5

Cotes's queries

Newton's empiricism and conceptions of matter

ZVI BIENER AND CHRIS SMEENK

5.1 Introduction

The relation of Isaac Newton's natural philosophy to his method of inquiry is of central importance to Newtonian scholarship. In this chapter, we investigate this relation as it concerns Newton's ideas about the nature and measure of matter. We argue that a conflict between two conceptions of "quantity of matter" employed in a corollary to proposition III.6 illustrates a deeper conflict between Newton's view of the nature of extended bodies and the concept of mass appropriate for the *Principia.* The conflict was first noted by the editor of the *Principia*'s second edition, Roger Cotes. His "two globes" objection demonstrates that Newton employed two different measures of "quantity of matter," related to competing views on the nature of matter. On what we call the "dynamical conception of matter" – dominant in the *Principia* – quantity of matter is measured through a body's response to impressed force. On the "geometrical conception of matter," quantity of matter is measured by the volume a body impenetrably fills. The discussion with Cotes reveals Newton's commitment to the geometrical conception: he assumes all atoms have a uniform specific gravity; i.e., that the inertia of completely filled bodies is proportional to their volume. On the dynamical conception of matter, there is no reason for this proportionality to hold. A purely dynamical conception is consistent with the inertia of completely filled bodies varying in proportion to their volumes, or with bodies treated as non-extended point particles. By analyzing the exchange with Cotes (and related texts), we show that before Cotes's

Many thanks to J. E. McGuire, who along with B. R. Goldstein, was our first guide to Newton, for insight and encouragement. Additional thanks to others who provided us with comments and discussion: Katherine Brading, Tyler Burge, John Carriero, William Harper, Andrew Janiak, Eric Schliesser, George Smith, Sheldon Smith, Howard Stein, Wouter Valentin and other participants in the "Newton and/as Philosophy" conference. Our treatment of Newton's method of inquiry is indebted to Howard Stein's work.

prodding in 1712, Newton held both conceptions of matter and apparently saw no conflict between them.

We trace Newton's failure to recognize the conflict between the two conceptions to Newton's allowance for the justification of natural philosophical claims by two types of a posteriori, empiricist methodologies, which both turn away from the a priori Cartesian approach Newton deplored. Although both proceed "from the phenomena," we argue that there are important differences between them. The first, underlying the dynamical conception, is mathematical and relies on a nuanced interplay between specific phenomena and their theoretical descriptions. Recent work by George Smith, Bill Harper, and Howard Stein has shown how this methodology was used in the *Principia* to justify the laws of motion. Drawing on their analyses, we briefly characterize this method, using Newton's reply to Cotes's better-known "invisible hand" objection as an illustration. The second empiricist method, underlying the geometrical conception, also proceeds from the phenomena, but does not draw on the technical resources used in the first. Instead, its conclusions are intended to follow from general features of our experience, in a way articulated most clearly in *De gravitatione* (hereafter, *DG*) and through certain of Newton's examples in Rule III of the *Regulae Philosophandi.* We argue that although both methods of inquiry are based on empirical considerations, the relationship of theory to evidence in each is distinct. Centuries of debate attest to the difficulty in extracting from Newton's methodological discussions a clear account of evidential warrant that spans all of his work. We do not tackle this general project here. Instead, we highlight two different types of arguments from the phenomena endorsed by Newton, and argue that he failed to clearly distinguish them. He thus failed to recognize that one was not as secure as the other. In the *Principia* and *DG*, the two conceptions of matter are justified by these different types of arguments, yet prior to Cotes's "two globes" objection, Newton treated the two conceptions as if on equal footing, without recognizing their different sources of warrant. Cotes's objection forced Newton to reconsider the status of the geometrical conception. Although he never drew general conclusions regarding the relation between his two methods of inquiry, he came to side with the more sophisticated method of inquiry and, in revisions prompted by Cotes, stated the geometrical conception hypothetically. Given the deep-seated Cartesian and atomistic roots of the geometrical conception in Newton's thought, this was a profound shift.

We begin (in Section 5.2) by introducing the geometrical and dynamical conceptions of matter and the associated measures of quantity of matter. To do so, we review the reasons that led Newton to abandon aether theories of gravitation and accept the existence of void spaces. Although these forced Newton to reject much of the Cartesian analysis of space and body, we show that the geometrical conception of matter present in *DG* and later work betrays a lingering debt to Descartes. At the close of Section 5.2, we explicate the a posteriori method of inquiry underlying the geometrical method of quantifying matter. In Section 5.3 we turn to the a posteriori method underlying the dynamical conception of matter and the argument for universal gravitation (AUG). Drawing on these accounts of the contrasting methods, we investigate the conflict between the two conceptions of matter in Section 5.4.

5.2 Newton's conceptions of matter

5.2.1 Against the mechanical aether and Cartesian body

In the years leading up to the *Principia*, Newton's natural philosophy underwent a "radical conversion" (borrowing Westfall's phrase); he abandoned the fundamentals of Cartesian natural philosophy and replaced them with novel conceptions of space, motion, and body. This radical conversion was motivated in large part by Newton's rejection of the idea that the planets are carried in their orbits by an aetherial vortex. We begin by elucidating two empirical reasons that led Newton to abandon a gravitational aether and clarifying how this a posteriori line of reasoning undermined Cartesian philosophy.

As early as 1664, Newton took the cause of terrestrial gravitation to be mechanical and formulated a mechanical aether theory akin to other contemporary theories. The central idea of these theories – that a body's weight could be explained in terms of an aetherial fluid exerting pressure on a body's inner surfaces – appears to have persisted in Newton's accounts through the 1670s, even as the gravitational aether became more intricately tied to active principles inspired by his alchemical studies.[1] The central role played by the gravitational aether in the 1670s makes its nearly complete (if temporary) disappearance from Newton's natural philosophy in the period preceding the composition of the *Principia* remarkable.[2]

In *E1* Newton gave two decisive empirical reasons for abandoning the aether. (*Principia* editions are abbreviated as *E1*, *E2*, and *E3*.) First, he became convinced that the planets and comets encounter negligible resistance to their motion. In the first two theorems of the *De motu corporum in gyrum*, Newton derived Kepler's area law and the harmonic law for a central force *with no resistance*. The accuracy of Kepler's law in describing planetary motions implied that there was no need to introduce a resisting force alongside the centripetal force holding the planets in their orbits.[3] Newton strengthened his case in later

1 See "Of Gravity and Levity," in Newton (1983, pp. 362–365, 426–431), and Wilson (1976, pp. 192–195) regarding Newton's early views, and Dobbs (1991, chapter 4) regarding the development of Newton's views through the 1670s.

2 Dobbs (1988) details Newton's abandonment of the aether at the time of *E1*; McGuire (1966) discusses later shifts in Newton's views.

3 Newton was aware that even if there is no resistance Kepler's laws fail to hold *exactly* for universal gravity due to the perturbing effects of each planet on the other planets'

drafts and in the *Principia*. The persistence of planetary motion over thousands of years is also incompatible even with slight aetherial resistance, which would lead to a steady decrease in quantity of motion (Herivel 1965a, p. 302). Although the negligible resistance encountered by the planets is compatible with an aetherial vortex in which the planets move with the aether, reconciling the motion of comets, especially retrograde and highly eccentric ones, with an aetherial vortex is difficult.

Second, Newton failed to detect aether resistance in a series of pendulum experiments reported in Book II. Based on the realization that a gravitational aether must penetrate to the inner surfaces of bodies – without such penetration, the aether's action could only depend on a body's surface area – Newton designed experiments to measure the *internal* resistance due to the aether.[4] Newton constructed a pendulum consisting of a "round firwood box" suspended from a cord. He measured the oscillations of the empty box, then filled the box with various metals, adjusting the cord to the same length. The metal-filled box weighed 78 times as much as the empty one; in the absence of internal resistance Newton expected the oscillations of the full pendulum bob (from the bob's increased inertia) would take 78 times as long to decay. Newton initially assumed that filling the box would not change its *external* resistance. Because the decay only took 77 times as long, Newton concluded that the internal resistance must be over 5,000 times less than the external resistance. In *E2* and *E3*, Newton interpreted this to mean that *aether resistance* caused the damping:

> This argument depends on the hypothesis that the greater resistance encountered by the full box does not arise from some other hidden cause but only from the action of some subtle fluid upon the enclosed metal.
>
> (Newton 1999, p. 723)

However, in *E1* (in a passage subsequently omitted), Newton proposed a different cause:

> But I suppose that the cause is very different [than the aether acting on the internal surfaces of the box]. For the times of the oscillation of the full box are less than those of the empty one, and therefore the resistance to the *external surface* of the full box is greater, by virtue of its velocity and the length of its oscillations, than to the empty box. From which it follows that the resistance [due] to the internal parts of the box is either zero or entirely insensible.
>
> (translated in Kuhn 1970b, pp. 106–107)

orbits (Herivel 1965a, p. 301). However, these departures from Keplerian motion differ in character from the departures Newton expected for a resisting medium.

4 See Newton (1999, pp. 722–723). Note that *external* resistance may arise due to the air alone or the air and the aether conjointly, but the experiment is designed so that external resistance (as well as buoyancy of the air) is held constant. Cf. Kuhn (1970b, pp. 106–108).

Newton's conclusion is phrased cautiously. He did not claim that the aether does not exist; instead, he inferred only that if there is an aether, then its *resistance* is either nil or negligible. However, he had also concluded from other experiments (reported in the same scholium) that the primary contribution to resistance is proportional to the *material density* of the fluid through which an object moves. Thus, the experiments gave Newton grounds to reject a *mechanical* aether, although with his usual care he did not claim that they rule out an aether altogether (cf. Smith 2001b).

Although these considerations triggered Newton's radical conversion, they were not decisive for his contemporaries and successors. In Newton's later treatment in the *Principia*, fluid resistance arises primarily from the inertia of the fluid, and the dominant component of the force of resistance is proportional to ρv^2 (ρ is the density of the fluid, and v is the relative velocity). Leibniz demurred to Clarke regarding this assumption, arguing that "it is not so much the quantity of matter as its difficulty in giving place that makes resistance" (Alexander 1956, p. 65). Leibniz had earlier distinguished between two sources of resistance, viscosity and density, and argued that they make distinct contributions to the overall resistance for different types of fluids. Drawing this distinction between different types of resistance opens up the possibility of an aetherial fluid with resistance not proportional to density – which would avoid Newton's arguments. In fact, the possibility is much easier to realize than Newton had anticipated (Smith 2001b). In 1752, d'Alembert showed that a fluid without viscosity has exactly zero resistance, undercutting Newton's proposal that the dominant contribution to fluid resistance arose from the fluid's inertia. This error does not detract from Newton's insight that a single force law was sufficient to account for planetary motions, but it does undermine Newton's empirical case against the aether. Yet the problem was not solely with *Newton*'s arguments against the aether. Contemporary versions of aether theory were also based on misconceptions regarding fluids and the nature of resistance, and any aether theorist faced the challenge of elucidating how the aether produced gravitational effects without causing appreciable resistance (cf. Aiton 1972).

Newton's rejection of a mechanical aether left him without a mechanical explanation of gravitation, along with an awareness of the obstacles to providing one. Here we focus on one fundamental consequence of this awareness for Newton's thought: he was forced to reconsider Descartes's doctrines regarding the nature of body and space, and replace them with ones compatible with the existence of void spaces.

Newton's most sustained critical discussion of Descartes appears in *DG*. The stated aim of the manuscript is the study of the gravitation and equilibrium of fluids. Written in the geometrical style, it begins with a series of definitions and closes with two theorems. Newton makes room for his own definitions of space, body, and motion with a long philosophical discussion expressly devoted to undermining the corresponding Cartesian definitions – to "dispos[ing] of [Descartes's] *figmenta.*" The main thrust of this digression is that an adequate definition of motion requires an appropriate structure relating locations over time.[5] Descartes's plenum lacked the necessary structure, leaving Descartes with a definition of motion that failed to support distinctions fundamental to his physics. Newton overcame this defect by introducing space as a distinct entity with a sufficiently strong structure, albeit an entity that did not fit neatly into traditional ontological categories.[6]

ether resistence

Even in this overtly philosophical context Newton supported his arguments against Descartes with empirical evidence in favor of void space. On the basis of pendulum experiments (that may have been either the experiments discussed above or precursors), Newton asserted that the resistance of the aether is "over ten or a hundred thousand times *less*" than the resistance of quicksilver (Newton 2004, p. 35, emphasis added).[7] Newton also took resistance to moving through a medium as a consequence of the material nature of the medium's parts. As he put it, "if we set aside altogether every resistance to the passage of bodies, we must also set aside the corporeal nature [of the medium] utterly and completely" (Newton 2004, p. 34). Because two bodies cannot simultaneously occupy the same region of space, one body resists the passage of another body through the region it occupies. Though controversial, if this view is accepted then the failure to detect resistance is decisive evidence against the Cartesian plenum.

D's "elimination" arg.

Rejecting the plenum posed a clear challenge to the Cartesian identification of extension as the principal attribute of body. In *DG*, Newton singled out Descartes's so-called "elimination argument" (*Principles of Philosophy*, II.4 & II.11) as the main argument for this thesis. According to Newton, Descartes argued that various sensory properties such as hardness, weight, and color can be abstracted from a body without endangering its status *as a body.* Only the elimination of extension can destroy a body's corporeality, and so extension alone constitutes body's principal attribute, or, as Newton put it, "pertain[s] to [body's] essence" (Newton 2004, p. 21). To Descartes's argument, Newton countered that to be recognized as body, a body had to possess not only extension, but "faculties," particularly the ability to stimulate perceptions and

5 What is actually required in the *Principia* is the distinction between inertial and non-inertial motion; this only requires an affine connection and not the stronger structure that would be provided by identifying the "same position" over time; see Stein (1967). However, Newton was apparently unclear on this issue in *DG*; some of his criticisms of Descartes presume a stronger structure than necessary.

6 See Stein (1967), Rynasiewicz (1995a,b), DiSalle (2002).

7 Based on this brief allusion it is unclear how these experiments relate to discussions in other texts. See Dobbs (1991, pp. 134–143), and Westfall (1971a, pp. 341, 375–77) for further discussion.

"transfer action" to other bodies.[8] The core of Newton's critique was the claim that "although philosophers do not define substance as an entity that can act upon things, yet everyone tacitly understands this" (Newton 2004, p. 21). Newton – *contra* Descartes – held that what we should primarily care about is not what a substance is, but what it does.

This difference of orientation is also evident in the stated aim of Newton's speculation regarding body. By contrast with Descartes, Newton's goal in *DG* was the development of an account of body *sufficient* to serve as a basis for physical theory and *sufficient* to capture the phenomenal properties of bodies, the properties of "a kind of being similar in every way to bodies, and whose creation we cannot deny to be within the power of God, so that we can hardly say that it is not body" (Newton 2004, p. 27). Newton was clear, however, that he could not establish more than the sufficiency of his account. In particular, he made no claims to reveal the necessary essence or nature of body.[9]

5.2.2 The geometrical and dynamical conceptions of matter

But we must not stress only the *differences* between Newton and Descartes. Although Newton's conception of body in *DG* differed from Descartes's both in its content and metaphysical pretensions, it still possessed vestiges of Cartesianism. While defining body in terms of regions of space endowed with additional attributes – attributes foreign to Descartes's account – Newton still followed Descartes by treating bodies as regions of space, as extended geometrical structures, albeit not geometrical structures *simpliciter*. In *DG*, the character of bodies is partially dependent on the character of space. Space, in turn, has geometrical structure – it is full of "all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular, elliptical, parabolical, and all other kinds of figures, and those of all shapes and sizes, even though they are not disclosed to sight" (Newton 2004, p. 22 ff.). Bodies, as regions of space, are consequently geometrical, although they admit non-geometrical properties as well.

8 Newton was also familiar with the predecessor of the argument of *Principles* II.4 in the *Second Meditation*, but does not address it explicitly in *DG*; see Harrison (1978, p. 132) and Newton (1983, p. 23).

9 The epistemological status of Newton's accounts of space and body differs. While Newton emphasized the tentative status of his account of body – as one possible account compatible with experience – he did not treat his account of space as similarly conjectural and tentative, as Stein (2002) emphasizes. Consequently, when we speak of the "nature" of body according to Newton, we do not mean to impute to him any form of essentialism or a conception of natural philosophy according to which the goal of philosophizing is to draw observable consequences from the natures of ontological primitives.

In *DG*, Newton treated bodies as "*determined quantities of extension which omnipresent God endows with certain conditions*," namely:

> (1) that they be mobile; and therefore I did not say that they are numerical parts of space which are absolutely immobile, but only definite quantities which may be transferred from space to space;
>
> (2) that two of this kind cannot coincide anywhere; that is, that they may be impenetrable, and hence that oppositions obstruct their mutual motions and they are reflected in accord with certain laws;
>
> (3) that they can excite various perceptions of the senses and the imagination in created minds.
>
> (Newton 2004, pp. 28–29)

Central to this account of body is the notion that bodies are primarily "determined quantities of extension." The reliance on a determinate spatial substratum as a precondition for the existence of bodies is one of the main features of Newton's account. After providing the above definition of body, Newton emphasized one of its main anti-Aristotelian implications; namely, that it does away with the need for a property-less substratum as the metaphysical support for properties and forms and instead makes do with space itself:

> [F]or the existence of [bodies] it is not necessary that we suppose some unintelligible substance to exist in which as subject there may be an inherent substantial form; extension and an act of the divine will are enough. Extension takes the place of the substantial subject in which the form of the body is conserved by the divine will; and that product of the divine will is the form or formal reason of the body denoting every dimension of space in which the body is to be produced.
>
> (Newton 2004, p. 29)

Newton's analogy between his account and hylomorphism demonstrates that extension was as central to his conception of body as the substantial subject was for the conception of body of his Aristotelian adversaries. On Newton's account, extension is necessary for the application of so-called "form" and thus for the existence of body. "Body," as defined in *DG*, "is that which fills space" (Newton 2004, p. 13). It is not necessarily that which gravitates, nor that which moves, nor that which is tangible and visible (although it may also be any of those).

The conception of body as impenetrable extension takes precedence in *DG* over the nascent conception of body defined by laws of motion. In *DG* Newton held that bodies must move "in accord to certain laws," but the phrase does not acquire special significance without the juxtaposition of *DG* against later texts. Taken by itself, *DG* defines body primarily as a region of filled-in extension and only secondarily in terms of laws governing motion. On Newton's account, tangibility, visibility, and other traits that constitute the "corporeality" of matter according to our senses all depend, primarily, upon the impenetrability of regions of space (Newton 2004, pp. 27–28). Motion has a secondary role in constituting that corporeality because motion only makes impenetrability manifest to our senses. Newton only introduced motion once regions of space were rendered impenetrable:

> [W]e may suppose that there are empty spaces scattered through the world, one of which, defined by certain [spatial] limits, happens by divine power to be impervious to bodies, and by hypothesis it is manifest that this would resist the motions of bodies and perhaps reflect them, and assume all the properties of a corporeal particle, except that it will be regarded as motionless. If we should suppose that that impenetrability is not always maintained in the same part of space but can be transferred here and there according to certain laws, yet so that the quantity and shape of that impenetrable space are not changed, there will be no property of body which it does not possess.
>
> (Newton 2004, p. 28)

Even if mobility has only a secondary status in this passage, it is still essential to Newton's account both here and in the "determined quantities of extension" passage. However, our point is that mobility *of impenetrable regions* is essential, not mobility taken by itself. The centrality of the impenetrability of the extensional substratum reveals Newton's residual Cartesianism: in *DG* he considers bodies to be essentially extended geometrical structures – geometrical structures made real by further conditions, but geometrical structures nevertheless.

Newton's manner of quantifying body in *DG* further illustrates his residual Cartesianism. Newton measured quantity of matter through a body's geometrical rather than dynamical properties. After defining the absolute quantity of force as a product of the force's intension ("the degree of its quality") and extension ("the amount of space or time in which it operates") Newton wrote:

> [M]otion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly. Again, motion is more or less extended as the body moved is greater or less, or as it is diffused through a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body.
>
> (Newton 2004, p. 37)

In modern terminology, Newton equated momentum (the "force of motion") to the product of the velocity (intension) and the "magnitude of the moving body" (extension). The latter is measured by the body's volume ("the amount of space in which [the force of motion] operates") rather than by the body's resistance to impressed forces (i.e., inertia). Given that (from the *Waste Book* onward) Newton equated force of motion with the product of velocity and *quantity of matter*, here Newton measures quantity of matter through its volume or quantity of extension (see Herivel 1965a, p. 26). We call this method of quantification, along with Newton's account of the nature of bodies as primarily determined quantities of extension, Newton's *geometrical conception of matter*.

Two caveats must be made regarding this geometrical conception. First, it is Cartesian in inspiration without being wholly Cartesian. Newton did not attempt to reduce all of a body's properties to geometrical properties, nor to treat any single property as a body's principal attribute. However, Newton did follow Descartes in considering extension as essential to our understanding of body *and* to the practice of physics vis-á-vis the measure of the quantity of matter associated with body. Second, although we have highlighted the geometrical conception's indebtedness to Newton's Cartesianism, the conception is also closely tied to Newton's atomism, particularly his belief in the uniformity of nature. This is an important aspect of Newton's thought, but we can only touch on it briefly in Section 5.4 below.

In *DG*, the geometrical measure of quantity of matter is not supplemented with a precise dynamical measure, as it is in the *De Motu* drafts and the *Principia*. According to what we call the *dynamical conception of matter*, a quantity of matter is measured by its response to impressed force, not by the volume of space which it impenetrably fills. As with the geometrical conception, the dynamical conception also incorporates a view regarding the nature of bodies, which we consider shortly. Newton introduced the dynamical measure of quantity of matter in Definitions I and III of the *Principia*. In Definition III, Newton states that the internal force of a body (its *vis insita*) "is always proportional to the body and does not differ in any way from the inertia [*vis inertia*] of the mass except in the manner in which it is conceived" (Newton 1999, p. 404). We are to understand that *vis insita* is also proportional to a body's *quantity of matter* since Definition I states that:

> I mean this quantity whenever I use the term "body" or "mass" in the following pages.
>
> (Newton 1999, p. 404)

Together with Law II, these two definitions establish a proportionality between a body's quantity of matter and the force responsible for the body's dynamical properties.[10]

Although Definition I also states that "Quantity of matter is a measure of matter that arises from its density and volume jointly," the quantification

10 See McGuire (1994) on the nature of *vis insita* and Bertoloni Meli (2006a) on its connection to *vis centrifuga*.

method implied by Definition III is used throughout the *Principia* almost exclusively. Even in Definition I Newton made explicit that quantity of matter "can always be known from a body's weight for – by making very accurate experiment with pendulums – I have found it to be proportional to the weight."

This is a far cry from *DG*. Of course, Newton did define force in *DG* as either "external" – "one that generates, destroys, or otherwise changes impressed motion in some body" – or "internal" – "by which existing motion or rest is conserved in a body, and by which any body endeavors to continue in its state and opposes resistance" (Newton 2004, p. 36). But, unlike in the *Principia*, the dynamical method is not used to *quantify* body.[11] Lacking a clear statement of Law II, the dynamical measure of a quantity of matter remains vague in *DG* and intertwined with the conception of body as that which fills space. *De gravitatione* adumbrates the dynamical conception of matter, but does not contain it fully and certainly does not contain its central element, the measurement of quantity of matter by a body's response to impressed force. In the *Principia*, the two methods of quantifying matter co-exist, but the geometrical conception is relegated to the wings while the dynamical conception takes center stage.

What account of the nature of bodies accompanies the dynamical measure of matter in the *Principia*? The *term* "body" appears in the definitions and laws, but Newton does not explicitly define "body" or provide an account of body's possible nature like the one provided in *DG*.[12] Nevertheless, in contrast to *DG*, in the *Principia* Newton characterizes material bodies almost exclusively by their dynamical properties. This suggests a transformation in Newton's view: the *Principia* provides clear formulations of the concept of force and the laws of motion, but bodies are defined only derivatively – as the entities subject to forces and for which the laws of motion hold. The nature of body in the *Principia* thus depends upon whatever constraints are implied by satisfaction of the laws. Furthermore, empirical support for this view of bodies derives from the empirical support for the laws of motion and the physical theory based on them.

example of approach in φ of special science f.

Yet the dynamics of the *Principia* place surprisingly weak constraints on the nature of body. In particular, "bodies" satisfying the laws of motion need

11 In *DG* Newton used the term *vis inertia* for the internal principle of motion (Newton 2004, p. 36) in much the same sense as he used the term in an excised portion of Definition 1 in the Lucasian lectures (1685, Newton 1967–1981: Vol. V). This contrasts with his use of *vis insita* in the early *De Motu* drafts (see Herivel 1965a, pp. 26–28). We thank George Smith for this point. Yet despite the appearance of the term *vis inertia*, *DG* is a transitional text which only hints at the concept of *vis inertia* developed in the *Principia*. This is not surprising, since in *DG* Newton is working out the metaphysics of "natural power" on which *vis insita* depends; see Stein (1990).

12 Newton did define "body" in unpublished definitions intended for *E3* of the *Principia*, but these only bolster the argument below, cf. McGuire (1966). We discuss them briefly in Section 5.4.

not have any geometrical properties whatsoever. This may appear to conflict with Newton's various theorems regarding extended bodies, such as the famous proofs to the effect that a spherical body can be treated as if the mass were concentrated at a point (Propositions I.71–75). However, even these proofs only require that the force acting on or produced by the whole body is the sum over forces related to its constituent parts. They do not require the attribution of geometrical properties to the parts of the spherical bodies, and are compatible with bodies treated as Boscovichian point-particles characterized by parameters such as "quantity of matter" that have no geometrical basis. The austerity of the dynamical conception of matter stems from the limited mathematical framework of the *Principia*. The generalization of Newtonian theory to continuum mechanics leads to a richer notion of body that does have implications for the geometrical properties of bodies.[13]

That said, *DG*'s geometrical conception of matter did not disappear from Newton's thought following the elaboration of the dynamical conception in the *Principia*. In drafts of corollaries to III.6 written in the 1690s, Newton assesses the connections between gravitational aethers, matter theory, and the existence of void (McGuire 1967). In doing so, he assumes that the appropriate measure of quantity of matter is the volume of the basic particulate constituents of matter. These manuscripts indicate that Newton continued to take the geometrical conception of matter seriously post-*Principia*. Finally, Newton's reply to Cotes's "two globes" objection – also concerning III.6 – relies on the geometrical conception, and took place over 20 years later, in 1711. We return to the two globes objection in §IV. We now turn to the two types of a posteriori, empiricist arguments we believe are associated with the two conceptions of matter.

5.2.3 The a posteriori character of the geometrical conception of matter

How did Newton establish his geometrical conception of body a posteriori? Two distinct a posteriori contributions can be discerned. First, the results of pendulum experiments and the accuracy of Kepler's "laws" pushed Newton to reject the Cartesian identification of body with extension. In this regard, his path towards a new conception of body is similar to his path towards a new conception of light in his optical work (Shapiro 2004; Stein ms.) In both, Newton took a narrow set of experimental results to be sufficiently crucial to warrant revision to a fundamental concept of natural philosophy.

But there is an important difference: whereas the prism experiments, the crucial experiments in Newton's research on colors, were used to both refute

13 For example, Cauchy's generalization of Newton's laws involves contact forces and the outward normal defined over the contact boundary; Boscovichian point particles lack boundaries and Cauchy's formulation does not apply, cf. Truesdell (1968) and Smith (2007).

the extant conception of light and suggest a new conception (i.e., that white light is not a natural kind but is composed of individually homogeneous rays of differing refrangibilities), the pendulum experiments were used only to refute the Cartesian doctrine. On our view, Newton rejected an account of gravity based on the results of pendulum experiments along with his success in modeling planetary motions using a single force law. This rejection undercut the associated Cartesian accounts of body and space. However, the constructive element of Newton's geometrical conception of body was not secured by an *experimentum crucis*; rather, it seems to have been secured by a different type of argument from the phenomena.

This argument proceeds from the experience of any body whatsoever. Newton attempted in *DG* to provide an account of body that is sufficient for capturing the "evidence of our senses" (Newton 2004, p. 28). The traits of body he aimed to save were all quite generic and are reflected in the overall character of our experience; for example, that body is visible, tangible, audible, etc. Newton's account of body as mobile, impenetrable, and sensible extension is able to save these traits because it is set against a framework of natural philosophical presuppositions – e.g., that an object is visible because it reflects light and audible because it can move adjacent air – but, given this framework, the evidential basis for Newton's account includes any and all experiences of body. Importantly, success within this framework does not rely on any quantitative notion of "strength of evidence" that can help arbitrate between Newton's account and possible competitors – where by "strength of evidence" we mean any measure of the fit between a given theory and its evidential basis that allows discrimination among competing theories according to degree of evidential warrant. Rather, it relies on a notion of warrant akin to the one mechanical philosophers used to justify their mechanical models, but one that does not appeal to first principles or privileged modes of explanation. Strikingly, it does *not* rely on the sophisticated notion of warrant used in the AUG. Note Newton's explicit reference to the underdetermination of *DG*'s account of body:

start here?

> [I]t is hardly given to us to know . . . whether matter could be created in one way only, or whether there are several ways by which different beings similar to bodies could be produced . . . [H]ence I am reluctant to say positively what the nature of bodies is, but I would rather describe a certain kind of being similar in every way to bodies, and whose creation we cannot deny to be within the power of God, so that we can hardly say that it is not body.
>
> (Newton 2004, p. 27)

Newton is explicitly open to the possibility that another hypothesis regarding the nature of body can save the phenomena equally well.

This a posteriori method of arriving at claims regarding the nature of matter resembles the one offered in Rule III and its drafts (McGuire 1968; 1970). In Rule

III, Newton claimed that certain qualities of bodies are "universal," qualities that can be attributed to any body whatsoever and so constitute the core of our understanding of body (Newton 1999, p. 796). Often, Newton referred to such claims of universality as being "deduced from phenomena" (e.g., Newton 1999, p. 943). Body's universal qualities are extension, hardness, impenetrability, mobility, and inertia. However, Newton's evidence for their universality is not homogeneous. One of our theses is that deducing or gathering propositions "from phenomena" does not have a univocal meaning for Newton, and so the resemblance of Rule III to *DG* concerns some universal qualities – more will be said about the others in Section 5.3.3.

my theme

First, Rule III, like *DG*, appeals to our general experience of bodies as the evidential basis from which claims regarding the extension, hardness, and impenetrability of bodies ought to be drawn. Newton wrote, echoing *DG*, that:

> The extension of bodies is known to us only through our senses . . . [and] because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard . . . [and] justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies that we handle to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally.
>
> (Newton 1999, p. 795)

In each case, our experience of bodies broadly conceived forms the evidential basis of the generalization. Still, the evidential basis recommended by Rule III is more restrictive than the one used in *DG*. According to Rule III, only those qualities found in "all bodies on which experiments can be made" *and* passing the intension and remission criterion may be "taken as qualities of all bodies universally" (Newton 1999, p. 795). Thus only *some* features of our experience of bodies remain relevant to generalization about body; visibility and audibility, for example, are eliminated. Nevertheless, the remaining features are those that are truly general and are present in *all* experiences of body. Achieving this generality is precisely the point of Newton's application of the intension and remission criterion. Any quality that is not *always* present in our experience of bodies – i.e., one that can be remitted to zero and thus disappear, or one that is not present in some bodies – is not universal.

Second, regarding the first four qualities mentioned, Rule III, like *DG*, does not invoke a notion of evidential warrant similar in complexity to the one used throughout the *Principia.* This is because, while the intension and remission criterion can be made precise, it is unclear when in the course of empirical investigation we can be content that it is satisfied for "all bodies on which experiments can be made." Newton's examples do not help. According to Newton, the extension of bodies is made manifest in *all sensible bodies.* However, we know by experience that hardness is only found in *some* bodies while impenetrability is only found in "those bodies that we *handle*" – presumably a smaller class than "all *sensible* bodies." Is the judgment of universality regarding one of these better than the others? Newton suggested, but did not elaborate, a notion of strength of evidence: "the argument from phenomena will be *even stronger* for universal gravity than for the impenetrability of bodies, for which . . . we have not . . . even an observation, in the case of the heavenly bodies" (Newton 1999, p. 796, emphasis added). Something like simple enumerative strength seems to be at work here: the more instances of a quality we have, the stronger the judgment of its universality. This is a far cry, however, from the sophisticated and more robust relation between theory and evidence implicit in the *Principia*.

The lack of a robust notion of evidential warrant or strength would not be bothersome by itself, but we argue in Section 5.4 that, on at least one occasion, Newton overstated the evidence in favor of the geometrical conception of matter. The reason, we argue, is that Newton failed to distinguish the type of argument given in *DG* for the geometrical conception of matter from the type of argument used in the *Principia*. In order to clarify the latter type of argument and its notion of evidential strength, we will use Cotes's invisible hand objection.

my theme

5.3 The invisible hand

As the editing of *E2* neared completion in 1713, Cotes began writing a preface contrasting Newton's "experimental philosophy" with Cartesian and Aristotelian approaches. To exemplify Newton's method he intended to present a "short deduction of the Principle of Gravity from the Phænomena of Nature, in a popular way" (Newton 1959–1977, V, p. 391). However, he encountered a difficulty.

Cotes accepted the first two steps of Newton's AUG: (1) the planets are held in their orbits by an inverse-square centripetal force directed towards the Sun, and (2) this force can be identified with terrestrial gravity, via the Moon test. What troubled him was the next step, discussed in III.5.c3 and III.7. In this step, Newton applied the third law to the centripetal force holding planets in their orbits, and concluded that a given planet also attracts the Sun. Newton argued that gravity is a *mutual interaction* between the Sun and planet. Cotes objected that this step requires further hypotheses about the nature of gravitation:

> ye Force by which they [the planets] are continually diverted from the Tangents of their Orbits is directed & tends towards their Central Bodies. Which Force (from what cause whatever it proceeds) may therefore not improperly be call'd Centripetal in respect of ye revolving Body & Attractive in respect of the Central. . . . But in the first Corollary of the 5th

> [proposition of Book III] I meet with a difficulty, it lyes in these words *Et cum attractio omnis mutua sit.* I am persuaded they are then true when the Attraction may properly be so call'd, otherwise they may be false. You will understand my meaning by an Example. Suppose two Globes A & B placed at a distance from each other upon a Table, & that whilst A remains at rest B is moved towards it by an invisible Hand. A by-stander who observes this motion but not the cause of it, will say that B does certainly tend to the centre of A, & thereupon he may call the force of the invisible Hand the Centripetal force of B & the Attraction of A since ye effect appeares the same as if it did truly proceed from a proper & real Attraction of A. But then I think he cannot by virtue of this Axiom [*Attractio omni mutua est*] conclude contrary to his Sense & Observation that the Globe A does also move towards the Globe B & will meet it at the common centre of Gravity of both Bodies. This is what stops me in the train of reasoning by which I would make out as I said in a popular way the 7th Prop. Lib. III. I shall be glad to have Your resolution of the difficulty, for such I take it to be. . . . For 'till this objection be cleared I would not undertake to answer one who should assert that You do *Hypothesim fingere.* I think You seem tacitly to make this Supposition that the Attractive force resides in the Central Body.
>
> (Newton 1959–1977, V, p. 392)

hmm, so there is an empirical test. Is revolution around the center of gravity?

There are two ways of reading Cotes. On the first, there is a stark *empirical* contrast between the "invisible hand" scenario and Newton's account of gravitation. According to Cotes, the invisible hand moves Globe B without moving Globe A. According to Newton, however, true interactions are mutual, and so the central body of a gravitational system (Globe A) is predicted to move, however slightly. The mismatch between prediction and observed motion is Cotes's problem: "I think [an observer] cannot by virtue of this Axiom [*Attractio omni mutua est*] conclude *contrary to his Sense & Observation* that the Globe A does also move towards the Globe B" (Newton 1959–1977, V, p. 392, emphasis added). Cotes presumed that in a gravitational system, like in the invisible hand case, "Sense & Observation" will show that the central body does not move. Determining the motion of a central body in a real-world case is not straightforward, but it is possible. For truly mutual interactions between two bodies there is a "two-body" correction to Kepler's third law (see, e.g., Smith 2002b, p. 44). Neither Newton nor Cotes could have made an empirical case in favor of this correction based on contemporary observations, but the question was open to empirical resolution.

Newton shifted the terms of the debate rather than treating this as an unresolved empirical question. To account for the subsequent exchanges we focus on a second reading. On this reading, we take Cotes to suggest that *two* invisible hands are acting jointly to move the globes in a way identical with the predictions of Newton's theory. This scenario is not empirically distinguishable from Newton's description of planetary motions; instead, it suggests that Newton's argument for extending the third law to the central body rested on an unacknowledged assumption about the nature of gravity.[14] Although Cotes claimed that Globe A does *not* move, Newton seems to have responded to Cotes as if he had posed this more telling objection.

Newton's replies to Cotes focused on two points, both ultimately reflected in changes to the *Principia*. First, Newton defended the third law as a crucial feature of his conception of force by showing that it is necessary for extending the first law to systems of interacting bodies. This reply missed the point (of the second reading). Cotes did not challenge the third law itself, but rather the identification of the bodies involved in its application. Nonetheless, Newton's discussion reveals the third law's importance in going from a mathematical characterization of force based on the first two laws to a physical characterization of forces treated as mutual interactions among bodies. Second, Newton responded to the charge of feigning hypotheses: he clarified the nature of hypotheses in his method and argued that objections of a certain kind, exemplified by Cotes's invisible hand, should simply be set aside. By following Newton's response to Cotes and considering the status of the laws of motion, we argue that Newton properly answered Cotes here. We contrast the limited sense in which the laws are "hypothetical" on Newton's account with the role of hypotheses for his contemporaries (and the a posteriori character of the geometrical conception of matter), and by doing so describe the sophisticated a posteriori reasoning of the *Principia*, the basis for the dynamical conception of matter.

5.3.1 Applying the third law

The invisible hand highlights the ambiguity in Newton's application of the third law in the third step of the AUG. Suppose that invisible hands and an attractive force produce indistinguishable motions of an orbiting and a central body. The third law implies that there is an equal and opposite force corresponding to the force holding the body in its orbit, but it does not specify the nature and location of this force. Should it be a reaction force acting on the central body, or a force pushing against the invisible hand?

Newton's argument depends on one of two assumptions. Either, first, gravity is a force of attraction causally residing in the interacting bodies alone, an attractive force "properly so call'd", as Cotes put it; or, second, whatever underlying mechanism is responsible for gravitation must itself produce a reaction force on the central body. Suppose, for example, that an aether mediates the gravitational interaction. Newton's application of the third law would be appropriate only if there is no net momentum transfer from the two gravitating bodies

14 See Densmore (1996), Koyré (1968), Harper (2002b), Stein (1990a), and Harper in this volume. Kant rediscovered the problem, apparently independently of Cotes; see Kant (2002, pp. 225–226) and Friedman (1992, pp. 149–159).

to the aether. In this case, even though the third law *properly applied* yields a reaction force on the aether pressing against the planet, the aether interacts with the planet and Sun in precisely the right way to produce a reaction force on the Sun. Either option conflicts with Newton's claim that the validity of the AUG does not depend upon "hypotheses" regarding the nature of gravity.[15]

Newton responded to Cotes by defending the validity of the third law of motion itself. He asked Cotes to consider two bodies A and B acted on by no net external forces, such that the forces *between* A and B do not satisfy the third law. Say, for example, that A exerts a greater force on B than vice versa. Newton emphasized that the resulting imbalance of forces would cause the bodies to accelerate off to infinity, a result that conflicts both with experience and the first law of motion (Newton 1959–1977, V, p. 397). Newton added text to the *Principia* to the same effect. In the scholium following the Laws, Newton considered sections of the Earth cut off by parallel planes equidistant from the center. As before, an imbalance of the gravitational forces felt by these two parts of the Earth would lead to the Earth accelerating off to infinity with no net external force.[16] These examples reveal the intimate connection between the third and first laws. In order for the first law to hold for the center of mass of a closed system of interacting bodies, the third law must hold for the interactions among the bodies, although Newton's examples only involve *contiguous* bodies pressing against one another.

This line of response highlights the importance of the "mutuality" of force, Newton's crucial novelty. As Stein (2002) emphasizes, speaking of *separate* forces acting on two bodies, which happen to come in an action–reaction pair, is misleading. In Newton's usage, the "force" corresponds to an interaction between bodies that is *not* broken down into separate "actions" and "reactions," except in our descriptions of it. Newton's own "popular" version of the third book, the *System of the World*, included a clear statement to this effect:

> It is true that we may consider one body as attracting, another as attracted; but this distinction is more mathematical than natural. The attraction resides in each body towards the other, and is therefore of the same kind in both.... In this sense it is that we are to conceive one single action to be exerted between two planets arising from the conspiring natures of both.
>
> (Newton 1934, p. 568)

15 There is a second, distinct objection that Cotes did not raise (discussed in Harper 2002b): are the motions sufficient to establish that the motive forces of two interacting bodies are equal in magnitude? The equality can fail if, in modern terms, one allows the gravitational constant G to vary rather than treating it as a universal constant. The assumption that G is a universal constant amounts to treating the various acceleration fields produced by the celestial bodies as instances of the same type of force.

16 Applying this line of reasoning to orbital motion is less straightforward; see Harper (2002b) for discussion.

This conception of force as an interaction manifested by equal and opposite impressed forces is built into the Laws of Motion. It also plays a crucial role in distinguishing apparent forces from real forces. Given a body in motion, the first two laws allow one to infer the existence of a force producing the motion that may be well defined quantitatively (given a definite magnitude and direction), without considering the question of what produces the force. But the third law further requires that the force results from an interaction between the body in motion and some other body. (Coriolis forces illustrate this distinction: the force is well defined quantitatively and can be inferred from observed motions, but there is no "interacting body" responsible for the force.) The first two laws figure primarily in treating forces from a mathematical point of view, whereas the introduction of the third law marks an important physical constraint. Although Newton famously abstained from requiring a full account of the "physical cause or reason" of a force as a precondition for establishing its existence, any further account of the physical nature of the force would have to satisfy the constraint imposed by the third law.[17]

But even granted this conception of force as mutual interaction, Cotes's query prompted justification for Newton's identification of the second "conspiring" body. The first two steps of the AUG established that the force producing orbits of the celestial bodies is closely related to their respective central bodies: it is directed toward the central body, and it varies as the inverse square of the distance from the central body. These features make plausible Newton's identification of the central body as the second body whose "conspiring nature" produces the interaction. If the list of candidates is limited to other *known* bodies, there are few plausible choices other than the central body. Cotes accepted the first two steps of the AUG, and so accepted those features of the force law apparently related to the central body. However, Cotes was correct to insist that this plausibility argument is inadequate. Newton's rivals pursuing a vortex theory of planetary motion aimed to recover both these aspects of the force without introducing a truly mutual interaction with the central body. They did so by introducing an analog of the "invisible hand" – an aether that was unobservable except for its gravitational effects.

heh, why didn't they, or we, consider geometrical equivalence? sun rotates around planet.

In sum, although this part of Newton's response emphasizes the viability of the third law, this clarification is not sufficient to answer Cotes without further stipulations regarding the bodies referred to by the law. However plausible these further stipulations may seem, they beg the question as a reply to Newton's contemporary critics. One may charge that these stipulations are inconsistent with

17 Smith (2002a: 150) argues that Newton imposes five further conditions that must be satisfied for a component of a mathematically characterized force to qualify as physical; cf. Janiak (2007, 2008), regarding Newton's mathematical and physical characterizations of force.

Newton's own method, that they are feigned hypotheses. Newton responded to this charge directly.

5.3.2 *Status of the laws and the dynamical conception*

Since Cotes concluded that Newton did indeed "feign hypotheses" in the AUG, Newton offered two clarifications. First, Newton reprimanded Cotes for applying the term hypothesis too broadly:

> as in Geometry the word Hypothesis is not taken in so large a sense as to include the Axioms & Postulates, so in experimental Philosophy it is not to be taken in so large a sense as to include the first Principles or Axiomes wch I call the laws of motion. These Principles are deduced from Phænomena & made general by Induction: wch is the highest evidence that a Proposition can have in this philosophy.
>
> (Newton 1959–1977, V, pp. 396–397)

To make this clear in the *Principia*, Newton added the well-known passage immediately following "hypotheses non fingo" to the General Scholium (Newton 1999, p. 943). Thus, apart from defending his application of the third law, Newton argued that the laws of motion had a distinctive status.

Characterizing this status is a delicate matter, but the exchange with Cotes and a comparison of Newton with his contemporaries sheds light on Newton's position. Newton regarded the laws of motion as having a more secure status than the hypothetical models pursued by "mechanical philosophers" such as Huygens. Huygens characterized the aim of physics as the construction of mechanical models that rendered various phenomena intelligible. Confidence in a hypothetical model was based on the caliber of explanations it offered, its ability to predict novel phenomena, and other theoretical virtues such as simplicity. The well-recognized problem with this approach was the possibility of alternative, yet equally satisfactory, models. Mechanical philosophers often addressed it by insisting that their models satisfied further constraints, such as the compatibility of the models with privileged first principles, particularly with the ontology of matter in motion. These constraints, however, did not eliminate underdetermination worries; they merely limited their scope.

Newton, however, did *not* conclude that certainty was unattainable in natural philosophy.[18] His criticisms of mechanical theories were combined with the assertion that his own method could establish results with as much certainty "as the nature of things admit," a certainty guaranteed by a criterion of evidential warrant distinct from that of the mechanical philosophers (cf. Harper and Smith 1995). Propositions that met this more stringent criterion qualified as

18 See, e.g., Newton's evaluation of his optical work; Newton (1978, p. 106).

"deduced from the phenomena" or "proved by experiments," and Newton claimed that they were not susceptible to the problems faced by mechanical hypotheses. Specifically (cf. Section 5.2.3), confidence in Newton's reasoning about natural phenomena did not depend on its conformity with first principles regarding fundamental natures.

But there is more to "deduction from phenomena" or "proof by experiment" than a disregard for first principles, particularly in the context of the *Principia*. Specifying the difference is no mean feat, but we are able to draw on prior work on the implicit methodology of the *Principia* by Howard Stein, Bill Harper, and George Smith.[19] Despite disagreements on several finer points, this line of work highlights two general contrasts between Newton and the mechanical philosophers. First, Newton's predecessors – such as Galileo – did not deal with the complexity of actual motions as Newton did. Although the consequences of Galileo's theory of uniformly accelerated motion were not taken to apply exactly to actual motions, a rough conformity between actual and theoretically described motions was taken as evidence in favor of the theory. More refined judgments of conformity, however, require an assessment of factors such as air resistance and measurement imprecision, and these are problematic precisely because they are not treated in the original theory.[20] By way of contrast, Newton had an elegant way of handling the complexity of actual motions. He took the care to prove theorems that could underwrite "robust inferences," that is, inferences whose conclusions (usually claims regarding forces) hold approximately if their antecedents (usually observational claims) hold approximately (Smith 2002b). For example, Newton's use of the precession theorem in the first step of the AUG makes it possible to infer properties of the gravitational force from actual motions even if they only *approximately* satisfy a simple mathematical description, such as Kepler's laws. An initial theoretical description is not blocked by the complexity of actual motions. The argumentative structure of the *Principia* further illustrates that Newton approached the complexity of actual motions piecemeal, building up to increasingly complicated descriptions in what Cohen called the "Newtonian Style" (Cohen 1980). As we shall see below, this style is also crucial for establishing the epistemological warrant of Newtonian mechanics.

Second, the Newtonian laws of motion by themselves do not entail specific predictions about directly observed motions – falling bodies, for example – but must be supplemented with assumptions regarding the forces at play. The laws

19 See Harper (1990, 2002b), Harper and Smith (1995), Smith (2001b, 2002a,b), Stein (1991, ms), and Harper and Smith in this volume.

20 Many mechanical philosophers expected that such effects could not be incorporated into theories of motion; Galileo, for example, doubted whether air resistance could ever be handled theoretically, but defended mathematical idealization nevetheless, see McMullin (1985).

like modern treatment of idealization.

are thus not *directly* "deduced from phenomena" on the basis of successful predictions. This claim apparently runs counter to Newton's defense of the laws in the scholium following the Laws and Corollaries (Newton 1999, pp. 424–430). There, Newton discussed phenomena (such as the ballistic pendulum) which could plausibly be taken as the basis for a "deduction" of each of the laws. For example, Newton defended the third law as a natural extension of the static treatment of forces to cases wherein the mutually balanced forces apply to different bodies. Although the successful treatment of these phenomena provides evidence in favor of the laws of motion, this is not a case of simple predictive success. For each, further assumptions regarding the forces at play are required. The scholium persuasively establishes that a variety of phenomena are compatible with the laws of motion when motion is characterized dynamically using Newton's definitions, but it does not uniquely entail the laws. The challenge in giving an account of the status of the laws – and the status of Newtonian mechanics more generally – is to clarify the sense in which *indirect* empirical support accrues to the laws, and thus the dynamical conception of matter, that is nonetheless stronger than that offered by mere predictive success of hypothetical models.

Harper's and Smith's reconstructions of the status of Newton's laws help here. Harper holds that empirical success is judged according to whether observed motions provide multiple agreeing measurements of theoretical parameters used to describe them (Harper 1990). This approach shifts the focus from the predictive success of a single model to the stability of parameter values across a set of theoretical descriptions. Smith emphasizes the importance of approaching actual motions by a series of approximations (Smith 2002b). An initial inference establishes the *approximate* validity of the gravitational force law as applied to actual motions, but one can further calculate trajectories on the assumption that the gravitational force holds *exactly* in a precisely specified situation – such as two point-masses interacting solely via the gravitational force. Discrepancies between this initial theoretical account and the actual motions may indicate that some idealizing assumptions are flawed, and the next step is to drop these assumptions and provide a more elaborate theoretical description. On Smith's account, the laws of motion accrue empirical support with each stage in a series of approximations when discrepancies between actual motions and a particular stage can be explained by relaxing idealizing assumptions in a way that is self-consistent and that identifies further physical details of the system.

This brief sketch is sufficient to contrast the methods of inquiry associated with the dynamical and geometrical conceptions of matter. Clearly, both Harper's and Smith's accounts of Newton's method depend crucially on the exactness provided by the mathematical framework of Book I. But Newton claimed to have established "from the phenomena" not only the laws of motion and gravity, but also the impenetrability and extension of bodies (see Sections 5.2.2–5.2.3). Can similar accounts be given for these claims?

Certainly, Newton does *not* introduce parameters characterizing impenetrability and show how various phenomena give agreeing measurements of them. Nor does he give controlled idealizations that can be utilized as first approximations in order to derive the properties of impenetrability and extension from observed motions, and then proceed to develop successively more detailed approximations. The evidential warrant for such inferences "from the phenomena" relies on a less sophisticated chain of reasoning than does the warrant provided for the laws of motion. We return to this issue shortly, when we consider a case in which a deduction from the phenomena that uses a sophisticated mathematical framework is pitted against one that does not.

Before doing so, however, we must note a feature that is *shared* by both types of inferences "from the phenomena." Newton also warned Cotes against overstating the certainty of any *a posteriori* deductions:

> Experimental philosophy proceeds only upon Phenomena & deduces general Propositions from them only by induction. And such is the proof of mutual attraction. And the arguments for ye impenetrability, mobility & force of all bodies & for the laws of motion are no better. And he that in experimental Philosophy would except against any of these must draw his objection from some experiment or phænomena & not from a mere Hypothesis, if the Induction be of any force.
>
> (Newton 1959–1977, V, p. 400)

Newton acknowledged that the laws of motion were "hypothetical" in the sense of being open to revision, but limited in how they may be revised. In modern terminology, "provisional" or "corrigible" are more apt for capturing Newton's meaning. For Newton, the laws of motion are not hypothetical due to the threat of underdetermination and alternative models. Rather, they are hypothetical – provisional, corrigible – because in establishing them one must generalize from a limited set of phenomena, and this necessarily inductive step may be overturned by new evidence. In an unsent draft of the letter above, Newton elaborated:

> One may suppose that God can create a penetrable body & so reject the impenetrability of matter. But to admitt of such Hypotheses in opposition to rational Propositions founded upon Phænomena by Induction is to destroy all arguments taken from Phænomena by Induction & all Principles founded upon such arguments. And therefore as I regard not Hypotheses in explaining the Phenomena of nature so I regard them not in opposition to arguments founded upon Phænomena by Induction or to Principles setled upon such arguments . . . This Argument holds good by the third Rule of philosophizing. And if we break that Rule, we cannot

> affirm any one general law of nature: we cannot so much as affirm that all matter is impenetrable.
>
> (Newton 1959–1977, V, p. 398)

By the time of this exchange, the earlier portions of Book III had already been printed and new material could not be added. In *E3*, however, Newton added Rule IV, a claim much to the same effect but now no longer treated as a consequence of Rule III (Newton 1999, p. 796). Rule IV clarifies that the uncertainty Newton associated with deductions from the phenomena was quite different than that associated with mechanical models. Taking the results of such a deduction to apply without exception introduced uncertainty, but merely the uncertainty of any inductive generalization. Newton further acknowledged the possibility that the results of a deduction may only be approximations to further, more exact theoretical descriptions. But in both cases, Newton held that the way to handle the associated uncertainty was to continue to compare observations and their theoretical descriptions, with the hope of turning up contrary phenomena indicating error. Pursuing "hypotheses" in the sense of the mechanical philosophy had no part in this effort.

5.3.3 *Gravity as an essential property*

How did Cotes respond to Newton's elaboration of his method? Cotes was tempted to bite the bullet and assert that the matter of the central body actively produces the gravitational force felt by the orbiting body, that it is the physical seat of the force of gravitation.[21] The third law applies in this instance because the central body, rather than some intermediary, is *directly* responsible for the force felt by the orbiting body. However, this suggests an intimate connection between matter and gravitation, and so a question arises about how to characterize this connection. In writing the preface to *E2* Cotes initially called gravitation an essential property of matter – a property "without which no others belonging to the same substance can exist" (Newton 1959–1977, V, pp. 412–413) – but was reprimanded by Clarke. In response, Cotes substituted "primary" for "essential," but still treated gravitation as on par with impenetrability, extension, and mobility; it has, he wrote, "as fair a claim to that title" as the other properties.[22]

21 There are two senses in which gravitation can be ascribed to matter (McMullin 1978, pp. 59–61). First, gravity causes deviations from inertial motion in accordance with the second law, and matter plays a *passive* role by responding to the impressed force (gravity). But a body must also produce the impressed force felt by other bodies, and this second, *active* sense is more problematic for Newton. For the third law to apply to an attractive force between two bodies, without any mediation, each body must respond to and also produce the force.

22 See also Newton (1999, p. 392).

Cotes did not elaborate, but he might have defended himself as follows. Inertia is taken to be essential to material bodies because the laws of motion – the laws detailing the relations between inertia, impressed force, and motion in bodies – require it. To be a body subject to the laws of motion is necessarily to be a body with inertial properties. Likewise, gravity has a "fair claim" to the title of an essential property because the understanding of attractive forces at work in the *Principia* requires it. The *Principia* demonstrates that all bodies attract one another according to a single force law, and so, taking this force law as his guide in determining the essential properties of matter, and having no indication that this force law could be explained by some deeper mechanism, Cotes is ready to claim that gravity is essential to material bodies. For Cotes, physical theory itself is the guide to determining essential properties. As promised in Section 5.2.3, we can now also see why the list of qualities generalized by Rule III of the *Regulae Philosophandi* is heterogeneous. The force of inertia, for example, is essential for the Newtonian theoretical description of actual motions. But extension, hardness, and impenetrability are not.[23] Moreover, gravity and inertia are established by the complex method outlined in the previous section, but extension, hardness, and impenetrability are not. As we shall see in Section 5.4, Cotes recognized that the qualities treated by Rule III are not on an equal footing and thus that not all "deductions from phenomena" generalized by Rule III are equally meritorious.

For Newton, however, responding to the objection by taking gravity as an essential property was a misstep. The physical characterization of gravity as a real rather than merely apparent force requires at least that it is a mutual interaction satisfying the third law. This is an important constraint on the nature of the force and it runs deeper than might be expected, but Newton does not follow Cotes in taking this to have direct implications for the ultimate cause of gravitation or the essential properties of matter. Newton's original reprimand – that Cotes applied the term "Hypothesis" too broadly – is instructive. For Newton, the application of the third law to the orbiting and central body is a crucial step in moving from a mathematical characterization of a force, as a well-defined quantity inferred from observed motions, to a characterization of the physical causes, species, and proportions of real forces. But taking this step does not require determining the cause of gravity or the relation of gravity to the essential properties of matter. The application of the third law has a "hypothetical" or provisional character, in the limited sense in which the overall framework of the laws of motion is "hypothetical." However, this sense is not analogous to the hypothetical character of mechanical models. The true nature of the gravitational force – i.e., whether or not it acts *immediately* as a force of interaction between the orbiting and central bodies – is a separate question,

23 Mobility has a curious status as an object of the intension/remission criterion, so we leave it aside here.

not directly related to the status of laws of motion, and Newton reserved judgment regarding it.[24] To speculate, as Cotes did, that the application of the third law is inconsistent with the true, yet unknown, cause of gravitation is to repeat a common mistake of the mechanical philosophers, namely to judge an experimentally established proposition on the basis of its compatibility with claims regarding the fundamental nature of bodies. Given his skepticism regarding such claims, Newton rejected the need for such a compatibility check, and this was one of the most distinctive aspects of his method.

In sum, in our opinion Newton's answers to Cotes only *seem* to fail to recognize the question of whether gravity is mutual *per se* because Newton *purposely* rejected any discussion of what gravity is, *per se*. Newton's reference to the conspiring nature of both orbiting and central bodies should not be taken to mean that gravitational attraction resides essentially in either. Had Newton explicated his own methodological tenets with enough clarity, he could have made it clearer to Cotes that he chose to remain agnostic about the implications of his own theory regarding the essential natures of bodies. However, his lack of explicitness on this occasion, the fact that he often entertained deeper explanations (albeit with sufficient caveats), *and* the fact that he was the sole natural philosopher endorsing this approach, all contributed to Cotes's confusion and willingness to consider such implications. The same pattern of misunderstanding recurs in Cotes's query about the proportionalities that hold between weight, inertia, and quantity of matter. There, however, Cotes shows Newton to be mistaken about the claims warranted by his own method.

5.4 Proportionalities

In III.6, Newton demonstrated that:

> All bodies gravitate toward each of the planets, and at a given distance from the center of any one planet the weight of any body whatever toward that planet is proportional to the quantity of matter which the body contains.
>
> (Newton 1999, p. 806)

The weight of a body does *not* depend on properties such as form or texture. This distinguishes gravity from forces such as magnetism, and also sets Newton's view apart from several contemporary accounts that left open the possibility that gravity could depend upon a wide variety of a body's properties.[25] In the text of the proposition, Newton described a pendulum experiment meant to establish that near the surface of the Earth the weight of a body is proportional

24 Newton famously denied that his characterization of gravitational force implied that brute matter could act directly at a distance; see, for example, the oft-quoted letter to Bentley (Newton 1959–1977, III, pp. 240–44).

25 See Westfall (1967, pp. 246–251), Koyré (1965, pp. 173ff., 185ff.).

to its quantity of matter, and further that the weight of Jupiter's moons is proportional to their quantities of matter.[26] The experiment was first mentioned in two manuscripts which follow the initial *De Motu* drafts.

Newton constructed two equal-length pendulums with wooden boxes as bobs and filled the wooden boxes with equal weights of gold, silver, lead, glass, sand, common salt, wood, water, and wheat. For each pair of materials, he measured the periods of oscillation. According to II.24, the mass of a pendulum bob is proportional to the product of its weight and the square of its period, $m \propto wp^2$. This proposition is based on two basic assumptions. First, $f_m \propto \frac{m\Delta v}{\Delta t}$, where f_m is the motive force, v is the velocity and t the time – a restatement of definition 8 (motive force). Second, $f_m \propto w$, motive force is proportional to the weight of the pendulum bob.[27] For a simple pendulum near the Earth's surface, the period depends upon both the length of the pendulum and the acceleration due to gravity. Since Newton used pendulums of equal length, the pendulums would only have different periods if the gravitational acceleration varied for different materials. Newton reported that the periods of two pendulums containing different materials were in fact the same, to within an accuracy of 1/1000, and so concluded that $m \propto f_m \propto w$ for all materials tested. Citing Rule III, he then generalized to "all bodies universally," even those composed of materials not tested in the experiment (Newton 1999, p. 809).

In corollary 3 of *E1*, Newton highlighted an important implication of this proportionality for matter theory; namely, that a vacuum exists:

> And thus a vacuum is necessary. For if all spaces were full, the specific gravity of the fluid with which the region of the air would be filled, because of the extreme density of its matter, would not be less than the specific gravity of quicksilver or gold or of any other body with the greatest density, and therefore neither gold nor any other body could descend in air. For bodies do not ever descend in fluids unless they have a greater specific gravity.
>
> (Newton 1999, p. 810)

Cotes objected that this argument implicitly assumes that completely filled regions of space possess identical specific gravities, which can be the case if and only if those regions contain identical quantities of matter. He illustrated the objection with a thought-experiment:[28]

26 See Harper's contribution to this volume.

27 As Newton notes in II.24.c5, the result also holds with "relative" (or buoyant) weight of the pendulum bob in place of w, because for a body immersed in a medium the motive force is the relative weight.

28 Cotes's Cambridge contemporary Robert Greene lodged essentially the same objection in Chapter VI of Greene (1712), albeit not nearly as perspicaciously as Cotes.

> Let us suppose two globes A & B of equal magnitudes to be perfectly fill'd with matter without any interstices of void Space; I would ask the question whether it be impossible that God should give different *vires inertia* to these Globes. I think it cannot be said that they must necessarily have the same or an equal *Vis Inertia*. Now You do all along in Your Philosophy, & I think very rightly, estimate the quantity of matter by the *Vis Inertia* & particularly in this VIth Proposition in which no more is strictly proved than that the Gravitys of all Bodys are proportionable to their *Vires Inertia*. Tis possible then, that ye equal spaces possess'd by ye Globes A & B may be both perfectly fill'd with matter, so no void interstices remain, & *yet* that the quantity of matter in each space shall not be the same. Therefore when You define or assume the quantity of Matter to be proportionable to its *Vis Inertia*, You must not at the same time define or assume it to be proportionable to ye space which it may perfectly fill without any void interstices; unless you hold it impossible for the 2 Globes A & B to have different *Vires Inertia*. Now in the 3rd Corollary I think You do in effect assume both these things at once.
>
> (Newton 1959–1977, V, p. 228)

Cotes emphasized that contrary to Newton's assumption in the third corollary, the two ways of quantifying matter – based on response to impressed force (*vis inertiae)* and volume filled – need not agree. If they do not, one can account for differences in specific gravity without postulating a vacuum. The implications for Newton's anti-Cartesian, anti-plenum arguments are clear.[29]

But Cotes's objection also has broader implications, implications that tie together our treatment of the geometrical and dynamical conceptions of matter. Cotes's objection shows that he recognized the possibility of measuring "quantity of matter" in the two distinct, but possibly conflicting, ways. If both the dynamical and geometrical measures are correct, i.e. if both *vis insita* and extension are proportional to quantity of matter, it should follow that both are proportional to one another. However, a proportionality between the dynamical and geometrical measures can be justified neither a priori nor empirically. First, nothing in the concepts of spatial impenetrability or force of inertia necessitates a determinate proportionality between them. Second, although the pendulum experiments are intended to prove that gravitation depends upon the quantity of matter, as Cotes indicated to Newton, "no more is strictly proved [in them] than that the Gravities of all Bodys are proportionable to the *Vires Inertiae*." Whether the gravities of bodies are further proportional to their *quantities of*

29 As with the invisible hand objection, Kant also criticized Newton on precisely this point. See Proposition XII of the *Physical Monadology* (Kant 1992, p. 64). A passage from the *Critique* (Kant 1998: (A173/B215–A174/B216)) more closely parallels Cotes's argument (we thank Kent Baldner for bringing it to our attention).

matter depends on how one defines "quantity of matter."[30] If one defines it to be proportional to the inertia of a body, then the experiments support the desired conclusion. But if one defines it to be proportional to the extension a body impenetrably fills, they do not. Cotes's objection reveals, although he does not say so directly, that the choice of an appropriate definition is crucial for the AUG: Assume that quantity of matter, defined geometrically, can vary in relation to *vis inertia*, as in Cotes's two globes. We can replace *vis inertia* with weight in the conclusion to III.6, since the pendulum experiments show that they are proportional at a given distance; thus, quantity of matter geometrically defined is not proportional to weight at a given distance. That is, if quantity of matter is defined to be proportional to quantity of extension, even at a given distance, *the quantity of matter of a body is not proportional to its weight.* Cotes's objection undermines not just the III.6.c3, but III.6 itself, and thus the AUG. If Newton wants to maintain that quantity of matter can be defined by *either* quantity of extension or quantity of inertia, he must assume that the two are determinately proportional, a claim for which he can offer no justification. This was Cotes's point.

Newton attempted to rebut Cotes by claiming that matter has inertial properties proportional to its quantity and geometrical properties due to its impenetrability, and that these two entail a fixed proportionality of inertia to extension. Yet this missed Cotes's point. The point was that these two facts, which Cotes did not dispute, do *not* entail the proportionality of inertia to extension.[31] Newton's second response to Cotes (after Cotes reiterated his reasoning) illustrates his misunderstanding and his continued commitment to *both* the dynamical and geometrical conceptions of matter and their a posteriori character. He wrote:

> I have reconsidered the third Corollary of the VIth Proposition. And for preventing the cavils of those who are ready to put two or more sorts of matter you may add these word[s] to the end of the Corollary: [1] From pendulum experiments it is established that the force of inertia is proportional to the gravity of a body. [2] The force of inertia arises from the quantity of matter in a body and so is proportional to its massiness [*massa*]. [3] A body is condensed by the contraction of the pores in it, and when it has no more pores (because of the impenetrability of matter) it can be condensed no more; and so in [completely] full spaces [the force

30 One might object that Newton and Cotes are conflating inertial and gravitational mass, see Densmore (1996, pp. 313–330). The problem is distinct from the objection under consideration.

31 Cotes's position shifted slightly during this exchange: whereas initially he objected to the implicit assumption of the proportionality of inertia to quantity of extension ("You *must not* at the same time define . . ."), he later allowed that the proportionality could be invoked as an unproved assumption. In either case, his objection is that Newton's explicit commitments do not entail that the proportionality holds.

> of inertia] is as the size of the space. Granted these three principles the corollary is valid.
>
> (Newton 1959–1977, V, p. 240)

Since Newton and Cotes explicitly agreed on [1], the source of their disagreement lies in [2] or [3]. In [2], Newton implicitly defined quantity of matter to be proportional to the force of inertia. Since Cotes had already written to Newton that "all along in your Philosophy, & I think very rightly, you estimate the quantity of matter by the *Vis Inertiae*," the source of conflict must be [3]. In [3], Newton deduced from [2] and the impenetrability of matter that the inertia of matter is proportional to the extension it solidly fills. Clearly, Newton took this to be a valid inference. According to Cotes, however, Newton's reasoning is circular: he implicitly assumed that the force of inertia is determinately proportional to the extension solidly filled by matter in order to deduce that, after condensation, the force of inertia would be determinately proportional to the extension filled by matter. Cotes wrote in his subsequent response:

> I am not yet satisfied as to the difficulty unless You will be pleased to add, That it is true upon this concession, that the Primigenial particles . . . have all the same *Vis Inertiae* in respect to their magnitude or extension in *Spatio pleno*. I call this a concession because I cannot see how it may be certainly proved either a Priori by bare abstracted reasoning; or be inferr'd from Experiments.
>
> (Newton 1959–1977, V, p. 242)

Cotes took Newton to be putting a uniformity constraint on the fundamental, "Primigenial" particles of matter, particles that are inaccessible to direct experimental investigation. The uniformity constraint is the claim that all fundamental particles have identical specific gravities; or, equally, that their quantity of matter is uniformly proportional to their extension. Newton had appealed to the uniformity constraint from his student days: in the *Certain Philosophical Questions*, in his draft and final revisions to Hypothesis III of *E1*, and in his considered arguments against the vacuum. It is a fixture of Newton's thought that had gone unchallenged until this exchange with Cotes, although Newton appears to have justified the constraint by subtly different means at different points in his career (McGuire 1970). At the beginning of this exchange with Cotes, Newton believed that the constraint could be justified a posteriori. His initial responses demonstrate that, by his own lights, the uniformity of primigenial particles followed from observable facts regarding the extension, impenetrability, and inertia of matter. Cotes's objection pointed to the conflict between the geometrical and dynamical measures of matter, both of which, according to Newton, were derived a posteriori.

However, the geometrical definition (and the conception of matter underlying it) is derived by a different sort of a posteriori argument than the dynamical definition (and the conception of matter underlying it). The geometrical definition is derived from the claim that, as Newton puts it in Rule III and as he articulated more elaborately in *DG*, "extension is found in all sensible bodies." This derivation is in some sense immediate – it rests on no sophisticated mathematical chain of reasoning, no process of approximation, and no fixing of causal parameters. Within Newton's broadly mechanical account of perception, it simply follows from our experience of any body whatsoever. The dynamical definition is a crucial part of Newton's account of force, developed and used to account for a variety of motions in the *Principia.*

Ultimately, Newton backed down. In III.6.c4 of *E2*, he rephrased the anti-plenum argument in the form of a conditional, acknowledging the assumption Cotes insisted on:

> If all the solid particles of all bodies have the same density and cannot be rarefied without pores, there must be a vacuum. I say particles have the same density when their respective forces of inertia [or masses] are as their sizes.

If the fundamental particles have a fixed ratio between inertia and volume, *then* a vacuum must be granted. Yet the interchange with Cotes shows that Newton's initial inclination was to positively maintain that all primigenial particles are uniformly extended in proportion to their quantities of matter, despite the fact that his pendulum experiments and the mathematical structure used to interpret them recommended no such steadfastness. In fact, the dynamical conception supported by the results of the *Principia* is compatible with treating matter as constituted by Boscovichian point-particles, with the quantity of matter appearing solely as a parameter of these points. The geometrical properties of matter play no role in physical explanations in this schema, since such explanations depend solely on the laws of motion and the further specification of inter-particle forces.

Newton's initial failure to see this point reflects, on our view, a failure to clearly distinguish the distinctive a posteriori methods described above. As we saw in the treatment of Rule III, Newton conceived of properties like extension and impenetrability as having the same status as inertia, despite the fact that they were supported by a distinctive line of argument that was not intertwined with the AUG or the deduction of the laws of motion from phenomena. Cotes, to his credit, was quite clear that Newtonian mechanics does not support a geometrical conception of matter. He even pointed out the precarious status of extension to Clarke:

> I understand by Essential propertys such propertys without which no others belonging to the same substance can exist: and I would not undertake to prove that it were impossible for any of the other Properties of Bodies to exist without even Extension.
>
> (Newton 1959–1977, V, pp. 412–413)

Cotes obliquely entertained the possibility that Boscovichian non-extended point-particles can constitute bodies and that our *experience* of bodies – even our experience of those qualities that seem immutable and invariably present – is no guide in questions of essentiality. For Cotes, to repeat a point made in Section 5.3.3 regarding gravity, physical theory itself is the guide to determining essential properties. It just so happens that within Newtonian mechanics inertia plays a central role in giving an account of observed motions whereas extension does not. Insofar as Cotes is concerned, so much the worse for extension.

Newton was not far behind. After Cotes's objection highlighted the incongruity between the two conceptions of matter, Newton began to doubt the geometrical conception more thoroughly. The change of mind for an astute and tenacious figure such as Newton is significant: Newton did not back down in response to the invisible hand objection because he was certain of his correctness. In response to the two globes objection, however, Newton modified his views. In a series of draft definitions intended for Book III of *E3* (dated by McGuire (1966) to 1716), Newton explicitly addressed his now-changed conception of body. He wrote:

> *Definition II* Body I call everything which can be moved and touched, in which there is resistance to tangible things, and its resistance, if it is great enough, can be perceived.
>
> (p. 115)

Lacking from this definition is any mention of the extension of bodies. The *only* definitional property of body here is its inertial resistance. A far cry indeed from *DG*'s definition:

> Definition 4. Body is that which fills space.[32]
>
> (Newton 2004, p. 13)

5.5 Conclusion

We have stressed two aspects of Newton's thought. The first is Newton's empiricist method, and the two approaches he took to justifying claims in natural philosophy. The approach exemplified by the AUG contrasts sharply with the method of the mechanical philosophers. Unlike the mechanical philosophers, Newton did not allow the satisfaction of intelligibility constraints (e.g., that only contact action is comprehensible) to serve as justification, even if partial, for a physical theory; the justification for the laws of motion and universal

32 We take the development of Newton's views on body to show that *DG* cannot be automatically taken to reflect Newton's mature metaphysical views. Rather, it is best taken as Newton's relatively early attempt to explicate the philosophical infrastructure in which his physics is embedded, but by no means the last word.

gravitation is their ability to serve successfully as a framework for describing motions. Newton's response to Cotes's invisible hand objection reflected this methodological stance. Cotes objected that Newton had inappropriately assumed that gravitational force must be produced by the orbiting and central bodies, despite his professed agnosticism regarding the underlying cause of gravity. Newton responded by clarifying that his characterization of gravity as a force obeying the three laws was hypothetical in the same limited sense that the laws of motion are hypothetical and did not entail further assumptions regarding the essential properties of matter or the underlying cause of gravity. The second approach to establishing results a posteriori is exemplified by the account of body in *DG* and some of Newton's statements in Rule III. It involves a more direct argument, essentially reading off the properties of matter from the general experience of bodies. It does not draw on a precise mathematical framework like that of the *Principia*, and so the ways of clarifying evidential warrant within the first approach apply. It is consequently unclear how to assess the strength of the conclusions derived from this type of reasoning.

Second, there is an uncomfortable union in Newton's thought between two competing conceptions of matter. The geometrical conception reflects Newton's Cartesian roots and was linked to the possibility of an aetherial explanation of gravitation. Although Newton decisively rejected several aspects of Cartesian thought in *DG*, he retained an account of bodies that took their geometrical properties to be fundamental. Consequently, he took a body's quantity of matter to be proportional to the volume it impenetrably fills. At the same time, Newton developed the distinctive dynamical conception of matter of the *Principia*, which measures quantity of matter by a body's response to impressed force. Newton apparently treated the two measures as aspects of a single, coherent account of matter. Cotes's second objection brought out the tension between these two conceptions. Cotes argued that Newton's claims could not be sustained without an explicit assumption regarding the fundamental constituents of matter, betraying Newton's professed agnosticism on such matters. Although Newton's response to Cotes reflects his failure to clearly distinguish the two approaches to a posteriori reasoning characterized above, there is evidence he took Cotes's criticism to heart and attempted to dispense with *DG*'s geometrical conception.