Linear Classifiers (Part 2)

CS114B Lab 3

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February 10, 2022

Linear Classifiers

- What is g?

Linear Classifiers

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  - Function with many names
    - In the context of generalized linear models, this is called a link function
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- A dot product of two vectors produces a scalar, but in general, we don't just want an arbitrary real number
  - Sometimes, we want a probability (logistic regression)
  - Sometimes, we just want the decision itself (perceptron)

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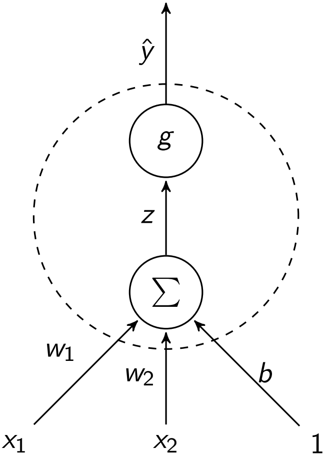
  - What if $z = 0$?
    - Set by convention (1, 0, or 1/2)

Graphical Representation of a Linear Classifier (1)



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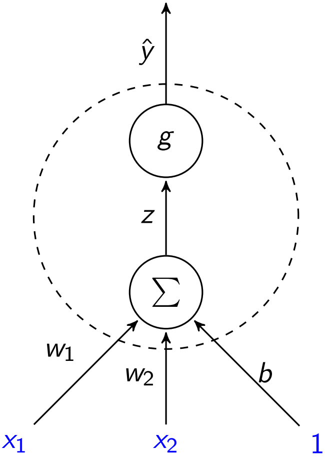

- Input (including dummy feature 1)

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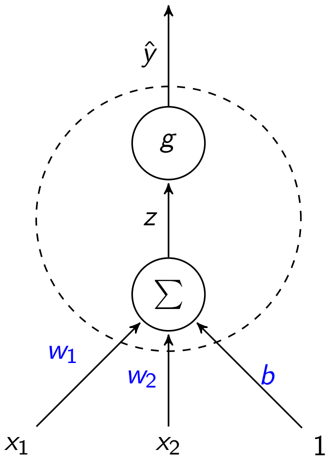

- Parameters (weights and bias term)

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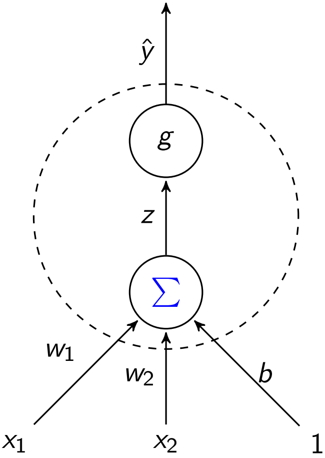

- Sum function $\sum$

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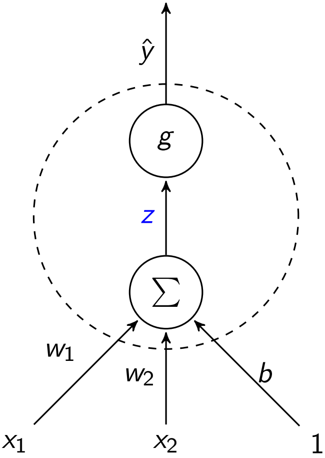

- "Score"

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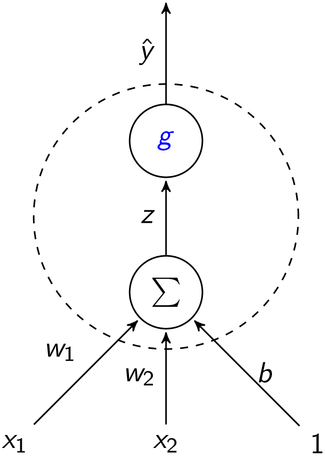

- Activation function g

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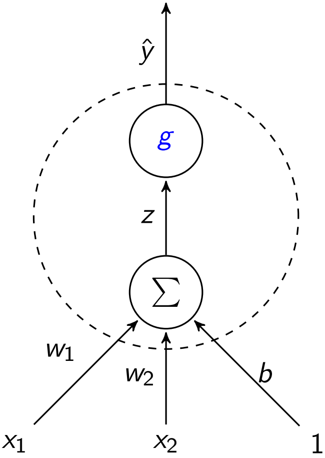

- Activation function g
  - Logistic, step, etc.

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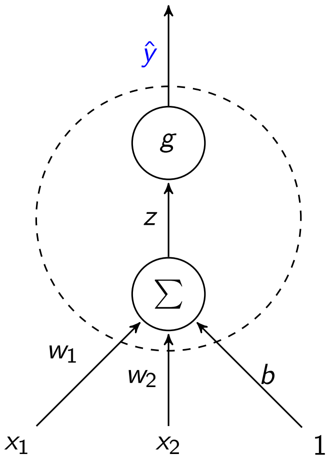

- Output

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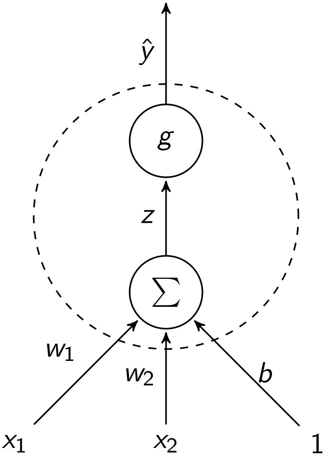

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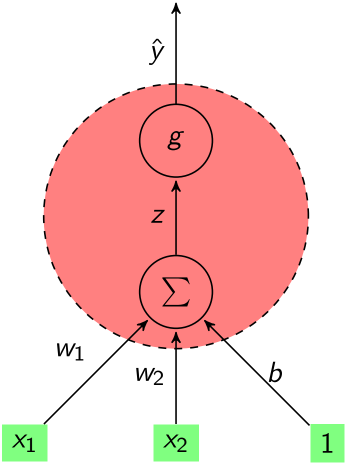

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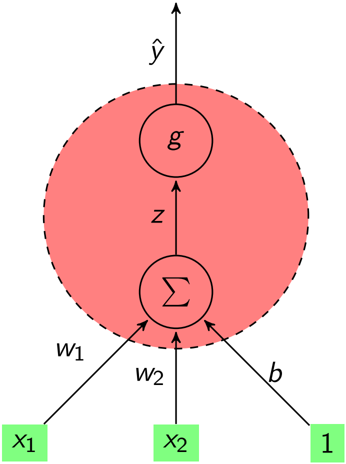

- Output
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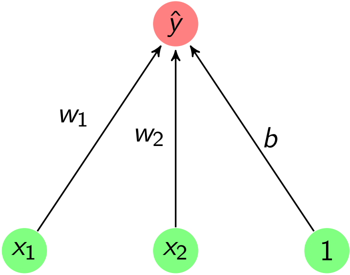
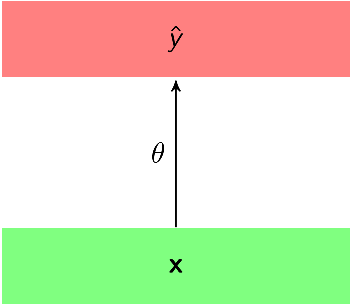


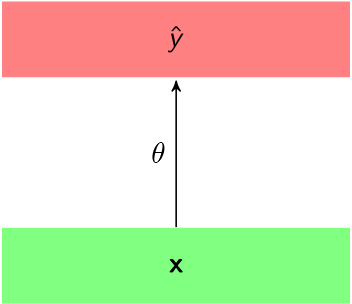
- Output
- Input

Graphical Representation of a Linear Classifier (2)



- Output
- Input

Graphical Representation of a Linear Classifier (2)



- Output
- Input
- $\hat{y} = g(\theta \cdot \mathbf{x})$
  - We will assume that the dummy feature 1 is part of $\mathbf{x}$