Galileo's First New Science: The Science of Matter

Zvi Biener
University of Pittsburgh

Although Galileo's struggle to mathematize the study of nature is well known and oft discussed, less discussed is the form this struggle takes in relation to Galileo's first new science, the science of the second day of the Discorsi. *This essay argues that Galileo's first science ought to be understood as the science of* matter*—not, as it is usually understood, the science of the strength of materials. This understanding sheds light on the convoluted structure of the* Discorsi*'s first day. It suggests that the day's meandering discussions of the continuum, infinity, the vacuum, and condensation and rarefaction establish that a formal treatment of the "eternal and necessary" properties of matter is possible; i.e., that matter as such can be considered mathematically. This would have been a necessary, and indeed revolutionary, preliminary to the mathematical science of the second day because matter itself was thought in the Aristotelian tradition to be responsible for the departure of natural bodies from the unchanging and thus mathematizable character of abstract objects. In addition, the first day establishes that when considered physically, these properties account for matter's force of cohesion and resistance to fracture. This essay closes by showing that this dual style of reasoning accords with the conceptual structure of mixed mathematics.*

1. Introduction

Although Galileo's *Discorsi e Dimostrazioni Matematiche, intorno a due nuove scienze* (1638) is often heralded as his greatest work, the dialogue's first day was largely neglected in Galileo's time and continues to be so in the historical literature. One cause for this neglect is surely the day's convoluted structure. In contrast to the clear-cut propositional style of the remaining

Many thanks to Peter Machamer, Paolo Palmieri, Erik Angner, Brian Hepburn, Christopher Smeenk, and Kate Sorrels for comments on previous drafts. Additional thanks to Daniel Garber and Steven Nadler, whose questions at talks in Delphi, Greece and Madison, WI, motivated important revisions to this paper.

Perspectives on Science 2004, vol. 12, no. 3

days, the first day appears as a rambling foray into the host of conceptual issues related to the problem of the cohesion of bodies, often only tangentially. What's more, the lack of an explicit statement as to the purpose of the day's numerous discussions suggests that perhaps there was no unifying purpose, only a mélange of related concerns.[1]

Despite this apparent incoherence, I will argue that the first day plays a crucial role in the overall argumentative structure of the *Discorsi.* The first day serves as a prelude to the science of the second day by establishing that certain mathematical properties belong to matter necessarily.[2] In the second day, Galileo derives a set of theorems concerning matter's resistance to fracture that rest on fundamental assumptions concerning the mathematical structure of matter. The role of the first day is to make these mathematical assumptions plausible. It has long been recognized, of course, that the first day also makes plausible a *physical mechanism* that causally accounts for matter's resistance to fracture. I would like to suggest, however, that what is more important about the first day is that it establishes that this mechanism can be described mathematically. In particular, by showing that matter's physical properties are such that its force of cohesion is finite and acts continuously and uniformly along a given cross-section of a body, the first day justifies the application of the law of the lever—the principle on which the science of the second day is based—to the phenomena of fracture. Once the application of the law of the lever is justified, the causal explanation of matter's resistance to fracture becomes irrelevant. In other words, the second day only requires that certain mathematical properties are true of matter, regardless of their underlying physical explanation. It is for this reason that Galileo can frame his causal account of cohesion as a mere conjecture (*fantasia*) (*Discorsi,* p. 27 [66]).[3] His attitude towards it echoes his attitude in the third day towards the (lacking) causal account of the heaviness of bodies. In both cases, he is committed to a quantitative description of the phenomena in question

1. In contemporary scholarship, the first day is usually discussed in the context of Galileo's matter theory, particularly his so-called mathematical atomism (see, e.g., Shea 1970; Smith 1976), and in the context of Galileo's role in the development of the infinitesimal calculus (for a concise introduction, see de Gandt [1995, p. 169–176]). Although many insights can be gleaned from these and similar studies, they ordinarily focus on particular aspects of the first day and do not attempt to explain the day's contents as a whole or its overall relation to the remainder of the *Discorsi.* Notable exceptions are Le Grand (1978) and Palmerino (2001).

2. I believe the same thesis can be made regarding the relationship of the first day to the third and fourth days, but I will focus my attention on the second day only. Palmerino (2001) studies the relationship between the first and last days.

3. References to the *Discorsi* cite the page number in Drake's (2000) translation, followed by the page number in Favaro's *Opere,* Vol. VIII.

while remaining agnostic about its underlying physical explanation. Given the structure of the first two days, I will argue that Galileo's first new science is an instance of mixed mathematics; or, in the language of medieval scholasticism, that it is a middle or subalternate science. Under this description, the first day appears not simply as an attempt to mathematize certain properties of matter, but as an attempt to establish that a natural kind—matter—possesses physical properties that can be understood mathematically.

Two caveats are required here, one concerning Galileo's relationship to the tradition of the subalternate sciences and another concerning terminology. The first caveat is that there is significant disagreement regarding Galileo's relationship to this tradition. For example, although Wallace (1992) holds that the young Galileo was an avid student of the theory of mathematical demonstration of the *Posterior Analytics* and Machamer (1978) and Lennox (1986) hold that Galileo's actual mathematical reasoning fit the mold of demonstrations in the subalternate sciences, Laird (1997) persuasively argues that Galileo was not engaged with (and could have only been discouraged by!) any traditional philosophical questions concerning these sciences. While it is beyond the scope of this paper to arbitrate between these lines of thought, it seems that disagreements as to the very meaning of "the tradition of the subalternate sciences" stand in the way of a clear understanding of Galileo's relation to it. The tradition can be understood as a philosophical one, concerned with the justification of certain modes of reasoning (as Laird [1997] understands it), or it can be understood as a practical one, concerned with the application of such reasoning to particular physical problems (as Lennox [1986] understands it). Moreover, Galileo's relation to it can be judged in light of his own understanding of his position in the history of thought (as in Machamer 1978 and Wallace 1992), or it can be judged in light of his contemporaries' and successors' reception of his work (as in Garber 2004). The goal of this essay is not to establish any historical claims along these or other possible axes, but to provide an analytic view of the first two days of the *Discorsi* based on internal textual evidence which can later serve as fodder for a more comprehensive, and needed, historical thesis.

The second caveat is that when claiming that the first day establishes that "certain mathematical properties belong to matter necessarily," I do not mean to be imputing to Galileo any belief in the *metaphysical* necessity of the properties he studied. Galileo's concern was only with matter as it *actually* is, and questions regarding whether God could have created it differently were foreign to his purposes in the *Discorsi.* I also do not mean to impute to Galileo a belief in "essentialism," the doctrine that "science is concerned with and is able to discover facts about the inner natures or real essences of things" (Osler 1973, p. 504). As Osler rightly emphasizes, Galileo did not see himself as investigating *essences,* but the more-or-less apparent properties of phenomena. I will argue, however, that in the case of matter Galileo considered certain properties to be immutable, to always accompany matter, and to belong to matter simply by virtue of matter being the sort of thing it is.[4] Although "proper attributes" captures this meaning and would have been a plausible rendering of Galileo's intention, using a term of art attributes to Galileo a precision in philosophical matters that is unwarranted. Although he was certainly aware of the philosophies of his time, in the *Discorsi* Galileo did not use a technical term to label the properties he studied. Rather, he only characterized them discursively as "eternal and necessary" (*Discorsi,* p. 13 [51]). I follow his lead and use these terms, but stress again that the more metaphysical implications of "necessary" are nowhere to be found in the *Discorsi.*

Caveats aside, Galileo's statements regarding the properties of matter suggests that the *Discorsi*'s first science ought to be understood as the science of matter—i.e., the science of the properties of bodies *insofar as they are enmattered objects*—not merely the science of the strength of materials.[5] The distinction is fine but important. As I will argue, by viewing Galileo's science as an instance of mixed mathematics we can see that the subject of that science is natural kind—matter. Labeling the science as "the science of matter" makes this fact explicit in a way that "strength of material" simply does not. More importantly, and without reliance on mixed mathematics as an analytical tool, it is clear that Galileo does not present the study of the strength of materials as a free-standing enterprise, but bases it on considerations regarding the nature of matter. In doing so he replaces the received Aristotelian conception of matter with a radically new one—one on which matter itself can have formal, mathematical properties. Thus, although the moniker "strength of materials" accurately represents the contents of Galileo's new science, it fails to capture the significance of his approach to it.

4. In other contexts Galileo does not focus on similar properties. In the *Letters on Sunspots,* for example, he is happy to examine sunspots by comparing their properties to those of vapors in a pan, with no regard to the properties' necessity and immutability.

5. The description of the first new science as one concerning the "strength of materials" can be found in, for example, Wisan (1978, pp. 37–8), Segre (1989, p. 228) and the translation of Crew and de Salvio (Galilei [1638] 1952, pp. 109, 245). This description seems to have originated with the Elzevirs, who in their original table of contents for the *Discorsi* described the first science as one "concerning the resistance of solid bodies to separation" (*Discorsi,* p. 9 [47]). The Elzevirs were also responsible for describing the contents of the second day as a "science" both in the table of contents and the title of the entire work, but with some justification: in the *Discorsi* Galileo himself referred to the contents of the second day as a science (*Discorsi,* pp. 15 [54], 143).

The plan of the work is as follows. In the following section (§2) I examine the main obstacle offered by Galileo to the mathematical treatment of matter. It is only against this backdrop that we can understand the radical nature of Galileo's project in the first and the second days. In section 3 I show that the law of the lever is the mathematical principle grounding the science of the second day and in section 4 I argue that the very application of this law depends on the contents of the first day. In Section 5 I will suggest that the first new science and Galileo's arguments for it embody the conceptual structure of the subalternate sciences. I will provide reasons for conceiving of Galileo's new science as the science of matter in both §2 and §5. I now turn to the first pages of the *Discorsi,* wherein Galileo sets out his plan for the next two days.

2. The Obstacle to the New Science

The announced purpose of the *Discorsi*'s first two days is the investigation of the *disproportionate* relation between the absolute size of machines and their ability to function properly, particularly their ability to resist fracture.[6] According to Galileo, the principal difficulty in investigating this relation is that Euclidian proportion theory (Galileo's main demonstrative tool in the investigation of mechanical problems) is indifferent to absolute magnitudes and concerns only *ratios.*[7] Galileo puts this difficulty in the mouth of Sagredo, who holds that since machines are at root geometrical, and since in geometry only proportions play an essential role, machines themselves *cannot* be sensitive to absolute size. At this point in the dialogue, the conversation concerns the breaking of scaffoldings:

> [W]hat we [are discussing] . . . is something commonly said and believed, despite which I hold it to be completely idle, as are many

6. A wide range of size-sensitive phenomena were well known since antiquity, and attempts to explain them can be found in the pseudo-Aristotelian *Mechanica* as well as the tracts of several of Galileo's predecessors. The very first question of the *Mechanica* deals with the size-dependence of machines by asking why larger balances are more accurate than smaller ones. Commentaries on the *Mechanica* consequently tackle the problem of size-sensitivity in this context; see, e.g., Niccolò Tartaglia's *Quesiti et inventioni diverse* (1546) and Giovanni Battista Benedetti's *Diversarum speculationum mathematicarum et physicarum liber* (1585), translated in Drake and Drabkin (1969, pp. 106ff., 180ff.). Similarly, the *Discorsi* states that larger clocks are more accurate than smaller ones before tacking the problem (*Discorsi,* p. 12 [50]). As Renn and Valleriani (mss.) convincingly argue, Galileo's was inspired to confront the problem of size-sensitivity not because of any desire to engage with the commentary tradition, but because of his involvement with Venetian ship-building efforts.

7. The very definition of "proportion" was under scrutiny by Galileo and his contemporaries, but in ways that do not impact on this problem; see Palmerino (2001) and references therein.

> things that come from the lips of persons of little learning . . . [This is that] one cannot reason from the small to the large, because many mechanical devices succeed on a small scale that cannot exist in great size. Now, all reasonings about mechanics have their foundations in geometry, in which I do not see that largeness and smallness make large circles, triangles, cylinders, cones or any other figures [or] solids subject to properties [*passioni*] different from those of small ones; hence if the large scaffolding is built with every member proportional to its counterpart in the smaller one, and if the smaller is sound and stable under the use of which it is designed, I fail to see why the larger should not also be proof against adverse and destructive shocks that it may encounter (*Discorsi,* pp. 11–12 [49–50]).

Although with characteristic rhetorical zeal Galileo initially presents this opinion as natural and self-evident, the first and second days of the *Discorsi* go to show that Sagredo is wrong; that is, that machines are sensitive to size and that this sensitivity can be demonstrated geometrically. The possibility of such a demonstration was of the utmost importance to Galileo, given his own methodological commitment to necessarily true, conclusions. If his solution to the problem of the size-sensitivity of machines could not be demonstrated with geometrical necessity, then, by his own lights, it would not amount to a *science.* It is for this reason that Galileo rejected any non-geometrical explanation of size-sensitivity. For example, he rejected the opinions of:

> [S]ome persons of good understanding when, to explain the occurrence in large machines of effects not in agreement with pure and abstract geometrical demonstration, they assign the cause of this to the imperfection of matter, which is subject to many variations and defects. Here I do not know whether I can declare, without risking reproach for arrogance, that even recourse to imperfections of matter, capable of contaminating the purest mathematical demonstrations, still does not suffice to excuse the misbehavior of machines in the concrete as compared with their abstract ideal counterparts. Nevertheless I do say just that . . . (*Discorsi,* p. 12 [50–51]).

Galileo was not attacking imagined opponents here. Niccolò Tartaglia, for example, a translator into the vernacular of Euclid and Archimedes and a mechanician in his own right, offered the view in his *Quesiti et inventioni diverse* (1546). In his comment on the first question of the pseudo-Aristotelian *Mechanica,* Tartaglia explained the size-sensitivity of balances in terms of virtual displacements. He held that a larger balance is more sensitive than a smaller one because a given weight generates a greater motion in a larger balance. Because Tartaglia did not consider the time of motion (or, more accurately, he treated motion in differently sized balances as taking place in the same time), he equated a balance's displacement from the equilibrium position with the strength of its motion. Since the extremities of a larger balance cover a greater distance than the extremities of a smaller one when the two go out of equilibrium, Tartaglia concluded that a larger balance is more sensitive. However, he also noted that a consideration of the phenomena "according to reason, all matter being abstracted—as . . . Euclid was accustomed to do" often contradicts "[a] test {of} that statement materially and with physical arguments . . . by the sense of sight and with a material balance" (Drake and Drabkin 1969, p. 106). Tartaglia believed there was an obvious reason for this: "[T]he cause of this contradiction stems simply from matter; for things constructed or fabricated thereof can never be made as perfectly as they can be imagined apart from matter, which sometimes may cause in them effects quite contrary to reason" (Drake and Drabkin 1969, p. 106). That is, Tartaglia believed that embodiment in matter invalidates geometrical reasoning. Clearly at work in this passage is a conception of matter as that which resists formal, mathematical treatment. Tartaglia makes no allusions to the philosophical underpinning of the conception, but it was a basic tenet of the larger framework of scholastic and renaissance hylomorphism.[8] This framework is crucial for understanding Galileo's view.

Galileo certainly rejected any appeal to matter akin to Tartaglia's in the explanation of the size-sensitivity of machines. But this is not to say that Galileo thought the matter of machines was unrelated to their size-sensitivity. Rather, it is to say that his response to Sagredo—and with it his solution to the seeming mismatch between abstract arguments and concrete machines—involved a rejection of the very conception of matter with which it was intertwined. By rejecting this conception, Galileo was also rejecting the very idea that embodiment in matter invalidates geometrical reasoning. He held that matter *as such*—not the accidental varia-

8. Although Tartaglia himself was not educated at a university and made sparse contact with the philosophical tradition of his time, a conception of matter similar to the one he invokes can be traced through the philosophical tradition back to the works of Aristotle (e.g., *Physics* II.8, 199A11, *De Generatione Animalium* IV.3, 767b13ff, 769b10ff. and IV.4). See also Meli (1992) for the differing attitudes of Tartaglia and his predecessors to the Aristotelian tradition. It is interesting to note that Tartaglia believed that the mismatch between mathematical arguments and real machines can be minimized by building machines that are as uniform as possible, but did not believe the mismatch can be entirely eliminated (Drake and Drabkin 1969, pp. 108–109).

tions that account for its resistance to formal treatment—was responsible for the size-sensitivity of machines. In the first day of the *Discorsi,* Galileo hypothesized that matter's tendency to cohere and finite capacity to resist breaking forces are responsible for fracture phenomena. Moreover, he held that these can be successfully submitted to mathematical analysis. Offering a full-blown anti-scholastic, single-element theory of matter, Galileo wrote:

> I affirm that abstracting all imperfections of matter, and assuming it to be quite perfect and inalterable and free from all accidental change, still *the mere fact that it is material* makes the larger framework, fabricated from the same material and in the same proportions as the smaller, correspond in every way to it except in strength and resistance against violent shocks; and the larger the structure is, the weaker in proportion it will be. And since I am assuming matter to be inalterable—that is, always the same—it is evident that for this [condition] as for any other *eternal and necessary property,* purely mathematical demonstrations can be produced that are no less rigorous than any others (*Discorsi,* pp. 12–13 [51], emphasis added).

In other words, in order to treat fracture mathematically, Galileo substitutes the conception of matter inspired by hylomorphism with a conception more amenable to finding the cause of fracture in the formal, geometrical character of matter. In this light, we can see that Sagredo's worry about the limits of abstract geometrical reasoning is a worry about the very constitution of matter. Galileo addresses it head-on by rejecting the conception of matter that leads to the dichotomy between abstract and concrete machines. He rejects the idea that abstraction from the imperfections of matter amounts to "[an] abstraction from *all* matter" (to use Tartaglia's words). Rather, he holds that such an abstraction allows us to arrive at those properties of matter that are "always the same" and belong to matter by "the mere fact" that it is matter.[9] These properties can be treated mathematically because, like purely mathematical properties, they are inalterable, eternal, and necessary.

Interestingly, although Galileo's statements regarding the limits of geometrical reasoning are given pride of place at the beginning of the *Discorsi,* they do not occupy much space in the work. There are few arguments, like the one above, appealing to broad philosophical and method-

9. For a similar problem and approach in relation to Galileo's study of motion, see Koertge (1977).

ological principles in the style of *Il Saggiatore* and they are passed over quickly. Rather, Galileo spends most of the first day establishing that the particular properties of matter responsible for the force of cohesion are amenable to treatment by a specific mathematical device, namely the law of the lever. This is perhaps the hallmark of the Galilean approach to the mathematization of nature. Unlike his close contemporaries Descartes and Hobbes, who attempted to justify the mathematical treatment of nature through broad philosophical systems, Galileo's approach was piecemeal, involving the solution to particular problems in particular contexts.

However, and this is the main point of the present section, Galileo's solution to the particular problem of cohesion was nevertheless based on a general conception of matter. That is, it was based on the idea that matter itself possesses a certain structure and this structure (to which I'll return in §4) has "eternal and necessary" properties that can submit to mathematical analysis. Galileo's science was profoundly new for precisely this reason: it was a science that not only treated physical bodies mathematically, it treated their very physicality, their matter, mathematically. To put matters more suggestively, if less accurately, Galileo's first science was truly revolutionary because it provided a *formal* treatment of the *material causes* of fracture phenomena. This would have been an outright impossibility on the Aristotelian conception of matter. In the next two sections I will show how this formal treatment is based on the law of the lever, how the law comes to gain its foundational status, and how its application to cohesion requires that matter's force of cohesion exemplify particular mathematical properties, namely continuity, uniformity, and finitude. These are the properties of matter which Galileo believes are immutable and belong to matter necessarily.

3. The Foundation of the New Science

That Galileo aimed to base his new science on secure foundational principles is clear. Early in the *Discorsi* he notes his dissatisfaction with the lack of clear foundations in mechanical treatises and promises to correct their failure. In the mouth of Salviati, he writes:

> I cannot refuse to be of service, provided that memory serves me in bringing back what I once leaned from our Academician [Galileo] who made many speculations about this subject, all geometrically demonstrated, according to his custom, in such a way that not without reason this could be called a new science. For though some of the conclusions have been noted by others, and first of all by Aristotle, those are not prettiest; and what is more important, they

> were not proved by necessary demonstration from their primary and unquestionable foundations. Since . . . I want to prove these to you demonstratively, and not just persuade you of them by probably arguments, I assume that you have that knowledge of the basic mechanical conclusions that have been treated by others up to the present which will be necessary for our purpose. First of all, we must consider what effect is at work in the breaking of a stick, or of some other solid whose parts are firmly attached together; *for this is the primary concept* [la prima nozione], *and it contains the first simple principle* [il primo e semplice principi] *that must be assumed as known* (*Discorsi,* pp. 15–16 [54], emphasis added).

We can learn three things from this passage. First, the fundamental principle of the new science concerns "what effect is at work in the breaking of a stick." Second, this fundamental principle is well known and, in some sense, taken for granted. Third, this principle relates to features of mechanics that are also supposed to be well known. Although the relation between the "first simple principle" and mechanics is not explicated here, other passages suggest that that the principle is a *mechanical* principle. For instance, in the opening to the second day, Galileo writes that: "In such speculations I take as a known principle one which is demonstrated in mechanics about the properties of the rod which we call the lever: that in using a lever, the force is to the resistance in the inverse ratio of the distances from the fulcrum to the force and to the resistance" (*Discorsi,* p. 151 [152]). Since this is the *only* non-purely-geometrical principle Galileo assumes at the beginning of the second day, it must be the very principle referred to in the above quotation.[10] The ubiquity of the law of the lever in mechanical treatises also explains why Galileo claimed it was a "known" principle related to known mechanical facts. However, questions still remain: first, what is the "primary concept" exemplified "in the breaking of a stick"? and, second, what does the law of the lever have to do with this primary concept?

Consider the "primary concept" first. After stating that it is necessary to consider what happens when a piece of solid is broken, Galileo describes a solid column from which a weight is suspended:

> To clarify this, let us draw the cylinder or prism *AB,* of wood or other solid and coherent material, fastened above at A, and hanging

10. In truth, Galileo does not assume the law of the lever, but derives it by considering the behavior of weights suspended from a balance in equilibrium. I will return to this shortly.

> plumb; at the other end, B, let the weight *C* be attached. It is manifest that whatever may be the tenacity and the mutual coherence of the parts of this solid, provided only that that is not infinite[ly strong], it can be overcome by the force of the pulling weight *C,* of which the heaviness [gravità] can be increased as much as we please and that this solid will finally break, just like a rope (See Fig.1) (*Discorsi,* p. 16 [54–55]).

Since after this passage the topic of discussion turns to the cause of the cohesion of ropes, it seems that the "fundamental concept" Galileo has in mind is simply that for any given solid, there is a weight which will break the solid when applied longitudinally; or in other words, that for all intents and purposes the "tenacity and mutual coherence of the parts of this solid" has a finite strength that can be overcome by a sufficiently strong counteracting force. Galileo later dubs this the "absolute resistance" of a body to fracture.[11]

Galileo returns to this "primary concept" in Proposition I of the second day. Here, he tries to analyze the resistance of a cantilever in light of what he takes to be the "absolute resistance" evident in the case of a column. Galileo's method for doing so relies on turning the column case—wherein a weight is applied to a solid *longitudinally*—into 'half' of a balance problem—wherein a weight is applied *transversely* to one arm of a balance. The weight acting transversely on the other arm of the balance—the other 'half' of the balance problem—is supplied by the weight of the cantilever. The shift is important not only because it signals yet another instance of Galileo's trademark reliance on the balance,[12] but because it explains the connection between the law of the lever and Galileo's new science. The law of the lever is the principle used to establish quantitative relationships in the balance problem, and thus, by analogy, is the principle used to establish quantitative relationships between the longitudinal case and the cantilever case. Since Galileo explains the size sensitivity of machines by a series of propositions derived from the cantilever case, the law of lever is the key for solving the problem framed at the outset of the first day. Moreover, since the law is a geometrical principle that licenses the demonstrative inferences Galileo seeks, it has good claim for being an adequate foundation for the new science.

11. It seems Galileo conceives of the case of the column as the "absolute" case because in it the force responsible for fracture is applied in the same direction as the resulting motion (*Discorsi,* p. 115 [156–157). See also Footnote 17.

12. See Machamer (1998b) for a general discussion of Galileo's extensive use of the balance as a model for physical problems.

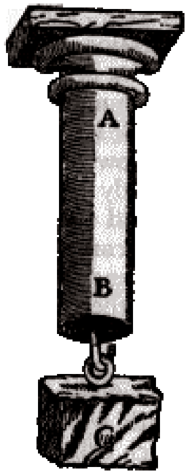

Fig. 1: Longitudinal pull case.

However, in order to establish the foundational status of the law of the lever, Galileo must justify the shift from the longitudinal and cantilever cases to the balance case—the lynchpin of the first two days. The shift occurs in Proposition I of the second day. I quote it here in full:

> Let us imagine the solid prism *ABCD* fixed into a wall at the part *AB;* and at the other end is understood to be the force of the weight *E* (assuming always that the wall is vertical and the prism or cylinder is fixed into the wall at right angles) [Fig. 2]. It is evident that if it must break, it will break at the place *B,* where the niche in the wall serves as support, *BC* being the arm of the lever on which the force is applied. The thickness *BA* of the solid is the other arm of this lever, wherein resides the resistance, which consists of the attachment that must exist between the part of the solid outside the wall and the part that is inside. Now, by what has been said above, the moment of the force applied at *C* has, to the moment of the resistance which exists in the thickness of the prism (that is, in the attachment of the base *BA* with its contiguous part), the same ratio that the length *CB* has to one half of *BA* [Fig. 3]. Hence the absolute resistance to fracture in the prism *BD,* (being that which it makes against being pulled [apart] lengthwise, for then the motion of the mover is equal to that of the moved) has, to the resistance against breakage by means of the lever *BC,* the same ratio as that of

> the length BC to one-half of AB, in the prism . . . And let this be our first proposition (*Discorsi,* pp. 114–115 [156–157]).[13]

The proposition states that an analogy can be constructed between the cantilever $ABCD$ of Figure 2 and the balance ABC of Figure 3, which I have constructed from other Galilean drawings to fit Galileo's description, but which is not in the *Discorsi.*

Although Galileo does not spell out the structure of the analogy in any detail, it is clear that he equates what happens "in the breaking of a stick" with what happens when a balance goes out of equilibrium. Intuitively, a piece of solid breaks only when the force of cohesion of its internal parts is overcome by an external pull. Analogously, a balance goes out of equilibrium only when the force on one of its sides is overcome by a force on its other side. Consequently, if the internal force of cohesion is modeled as the force on one side of a balance and the external pull is modeled as the force on the other side, a piece of solid will break only when the (model) balance tilts to the "external pull side," i.e., when it goes out of equilibrium in the appropriate direction. This is precisely the chain of reasoning behind Galileo's (already quoted) statement that: "It is evident that if it must break, it will break at the place B, where the niche in the wall serves as support, BC being the arm of the lever on which the force is applied. The thickness BA of the solid is the other arm of this lever, wherein resides the resistance, which consists of the attachment that must exist between the part of the solid outside the wall an the part that is inside" (*Discorsi,* pp. 114–115 [156]). The problem with this analogy is that it should to be unclear where to place the weight hanging from the AB arm of the model balance. To see the problem, consider the actual case (Fig. 2) and the model case (Fig. 3). Since in the actual case weight E is *literally* hanging from the endpoint of one side of a cantilever (Fig. 2), it is rather obvious to "model" it as a weight hanging from the endpoint of one arm of a balance (Fig. 3). However, since the resistance at AB acts along the *continuous finite line* AB, it is not immediately clear that it can be represented as a weight, $W(AB)$, hanging *at a point* of the other balance arm, let alone the *mid-point* of that

13. Galileo first introduces this scenario at the outset of the first day, but in a strictly qualitative manner. Not surprisingly, he does so immediately after framing the problem of the applicability of geometry to fracture problems. See (*Discorsi,* p. 13 [52]). In modern notation, the proposition states that $f(C)/f(AB) = 2CB/AB$, where $f(C)$ is the force applied to the cantilever at point C and $f(AB)$ is the force of resistance at AB. Since $f(AB)$ is modeled as a weight hanging from a balance, I label it in the diagrams as $W(AB)$. The proposition's last line further states that $W(AB)$ is equal to the force of a weight that can overcome the absolute resistance of the column $ABCD$. I follow Galileo's treatment by suppresses the depth of the beam.

Fig. 2: Cantilever case.

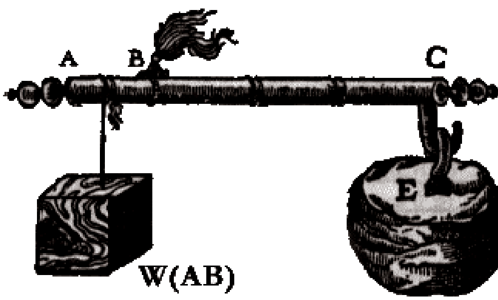

Fig. 3: Balance Case I: The Resistance to fracture in the cantilever case modeled as a weight applied transversely at the mid-point of balance arm *AB.*

balance arm.[14] To see this, consider as an intermediary another figure I have constructed from Galilean drawings, but that is not supplied in the *Discorsi,* Figure 4. In Figure 4, the force of resistance at *AB* (Fig. 2) is represented as a weight *W*(*AB*) that hangs from *every point* of one arm of a balance. This is the proper analogue of Figure 2. Is it reducible to the

14. I am assuming with Galileo that the resistance of the cantilever can be treated as if localized in *AB.* In both the longitudinal and the cantilever cases, Galileo believes that the resistance to fracture can be treated if acting along a single cross-section of the body. This assumption is false. See Truesdell (1968, pp. 200–3) and Timoshenko (1953, p. 12).

mid-point case (Fig. 3)? It turns out that it is, but only under some strict assumptions. These assumptions are implicit in Galileo's proof of the law of the lever.

Galileo's proof of the law of the lever need not concern us in its entirety here. It begins by asking the reader to imagine a beam of material *AB* hanging by its end-points from a balance in equilibrium *HI;* i.e., hanging by threads at *HA* and *IB* (Fig. 5). Clearly, Galileo writes, if the beam were to be cut at *any* point (say, *D*) and a thread *ED* attached from the balance to that point, "[t]here is no question that since there has been no change of place on the part of the prism with respect to the balance *HI,* it will remain in the previous state of equilibrium" (*Discorsi,* p. 111 [153]). Even while ignoring the remainder of the proof, it is clear that the only way to guarantee that *D* can be chosen arbitrarily is to assume that the weight of the beam is continuously distributed along the segment *AB;* that is, that the beam has no gaps.[15] Moreover, the only way to guarantee that the beam will maintain its original configuration after the cut, given that *D* may be chosen arbitrarily, is to assume that the weight is distributed uniformly. With these assumptions—continuity and uniformity—and the law of the lever itself is easy to show that *W*(*AB*) in Figure 4 can be reduced to *W*(*AB*) in Figure 3. Although Galileo does not make these assumptions explicit, it seems they must support his (already quoted) claim that: "[T]he moment of the force applied at *C* has, to the moment of the resistance which exists in the thickness of the prism (that is, in the attachment of the base *BA* with its contiguous part), the same ratio that the length *CB* has to one-half of *BA*" (*Discorsi,* p. 115 [156]). What is additionally important about these assumptions (as well as the whole of the balance analogy) is that they make clear that Galileo treats the force of resistance at *AB* as a *weight,* uniformly and continously distributed through a beam.[16]

But the proposition does not end here. Its aim, recall, is to relate the cantilever case to the case of "absolute resistance"; i.e., the longitudinal case. Once again, Galileo does not make any intermediary reasoning explicit, but merely notes after the quote above that: "Hence the absolute resistance to fracture in the prism *BD,* (being that which it makes against being pulled [apart] lengthwise, . . .) has, to the resistance against breakage by means of the lever *BC,* the same ratio as that of the length *BC* to

15. I will discuss Galileo's conception of the continuum in the following section.

16. In truth, the stress distribution along the cantilever section *AB* is not uniform at the time of fracture (Timoshenko 1953, p. 12). However, I will assume with Galileo that it is and that the analogy with weight stands.

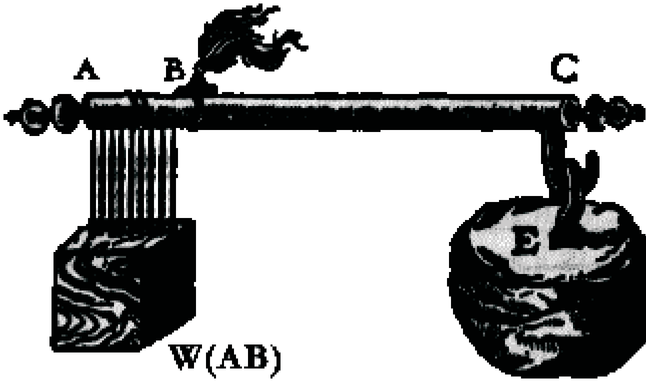

Fig. 4: Balance Case II: Resistance to fracture in the cantilever case modelled as a weight applied transversely and continuously along balance arm *AB.*

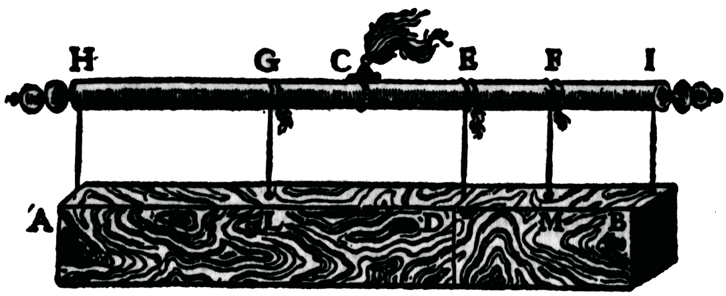

Fig. 5: Galileo's proof of the law of the lever: A continuously and uniformly distributed weight hanging from a balance.

one-half of *AB,* in the prism . . . " (*Discorsi,* pp. 114–115 [156–157]). However, the move to absolute resistance requires another crucial assumption; namely, that the force of resistance at *AB* to a longitudinal pull along *ABCD* is equivalent to the force of resistance at *AB* to a transverse pull across *ABCD.* Although this is false, it explains how the "primary notion" enters into Galileo's new science.[17] Galileo believes that all fracture cases can ultimately be related to the longitudinal fracture of a column. The "primary concept" in the longitudinal case was that for any given solid,

17. Although it is an open question why Galileo endorses this assumption to begin with, I can offer the following hypothesis: Galileo conceives of fracture as the motion of

there is a finite weight that can cause it to fracture. Since all cases of fracture can be understood in terms of the longitudinal case, in all cases the force of resistance can be overcome by a sufficiently large, finite weight. That is, the force of resistance to fracture is itself always finite.

By this chain of reasoning, Proposition I establishes the relationship between the case of absolute resistance and the case of the cantilever. What the proposition demands, however, is that the assumptions made regarding the figures above are physically plausible; that is, that the force of cohesion is finite and can be unproblematically described as acting uniformly and continuously along line segment *AB* of Figure 2, just as a finite weight can be described as being uniformly and continuously distributed along the hanging beam *AB* of Figure 5.

4. Justifying the Foundation

We come now to the question with which this essay began: what is the role of the first day of the *Discorsi?* I claim that the first day shows that in all formal, geometrical respects, matter's force of resistance to fracture is like the force of a uniformly distributed, continuous, and finite weight hanging from a balance. In other words, the first day establishes that the mathematical properties necessary for the balance analogy to be applied are true of the force of cohesion. Galileo's physical explanation of the force of resistance is important to keep in mind here. Galileo hypothesizes that this force is caused by indivisibly small vacua that are interspersed among the indivisibly small particles of matter. Because of the *horror vacui,* these vacua pull together the particles of matter; that is, they resist the particles' separation. In essence, Galileo explains the cohesion of matter through nature's abhorrence of a vacuum and his peculiar conception of the structure of matter. However, his main task is to show that this structure has the mathematical properties discussed in the previous sections. Although I cannot examine the first day in detail here, a sampling of its contents and a brief description of their relation to the day's larger purpose are sufficient to make my case.

---

one part of a body away from another. In the cantilever case, he believes fracture occurs along cross-section *AB.* Consequently, he believes that the force responsible for fracture is equal to the force required to move the portion of the cantilever to the right of *AB* away from the part embedded in the wall. In the longitudinal case, he also believes fracture occurs along a single cross-section. Consequently, he believes that the force responsible for fracture is equal the force required to move the portion of the column below the breaking point away from the portion of the column above that point. If the prism and cantilever are constructed to have appropriately similar dimensions, the resulting motions will only be different in orientation. It stands to reason that the forces responsible for them would be identical in all but orientation.

First, resistance to fracture must be finite. This is not only because reasoning concerning an infinite quantity would be impossible by Galileo's lights, but because in order to model resistance on weight, it should be possible to correlate any given measure of resistance to a measure of weight. Among others, Galileo's "hanging bucket" experiment is meant to show that such a correlation is possible (*Discorsi,* p. 23 [62]). In this experiment, Galileo asks the reader to imagine a piston inserted into a cylindrical cavity that has been evacuated of air and turned such that its open end is pointing downwards. Galileo holds that the piston will be held in place by the force of the vacuum within the cylinder and that this force could be overcome by hanging a bucket on the piston and placing within it a sufficiently large weight. Since in this experiment resistance is not simply *correlated* to weight, but is *directly measured* by it, the experiment allows for a straightforward substitution of a weight-magnitude for a resistance-magnitude—precisely as done in Proposition I. Also in support of the claim that the force of resistance is finite is Galileo's well-known discussion of Aristotle's wheel—the centerpiece of the first day (*Discorsi,* pp. 25–58, [68–97]).[18] Galileo explains by means of Aristotle's wheel how an infinite number of indivisibles can sum to a finite quantity. The first day's remaining discussions on the nature of infinity, the continuum, and indivisibles go to support Galileo's treatment of Aristotle's wheel and bolster his argument that, despite his unique explanation of cohesion by an infinity of indivisible vacua, on a macroscopic level the force of cohesion can be treated as finite and measurable.

Second, the force of resistance must act continuously and uniformly through any given cross-section of a body. Continuity and uniformity are perhaps the most important properties of resistance, since without them Galileo would not be able to make the analogy between the force of resistance along line segment *AB* (Fig. 2) and a weight hanging from a balance arm (Fig. 4). Continuity and uniformity are established primarily through Galileo's discussion of Aristotle's wheel. There, Galileo shows that his physical explanation of cohesion is such that regardless of the density of a material, the distribution of indivisible vacua and material particles will always be its uniform. Thus, the force of resistance to fracture will always be uniform. Moreover, Galileo provides an extended discussion of the nature of the continuum meant to show that the continuum is comprised of an infinite set of indivisible full and empty points (*Discorsi,* pp. 28–58 [68–97]). Of course, Galileo's theory of the continuum is subtle and mer-

18. For Galileo's treatment of Aristotle's wheel see Drabkin (1950), Wallace (1989), and Palmerino (2001).

its an extended discussion, which I cannot provide here. Suffice it to say that the theory supports Galileo's account of cohesion and illustrates that it is mathematically tractable.[19]

The remaining portions of the first day go to make the physical properties of cohesion plausible in themselves and immune to objections from the study of motion. Particularly, since Galileo argues that cohesion is due to the *horror vacui,* he must argue that a vacuum is physically possible (*Discorsi,* pp. 26–28 [66–67]). It is to this end that he cites the example of two blocks of marble that slide easily across one another but can only be separated by a great force. The example provides experimental verification that a vacuum can be found in nature, if only briefly (*Discorsi,* pp. 19–20 [59]), as does the aforementioned "hanging-bucket" experiment. It is also to this end that Galileo undertakes the long discussion of bodies falling in resistive media—a discussion that takes up more than half of the first day. The discussion is necessary since the impossibility of motion in a vacuum was traditionally taken as evidence against its existence. In order to show that a vacuum *can* exist, Galileo must therefore frame a new theory of motion.[20]

Although extremely sketchy, I take this brief roster to suggests that there exists a connection between the discussions of the first day and the application of the law of the lever in the first proposition of the second day. Given this connection, however, more could be said about the seemingly disparate character of the discussions, particularly the fact that some seem purely mathematical (e.g., the nature of the continuum), while others seem to involve a good deal of physical reasoning (e.g., the explanation of condensation and rarefaction). Clarifying the nature of these problems will lead us back to one of this essay's original concerns and the more precise reason Galileo's first new science is best understood as the science of *matter.* This reason concerns the nature of reasoning in the subalternate, middle, or mixed sciences.

19. Galileo addresses several issues of great importance in the medieval literature on infinity and infinitesimals; namely, the paradoxes of unequal infinities, the existence of first and last instants, the nature of the verbs "to begin" and "to cease," and the problem of maxima and minima. See Ketzmann (1982).

20. The structure of this portion of the first day can be found in Galileo's manuscripts in as early as 1590, see Drake and Drabkin (1969). The first day's treatment of the nature of motion in a vacuum can also be understood as showing that *weight* is a necessary property of matter, and that it can be treated mathematically. I have not stressed this in this essay since my focus is on those features of the first day that are necessary for the second day, not for the third and forth days. Palmerino (2001) focuses on the relevance of elements of the first day to Galileo's science of local motion.

5. The Science of Matter

The subalternate, middle, or mixed sciences are sciences in which physical objects are considered *qua* mathematical. Mechanics, optics, astronomy and harmonics were traditionally understood to be such sciences. Their common appellatives refer either to their middle position in the disciplinary hierarchy (in which they were subordinate to higher, mathematical sciences and superior to lower, purely empirical ones) or to the fact that in their syllogistic explanations the genus in question—the middle term—was of a mixed character, i.e., it could be taken both mathematically and physically.[21] My contention in this section is that Galileo's first new science is a mixed science. I do not contend, however, that Galileo saw it as such, even though I have argued that his purpose in the first day was to establish those mathematical properties of matter required for the application of the law of the lever in the second day.[22] My aim is only to show that the reasoning of the first two days of the *Discorsi* does in fact fit the structure of arguments in the mixed sciences. I will use a characterization of these sciences by Lennox (1986), based on a study of Aristotle's *Posterior Analytics.*[23]

The goal of a science in the Aristotelian tradition was to supply demonstrably true claims regarding its subject matter by showing what properties belong to that subject by virtue of the type of thing it necessarily is,

21. For characterizations of the middle sciences, see McKirahan (1978), Livesay (1982) and Laird (1983).

22. Although Galileo has been portrayed as a Platonist (e.g., Koyré 1939), a positivist (e.g., Mach 1960, pp. 151–191), and everything in between, (see Wallace 1992 and Feldhay 1998 for overviews), in recent years a leading interpretation based mostly on the work of William Wallace has characterized him as following an Archimedean inspired version of the philosophy of science outlined by Aristotle in the *Posterior Analytics* and elaborated by the Jesuits of Galileo's time—the same philosophy of science that gave rise to the notion of the "subalternate sciences." Of course, there is no overall consensus regarding Wallace's interpretation, and disagreements still abound about whether Galieo's method constituted proper apodictic *scientia,* whether it followed the medieval *regressus* more than Aristotle's original proclamations, whether it constituted Galileo's *juvenilia* more than his mature thought, etc.; see Jardine (1976), McMullin (1983), Wallace (1976), McMullin (1978) and Pitt (1978). For the purposes of this discussion, I put aside these larger historical questions and take for granted that some light can be shed on Galileo's work from the perspective of the tradition of the subalternate sciences. See also the second caveat made in the introduction to this paper.

23. Although medieval commentators expanded and revised Aristotle's opinions on the subalternate sciences, sufficient similarities exist between the two bodies of thought that no damage is done by using Aristotle's remarks directly. For an evolution of the philosophical discussions regarding the subalternate science, see Laird (1983). See also Dear (1995, ch. 6) for the characterization of the mixed sciences given by the Jesuits of Galileo's time.

not by virtue of any accidental features that may be true of it. As Aristotle writes in the *Posterior Analytics* I.9: "We understand a thing nonincidentally when we know it in virtue of that according to which it belongs, from the principles of that thing as that thing. For example, we understand something's having angles equal to two right angles when we know that to which it belongs in virtue of itself, from that thing's principle" (translated in Lennox 1986, p. 40, [76a4–8]). Taking the triangle example, Aristotle holds that we understand a particular isoceles (or any other instance of) triangle by coming to understand it as belonging to the class of triangles *in general* (i.e., the genus "triangle"); and by the principles of triangularity (i.e., geometrical principles) we come to understand that *any* triangle's angles are equal to two right angles. We thus come to know that an *isoceles* triangle's (or any other triangle's) angles are equal to two right angles. Even without explicitly putting the inference in syllogistic form, it is plain that the middle term here is "triangle": it serves to connect a particular isoceles triangle, through the geometrical features of triangularity, with the features of all triangles. In the case of the subalternate sciences, however, the middle term takes on a dual character. The quote above continues:

> Hence if that too [the thing's principle] belongs in virtue of itself to what it belongs to, the middle term must be in the same kind. If this isn't the case it will be as the harmonical properties are known through arithmetic. In one sense such properties are demonstrated in the same way, in another sense differently; for that it is the case is the subject of one science (for the subject-kind is different), while the reason why it is so is of a higher science, of which the per se properties are the subject (translated in Lennox 1986, p. 40, [76a8–13]).

Aristotle outlines two cases: one in which the thing studied and the principles used to study it are of "the same kind" and one in which they are not, as in the subalternate science of harmonics. In the latter case, the thing studied is natural, and the principles used to study it are mathematical. The middle-term in these cases, as Lennox writes, "picks out the description of the natural object in virtue of which it has a certain mathematical property; that property is a per se property of a natural kind qua being a mathematical kind" (Lennox 1986, p. 41). In other words, in a subalternate science natural objects must be described in a way that attributes to them via their (natural, physical) genus mathematical properties, which they have simply in virtue of being described mathematically. This is the essential feature of a subalternate science.

We can see that the first two days of the *Discosi* fit the mold of a subalternate science. In the first day, a natural kind—matter—is described in a way that attributes to it mathematical properties that belong to it by its very structure, simply because of the kind of thing it is. As argued above, the uniformity, continuity and finitude of matter's force of cohesion are such properties. The overall task of the *Discorsi*'s first two days is to argue 1) that bodies fracture because they are en-mattered (not because of their accidental properties), 2) that because they are enmattered they possess certain physical properties that can be described mathematically, and that these mathematical properties entail additional mathematical properties of fracture. The first of these is accomplished by Galileo's rejection of the Aristotelian conception of matter and his framing of a new conception on which fracture occurs even when matter is "free from all accidental change" and because of "the mere fact that it is material" (*Discorsi,* p. 12 [51]). The second of these is accomplished throughout the first day, as Galileo establishes that his conception of matter entails that matter can be described mathematically and treated geometrically. Galileo's *physical* arguments in support of his new conception show that the mathematical properties of matter are in fact properties of a natural, physical kind that is "always the same" (*Discorsi,* p. 12 [51]). It is for this reason that the first day seems to oscillate between purely mathematical considerations (e.g., regarding infinity and indivisibles) and purely physical ones (e.g., regarding condensation and rarefaction). The third of these is accomplished by the second day. In the first proposition of the second day, the law of the lever is applied to the phenomena of fracture given the mathematical properties of matter justified by the first day. The remainder of the second day draws out the implications of the first proposition and establishes the theory of fracture.

A further clue to the character of Galileo's first new science is supplied by the fact that the question with which the first day begins—why large machines are different than small ones—is solved only in Propositions VI and VII of the second day (*Discorsi,* pp. 12, 120–124 [50, 163–166]). In the first day, only the *fact itself* is mentioned, and it is noted as a known mechanical fact. The explanation of the fact—"the reason why it is so"—is delayed until the second day. In other words, although Galileo began the first day with the promise that "it can be demonstrated geometrically that the larger [machines and structures] are always proportionately less resistant than the small" (*Discorsi,* p. 13 [51]), it is only at the closing of Proposition VI of the second day that he admits the desired demonstration had been reached:

> *Simp.* This proposition strikes me as not only new but surprising . . . I should have thought it certain that their moments [i.e.,

> the moments of large and smaller cylinders and prisms] against their own resistances would be in the same ratio [as their sizes].
>
> *Sagr.* This demonstrates the proposition which, as I said at the *beginning of our discussions,* seemed then to reveal itself to me through shadows (*Discorsi,* p. 122, [164], emphasis added).

The proof is not available until the second day because Galileo had first to establish the relevant mathematical properties of matter on which the mathematical demonstrations of the second day rest. In this sense, the convoluted discussions of the first day enable the transition between the *mere* fact of fracture and the *reasoned* fact concerning it. It is crucial to note that only the second day, as a series of propositions, properly constitutes Galileo's new *science,* but its formulation depends on the deliberations of the first day. The expository structure of this new science fits well with Aristotle's statement that "that it is the case is the subject of one science . . . , while the reason why it is so is of a higher science" (translated in Lennox 1986, p 40). In this case, Galileo's higher science is used to explain facts about fracture that are known from mechanical practice. The higher science is a geometrical science based on the law of the lever, but its subjects are the "eternal and necessary" properties of matter. Since this science is framed in response to worries about the applicability of geometrical reasoning to matter and since it is concerned with objects insofar as they are enmattered, it should be understood as the science of matter *as such.*

6. Conclusion

Although I have spent the bulk of this essay making narrow points concerning the mathematical assumptions underlying Galileo's first new science and the nature of reasoning in the subalternate sciences, my main point is rather general. It is that the first day of the *Discorsi* frames a new theory of matter in order for the second day to expand the limits of mathematical reasoning to include the phenomena of fracture. Until we recognize the central role of Galileo's new conception of matter in justifying his new science, we will not be able to properly place him in the context of the early modern transformation of natural-philosophy. Particularly, his relation to the mechanical philosophy will be obfuscated unless we can see how in the *Discorsi*'s four days he offered a truly *mechanical* model of explanation; that is, a model of explanation based on the twin pillars of *matter* and *motion.*

Moreover, until we recognize the central role of Galileo's new conception of matter in justifying his new science, we will not be able to understand the *Discosi* as a whole. Galileo authored masterfully argued and carefully structured treatises throughout his life, even suppressing the publication of treatises he considered incomplete. Thus, before we resign to consider the *Discorsi*'s first day as the ramblings of an old man, intent on publishing whatever is left in his arsenal of researches, we ought to look for his plan in writing it. Although I have not offered any direct evidence regarding Galileo's intention, I hope my analysis will prompt others to formulate more historical theses.

References

Dear, Peter. 1995. *Discipline & Experience: The Mathematical Way in the Scientific Revolution.* Chicago: University of Chicago Press.

Drake, Stillman and I. E. Drabkin. 1969. *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, & Galileo.* Madison: University of Wisconsin Press.

Feldhay, Rivka. 1998. "The Use and Abuse of Mathematical Entities: Galileo and the Jesuits Revisited." Pp. 80–145 in *The Cambridge Companion to Galileo.* Edited by Peter Machamer. Cambridge University Press.

Galilei, Galileo. (1638) 1952. *Dialogues Concerning Two New Sciences. Discorsi e Dimostrazioni Matematiche, intorno à due nuove scienze.* Translated by Henry Crew and Alfonso de Salvio. New York, N.Y.: Dover.

———. (1638) 2000. *Two New Sciences, including Centers of Gravity and Force of Percussion.* Translated by Stillman Drake. Toronto, Ontario: Wall & Emerson.

de Gandt, François. 1995. *Force and Geometry in Newton's Principia.* Translated by Curtis Wilson. Princeton: Princeton University Press.

Garber, Daniel. 2004. "On the Frontlines of the Scientific Revolution: How Mersenne Learned to Love Galileo." *Perspectives on Sciences,* 12: 135–163.

Jardine, Nicholas. 1976. "Galileo"s Road to Truth and the Demostrative Regress." *Studies in History and Philosophy of Science,* 7: 227–318.

Koertge, Noretta. 1977. "Galileo and the Problem of Accidents." *Journal of the History of Ideas,* 38: 277–318.

Koyré, Alexandre. 1939. *Études Galiléennes.* Paris: Hermann.

Laird, W. Roy. 1983. *The Scientiae Mediae in Medieval Commentaries on Aristotle"s Posterior Analytics.* PhD thesis. University of Toronto.

———. 1986. "The Scope of Renaissance Mechanics." *Osiris,* 2: 43–68.

———. 1997. "Galileo and the Mixed Sciences." Pp. 253–270 in *Method and order in Renaissance philosophy of nature: the Aristotle commentary tradition.* Edited by Daniel A. Di Liscia, Eckhard Kessler, and Charlotte Methuen. Aldershot; Brookfield, VT.: Ashgate.

Le Grand, H. E. 1978. "Galileo"s Matter Theory." Pp. 197–208 in *New perspectives on Galileo.* Edited by Robert E. Butts and Joseph C. Pitt. Dordrecht; Boston: D. Reidel.

Lennox, James G. 1986. "Aristotle, Galileo and 'Mixed Sciences'." Pp. 29–52 in *Reinterpreting Galileo.* Edited by William A. Wallace. Washington, D.C.: Catholic University of America Press.

Mach, Ernst. 1960. *The Science of Mechanics; a Critical and Historical Account of Its Development.* Translated by Thomas J. McCormack. LaSalle, Ill.: Open Court Pub. Co.

Machamer, Peter. 1978. "Galileo and the Causes." Pp. 161–180 in *New perspectives on Galileo.* Edited by Robert E. Butts and Joseph C. Pitt. Dordrecht; Boston: D. Reidel.

———. 1998. "Galileo"s Machines, His Mathematics, and His Experiments." Pp. 53–79 in *The Cambridge Companion to Galileo.* Edited by Peter Machamer. Cambridge University Press.

McKirahan, Richard D. 1978. "Aristotle"s Subordinate Sciences." *British Journal for the History of Science,* 11: 197–220.

McMullin, Ernan. 1978. "The Conception of Science in Galileo"s Work." Pp. 209–257 in *New perspectives on Galileo.* Edited by Robert E. Butts and Joseph C. Pitt. Dordrecht; Boston: D. Reidel.

———. 1983. "Review of Wallace, *Prelude to Galileo.*" *Philosophy of Science* 171–73.

Meli, Domenico Bertoloni. 1992. "Guidobaldo dal Monte and the Archimedean Revival." *Nuncius Annali Di Storia Della Scienza,* 7: 3–34.

Osler, Margaret J. 1973. "Galileo, Motion, and Essences." *Isis,* 54: 504–509.

Palmerino, Carla Rita. 2001. "Galileo's and Gassendi's Solutions to the *Rota Aristotelis* Paradox: A Bridge Between Matter and Motion Theories." Pp. 381–422 in *Late Medieval and Early Modern Corpuscular Matter Theories.* Edited by Christoph Lüthy, John E. Murdoch and William R. Newman Leiden; Boston: Brill.

Palmieri, Paolo. 2001. "The Obscurity of Equimultiples: Clavius' and Galileo's Foundational Studies of Euclid's Theory of Proportions." *Archive for the History of Exact Sciences,* 55: 555–597.

Pitt, Joseph C. 1978. "Galileo: Causation and the Use of Geometry." Pp. 181–195 in *New perspectives on Galileo.* Edited by Robert E. Butts and Joseph C. Pitt. Dordrecht; Boston: D. Reidel.

Segre, Michael. 1989. "Viviani's Life of Galileo." *Isis,* 80: 206–231.

Shea, William. 1970. "Galileo"s Atomic Hypothesis." *Ambix,* 17: 13–27.

Smith, Mark A. 1976. "Galileo"s Theory of Indivisibles: Revolution or Compromise." *Journal of the History of Ideas,* 37: 571–588.

Timoshenko, Stephen. 1953. *History of Strength of Materials: With a Brief Account of the History of Theory of Elasticity and Theory of Structures.* New York, N.Y.: McGraw-Hill.

Truesdell, Clifford. 1968. "'The Creation and Unfolding of the Concept of Stress." Pp. 184–238 in *Essays in the History of Mechanics.* Berlin, New York: Springer-Verlag.

Wallace, William A. 1976. "Galileo and Reasoning *Ex Suppositione:* The Methodology of the *Two New Sciences.*" Pp. 79–104 in *Proceedings of the 1974 Biennial Meeting of the Philosophy of Science Asociation.* Edited by Robert S. Cohen and Marx Wartofsky. Dordrecht, Boston: D. Reidel.

———. 1992. *Galileo's Logic of Discovery and Proof: The Background, Content, and Use of His Appropriated Treatises on Aristotle*"*s Posterior Analytics.* Dordrecht; Boston: Kluwer Academic.

Wisan, Winifred Lovell. 1978. "Galileo's Scientific Method: A Reexamination." Pp. 1–57 in *New Perspectives on Galileo.* Edited by Robert E. Butts and Joseph C. Pitt. Dordrecht; Boston: D. Reidel.