Neural Networks

CS114B Lab 5

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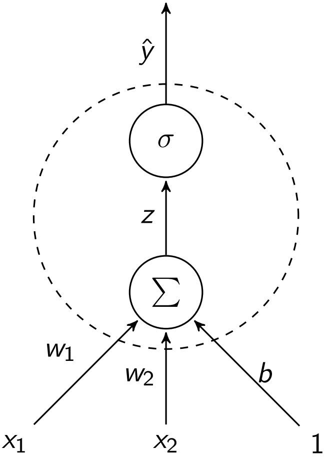
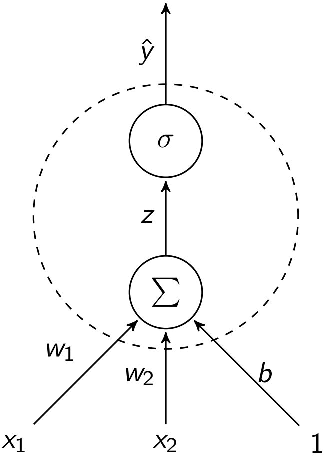
March 3, 2022

Graphical Representation of a Linear Classifier



Graphical Representation of a Neuron



Neural Networks

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- (Artificial) neurons are basically linear classifiers!
  - Neurons compute their output based on a linear combination of inputs
  - Outputs are not necessarily classification decisions ($\hat{y}$), but can be inputs to other neurons

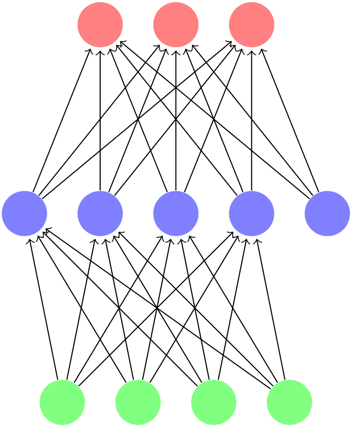
Feedforward Neural Networks

- Suppose that our neurons are grouped into a sequence of layers

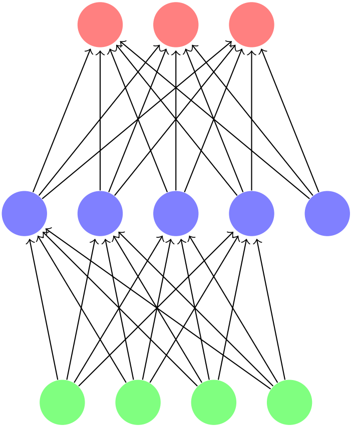
Feedforward Neural Networks

- Suppose that our neurons are grouped into a sequence of layers
- Also suppose that these layers are fully connected (every neuron in one layer is connected to every non-bias neuron in the next layer, and no others)

Feedforward Neural Networks

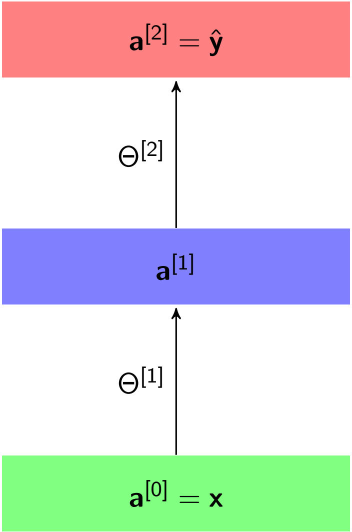


Feedforward Neural Networks



- Output layer
- Hidden layer(s)
- Input layer

Feedforward Neural Networks



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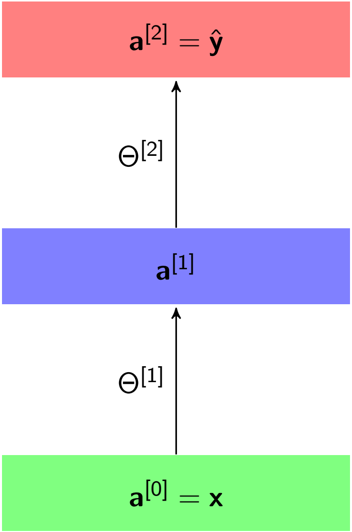
Feedforward Neural Networks



- Output layer
- Hidden layer(s)
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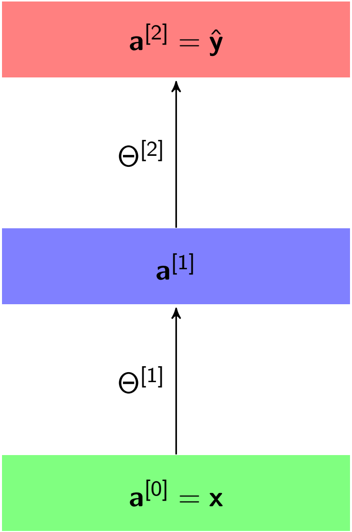
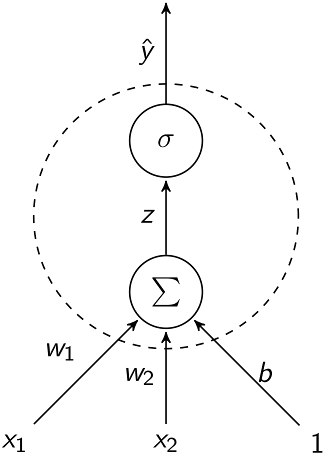
  - Outputs of the hidden layers are vectors

Feedforward Neural Networks

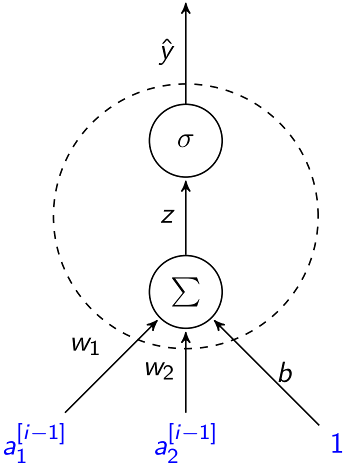


- Output layer
- Hidden layer(s)
- Input layer
  - Outputs of the hidden layers are vectors
    - In particular, they can be seen as intermediate representations of the input

Graphical Representation of a Neuron

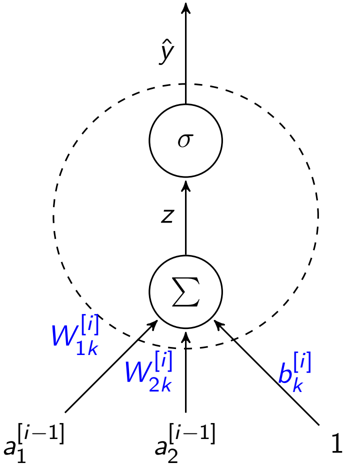


Graphical Representation of a Neuron



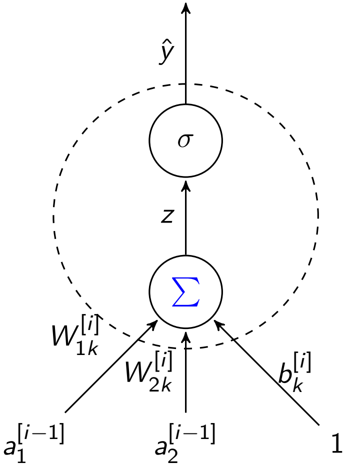
- Inputs to neuron k in layer i = outputs of neurons in layer $i-1$ (and dummy node 1)

Graphical Representation of a Neuron



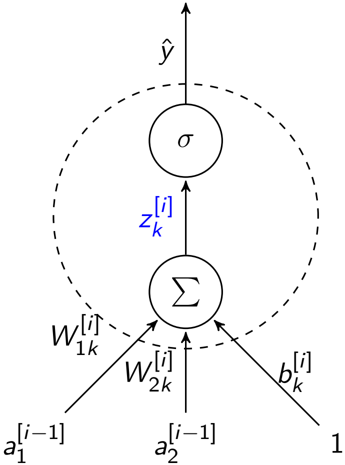
- Weights (and bias term)

Graphical Representation of a Neuron



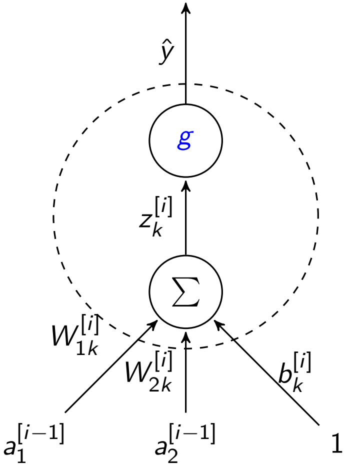
- Sum function $\sum$

Graphical Representation of a Neuron



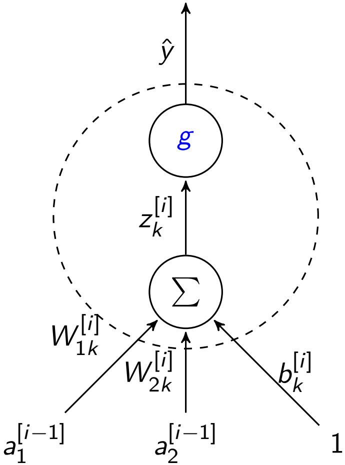
- "Score"

Graphical Representation of a Neuron



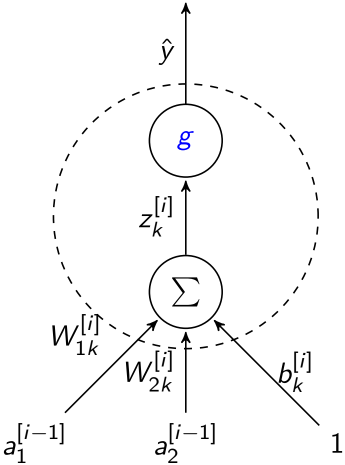
- Activation function g

Graphical Representation of a Neuron



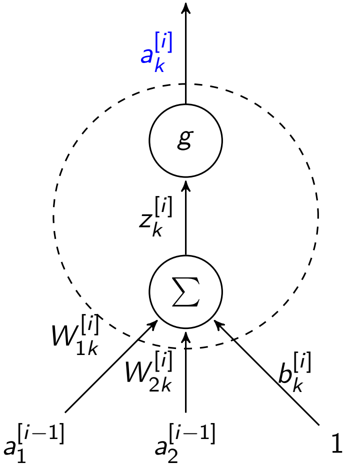
- Activation function g
  - Output neuron: logistic or softmax

Graphical Representation of a Neuron



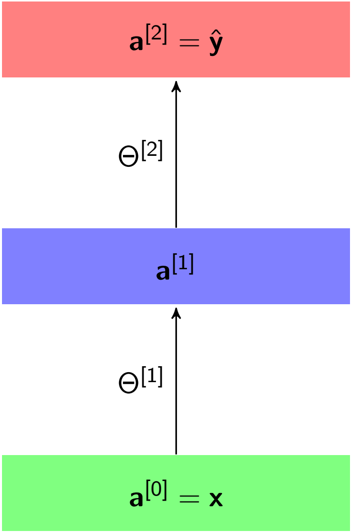
- Activation function g
  - Output neuron: logistic or softmax
  - Hidden neuron: typically logistic, tanh, ReLU, etc.

Graphical Representation of a Neuron



- Activation (output of neuron k in layer i)

Feedforward Neural Networks



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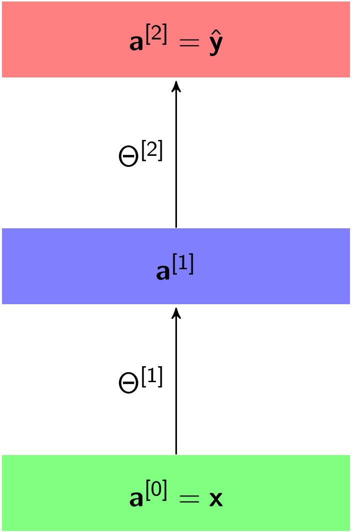

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- $\mathbf{a}^{[1]} = g(\mathbf{a}^{[0]} \cdot \Theta^{[1]})$

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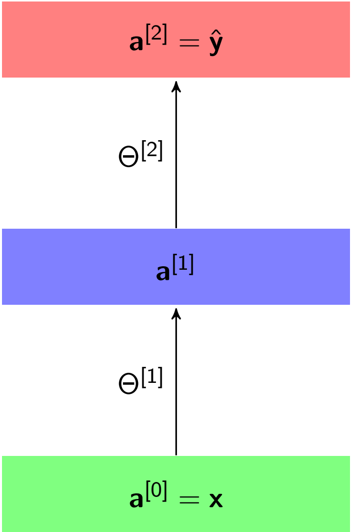

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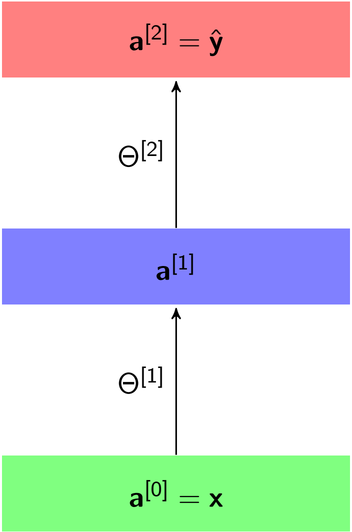

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- $\mathbf{a}^{[1]} = g(\mathbf{a}^{[0]} \cdot \Theta^{[1]})$
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- $(\nabla L)^{[2]} = \frac{1}{m}\Big((\mathbf{a}^{[1]})^T \cdot (\hat{\mathbf{y}} - \mathbf{y})\Big)$
  (for the cross-entropy loss, logistic or softmax function)

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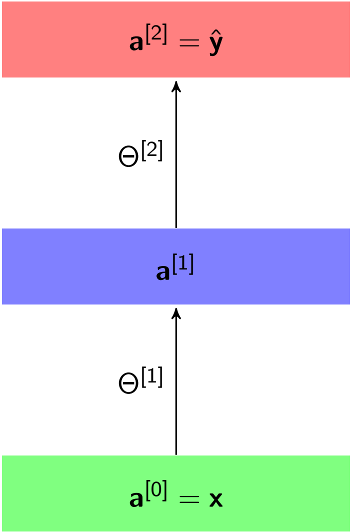

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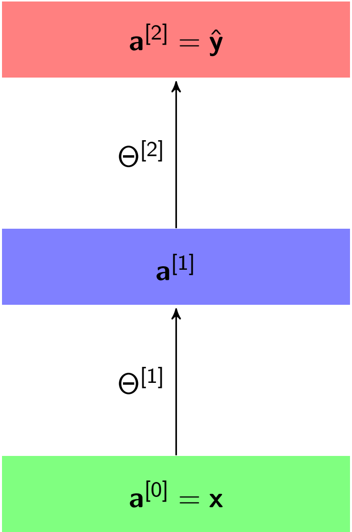

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  - Same as for logistic regression, except replace $\mathbf{x}$ with $\mathbf{a}^{[1]}$
- $(\nabla L)^{[1]} = ?$

Backpropagation

- ...
- (calculus–see supplement slides)
- ...

Backpropagation

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- (calculus–see supplement slides)
- ...
- For all layers i:
  - $(\nabla L)^{[i]} = \frac{1}{m}\left((\mathbf{a}^{[i-1]})^T \cdot \delta^{[i]}\right)$

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    - $\mathbf{W}$ does not include the bias $\mathbf{b}$

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- We can compute $\delta^{[\mathcal{L}]}$ for an output layer $\mathcal{L}$

Backpropagation

- We can compute $\delta^{[\mathcal{L}]}$ for an output layer $\mathcal{L}$
- Key idea: if we have already computed $\delta^{[i+1]}$ for some layer $i+1$, then we can use it to calculate $\delta^{[i]}$ for the previous layer i

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  - Initialize parameters $\Theta^{[i]} = \mathbf{W}^{[i]}, \mathbf{b}^{[i]}$ randomly (note: not $\mathbf{0}$)

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  - For each layer i, starting from the output and working backwards:
    - Compute gradient $(\nabla L)^{[i]}$

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  - For each layer i:
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- $\Theta^{[i]}_{t+1} = \Theta^{[i]}_{t} - \eta(\nabla L)^{[i]}$
- Because L is not necessarily convex anymore, we are not guaranteed to reach a global minimum
  - But it works well enough in practice

Further Reading

- Goodfellow, I., Bengio, Y., and Courville, A. (2016). *Deep Learning*. MIT Press.
- Nielsen, M. A. (2015). *Neural Networks and Deep Learning*. Determination Press USA.