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On The Integration of Mathematical and Physical Considerations in Aristotle's Subordinate Sciences

Abstract

Aristotle held that nature can be studied mathematically by sciences that stand "beneath" arithmetic or geometry: optics, harmonics, astronomy, and nascent forms of mechanics. On an orthodox way of reading him, these 'subordinate' sciences differ from pure mathematics only in their restricted subject matters, which are nonetheless treated wholly mathematically. I argue for an alternate reading. I show that while a subordinating science's way of treating its subject matter is wholly mathematical, a subordinate science's is defined by the essential inseparability of physical and mathematical considerations. My textual case focuses on Aristotle's dialectic use of the subordinate sciences in *Physics* 2.2. I show that Aristotle uses the subordinate sciences to exemplify a mode of thought that he believes ought to be definitional of 'physics,' one in which material and formal considerations are inseparable. However, I argue that on the orthodox reading of Aristotle the subordinate sciences cannot exemplify this. For context, I appeal to Euclid's *Optics* as well as Plato's *Republic*. I further show that ancient optical practice involved an inseparable mélange of physical and mathematical considerations. I conclude that my alternate reading makes better sense of the *Physics* and better represents the scientific context in which it was written.

Introduction

Aristotle famously held that nature can be studied mathematically by sciences that stand "beneath" arithmetic or geometry.[1] These so-called subordinate sciences included, for Aristotle, optics, harmonics, astronomy, and nascent forms of mechanics.[2] Although his account of them is somewhat enigmatic, many

*. Many thanks to X and Y. I am especially indebted to Z for his ZZ and conversation.

1. *Posterior Analytics* I.7, 75b14–16.

2. For the state of these sciences in Aristotle's time, see Mueller (2004), Burnyeat (2005), Burnyeat (2000), and McKirahan (1978). Although Aristotle allows for subordination outside

scholars agree on the following: Subordinating and subordinate sciences study their objects by means of the same mathematical concepts and principles. Their difference pertains only to their subject matters. Subordinating sciences investigate abstracted mathematical attributes, while subordinate sciences investigate those same mathematical attributes in particular natural domains. Geometry, for example, investigates straight lines *simpliciter*, while mathematical optics investigates straight lines in the domain of vision.

This account of subordination and 'applied mathematics' has venerable proponents. Elements of it can be found in Thomas Aquinas, for example. Using an altered spatial metaphor, Thomas writes:

> Those sciences are called intermediate sciences [*scientiae mediae*] which take principles abstracted by the purely mathematical sciences and apply them to sensible matter. For example, optics [*perspectiva*] applies to the visual line those things which are demonstrated by geometry about the abstracted line; and harmony… applies to sound those things which arithmetic considers about the proportions of numbers…[3]

Jacopo Zabarella was most explicit in his commitment to the methodological identity of subordinating and subordinate sciences. Adopting terminology that originated in Ibn Rušd (Averroës), Zabarella claimed that subordinating and subordinate sciences exemplify the same mathematical *modus considerandi*, the same way of treating their subject matters.[4] What distinguishes them is that a subordinate science treats a mathematical subject matter conjoined to a sensible quality, while a subordinating science treats that subject matter divorced from

of mathematics, I use "subordinate sciences" to refer only to the mathematically subordinate sciences.

3. Aquinas (1963, 80); translation modified from Aquinas (1884, §2.3.8, 36). Aquinas also suggests a contrasting characterization of the subordinate sciences, one which I will defend in this essay. See Aquinas (1888, I.1.1.2, 7).

4. See Laird (1983, 216ff.). As Laird notes, Zabarella also called the *modus considerandi* a *ratio formalis* or *forma constituens*; it is the formal aspect of a science, distinguished from its material aspect or *res considerata*.

sensible quality. The presence or absence of sensible quality, however, does not alter the two sciences's shared *modus considerandi*, which is their primary defining characteristic. A modern commentator writes:

> From the point of view of the manner of treating the subject matter, this difference [in sensible quality] is irrelevant—what is added is accidental. It is the *modus considerandi* that determines the structure, organization, and approach used in the proofs of the science[s].[5]

This interpretation of Aristotle can be found to varying degrees in a host of contemporary works.[6] It holds that, apart from their differing subject matters, subordinate and subordinating sciences are the same; they treat their subject matters in the same way. Put differently, it holds that subordinate sciences are just mathematical sciences, but their subject matters are restricted to specific natural domains. The interpretation is premised on the idea that physical considerations related to a subordinate science's subject matter (other than the fact that it can be treated mathematically) are essentially separate from its proper mathematical treatment. I call this the 'Identity Interpretation' of subordination.

In this paper, I argue for an alternate reading. I show that the Identity Interpretation fails to provide a proper textual analysis of Aristotle, particularly that it fails to capture what is distinctive about the subordinate sciences from Aristotle's point of view. I argue instead that subordinate and subordinating sciences do *not* treat their subject matters in the same way, that they use *different modi considerandi*. While a subordinating science's *modus* is thoroughly mathematical, a subordinate sciences's *modus* is defined by the essential inseparability of physical and mathematical considerations.[7] In a subordinate science both physical and mathematical

5. McKirahan (1978, 202).
6. More on this in §4.
7. This claim is superficially similar to one made by proponents of the Identity Interpretation; namely, that it is essential for a subordinate science to treat some physical kind, albeit mathematically. My claim will turn out to be significantly stronger and incompatible with the Identity Interpretation.

considerations determine the "structure, organization, and approach" used. Put differently, there is more to a subordinate science than pure mathematics, but restricted to some specific natural domain. I call this the 'Difference Interpretation.' This interpretation shows that Aristotle was far more accepting of what came to be called 'physico-mathematics' in the scientific revolution, and that these combined physical/mathematical investigations fit his vision of natural science far more thoroughly than we have previously supposed.

My textual case focuses on Aristotle's seeming 'aside' on mathematics and the subordinate sciences at the beginning of *Physics* 2.2. This text provides a unique opportunity for studying the subordinate sciences because it is not Aristotle's purpose in it to shed light on the nature of subordination. Rather, his purpose is to argue that a proper science of nature—the articulation of which is the work's task—is a unified investigation of matter and form. Aristotle only appeals to the subordinate sciences in order to further that purpose. Consequently, the *Physics* allows us to see the subordinate sciences at dialectical work, so to speak. The crux of my case is that, if we believe the Identity Interpretation, the subordinate sciences cannot carry out the dialectical task Aristotle set for them. To respect the argumentative structure of *Physics* 2.1–2.2, we must endorse the Difference Interpretation.

The paper runs as follows. I begin in §2 by asking what the dialectical role of Aristotle's 'aside' on mathematics is. I consider a standard answer and argue it is deficient. In §3, I offer an alternate answer. I claim that the 'aside' is intended to provide an example familiar to Aristotle's Platonic audience of how formal *inseparability in thought* is compatible with formal *priority* over them. This set of characteristics would have likely seemed incompatible to Aristotle's Platonic audience, but their juxtaposition is central to the hylomorphism Aristotle wishes to defend. In §4, I argue that *on the Identity Interpretation* it is hard to see how the subordinate sciences can provide such an example, at least to the extent Aristotle suggests. Either Aristotle exaggerated their illustrative nature or the Identity Interpretation is wrong. In §5, to articulate my alternative 'Difference Interpretation' and to provide contextual support for it, I appeal to the practice of ancient optics as well as to Plato's treatment of mathematics in *Republic* VII. These show that Aristotle's audience would have likely understood the subordinate sciences to involve a method in which mathematical (formal) and physical (material) considerations are essentially inseparable, one that is distinct from the method of pure mathematics. In other words, they would have likely endorsed something like the Difference Interpretation. This explains why Aristotle used the subordinate sciences as he did. They exemplified to his Platonic audience his vision for the study of nature: as a science in which formal and material considerations are inseparable. Since the Difference Interpretation is preferable both textually and contextually, I conclude (§6) that we must reject the Identity Interpretation.

1 What is the Purpose of Aristotle's Discussion of Mathematics in *Physics* II.2?

To discover the role of the subordinate science in the overall dialectic of *Physics* 2.1 & 2.2, we begin with a textual problem. In *Physics* 2.1, Aristotle distinguishes things that are due to nature from those that are due to other causes. By considering items of both sorts, he uncovers the implicit standard according to which they are distinguished; namely, that things that are due to nature have within them "a source of change and staying unchanged" primarily, by virtue of the sorts of things they are (192b9–27). With this at hand, Aristotle continues to consider ways in which "nature" can be understood; that is, ways in which the "source of change and staying unchanged" can be attributed to a thing given that thing's account. Two options are offered: nature can be a source of change in virtue of a thing's "primary underlying matter" (193a9–30), and/or it can be a source of change in virtue of a thing's "shape and form" (193a28–193b18). Aristotle considers which has the *better* claim to grounding a thing's nature, and comes down decisively on the side of form (193b7–9). At the end of Chapter One, he adduces additional instances in which "nature" is best spoken of as form (193b10–18).

This is where the problem arises. At the beginning of Chapter Two (193b22–26) the focus of discussion abruptly shifts, only to double back to the topic of Chapter One some lines later (194b12). At the beginning of Chapter Two, instead of continuing to consider the relations of matter and form, Aristotle turns to consider whether/how an inquiry into nature differs from mathematics, and whether/how astronomy, in particular, may differ from both (193b22–194a12). Scant mention is made of matter and form:

> Since the number of ways in which nature is spoken of has been distinguished, we should next consider in what way the mathematician differs from the physicist. For natural bodies have planes, solids, lengths, and points, and the mathematician investigates these things. Furthermore, is astronomy different from or a part of physics? For it would be strange if it were for the physicist to know what the sun or the moon is but not for him to know any of their essential properties especially because it is apparent that physicists speak about the shape of the sun and the moon and whether or not the earth and the cosmos are spherical in form or not… (193b22–31).[8]

By the middle of Chapter Two, however, Aristotle returns to the interrelations of matter and form. In the spirit of Chapter One, he argues that nature consist of both, but primarily form, and so the natural scientist should study both matter and form, but primarily form. Thus the problem: What is the purpose of the

8. I largely follow the translation in Mueller (2006) for 193b22–194a12, Charlton (1970) for the remainder of the *Physics*, and Barnes, Schofield, and Sorabji (1975) for other texts. I use both "natural" and "physical" for the adjectival form of φύσις, as well as "natural scientists" and "physicists" for the *physikoi*.

interposing discussion of mathematics and astronomy? Why would it be needed before drawing, by the end of Chapter Two, the seemingly obvious conclusion that the study of nature should have as its subject *nature*, precisely as defined in Chapter One?

In one way, Aristotle's discussion of mathematics seems relevant enough. If one is out to articulate a science of nature—as Aristotle intends to do in the *Physics*—how better to start than by outlining its *differentiae* against those of mathematics, *differentiae* that include the sorts of objects it treats? Since Aristotle ends the chapter by contrasting physics with first philosophy (194b14–15), there seems to be a *prima facie* good reason for the aside: it is needed at the beginning of *Physics* 2.2 because the chapter's purpose is to draw boundaries between Aristotle's three theoretical sciences.[9]

But this can't be enough. First, the goal of disciplinary classification may explain why Aristotle discusses mathematics in general, but it does not explain why he discusses *astronomy*, in particular. What makes astronomy relevant? What issues does it raise that cannot be addressed by treating mathematics *simpliciter*?[10] Second, the goal of disciplinary classification does not explain why

9. In the commentary tradition, the limning of disciplinary boundaries is often cited as the primary purpose of *Physics* 2.2. Philoponus, for example, reads the the above passage against Aristotle's division of sciences in *Topics* 145a12–18 and *Metaphysics* 1026b18–19, 1064b1–3. He writes:

> Now since the study of nature is a section of the theoretical part of philosophy, and the theoretical [part] is divided into the study of nature, mathematics and theology, [Aristotle] wants next to distinguish the study of nature from mathematics and theology; for it belongs to the man with special knowledge to set apart, when delineating the matters which are relevant to him, those which seem to be relevant but are not really so (Philoponus 1993, 218,25–219,4).

Aquinas also focuses on disciplinary classification. Inspired by a mistranslation, see Mueller (2006), he notes that the purpose of the discussion of mathematics is to show that astronomy is a proper part of physics. Ross (1936, p. 506) and Charlton (1970, p. 93) also use disciplinary terms to state the rationale behind the passage.

10. The goal of disciplinary classification is often cited by those who endorse the Identity Interpretation (see works in previous note). They do not see a methodological difference between mathematics and astronomy, and thus find no need to explain why astronomy, in particular, is relevant at this point. As we shall see, however, Aristotle himself makes clear that mathematics

the discussion of mathematics should occur where it does; i.e., why it should interrupt a seemingly continuous chain of reasoning about matter and form.[11] With only disciplinary classification in mind, *Physics* 2.1 & 2.2 seem rather scattered, a sequence of not-entirely-connected thoughts about natural science. The problem only becomes more acute when we note that the ancient text lacked chapter divisions, and so did not contain what may now seem like natural breaks in the text. For these reasons, we should be able to find a better explanation for the arc of Aristotle's deliberations.

2 Carving a Path between Priority and Inseparability

To explain the broad arc of *Physics* 2.1–2.2, we need to return momentarily to *Physics* 2.1. As we saw, after showing that nature could be spoken of as matter and form, Aristotle argues that form has a better claim to grounding a thing's nature; i.e., that it is in some way prior to matter vis-à-vis the source of change and staying unchanged (193b15–19).[12] To argue for form's priority, he offers an analogy between art and nature (193a32) and analyses the application of the locutions 'is art' and 'is in accordance to art' (193a33–b3). The crux of the argument is that since locutions related to art apply to objects whose matter possess artful forms in actuality (not merely possibly), locutions related to nature ought to apply to objects whose matter possess natural forms in actuality (not

and astronomy are relevant for different reasons, a fact the Identity Interpretation has no resources to explain.

11. The placement of Aristotle's discussion of mathematics has not bothered many. Charlton (1970, 93), for example, takes the discontinuity in *Physics* 2.2 as intended, calling the latter half of the chapter "a fresh approach to the question whether the form of a thing can be called its nature". I'll argue, as Lennox (2008) does, that there is no fresh approach here, but a single line of argumentation.

12. I use the language of "priority" following *Metaphysics* XIII 1077b1–11, where Aristotle balances the priority of mathematicals in definition/account with their inseparability in reality/substance. My argument in this section can be stated thus: *Physics* 2.2 attempts to temper the very same platonic intuitions regarding formal priority as the *Metaphysics*, but focuses on the balance between priority in definition with inseparability, not in reality, but also in definition.

merely possibly). The chain of reasoning is summed thus:

> So there is another way of speaking [other than speaking of 'nature' as matter], according to which nature is the shape and form of things which have in themselves a source of their changes, something which is not separable except in account. (193b3–6)

Note, however, that this is not merely a summation. It introduces an idea not present in the previous argument: that form is "not separable except in account." Although inseparability had been discussed as part of the argument against the Eleatics in *Physics* 1.3 and quickly mentioned in the opening paragraph of *Physics* 2.1 (192b27), this central concept of Aristotle's hylomorphism has yet to be related to questions of matter and form. Nevertheless, Aristotle does not elaborate on it. He continues straightaway to state the main thesis for which the analogy with art supplied the initial argument: that form "has a better claim than matter to be called nature" (193b7). The notion of inseparability has thus been introduced as crucial for the characterization of form, but remains unexplained. In fact, even what form is inseparable from is not explicitly discussed, but must be gathered from context.[13] Aristotle's focus remains form's priority, for which he supplies additional arguments (from generation, 193b8–12; from growth, 193b13–18).

A problem thus arises by the close of *Physics* 2.1: given that form has better a claim than matter to be "nature" but is nevertheless inseparable from matter and change, how can one understand natural change in a way that reconciles form's seemingly opposing characteristics: its priority and inseparability?

13. Aristotle's main target is clearly change (i.e., κίνησις). However, given the overarching contrast in *Physics* 2 between matter and form and the availability of a thorough discussion of the underlier of change in *Physics* 1, it is curious that Aristotle does not mention separation from *matter* explicitly in this context. His other discussions of separation, particularly those regarding the snub, do focus on separation from matter (e.g., *Metaphysics* 1025b30–33), and so many have read "matter" and "change" interchangeably here. So will I. The straightforward justification is provided by, for example, Charlton (1970, p. 96). Charlton notes that since matter is the underlier of change, it is implicitly, but necessarily, referred to in the discussion of change. See also *Metaphysics* 1026a2–4, 1036a9–12 1059b12–16. The question of "intelligible" matter introduces further complications I will not address.

I propose that the purpose of Aristotle's aside is to answer this question by means of examples from mathematics. The aside provides a more nuanced understanding of form's seemingly janus features by supplying the missing treatment of inseparability at that point in the *Physics* where Aristotle's Academic audience would have expected priority and inseparability to be irreconcilable. In fact, they would have expected form's priority to trump its inseparability, and might have easily taken Aristotelian form to be no different from Platonic form. More precisely, they might have misinterpreted the idea that there is a way in which form is separable in account (193b6) to suggest that form is separable *simpliciter.* As we shall see, the aside on mathematics shows that Aristotle expected the inference and attempted to block it. By using examples familiar to Platonists, he shows that priority over matter does not entail separability from it and that separability even *in account* is highly circumscribed.

Aristotle's dialectical strategy is crucial here. He first highlights the ways in which he agrees with his Academic audience and then, using their common ground to set the terms of debate, show hows they err *by their own lights.* To see this, consider what sorts of mathematical investigations would have been familiar to Aristotle's platonic auditors. Minimally (we'll return to the question in §5), they would have been the ones described in middle dialogues like *Meno* 81a–86a, *Pheado* 73a–77a, 103–106, *Euthydemus* 290b–c, and *Republic* 527a-d. In these, mathematicals are said to have metaphysical priority over material things: they are genuinely existing, non-perceptual entities that are exemplified in material things only deficiently.[14] Accordingly, the job of the mathematician is to discover (perhaps recollect) them while taking no account of physical embodiment.

The beginning of *Physics* 2.2 trades on this conception. It insinuates that there is a way in which the physicist is rather like the mathematician (as Platonists

14. On whether Plato conceived mathematicals on par with Forms or as 'intermediaries,' see Annas (1976, p. 13ff.).

understands her), but with a different purview. Returning to our passage:

> [W]e should next consider in what way the mathematician differs from the physicist. For natural bodies have planes, solids, lengths, and points, and the mathematician investigates these things. Furthermore, is astronomy different from or a part of physics? For it would be strange if it were for the physicist to know what the sun or the moon is but not for him to know any of their essential properties especially because it is apparent that physicists speak about the shape of the sun and the moon and whether or not the earth and the cosmos are spherical in form or not… (193b22–31).

An overlap in the physicist's and the mathematician's investigations would have seemed natural to Platonists. As Charlton (1970) notes, "[t]he tendency of the Academy was to confine the study of nature to the study of the forms of natural things." Accordingly, to Academicians, the difference between mathematicians and physicists would have seemed one of scope: mathematicians study mathematical forms while physicists studies the forms of natural things, some of which are also mathematical. Put differently, to Academicians, mathematics could provide a model for natural science because (given the priority of form over matter) mathematicians and physicists both study their objects independently of material embodiment.

Aristotle goes on to object to this idea, but the mathematician's model is useful because it introduces the idea that the physicist *is* a student of form and that natural form *does have* priority over matter—a claim from *Physics* 2.1 Aristotle needs to stress to Platonists. However, the *way* in which form is prior—without being separable—requires further qualification. The case of mathematics is dialectically useful because it presents a clear case of justified (and soon to be qualified) priority and, as we shall see in §4, a set of further caveats concerning the subordinate sciences that Platonists were likely to accept. Aristotle describes the case of mathematics thus:

> Now the mathematician also treats these things [essential properties like the shape of the sun and the moon], but does not treat each as

> the limit of a natural body; nor does he study their properties as the properties of such bodies. Therefore he makes a separation; for they are separate from change in thought, and it makes no difference, nor does anything false result when they [*sic*] make the separation. (193b32–194a2)

Aristotle and his Platonic audience agree here. For both, the objects of pure mathematics are cognitively separable from matter and change. Their reasons differ, of course. Platonists believes that the objects of mathematics are separable in thought *because* they are separate in reality, whereas for Aristotle those objects are fundamentally attributes of physical things.[15] Each agrees, however, that under certain conditions the formal characteristics of physical objects can be cognitively separated from material embodiment, and no error results.

Yet Aristotle's point is *not* that mathematical objects are fundamentally attributes of physical things. In *Physics* 2, this goes with saying. His point is that, even granted this *ontological* commitment, the mathematician's manner of separation can't be that of the physicist, since the latter is even more limited than Platonists presume: some objects of study are not even *cognitively* separable. Aristotle had not addressed the topic in the closing passages of *Physics* 2.1 because the dominant theme there—that form has priority over matter—would already have been attractive to Platonists. In *Physics* 2.2, Aristotle is trying to reign in the platonic intuitions of his audience, and so must make clear that form's priority does not entail separability, even in thought.

The following oft-quoted passage serves as a deterrent against an over-emphasis of form's separability:

15. Because of this difference, Aristotle's justification of mathematical separation is more complex than the Platonist's, but it is *not* discussed here as it is discussed in, e.g., *Metaphysics* XIV 3. This is a significant fact. It indicates that in our passages Aristotle is merely assuming the example provided by the mathematician and is not worried about the underlying metaphysics that justify her practice. Put differently, although the *Metaphysics* is surely in the background here, the argumentative point is not about the ontological commitments of the pure mathematician, but about her cognitive practice and the extent to which it can shed light on the practice of physicist.

> Those who say there are forms do this unawares [i.e., separate from matter and change like the mathematician], since they separate natural things, which are less separable than mathematicals. This would become clear if one were to try to give definitions of each of them and of their properties. For odd and even and straight and curved and also number and line and figure will not involve change, but flesh and blood and man do involve it: these terms are used like snub and not like curved. (194a1–7)

This *cognitive* inseparability—the inability to define the objects of physics and their properties without reference to matter and change—is the central point of the hylomorphism Aristotle defends in the *Physics.* It entails that natural investigation must be an investigation of both matter and form, even if it is primarily an investigation of form. Aristotle's challenge is to make Platonists see its merits. As we shall see in §4, it is to this end that Aristotle invokes the subordinate sciences. They demonstrate that even within the familiar, platonically attractive, realm of mathematics—where separability in thought of forms from their material embodiment seems unquestionable—certain formal features *cannot* be cognitively separated from matter and change. And so, as we shall see in §5, they show to Platonists that they are *already* familiar with the idea that formal priority does not entail separability, and that they *already* accept it in certain contexts.

There are two further items to note before we get to Aristotle's treatment of the subordinate sciences. First, note that the scope of 'things' Aristotle believes cannot be defined without change includes both objects and "their properties." That is, his claim is not only that physical substances (as unities) cannot be defined without reference to matter and change, but that their properties (taken singly) cannot be so defined. And this is true even when those properties, like mathematical properties, are not constitutive of the definitions of the substances themselves. We will return to this point in the next section.

Second, note that Aristotle's attempt to temper platonic intuitions could also open the closing problem of *Physics* 2.1—how to properly balance form's priority and inseparability—to an an equally extreme misinterpretation. Aristotle's claim that a natural scientist cannot study form in complete separation from matter and change (because falsity *does* result) may suggest that insofar as a natural scientist is a student of *nature*, it is because she studies *enmattered* form. In other words, one may be pushed back into the Empedoclean/Democritan position of locating a thing's nature and "source of change and staying unchanged" solely in matter. On this interpretation, the natural scientist is, strictly speaking, concerned with matter, and so to the extent she studies form, she still does so in the same manner as the mathematician—independently of matter and change. Consequently, although Aristotle has already argued against a materialist physics in 2.1, the example of the pure mathematician leaves room for its reintroduction: "it might seem that the study of nature is the study of matter, for Empedocles and Democritus touched only very superficially on form and what the being would be" (194a18–21).

We are now in a position to resolve one of the puzzles of §2; namely, why the closing theme of *Physics* 2.1 is recapitulated in the latter half of *Physics* 2.2. Aristotle returns to questions of matter and form after the discussion of mathematics in order to block the Empedoclean/Democritan over-reaction to his anti-Platonic arguments. Aristotle must once again stress that the "source of change and staying unchanged" can be grounded in form *qua* form, not merely *qua enmattered* form. His point is that form can be studied as a source of change *insofar as it is form*, an activity that involves a cognitive separation, but one that is not as radical as the mathematician's. The closing lines of 2.2 go to stress just this by noting the ways in which the study of matter, insofar as it is relevant to the study of change—i.e., "up to a point" (194a22)—is not in itself a study of change, but is incumbent on the study of form as the source of change. Aristotle offers several examples from art here, stressing either that change is driven by form (e.g., 194a27–33) or that matter can only be understood relatively to form (194a33–b9).

The structure of *Physics* 2.2 as a whole is therefore contrapuntal, with an original anti-Platonic theme sounded by the discussion of mathematics and an anti-Empedoclean/Democritan counter-theme sounded by lessons drawn from art. Both themes are attempts to elaborate the leitmotif of *Physics* 2.1: that nature must be conceived of as both matter and form, but primarily form. At the same time, they go to constrain extreme positions which hold on to form's priority over matter or form's inseparability from matter, but not both. Put differently, we might say that the overall argument of *Physics* 2.2 seeks to center the proper interpretation of form between the two extremes just mentioned. That interpretation is most helpfully illustrated by means of the subordinate sciences, to which we now turn.

3 The Anti-Platonic Nature of Subordinate Sciences

Immediately after the passage last block-quoted, Aristotle asserts:

> This is also made clear by the more natural of the mathematical sciences, such as optics, harmonics, and astronomy; for they are in a certain way the reverse of geometry. For geometry investigates natural line, but not as natural, whereas optics investigates mathematical line, not as mathematical, but as natural. (194a7–12)

The connecting phrase—"[t]his is also made clear"—indicates that optics, harmonics, and astronomy supply additional evidence for the anti-Platonic position just articulated. The mode of investigation of optics, harmonics, and astronomy must be importantly *unlike* the mode of investigation of pure mathematics and must be in some way like an investigation of flesh, bone, man, and snubness: it must be impossible "to give definitions of each of them and of their properties" without reference to matter and change.

This difference is crucial. Although some consider the subordinate sciences to be species of mathematics (often "branches" or "parts"), Aristotle's discussion of them is *not* a finer illustration of his point regarding pure mathematics.[16] It contains an inversion: pure mathematics provides a negative example, illustrative of the platonic error to be avoided; while optics, harmonics, and astronomy provide a positive example, illustrative of a solution. The inversion is emphasized by the claim that the subordinate sciences are "the reverse" of pure mathematics; they are opposite in respect of the discussion just concluded. To return to another of the puzzles of §2, this is why astronomy is singled out for special treatment. While geometry does not treat its objects as natural, the subordinate sciences do, at least in some sense to be specified; they are "more natural." Their role in *Physics* 2.2 is thus to provide an example that is instructive for understanding what "nature" is.

Given what we have said about Aristotle's dialectical strategy vis-à-vis priority and inseparability in §3, we should therefore expect the subordinate sciences to somehow exemplify the idea that a study of nature must involve both form and matter, but primarily form; or, put differently, that form is prior to matter and change, but ultimately inseparable from them, *even in thought.*[17]

To see how they may do this, we must quickly review Aristotle's account of demonstrative knowledge. Aristotle held that properly scientific demonstrations

16. This is a standard topic in commentaries on 193b22–194a12. The issue is whether Aristotle intended chapter two's opening questions—"in what way [does] the mathematician differ from the physicist[?]" and "is astronomy different or a part of physics[?]"—as increasingly accurate specifications of the same question or as two separate questions. Mueller (2006) and Ross (1936) endorse the former option. Ross (1936, 506), for example, claims that "both questions are treated together, i.e., the general question [regarding mathematics] is treated with special reference to astronomy." For this reading to make sense, however, one needs to understand the method of the subordinate sciences as either identical to, a subset of, or as standing as species to the genus, pure mathematics. On the latter, see Distelzweig (2007, 94). In contrast, I agree with Lennox (2008), who reads the questions as distinct, although his focus is not on the methodological dissimilarity of the sciences—what I'm arguing for in this essay. Charlton (1970) and Philoponus (1993) do not even raise the question.

17. Again, I use "priority" here in the sense of Metaphysics XIII 1077b1–11. See note 12.

reveal the *per se* relations that hold between some subject and its properties. These relations must hold of the subject in virtue of its being of a certain kind.[18] For these requirements to hold, demonstrations must capture their subject at the proper level of generality. So, for example, a mathematical demonstration is properly scientific if it reveals the *per se* relations that hold between, say, a triangle and, say, the property of having internal angles equal to two right angles, and it can only reveal those if it treats the triangle *as a triangle*, not as isosceles or as shape.[19] Since Aristotle denominates sciences by the kinds they treat, this account entails that each science must treat only one kind, considered insofar as it is that kind.[20] Moreover, since demonstrations are paradigmatically formulated as universal affirmative syllogisms, it entails that all terms in a demonstration must pertain to the same kind and must be derived from *archai* that pertain only to that kind.

One exception to this broad schema is subordination. Some demonstrations may mix terms that strictly speaking pertain to different kinds, terms that are only of the same kind "in some respect."[21] This 'kind-crossing' is allowed in cases where one science stands 'beneath' another. The paradigmatic cases are the mathematically subordinate sciences.[22] In kind-crossing, truths regarding one kind can be used in reasoning about another. So, for example, optics can use geometrical principles to reason about visual rays, although geometrical principles, strictly speaking, pertain only to continuous magnitude. The exception is allowed because a mathematically subordinate science only reasons about a natural kind whose *mathematical* kind is such that some mathematical properties are true of

18. *Posterior Analytics* 1.2; 1.10, 76a38; 1.28, 87a38; *Metaphysics* VII, 1077b34–1078a2. Thanks to [Anonymized…].

19. *Posterior Analytics* 1.5, 74a26–30.

20. *Posterior Analytics* 1.7, 75b7–14.

21. *Posterior Analytics* 1.7, 75b9

22. The main passages on subordination are *Posterior Analytics* 1.7, 75b14–17; 1.9, 76a4–15; 1.13, 78b32–79a16; and 1.27. Hankinson (2005) provides a most thorough treatment of genera-crossing.

it *per se*.[23] In other words, because certain mathematical properties are true of a visual ray in virtue of it being of the kind 'visual ray,' 'visual ray considered mathematically' is the subject matter of a science, mathematical optics.[24] The *per se* attribution guarantees that the demonstration captures the visual kind at the right level of generality, although it ascribes to the kind mathematical properties that are also descriptive of other kinds.[25]

In what way do the subordinate sciences exemplify what Aristotle wishes to show about nature; namely, that its study involves primarily form, but that this form is inseparable from its physical embodiment, even in thought? Here is a standard answer which I will argue is deficient. First, the subordinate sciences exemplify the priority of form insofar as they are primarily concerned with the formal, mathematical features of bodies. Second, they exemplify the inseparability of those features from matter and change insofar as the kinds with which they are concerned are particular *natural* kinds, albeit considered mathematically. This latter point is a necessary condition for subordination. Without the restriction to particular *natural* kinds, subordinate sciences collapse into pure mathematics. Take our optical example. Optical demonstrations make use of geometrical principles to reason about visual rays. If, however, one ignores the fact that these demonstrations concern the kind "visual ray" (albeit described mathematically), one cannot practice geometrical *optics*. This is the case even though the geometrical principles of optics can themselves be defined without matter and change. Optical reasoning, in order to be *optical*, cannot be separated from the natural kind 'visual ray.' In general, in order to practice a subordinate science, one must always remember that one's mathematical proofs are ultimately about a particular

23. *Metaphysics* XIII, 1078a13–16. This point is emphasized by Lennox (1986, 41).
24. Distelzweig (2007) provides a systematic account of this "intersection" of mathematical and physical considerations by means of what Lear dubbed *qua* "operators". I thank X for discussion on this point. See also Lear (1982).
25. See Hankinson (2005) for how the types of *per se* attributions bear on kind-crossing.

natural kind. And since natural kinds cannot defined without reference to matter and change, a subordinate scientist cannot completely separate his/her objects from matter and change.[26]

Before I argue against this view, note that it is committed to the formal identity of subordinating and subordinate sciences, what I earlier called the "Identity Interpretation." It holds that subordinate sciences are *mathematical* sciences, formally like their subordinating partners, but they differ in that they treat specific natural domains and are thus 'tethered' to the natural world. Richard McKirahan endorses this view most explicitly. Using Zabarella's language, he claims that pairs of subordinating and subordinate sciences share their mathematical *modus considerandi*, but that in the subordinate sciences this *modus* is applied to a concrete subject matter. McKirahan's interpretation is influential. It is suggested, perhaps inadvertently, by several modern interpreters. For example, we find in recent works statements such as "Mathematical optics is the use of the relevant principles of pure geometry in the explanation of the restricted class of geometrical properties instantiated in the patterns of the optical array,"[27] and "Aristotle thinks the subordinate sciences are *mathematical* sciences, yet have a specific natural domain as their subject."[28] Some translations also promote the Identity Interpretation by invoking the idea of "branches" or "parts" to translate τὰ φυσικώτερα τῶν μαθημτων (194a7–8, roughly: the more physical of mathemat-

26. Simplicius makes the same point thus: "It becomes apparent that scientists engaged in optics, harmonics, and astronomy cannot separate their subject matter from natural bodies when each of them is actively engaged in research and applies himself *to his proper business* [i.e., does not become a pure mathematician]" (Simplicius 1997, 294,34–295,11, emphasis added).

27. Lennox (1986, 47). Importantly, Lennox (2008) does *not* make such a suggestion. Lennox's later essay agrees with my position here, asserting that the subordinate sciences "use a distinct *method*" (169, original emphasis).

28. Distelzweig (2007, 90). Distelzweig follows Lear (1982) in holding that the subordinate sciences select their specific natural domains by applying a physical "*qua*-operator," whereas their subordinating sciences apply only a mathematical "*qua*-operator." Nevertheless, it seems that for him the physical operator is used only to specify the relevant natural domain, while the mathematical operator enables inference and proof structure.

ics) thus implying that the subordinate sciences are the subsets of mathematics that deal with specific natural domains, but that, methodologically speaking, they are still just mathematics.[29]

But this view is erroneous. If it were right, the subordinate sciences would provide a poor illustration of the error of "those who say there are forms:" they would only exemplify inseparability from matter and change in thought rather weakly. In other words, they would not execute the dialectical task Aristotle set for them, as articulated in §3. The reason is that on the Identity Interpretation the subordinate sciences use *mathematical* properties and principles to reason about their subject matters, properties and principles that, in themselves, can be defined with no reference to matter and change. These are separable in thought just like the objects of pure mathematics, and so the *per se* relations that hold between them are *mathematical* relations, true independently of the natural kinds to which they are attributed. As McKirahan writes: a subordinate science "proceed[s] precisely like the associated branch of pure mathematics… the properties of optical lines that are abstracted for the sake of geometrical optics are precisely those abstracted for the sake of geometry."[30] In other words, the properties studied in optics are just geometrical, and thus separable, properties. Only the definitions of the natural kinds themselves are inseparable from matter and change. Yet Aristotle's point regarding the error of the Platonists is much stronger. I quote again:

> Those who say there are forms do this [separate from matter and

29. See, for example Charlton (1970) and Pellegrin (1999). For this point see also Lennox (2008, note 38). Mueller (2006) translates the phrase as "the more physical branches of mathematics" and seems to endorse the Identity Interpretation. After defining an applied mathematical science as one that must have a different "*modus cognoscibilis*" (Aquinas's term) from a purely mathematical science, Mueller cryptically asserts that "I believe, but cannot argue here, that Aristotle did not have the notion of an… applied mathematical science" (190). I take him to mean that Aristotle did not distinguish the *modus cognoscibilis* of an applied science from that of its subordinating partner, and so distinguished those sciences only by their domains. The Identity Interpretation fits well with Mueller's account of Aristotle's philosophy of mathematics in Mueller (1970).

30. McKirahan (1978, 213–214).

> change] unawares, since they separate natural things… This would become clear if one were to try to give definitions of each of them and *of their properties*. For odd and even and straight and curved… will not involve change, but flesh and blood and man do involve it: these terms are used like snub and not like curved. This is also made clear by the more natural of the mathematical sciences… (193a35–194a7, emphasis added)

Aristotle's defense of inseparability extends to properties, of which the ones used in purely geometrical demonstrations (e.g., straight and curved) are *not* appropriate examples, since they are separable in thought and no falsity results. This is not what Aristotle seeks to show is true of natural science. He seeks to model natural science on the study of snubness, which *cannot* be investigated apart from its material embodiment in a nose.[31] His point is that Platonists do not see how *thoroughly* natural science involves matter and change. The problem with the Identity Interpretation is that it doesn't allow the subordinate sciences to be illustrative of *this* point, a point Aristotle clearly asserts they are illustrative of. On the Identity Interpretation, the subordinate sciences study precisely those mathematical features of objects that *can* be investigated apart from their material embodiment. And so, either Aristotle's claim about the illustrative nature of the subordinate science is exaggerated, or the Identity Interpretation must be wrong.

In the following section, I'll show that the Identity Interpretation is in fact wrong and that the ancient practice of the subordinate sciences *did* provide Aristotle with a proper model for formal inseparability from matter and change *in thought*.

31. "Since there are two sorts of thing called nature, form and matter, we should proceed as if we were inquiring what snubness is" (194a13–14).

4 Physics and Mathematics in The Subordinate Sciences

To see why Aristotle's use of the subordinate sciences was both accurate and insightful, we first need to return to a question asked in §3: what sorts of mathematical investigations were familiar to Aristotle's audience? In §3, we focused on the purely mathematical sciences. But Aristotle's audience was also quite familiar with the subordinate sciences, which had already achieved a relatively advanced state. Miles Burnyeat notes, for example, that "optics in [Aristotle's] day looked much like the optics we know from the Hellenistic period and later": it admitted a proof-structure as well as a recognized body of findings.[32] The *archai* of that science, in Euclid's *Optics*, are:

> [A1][T]hat lines drawn… from the eye pass through a space of great extent… [A2] that the form of the space within our vision is a cone… [A3] that those things upon which the vision falls are seen and… those things upon which the vision does not fall are unseen; [A4] that those things seen within a larger angle appear larger, and those seen within a smaller angle appear smaller… [A5] that things seen within the higher visual range appear higher, while those within the lower range appear lower… [A6] that those seen… on the right appear on the right, [etc.]… [A7] that things seen within several angles appear to be more clear.[33]

All seven principles correlate geometrical concepts to non-geometrical ones. McKirahan (1978) calls them "determinations" or "translation rules" and holds that they allow mathematical concepts to be interpreted optically, and vice versa. For him, they play no role in the substance of an optical demonstration. They are only used to translate optical ideas into geometrical language at the beginning of a proof, and then translate back geometrical language into optical ideas at the proof's end. Viewing the *archai* of optics as "translation rules" is part and parcel

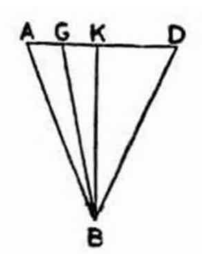
32. Burnyeat (2005, 37). Burnyeat also reviews the state of other sciences. He shows that harmonics was thought of somewhat differently than optics and astronomy, but that difference is not essential for us, see also Burnyeat (2000).

33. Euclid (1943, 357).

of the Identity Interpretation: it entails that pure geometry furnishes all concepts and inferences used in the non-translational portions of an optical proof. It allows pure geometry to wholly "determine… the structure, organization, and approach [of a subordinate science]."[34]

But this view does not align with the practice of ancient optics. First, there were patently non-geometrical considerations embedded in optical proofs that cannot be formalized by "translation rules." In fact, the very first proposition of the *Optics* uses a geometrically *false* premise, for physical reasons:[35]

> *Nothing that is seen is seen at once in its entirety.*
> (Fig. 1)
> For let the thing seen be *AD* and let the eye be B, from which let the rays of vision fall, *BA*, *BG*, *BK*, and *BD*. *So, since the rays of vision, as they fall, diverge from one another, they could not fall in continuous line upon AD*; so that there would be spaces also in *AD* upon which the rays of vision would not fall. So *AD* will not be seen in its entirety at the same time. But it seems to be seen all at once because the rays of vision shift rapidly.



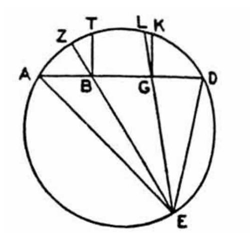
That lines drawn from *E* cannot intersect every point in *AD* runs counter to a purely geometrical understanding of Figure 1. It thus attributes to visual rays a property not captured by their geometrical representation. Yet it provides the inferential justification for the claim that nothing is seen in its entirety. Moreover, it is followed by another physical consideration—that the rays of vision move—also a property not captured by the geometrical representation. The attribution of non-geometrical but argumentatively crucial physical properties to visual rays was

34. McKirahan (1978, 202). He also writes: "The optician's conclusions are quite generally geometrical and then are given the relevant optical interpretation in accordance with the[translation rules]." It seems McKirahan views these on the model of Nagel's "bridge principles," see Nagel (1961, Ch. 11). Although I do not have the space to argue the point here, I believe the Identity Interpretation reads into Aristotle the vision of intertheoretic relations offered by Nagel and his followers. Newer work in philosophy of science might lead us to question its applicability, even in Aristotle's universe. See Mitchell (2002), but cf. Klein (2009).

35. Euclid (1943, 357; second emphasis added)

not unusual: Euclid thought that only a finite number of rays can occupy a given angle, Ptolemy attributed to them strength and weakness, and rays were almost always understood to move in some way.[36] Non-geometrical optical properties were an inseparable part of ancient mathematical optics.

There is also another, more subtle way in which the practice of mathematical optics involved physical considerations in a way not appreciated by the Identity Interpretation. Consider proposition 7 of the *Optics*:

> *Objects of equal size upon the same straight line if not placed next to each other and if unequally distant from the eye, appear unequal.* (Fig. 2)
>
> 
>
>
> [1] Let there be two objects of equal size, *AB* and *GD*, upon the same straight line, but not next to each other, and unequally distant from the eye, *E*, and let the rays *EA* and *ED* fall upon them and let *EA* be longer than *ED*. I say that *DG* will appear larger than AB. [2] Let the rays *EB* and *EG* fall upon them, and let the circle, *AED*, be circumscribed about the triangle, *AED*. [3] And let the straight lines *BZ* and *GL* be added as a continuation of the straight line *EB* and *EG*, [4] and from the points *B* and *G* let the equal straight line *BT* and *GK* be drawn at right angles. [5] And *AB* is equal to *GD*, but also the angle *ABT* is equal to *DGK*. [6] And so the arc *AT* is equal to *DK*. [7] Thus, the arc *KD* is greater than the arc *ZA*. [7] So the arc *LD* is much greater than *ZA*. [8] But upon the arc *ZA* rests the angle *AEZ*, and upon the arc *LD* rests the angle *LED*. [9] So the angle *LED* is greater than *AEZ*. [10] But within *AEZ*, *AB* is seen, and within *LED*, *GD* is seen. [11] Therefore *GD* appears larger than *AB*.

The demonstration begins with the premise that point *E* is an eye, as well the relevant placement of objects. It then uses geometrical constructions and propositions exclusively from Euclid's *Elements* to deduce [9] that angle *LED* is greater than *AEZ*. The last two steps 'translate' the purely geometrical result into an optical one by means of [A3] and [A4]. It indeed seems as if optical

36. Burnyeat (2005, 36–37).

considerations only impinge on the purely geometrical treatment at two points: in setting up the problem and in interpreting the result.[37]

But consider this demonstration without any of the relevant optical facts; that is, consider only Figure 2 and steps [2]–[9]. Clearly, an optical interpretation is highly underdetermined. Do lines EA, BT, or BZ correspond to visual rays? Is circle *AED* the bottom of a visual cone? This underdetermination is important because it lets us see that for a visual interpretation of the proof, one must not only supply the initial visual information in [1], one must *track that information through the proof* in order to determine which of the myriad geometrical properties inherent in Figure 2 are optically meaningful and can be leveraged in order to extract additional optically meaningful information. For example, one must to know that although arcs *AT* and *KD* comprehend objects *AB* and *DG*, the angles that include them are not *visually* significant and so are not relevant to the final 'translation.' One also must know that searching for a proportion between arcs

37. McKirahan holds that this division of considerations explains Aristotle's distinction between the fact (τὸ ὅτι) and the reason why (τὸ διότι). Indeed, the division is the most compelling evidence for the Identity Interpretation, as it fits well with Aristotle's statements in *Posterior Analytics* I.9, I.13 and I.27 that, e.g., "the fact falls under one [lower] science (for the underlying kind is different), while the reason falls under the higher science which is concerned with the attributes which hold of it in itself" (I.9, 76a5–13). The problem with this reading is that it requires mathematical optics to be merely a catalog of facts, with their proofs supplied wholly by pure geometry. In a sense, it allows for 'optics' (a body of finding), 'geometry' (a science of the interrelations of mathematical properties), but not the 'mathematical optics' we find in Euclid, the science that explains optical phenomena through their mathematical interrelations. A most compelling defense of this reading is Distelzweig (2007). While I do not have space to defend my alternate account, it relies on an idea defended by McKirahan (1978, 206–211), namely, that subordination involves triplets of sciences: e.g., arithmetic/mathematical harmonics/acoustical harmonics or geometry/optics/science of the rainbow. In these, a catalog of facts is generated by the lowest science, mathematical relations are revealed by the highest science, and mathematical relations are applied to the explanation of facts in the middle science. Consequently, Aristotle's appeals to the "higher science" are not necessarily always to pure mathematics, but can be to the middle science, in comparison to the lowest. This scheme allows the middle science to provide the reason why for the lower (e.g., by providing geometrical proofs for optical facts), and for the higher science to provide the reason why for the middle (e.g., by providing an explanation from the *archai* of geometry why the geometrical relations used in mathematical optics are true). The latter is *not* something found in mathematical optics, which does not show how its geometrical propositions are deduced from the axioms of geometry. This scheme allows the middle science to intermingle geometrical and physical considerations in the way I defend above.

ZT and *LK* is a fool's errand, since it doesn't alter any relevant *visual* relations.

Generally, the entities and properties in an optical demonstration are heterogenous: there are visual lines and avisual lines; there are seen-angles, unseen-angles, and avisual angles; there are circles that represent the base of the visual cone and avisual circles; there are lines that merely bisect incident visual rays, and lines that block vision. This information is essential: it determines which geometrical constructions and relations are relevant for an *optical* understanding of the represented situation. An optician must track it to prove her propositions. She *cannot* become a pure geometer.[38] This sort of tracking is entirely lost on the Identity Interpretation, which entails that optical information is simply irrelevant from steps [2]-[9]. To put it more colorfully, the idea that the optician can take off her optician's hat in steps [2]-[9] and then put it on again in steps [10]-[11] is flawed, because once the hat is off, it's not clear how to put it back on.

We can now appreciate how thoroughly the practice of the "more natural of the mathematical sciences" exemplifies the inability to cognitively separate "things and their properties" from their natural embodiment. Not only do some proofs rely (essentially) on *physical properties* that are not captured by the underlying geometry, the optician must discriminate between the purely geometrical and the physico-geometrical entities and properties within her proofs. Some are indeed like 'curved,' but some are like 'snub': their significance for the optical demonstration is lost if one looses sight—at any point in the demonstration!—of their physical nature. We can now also appreciate what I call the 'Difference Interpretation': the idea that subordinating and subordinate sciences do *not* treat their subject matters in the same way. The mathematically subordinate sciences take thorough heed of material embodiment. The mathematically subordinating sciences fully abstract from it.

38. Merely *stating* an optical proposition does not involve similar tracking. But without such tracking, there could be no optical *proof*. I'll return to this issue briefly in note 37.

This is why Aristotle treated the subordinate sciences as exemplars for a mode of investigation that is firmly grounded in the material world. Importantly, he was not alone in this.[39] Plato treated the subordinate sciences similarly. In the *Republic*, he excluded a number of mathematical sciences from the curriculum required for future, ideal leaders on the grounds that they kept the mind focused on the wrong things:

> [What concerns us is to] prevent our fosterlings from attempting to learn anything that does not conduce to the end we have in view [i.e., the investigation of the beautiful and the good]. . . , as we were just now saying about astronomy. Or do you not know that they repeat the same procedures in the case of harmonics? They transfer it to hearing and measure audible concords and sounds against one another, expending much useless labor just as astronomer do. (530e–531a)

Plato held that harmonics and astronomy do *not* properly direct the mind to the intelligible realm. Harmonicists, he claimed, "do not ascend to generalized problems and the consideration which numbers are inherently concordant and which not and why in each case" (531c). Astronomers, for the same reason, should "let be the things in the heavens" in order to focus on "the study of geometry" (530b). Plato's "black list" also likely included mathematical optics.[40] In each of these cases, scientific practice directed the mind to the wrong things, to sensible things. Consequently, they were "useless labor" for grasping the good and beautiful.[41] Only pure mathematics was good for the soul.[42] Yet if

39. I am indebted here to Burnyeat (2005) and Burnyeat (2000).

40. Burnyeat (2000, 17). Plato's complaint might seem to be directed at the Pythagorean belief that the fundamental constituents of the material world are mathematical. But this is not the case, as such a complaint would apply equally well to pure mathematics. Rather, Plato's target, like Aristotle's in the *Physics*, is the practice of the subordinate sciences and the way it directs attention to the natural world independently of the ultimate ontological status of mathematical entities. Ontological issues are not explicitly addressed in these passages as they are address by Aristotle in, for example, *Metaphysics* XIV a20–30.

41. In 527de, Plato even suggests that the soul's "instrument" for grasping the intelligibles cannot be "purified and rekindled" once "destroyed and blinded" by practical pursuits.

42. Burnyeat (2000).

astronomy, harmonics, and optics were understood by Plato and Platonists to be methodologically identical to their respective forms of pure mathematics, there would be little reason for Plato's dissatisfaction. His dissatisfaction suggests that he believed the subordinate science were *essentially* tied to sensible things, that they were thoroughly and inseparably physical. So much so that it was better not to engage in them at all.[43]

We can now see the brilliance of Aristotle's dialectical strategy. He uses the subordinate sciences as exemplars for his physics because they demonstrate in a way *that the Platonist was already likely to accept* that in certain *sciences* formal considerations cannot be separated, even in thought, from their physical embodiments. As so, as the subordinate scientist keeps mathematical (i.e., formal) and physical properties inseparably in mind, the natural scientist must keep form and matter inseparably in mind. Moreover, since formal considerations certainly play the dominant role within demonstrations of the subordinate sciences, the analogy also suggests it is on form that the student of nature must focus her attention, although not exclusively.[44]

5 Conclusion

I've tried to make sense of Aristotle's recommendation to the reader: keep in mind the subordinate sciences in order to understand something of the character of natural science. To begin with, I argued that the structure of *Physics* 2.1–2.2

43. Compare this with the subordinate sciences on the Identity Interpretation, which "proceed precisely like the associated branch of pure mathematics," McKirahan (1978, 213).

44. Of course, there are still loose ends here. Most significantly, although the subordinate sciences are highly illustrative of the character of natural science, they cannot give Aristotle *all* he wants. In particular, they do not demonstrate the central role of teleology in natural science. Consequently, after the passage on mathematics, Aristotle begins stressing how "it belongs to the same study to know the end of what something is for, and to know whatever is for that end" (194a27–28). He supplies some arguments from art (194a32–194b8), but his most significant evidence will come in later books, in his investigations of living things. See, e.g., Gotthelf (1997).

shows that what Aristotle wishes to illustrate about natural science is that it involves the cognitive inseparability of form from its material embodiment. I then showed that, on the Identity Interpretation—on the idea that the subordinate sciences differ from pure mathematics only in the subject matters to which they are applied—it is hard to see how the subordinate sciences could be illustrative of this. Only if we reject the Identity Interpretation, I argued, can we make sense of Aristotle's argumentative strategy. I offered instead a 'Difference Interpretation' by means of examples from the *Optics*. On this interpretation, the subordinate sciences are different from pure mathematics in their essential intermingling of mathematical and physical considerations. The Difference Interpretation is preferable both textually and contextually. It makes senses not only of Aristotle's dialectical strategy, but of the practice of the subordinate sciences, and of the way they were likely understood by Aristotle's Platonic audience.

Bibliography

Annas, Julia. 1976. *Aristotle's Metaphysics, Books M and N.* Oxford: Clarendon Press.

Aquinas, St. Thomas. 1884. *Opera omnia iussu impensaque Leonis XIII P. M. edita ..., t. 2: Commentaria in octo libros Physicorum Aristotelis.* Rome: Ex Typographia Polyglotta S.C. de Propaganda Fide.

———. 1888. *Opera omnia iussu impensaque Leonis XIII P. M. edita ..., t. 4: Pars prima Summae Theologiae a quaestione I ad quaestionem XLIX...* Rome: Ex Typographia Polyglotta S. C. de Propaganda Fide.

———. 1963. *Commentary on Aristotle's Physics.* Translated by Richard J. Blackwell, Richard J. Spath, and W. Edmund Thirlkel. Introduction by Vernon J. Bourke. New Haven: Yale University Press.

Barnes, Jonathan, Malcolm Schofield, and Richard Sorabji. 1975. *Articles on Aristotle: Metaphysics.* London: Duckworth.

Burnyeat, M. F. 2000. "Plato on Why Mathematics is Good for the Soul." *Proceedings of the British Academy* 103:1–82.

———. 2005. "Archytas and Optics." *Science in Context* 18 (1): 35–53.

Charlton, William, ed. and trans. 1970. *Aristotle's Physics. Books I & II.* Oxford: Clarendon Press.

Distelzweig, Peter. 2007. "The Intersection of Mathematics and Natural Science: The Subordinate Sciences in Aristotle." *Apeiron* 44 (2): 85–105.

Euclid. 1943. "Optics." Translated by Harry Edwin Burton. *Journal of the Optical Society of America* 35 (5): 357–372.

Gotthelf, Allan. 1997. "Understanding Aristotle's Teleology." In *Final Causality in Nature and Human Affairs,* edited by Richard F. Hassing, v. 30:71–82. Washington, D.C.: Catholic University of America Press.

Hankinson, R. James. 2005. "Aristotle on Kind-Crossing." In *Philosophy and the Sciences in Antiquity,* edited by R. W Sharples, 22–54. Aldershot, Hampshire, England: Ashgate.

Klein, Colin. 2009. "Reduction without Reductionism: A Defence of Nagel on Connectability." *The Philosophical Quarterly* 59 (235): 39–53.

Laird, W. Roy. 1983. *The* Scientiae Mediae *in Medieval Commentaries on Aristotle's* Posterior Analytics. PhD Thesis: University of Toronto.

Lear, Jonathan. 1982. "Aristotle's Philosophy of Mathematics." *Philosophical Review* 91:161–91.

Lennox, James G. 1986. "Aristotle, Galileo and 'Mixed Sciences'." In *Reinterpreting Galileo,* edited by William A. Wallace, 29–52. Washington, D.C.: Catholic University of America Press.

———. 2008. "'As If We Were Investigating Snubness': Aristotle on the Prospects for a Single Science of Nature." *Oxford Studies in Ancient Philosophy* XXXV:149–186.

McKirahan, Richard D. 1978. "Aristotle's Subordinate Sciences." *British Journal for the History of Science* 11:197–220.

Mitchell, Sandra D. 2002. "Integrative Pluralism." *Biology and Philosophy* 17:55–70.

Mueller, Ian. 1970. "Aristotle on Geometrical Objects." Reprinted in Barnes, Schofield, and Sorabji 1975, pp. 96–107, *Archiv für Geschichte der Philosophie* 52:156–171.

———. 2004. "Remarks on Physics and Mathematical Astronomy and Optics in Epicurus, Sextus Empiricus, and Some Stoics." *Apeiron* 37 (4): 57–87.

———. 2006. "Physics and Astronomy: Aristotle's *Physics* II.2.193b22-194a12." *Arabic Sciences and Philosophy* 16:175–206.

Nagel, Ernest. 1961. *The Structure of Science: Problems in the Logic of Scientific Explanation.* New York: Harcourt, Brace World.

Pellegrin, Piere, trans. 1999. *Physique.* Flammarion.

Philoponus, John. 1993. *On Aristotle's Physics 2.* Translated by A.R. Lacey. Ithaca, N.Y.: Cornell University Press.

Ross, W. D., ed. and trans. 1936. *Aristotle's Physics: A revised text with introduction and commentary.* Oxford: Clarendon Press.

Simplicius. 1997. *On Aristotle's Physics 2.* Edited and translated by Barrie Fleet. Ithaca, N.Y.: Cornell University Press.