Take, for example, the idea of studying the dimensions of the earth by treating it as a cube. The idea sounds absurd because that sort of 'accommodation' departs from natural-philosophical truth too radically. But treating the earth as a sphere, although an abstract mathematical representation and still physically untrue, remains closer to the truth; perhaps, for certain purposes, as close as possible. The point is that treating the earth as a sphere works because of the way the world is and is evaluable as more-or-less correct, even if the earth is not actually a sphere. In this way, *DG*'s distinction suggests that formal definitions can be

[21] One may object at this point that what I'm calling 'falsehoods' are merely approximations, and rather innocuous ones. Since Rule 4 allows approximations to be "considered either exactly or very nearly true," Newton could adopt them without threatening his commitment to physical truth. But this isn't quite right. Rule 4 does not license any approximation whatsoever. Rather, it licenses only approximations that submit to Newton's now-famous *quam proxime* proof procedure (Smith, 2014, 2016, 2002). That procedure demonstrates that small deviations from some ideal motion entail only small deviations in the properties of the ideal force that causes that motion (and vice versa). Approximations that submit to *quam-proxime* reasoning are therefore special because they are guaranteed not to change the fundamental, underlying physics of the situation they concern. For example, Newton shows that Kepler's area law holds *quam proxime* iff the force acting on the body in question is centripetal *quam proxime*. This means that small deviations from Kepler's law can only be caused by small directional deviations from a centripetal force. They *cannot* be caused by forces that point in radically different directions. Thus, the underlying physics of small deviations from Kepler's law must remain largely similar to the underlying physics of Kepler's law. Some approximations are like this. But not all. The approximation that bodies are continuous at the micro-level may seem innocuous because, for the most part, we can ignore the micro-level when reasoning about macro-level phenomena. However, as we'll see in §6, some macro-level claims necessitate consideration of their underlying micro-physics; specifically, Newton's proof of the existence of vacuum relies on assumptions about the fundamental constitution of matter. In this case, the continuity

mathematical and still true to the world, without capturing it fully. We can also recover how these definitions are stipulative, but not wholly so. They are stipulative because *from the perspective of the mos geometrarum*, there's no sense in asking whether the earth is really a sphere or not. It is treated as a sphere by fiat, so that mathematical treatment becomes possible. But the choice is not wholly arbitrary and unconstrained, since whether it is a good choice or not depends on the way the world is. Formal definitions are stipulative in the sense that they embody (or follow from) some enabling assumptions that can be freely chosen for some particular purpose; but not entirely freely, since whether they meet their purpose depends on the nature of reality.

In the case of *DG*, the enabling assumption is that bodies are continuous and absolutely hard or fluid. Consequently, "body" is defined as "that which [homogenously] fills place." There is no sense in asking whether this is true from the vantage point of *DG*'s mathematical treatment of bodies; the definition is just stipulated. But the enabling assumption it embodies (or follows from) is false from the vantage point of natural philosophy. Nevertheless, the definition is stipulated to enable further mathematical treatment, "for one cannot reason mathematically concerning … the innumerable figures, motions, and connections of the least particles." To be clear, formal definitions *may* be arbitrary and bear no relation to reality; Newton never suggests otherwise. But they are not constrained to be so. In contrast, definitions in the natural philosophical manner — i.e., in scholia — lack this flexibility; they are constrained to capture reality as it actually is.

As we shall see in the next section, this methodological difference allows us to explain why Newton's discussion of space takes place in a scholium (question A), as well as the sense of his *non definio* caveat (B), in a way that is consonant with Huggett's and Rynascienwicz's explanations. Moreover, it will allow us to answer C and D. This will require taking *DG*'s methodological distinction to apply to texts far removed from *DG*. The extrapolation may seem unjustified given the clear differences between *DG* and later works, but we shall see that in this regard Newton views remained largely unchanged (§6).

5. Treating space natural-philosophically (pre-1710s)

The distinction between the *mos geometrarum* and the natural-philosophical manner suggests an answer to A; namely, that by treating space, time, and motion in a scholium Newton meant to treat them natural-philosophically. Let's focus on space. It is clear that at least in *DG*, *E1*, and *E2* (more on *E3* shortly), Newton thought he was treating space as it really is, with all attendant claims to truth.

In *DG*, the asymmetry between Newton's account of space and his account body bears this out. Although both are found in *DG*'s "note," Newton explicitly despairs of offering a natural-philosophical account of body, thus differentiating it from the note's other contents. He is frustrated by the inexhaustibility and inscrutability of divine will, and so offers only a possible account, not one he affirms to be true: "I have no clear and distinct perception of this matter … I am reluctant to say positively what the nature of bodies is" (p. 27). Instead of describing real bodies, he settles for describing "a certain kind of being similar in every way to bodies" (p. 27, see also Stein, 2016, p. 274; McGuire, 1983). Newton explicitly disavows natural-philosophical treatment:

> I have not defined [body] *in a philosophical manner* … So that instead of physical bodies you may understand abstract figures *in the same way that they are considered by geometers*. (13, emphasis added).[23]

Newton's definition abstracts from bodies only those features needed for enabling further inquiry and treats body only as "that which fills place" (p. 13). Importantly, although "body" is discussed in *DG*'s note, the above definition is first provided in *DG*'s list of *formal* definitions. This is exactly where we would expect to find definitions that are "accommodated … not to physical things but to mathematical reasoning, after the manner of geometers" (p. 39).

Space is treated differently. Newton thought the nature of space was "exceptionally clear," "immutable," and necessary (pp. 22, 26). His account aims to capture the real being as it is and has to be, not a "kind of being" fabricated for mathematical tractability. Consequently, space is *not* defined in *DG*'s list of formal definitions. This is despite the fact that with within pages of that list and Newton's claim that space is too well known to be defined, he writes that "it may be expected that I should define extension as substance or accident, or nothing at all … [b]ut by no means," and follows with an extended definition (*DG*, p. 21). But given his all his explicit comments on the nature of definition in *DG*, his intent should be clear: he does not define space in the *manner of geometers*, although he does define it natural-philosophically by explaining what it is in and of itself.

Newton believed he was able to provide a natural-philosophical account of space because of a complicated chain of reasoning. Briefly: Newton held that space has "its own manner of existing," that it is "as it were an emanative effect of God and an affection of every kind of being" (*DG*, p. 21). Consequently, space exists necessarily. Because space exists necessarily, Newton was confident that his "exceptionally clear" account of it was certain. This is an odd inference: from the necessity of space, Newton inferred the certainty of the idea of space. He was clear about this, although he put the point negatively, in comparison with his account of body. Of body, he wrote, "the explanation must be more uncertain [than of space], for it does not exist necessarily but by divine will" (*DG*, p. 27). The contrapositive, we are to infer, is that since space exists necessarily, we can be more certain in our 'explanation' of it.[24] In fact, Newton treats this explanation as fully certain.[25] Space's unique manner of existing thus entails its modal status, and its modal status entails the certainty of its account. Newton believed that he was defining the necessary thing in itself, natural-philosophically.

Why Newton believed that space has its own manner of existing is a touchier point. Newton held that our knowledge of space's manner of existing follows from the fact that it is causally inert; that is, from the fact that "there is no force of any kind present in [space]," no "causal principle" of either a physical or mental nature (*DG*, pp. 26, 36, 33). He wrote:

> [I]t may be expected that I should define extension as substance, or accident, or else nothing at all. But by no means, for it has its own manner of existing which … fits neither substances nor

[24] The inference is odd, but less so when we consider that it was underwritten by the Cartesian theory of ideas that lurks in *DG*'s background. See McGuire (1983) and Gorham (2011).

[25] As Stein (2016) notes: "Newton distinguishes between the epistemological

accidents. It is not substance … because it is not among the proper affections that denote substance, namely actions, such as are thoughts in the mind and motions in body. [Similar reasoning follows regarding accidenthood]. (*DG*, p. 21).

Crucially, Newton thought he could establish the latter claim *experimentally*, by showing that spaces void of all (physical) activity, and thus void of substance, exist. On the basis of pendulum measurements of aetherial resistance and the phenomena of rise and descent, he argued:

> [T]here are empty spaces [*inania*] in nature. For if the aether were a corporeal fluid entirely without vacuous pores, however subtle its parts are made by division, it would be as dense as any other fluid, and it would yield to the motion of passing bodies with no less inertia … Since the resistance of the aether is on the contrary so small … there is all the more reason for thinking that by far the largest part of the aetherial space is empty … [Empty space] may also be conjectured from the various gravities of [quicksilver, air, aether, etc.], for the descent of heavy bodies and the oscillations of pendulums show that these are in proportion to their densities, or as the quantities of matter contained in equal spaces. (*DG*, 35).

He made the same point in *E1*, in a corollary to proposition 6 of Book III:

> And thus a vacuum is necessary. For if all spaces were full, the specific gravity of the fluid with which the region of the air would be filled, because of the extreme density of its matter, would not be less than the specific gravity of quicksilver or of gold or of any other body with the greatest density, and therefore neither gold nor any body could descend in air. For bodies do not ever descend in fluids unless they have a greater specific gravity (Newton, 1999, p. 810).

Newton argued that differences in inertial resistance and specific gravity show that "'[different] quantities of matter [are] contained in equal spaces," and that this can only happen if equal spaces contain different proportions of full parts (bodies) and empty parts (void space). Space itself must therefore be void.[26] From the phenomena, Newton reasoned to the existence of empty space, to space's unique "manner of existing," and thus to the certainty and natural-philosophical status of his account of space.

For the same reasons, Newton treated space natural-philosophically in the *Principia*.[27] This is why he provided his account of space in a scholium, not through the apparatus of formal definitions. In this light, Newton's *non definitio* caveat seems quite apropos. As we saw, the *Principia*'s formal definitions conclude with the warning: "[I consider] these forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by words of this kinds I am anywhere defining a species or mode of action or a physical cause or reason …" (Newton, 1999, p. 408). Reading this, one may easily assume that the same caveat applies to the subject of the very next paragraph; namely, words of the kind "time, space, place, and motion." In order to ward off this misinterpretation, Newton asserts *non definio*. Like Huggett and Rynasiewicz, I read this to mean that he does not define time, place, etc. *in the manner in which he defined the foregoing definitions*, which, on my account, is the "manner of geometers." Instead, he supplies his definitions in a scholium, precisely because he intends them to be taken natural-philosophically.

Unlike Huggett and Rynasiewicz, however, I do not believe the story ends there. To see this, and to answer questions C and D, we must return to Newton's argument for the emptiness of space, as well as introduce a set of events that I believe prompted Newton to come face-to-face with the methodological limitations of his account of space. To be clear: one can reject the following developmental view and still accept the *mos geometrarum*/natural-philosophy view just offered of the scholium, alongside the broader interpretation of the PMD. However, I believe the latter is most revealing when set against Newton's later development.

6. Interlude: draft definitions of body and void

We have just seen that the natural-philosophical status of Newton's account of space relied on an intricate chain of reasoning that bottomed out in empirical evidence concerning the existence of empty spaces. That empirical evidence, however, was not without objection. Newton reasoned that differences in resistance and specific gravity show that "[different] quantities of matter [are] contained in equal spaces," and that this can only happen if equal spaces contain different proportions of full parts (bodies) and empty parts (void space). However, the inference to empty parts relied on Newton's belief that a body's quantity of matter is proportional to the volume it impenetrably fills and that this proportion is the same for all bodies. Without this measure of quantity of matter — what Smeenk and I (2012) call Newton's *geometrical conception of matter* — one need not resort to empty space to explain differential specific gravity or differential resistance. For example, an Aristotelian could hold that all volumes are entirely full, but explain differences in specific gravity through the presence of non-porous, continuous substances whose quantities of matter vary per unit volume. A Cartesian could similarly hold that all volumes are full (and in fact, identical with body), but explain observable differences through high-level physical mechanisms, not the fundamental nature of body and extension (see Descartes, 1983, Part III, §122). Newton's evidence is open to several interpretations, not all of which require the existence of essentially void space.

Newton did not address this problem until 1712, when he was pushed to do so by Roger Cotes. Cotes — the editor of the second edition of the *Principia* — took issue with the corollary quoted in the previous section. He pointed out that, in the *Principia,* a body's quantity of matter is always measured by its *vis inertia,* which is defined as an inherent force that "is always proportional to the body" (Newton, 1999, p. 404). The laws of motion detail how a body with this sort of inertia behaves in response to impressed forces, but they are silent about the relationship of inertia or impressed forces to the volume that a body impenetrably fills. Newton, however, had held that impenetrably-full, equally-sized volumes possess identical quantities of matter, and so had assumed that

[26] Newton also determined in the very first *De motu* draft that Kepler's laws hold exactly for a body moving under a centripetal force iff no resistance is supposed. This entailed that aethereal *resistance* is virtually nil. However, Newton thought he needed further arguments to show that the heavens are *empty*. These are the arguments above. As we shall shortly see, the sea-change in Newton's treatment of empty space was his recognition that these additional arguments were suspect, but also superfluous, for the purposes of the *Principia.* For the interplay between arguments concerning activity and arguments concerning fullness/emptiness, see

define or assume the quantity of Matter to be proportionable to its *Vis Inertiae*, You must not at the same time *define or assume* it to be proportionable to ye space which it may perfectly fill without any void interstices" (Newton, 1959, V p. 228, emphasis added). The implications for the existence of empty space are straightforward. Cotes wrote:

> [A vacuum is only proved] upon this concession, that the Primigenial particles … have all the same *Vis Inertiae* in respect to their magnitude or extension in *Spatio pleno*. I call this a concession because I cannot see how it may be certainly proved either a Priori by bare abstracted reasoning; or be inferr'd from Experiments. (Newton, 1959, V p. 228).

Although Newton initially disagreed, he came to see Cotes's point after several exchanges. Consequently, in *E2* he replaced the previously quoted corollary with a conditional, acknowledging the hidden assumption:

> If all the solid particles of all bodies have the same density and cannot be rarefied without pores, there must be a vacuum. I say particles have the same density when their respective forces of inertia are as their sizes. (Newton, 1999, p. 810).

But matters did not end there. The same complaint was also lodged against Newton by Leibniz. In a letter to Abbé Conti (November 1715), Leibniz asserted that "M. Newton adduces no experiment or sufficient reason for the existence of a vacuum" (Newton, 1959/1977, VI p. 252). In the same month, Leibniz also initiated his famed correspondence with Clarke. He repeated similar criticisms in the postscript to his fourth letter (Biener, 2017, p. 5). What's important for present purposes is that there is good reason to think that Leibniz's grievance — prefigured by Newton's dialogue with Cotes — elicited a series of *Principia* revisions aimed at clarifying the foundations of Newtonian philosophy.[28] Among them were a set of new *formal* definitions intended to precede Book III.[29] They include definitions of "body" and "vacuum" — the very subjects of Cotes critique. They fall in line with the account of formal definition offered in §4 and repeat some of the key ideas we saw in *DG*.

We saw Newton's definition of body in §2. Instead of repeating it, here is his correlative definition of "vacuum":

> Definition II: Vacuum I call every place in which bodies are able to move without resistance … But it follows from the first Definition [of "Body"] that vacuum is that which is not a tangible thing and does not impede the motion of bodies. [UCL add 3695 422r][30]

Before commenting on the definition's significance, a word about its content. The definition is a profound shift in Newton's conception of empty space. Earlier, Newton had tied the vacuum to questions of fullness and emptiness. He argued from experimental evidence to the idea that some volumes had to be entirely empty, taking it as given that those volumes corresponded to "vacuum", "inania," or just "empty space." Here, he leaves questions of underlying fullness/emptiness aside. What matters for establishing the existence of "vacuum" is not whether volumes are full or empty, but whether bodies encounter opposing *vis inertia* when moving through them. He thus skirts Cotes's problem: because he separates dynamical questions about the presence/absence of inertia from questions about fullness/emptiness, he needn't presume that "forces of inertia are as [bodies's] sizes" in order to show that "vacuum" — as defined above — exists.

The methodological significance of the definition is also profound. We can see this by comparing it to the scholium's definitions, keeping in mind DiSalle's characterization of the latter from §1. The exchange with Cotes and III.6.3 both suggest that before Newton authored this definition of "vacuum", what he meant by the term was "established on purely philosophical grounds," something akin to 'absolutely empty space; ' or, literally, 'the empty.' Whether this sort of empty existed was a problem that had occupied British natural philosophers for decades, and Newton, for his part, was showing "what physics has to say about [this] philosophical concept."[31] But with the definition above, Newton was no longer "taking any such meanings for granted, but defining [a] new theoretical concept within a framework of physical laws" (DiSalle, 2016, p. 37). He offered a precise concept whose empirical content was fully specifiable within the bound of Newtonian theory, but by so doing divorced "vacuum" from much of its traditional meaning. Newton's strategy here is the same intra-theoretic definitional strategy he used in the scholium on space and time. In both texts, he defined the key concepts of his natural philosophy from *within* that natural philosophy — in particular, from within the inferential framework of the laws of motion — not prior to it.

What makes these drafts special is that in them, unlike in the scholium, Newton explicitly reflected on the nature of this definitional strategy. He told us how he understands definitions of this sort. His comments thus provide a cypher for interpreting the scholium, as well as an answer to D. Here is the definition in full:

> Definition II: Vacuum I call every place in which bodies are able to move without resistance … But it follows from the first Definition [of "Body"] that vacuum is that which is not a tangible thing and does not impede the motion of bodies. *For just as geometers define a line that has length without breadth*, so that their propositions concerning lines of this sort are only understood, and in mechanics, however, and other sciences the line having breadth has a place; *thus body and vacuum are here defined* so that these words may be understood in the sense defined in what follows. About other sorts of bodies and other sorts of void let authors in other sciences dispute. [UCL add 3695 422r, emphasis added].[32]

The central analogy appears, in various formulations, in every version of the draft definitions: as geometers can abstract from physical lines and by so doing *define* a line without breath, so Newtonians can abstract from real-world bodies and by so doing *define* "body" and "void" as they are defined here. The new

[28] There were other possible causes, although the record is less clear. For example, Cotes's objection had been published by 1712 by Robert Greene, a fellow of Clare Hall, Cambridge. He was certainly known to Newton and Cotes, and Cotes very likely read his (1712). Cotes did not hold Greene in high regard (Algarotti, 1772, p. 132), but Greene did present an astute analysis of some of Newton's unjustified commitments.

[29] McGuire (1966) dates the definitions to 1716. They were never published. They did, however, reach stable form, suggesting that Newton had settled on their content. The definitions also echo statements made in the *Opticks*, see Newton

definitions do not speak to the natures of body and void *in rerum natura*, but to the concepts required in order to make sense of a world using the tools of Newtonian physico-mathematics. They are not *the* definitions of body and void, but are the definition that enable the investigation at hand, the definitions consonant with the treatment of bodies according to the principles of the *Principia*.

An immediate consequence is that although Newton's strategy allows him to precisely define the key concepts of his natural philosophy, it also throws into doubt their status *as* natural philosophy. This is why they are offered as formal definitions; and thus, this is the answer to D. In fact, Newton signals their stipulative nature *by* offering them as formal definitions. Compare the above passage to the already quoted passage from *DG*, written at least 30 years earlier:

> And thus I have accommodated these *definitions* … to mathematical reasoning, after the manner of the geometers … And just as the dimensions of physical bodies are best determined by their geometry – as with the dimension of a field by plane geometry, although a field is not a true plane; and the dimensions of the earth by the doctrine of the sphere, even though the earth is not precisely spherical – so the properties of physical fluids and solids are best known from this mathematical doctrine, even though they are not … as I have *defined* them here. (*DG*, pp. 38–39, emphasis added).

The message is the same. Both passages concern definitions that may not be true from a natural-philosophical viewpoint, but are made to enable further study. In *DG*, the definitions embody the assumption that matter is continuous, although, natural-philosophically speaking, it is really atomic. In *E3*'s drafts, the definitions embody the assumption that only entities possessed of *vis* inertia — entities governed by the laws of motion — matter for the *Principia*'s universe. *About other sorts of bodies and other sorts of void let authors in other sciences dispute.* Like geometrical definitions, these definitions are stipulative, but not in the sense of being arbitrary. Rather, they are stipulative because they embody and rely on assumptions chosen in order to enable further study of their subject matter. Whether they are also true is not a question that can be asked from within the *mos geometrarum*. They are *definitions*. Newton's methodological point is that geometers can offer such definitions licitly.

Before circling back to question C and the scholium on space and time, I should note that the stipulative nature of *E3*'s definitions may explain why they were never published. Recall, they were intended to preface Book III of the *Principia*, the book that "come[s] down to physics" from the mathematical peaks of Books I and II (Newton, 1999, p. 588). However, the most enduring criticism of Newton's work — first articulated in the *Principia*'s first review — was that "the work of M. Newton is a mechanics, the most perfect that one could imagine … [yet] he has not considered [its] Principles as a Physicist, but as a mere Geometer" (in Koyré, 1965, p. 115). Publishing definitions that on their surface stated the same — "for just as geometers define a line that has length without breadth … thus body and vacuum are here defined" — would have been too great a cost. Other prominent changes to Book III also qualified the book's status as traditional natural philosophy. For example, Rule 2 was changed from "the causes of natural effects of the same kind *are* the same" to "the causes *assigned* to natural effects of the same kind must be, so far as possible, the same; " i.e., from an ontological claim to an epistemological one (p. 795, emphasis added; see also requested in Newton's handwritten *Corrigenda et addenda* page immediately below the draft definitions of "body" and "vacuum," and are clearly consonant with them. However, the definitions are far more explicit about the methodological direction in which all changes point.[33] Newton was creating a novel method for asking and answering philosophical questions, catered for "philosophical geometers and geometrical philosophers" (Newton, 1984, p. 87). But its articulation proved difficult and was often met with opposition and misunderstanding. He had good reason to expect that his new definitions would be met with the same.

7. Treating space *more geometrarum* (post-1710s)

Having answered A, B, & D (§§4, 5, & 6), we can hazard an answer to C; why did Newton remove *non definio* from the scholium? The answer relies on the tight-knit connection between Newton's conception of space and his experimental evidence concerning its emptiness. As we saw in §5, Newton reasoned from the phenomena of rise and descent and differential resistance to the existence of empty space, to space's unique "manner of existing," to the certainty and natural-philosophical status of his account of space. However, his draft definition of "body" and "vacuum" show that he had come to think differently about the first inference in that chain. The new definition entailed that the phenomena of rise and descent and differential resistance do *not* imply that space as such is empty. They only imply that it is empty of resistance and the entity defined through that single power. But perhaps it is full of "others sorts of bodies" with other sorts of *vires*. Perhaps it is *identical* with one of them and a substance in its own right. Perhaps space is identical with the familiar sort of body, but differentially resistive. Perhaps it is negligibly resistive, but high-level physical mechanisms are responsible for higher differential resistance. Some etherial explanations of gravity traded on the possibility of different sorts of matter; and both Aristotelian and Cartesian sympathizers explained differential resistance with no recourse to empty space (Greene, 1712, §1.1). With these possibilities on the table, Newton could not claim with certainty, on the basis of empirical evidence, that space as such is absolutely empty. This much he admits in his definition of "void."[34]

Whether Newton followed the consequences this admission is less certain. If he had, he would have realized that since he could not argue with certainty to space's emptiness, he could not argue for its inertness; consequently, he could not claim that it has its own manner of existing, that it is ontologically necessary, and that his own account of it is the unqualified natural-philosophical truth. All he could claim is that space is empty *on the assumption that body and void are defined as their treatment in the Principia suggests*. His account of space would therefore not be natural-philosophical, but only *more geometrarum*, as his treatment of body and void. Did Newton actually make these connections? There is some reason to suspect that he did. In the 1710s, especially 1715–1716, Newton was engaged in debate about the fundamental concepts of Newtonian philosophy, both directly and through proxies. These concepts included body and vacuum, but, more famously, space, time, and

[33] Newton also modified the General Scholium in a similar vein. In *E2*, he asserted that "to treat of God from phenomena is certainly a part of experimental philosophy." In *E3*, the latter two words were changed to "natural philosophy" (p. 943). This change also curtails the ambitions of the *Principia*'s experimental method and leaves matters of unqualified truth to a somewhat different enterprise, that of natural philosophy.

motion. They elicited several high-profile clarifications of his philosophical position in letters and published material, including the drafts we just examined. That Newton would have attended to his concepts of body and vacuum and nevertheless failed to recognize their consequences for the metaphysics of space seems to me unlikely.[35]

This is a far less radical claim than it might seem. I am not suggesting that Newton stopped believing in his account of space *simpliciter*. Rather, I am suggesting that he stopped believing in it an item of traditional natural philosophy. Instead, he came to see it as framed "in the manner of geometers." He took it to be dependent on, and enabling of, the *Principia*'s physical theory, just like his definition of "body" and "vacuum."

Stein's reading of the scholium is useful here. On Stein's reading, Newton uses the fundamental suppositions of the *Principia*'s dynamics to show that space and time must be understood as essentially geometrical structures with properties that allow for the appropriate characterization of inertial motion and acceleration. For Stein, space and time's modal and ontic properties — are they necessary? substantial? emanative? real? — are besides the point. As he puts it:

> If the [mathematical] structure [relevant for understanding Newtonian physics] … is in some sense really exhibited by the world of events; and if this structure can legitimately be regarded as an explication of Newton's "absolute space and time"; then the question whether, in addition to characterizing the world in just the indicated sense, this structure of space-time also "really exists," surely seems to be supererogatory (Stein, 1967, p. 277).

According to Stein, asking whether the scholium's definitions capture space and time as they really are — what Newton would have called natural-philosophically — is a fool's errand. It is the wrong question to ask. They are *defined* so that further questions about the world make sense. In *E3*'s drafts, Newton treated "body" and "void" in precisely this way and, moreover, articulated this approach in terms of "the manner of geometers." By claiming that Newton came to treat space in the manner of geometers, I am only claiming that Newton came to see it as Stein sees it. Articulating the nature of his definitions of "body" and "void" would have revealed, first, that his new definitions and the scholium's definition are methodologically parallel and, second, that the former bear on the latter in a way that only further suggests that his previous arguments for space's ontological and modal properties are suspect, and so that his account of space cannot be taken natural-philosophically, but only *more geometrarum*, as his definitions of "body" and "void."

Of course, it is possible that Newton already saw things this way. However, I think it is more likely that he came to this view in the mid-1710s (cf. DiSalle, 2013). Although the scholium's definitions remained identical in all editions of the *Principia*, if Newton had initially conceived of them as Stein (1967) does, he should have rebuffed existential questions already in the 1680s. But, on the contrary, he took them quite seriously, as the *Principia's* 1690s revisions and the General Scholium suggest. After all, he had empirical evidence to support what he thought were space's modal and ontic properties, and this evidence stood independently of the *Principia*'s fundamental assumptions concerning the description of motion; they followed directly, as he saw it, from the phenomena of rise and descent and differential resistance. My conjecture is that only when that evidence came under attack that Newton began to question his account of space's modal and ontic properties. And so, only then did he begin to see that whether his account of space was properly natural philosophical or not was besides the point for the *Principia*, in the same way that questions about whether geometrical entities are really as they are defined are besides the point in geometrical works. Of course, this is just a conjecture. I believe it is clear, however, that *E3*'s draft definitions and the scholium's definitions are methodologically parallel; and so, Newton's reflection on the former, which echoes *DG*'s characterization of the *mos geometrarum*, tells us how he thought about the latter. Independently of my conjecture, "defining *more geometrarum*" provides us with an actor's category by which to articulate what Newton thought he was doing in the scholium, without the need to veer into neologism or backwards looking Kantianism.

And this, finally, brings us back to *non definio*. In §5, I argued that the caveat was present in *E1* and *E2* to ward off a possible misinterpretation: since the *Principia*'s formal definitions were to be understood "not from a physical, but only mathematical point of view," Newton wanted to stress that "time, space, place, and motion" were not to be understood in the same way (Newton, 1999, p. 408). The latter concepts were placed in a scholium because they were to be understood natural-philosophically. However, if Newton began to believe that his treatment of space was more *more geometrarum* than he once thought, perhaps he chose to correct his earlier overreach. Completely altering the *Principia*'s introductory sections would have come with significant polemical costs, and would have required the sort of methodological commentary Newton's was reticent to offer. But removing *non definio* as he did in *E3* could change the sense of the scholium's introductory paragraph with minimal interruption. Without the caveat, the paragraph can be more naturally read as distinguishing between the definitions of "less familiar words" thus far given (in the "Definitions") and the definitions of "very familiar" words to follow (in the scholium), exactly as we now read it. And, to reiterate, this doesn't mean that the scholium *merely* articulates the meaning of words. Rather, it articulates the meaning of words *more geometrarum*; that is, in a way that is both conceptually rich and well circumscribed, as described in §5.

8. Conclusion

To sum, this essay has weaved together a story that touches on several interpretive issues: Newton's use of "definition," his physical/mathematical distinction, the nature of his methodological suppositions in *E3*'s draft definitions and the scholium on space and time, and their interrelations. It has also provided a developmental account of Newton's thought in order to answer the four main questions that made up this essay's 'skeleton.' To reiterate: I've argued that Newton drew a principled methodological distinction between the *mos geometrarum* and natural philosophy, one that is present in both *DG* and *E3*'s draft definitions. He associated the two methods primarily with formal definitions and scholia, respectively. In *DG*, *E1*, and *E2*, he thought his treatment of space fell on the natural-philosophical side of the divide. He signaled this by treating space in a scholium and by his *non definio* caveat. I've also argued that by *E3*, he had come to see his treatment of space as more in the manner of geometers, although the evidence that shows this went unpublished. In the published work, Newton signaled his more qualified stance by removing the caveat. The last two claims are conjectural. But even if they are wrong, I've tried to methodological stance is best understood through the mathematical/physical distinction drawn in *DG*, not the one drawn in the *Principia*.

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