

# **Text classification with Naive Bayes**

**COMPSCI 485, Applications of  
Natural Language Processing**

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# Roadmap

- Machine learning and text classification
- Classification method #1: Manually-defined rules and keywords
- Classification method #2: Supervised learning
  - Naive Bayes
  - Next week: Logistic regression

# What is machine learning?

From Wikipedia:

Arthur Samuel:

gives “computers the ability to learn without being explicitly programmed”

Tom Mitchell:

“A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E.”

# Classification

- Classification: putting a label on each datapoint
  - Is this a noun, an adjective, a verb?
  - Is this a person's name, a city name, a company name, or neither?
  - Does this text convey a positive or a negative opinion?
  - Is the domain of this text politics, financial, entertainment, sports?

# Classification

- input: some data point  $x$  (e.g., sentence, document)
- output: a label  $y$  (from a finite label set)
- goal: learn a mapping function  $f$  from  $x$  to  $y$

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Many NLP problems reduce to learning  
a mapping function with various  
definitions of  $x$  and  $y$ !

problem	x	y
sentiment analysis	text from reviews (e.g., IMDB)	{positive, negative}
topic identification	documents	{sports, news, health, ...}
author identification	books	{Tolkien, Shakespeare, ...}
spam identification	emails	{spam, not spam}
... many more!		

input  $\mathbf{x}$ :

From European Union <info@eu.org>☆  
Subject  
Reply to [REDACTED] ☆

Please confirm to us that you are the owner of this very email address with your copy of identity card as proof.

YOU EMAIL ID HAS WON \$10,000,000.00 ON THE ONGOING EUROPEAN UNION COMPENSATION FOR SCAM VICTIMS. CONTACT OUR EMAIL:  
CONTACT US NOW VIA EMAIL: [REDACTED] NOW TO CLAIM YOUR COMPENSATION

label  $\mathbf{y}$ : **spam** or **not spam**

we'd like to learn a mapping  $f$  such that  
 $f(\mathbf{x}) = \mathbf{spam}$

# Demo: Keyword count classifier

- Task: sentiment classification of movie reviews
- Can this be done with *manually defined* keyword lists?
  - For each category, define set of words
  - Predict a category if many of its words are used
- Let's try manually defined keywords!
  - Sending link on Piazza

# *f* can be hand-designed rules

- if “won \$10,000,000” in **x**, then **y = spam**
- if “CS485” in **x**, the **y = not spam**

what are the drawbacks of this method?

# $f$ can be learned from data

- Given training data (already-labeled  $x,y$  pairs) learn  $f$  by maximizing the likelihood of the training data
- this is known as supervised learning

## training data:

x (email text)	y (spam or not spam)
learn how to fly in 2 minutes	spam
send me your bank info	spam
CS585 Gradescope consent poll	not spam
click here for trillions of \$\$\$	spam
<i>... ideally many more examples!</i>	

## heldout data:

x (email text)	y (spam or not spam)
CS485 important update	not spam
ancient unicorns speaking english!!!	spam

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<i>... ideally many more examples!</i>	

## heldout data:

x (email text)	y (spam or not spam)
CS485 important update	not spam
ancient unicorns speaking english!!!	spam
learn mapping function on training data, measure its accuracy on heldout data	

# Supervised learning: Training data, test data

- Classifier uses training data to learn, generalize to new example
- Test performance on new data, with respect to measure P. need **test data**:
  - ? ancient unicorns speaking English!!
  - We know the true label of the test data, but the machine does not.
  - We make the machine “predict” (guess) a label
  - We can then see how many of the machine’s predictions are correct
- **Why is it important to have held-out test data?**

# Supervised learning: Training data, test data

- Train on training data
- Test performance on test data
- Possible third dataset: **development data.**
  - train on training data, test on development, calibrate training to do better, repeat. Then, at the end, test on completely unseen test data

# Naive Bayes classifiers

# Probability review

- random variable  $X$  takes value  $x$  with probability  $p(X = x)$ ; shorthand  $p(x)$
- joint probability:  $p(X = x, Y = y)$
- conditional probability:  $p(X = x \mid Y = y)$ 
$$= \frac{p(X = x, Y = y)}{p(Y = y)}$$
- when does  $p(X = x, Y = y) = p(X = x) \cdot p(Y = y)$  ?

# bag-of-words representation

i hate the actor i love the movie

# bag-of-words representation

i hate the actor i love the movie

word	count
i	2
hate	1
love	1
the	2
movie	1
actor	1

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equivalent representation to:  
actor i i the the love movie hate

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Why is this representation convenient?  
What do we lose with this representation?

equivalent representation to:  
actor i i the the love movie hate

# A probabilistic classifier

- Collection of labels:  $C = \{c_1, \dots, c_n\}$
- Classifier predicts some  $c \in C$
- True (gold) class:  $\hat{c}$
- **Given a datapoint (document)  $d$ , we want to assign the likeliest label:** the label  $c$  that has the highest probability  $P(c | d)$ :

$$c = \operatorname{argmax}_{c' \in C} P(c' | d) \quad \text{“the } c' \text{ with the maximum } P(c'|d)”$$

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# A probabilistic classifier

- How do we compute the likeliest label for a review?  $c = \operatorname{argmax}_{c' \in C} P(c' | d)$ 
  - What factors make a label  $c$  likely?

# A probabilistic classifier

- How do we compute the likeliest label for a review?  $c = \operatorname{argmax}_{c' \in C} P(c' | d)$ 
  - What factors make a label  $c$  likely? Same as before:
    - Words that appear often in positive reviews make the label “pos” likelier
    - Words that appear often in negative reviews make the label “neg” likelier
  - But additionally, we make use of overall label frequencies:
    - Do we mostly see negative reviews? Then we want to say “neg” is more likely a priori

# A probabilistic classifier

Taking conditional probabilities apart: Bayes' rule

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

or in our case:

$$P(c | d) = \frac{P(d | c)P(c)}{P(d)}$$

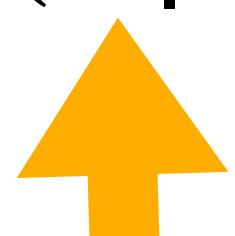
# A probabilistic classifier

Taking conditional probabilities apart: Bayes' rule

$$P(c | d) = \frac{P(d | c)P(c)}{P(d)}$$

We can drop  $P(d)$ : We want to classify review  $d$  as either “pos” or “neg”. In both  $P(\text{pos} | d)$  and  $P(\text{neg} | d)$ , the denominator  $P(d)$  is exactly the same number.  
So we get:

$$P(c | d) \sim P(d | c)P(c)$$



How likely are the words in the review to appear with this label?

# A probabilistic classifier

Taking conditional probabilities apart: Bayes' rule

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How likely, in general, is this label?

# The probability of a review given label “pos”

- We need, for example,  $P(\text{"I love this movie"} \mid \text{pos})$
- Goal: assign a probability to a sentence
  - Sentence: sequence of *tokens*  $\langle \text{"I"}, \text{"love"}, \text{"this"}, \text{"movie"} \rangle$
  - Every word  $w$  is from a set  $V$ , the vocabulary
  - How do we compute the probability of a word sequence?

# Word probabilities

- The “Bayes” part of Naive Bayes: Bayes’ rule, taking apart the probability  $P(c | d)$
- The “naive” part: The probability of any word  $w$  is independent of all other words in the document. (That’s quite naive, but it works.)  
Say  $d$  is the word sequence  $d = \langle w_1, \dots, w_n \rangle$ . Then this assumption gives us:

$$P(d | c) \approx \sum_i P(w_i | c)$$

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Independence of  $w_1, w_2$ : Then  
 $P(w_1 \text{ and } w_2 | c) = P(w_1|c) P(w_2|c)$

# Word probabilities

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# Toy sentiment example

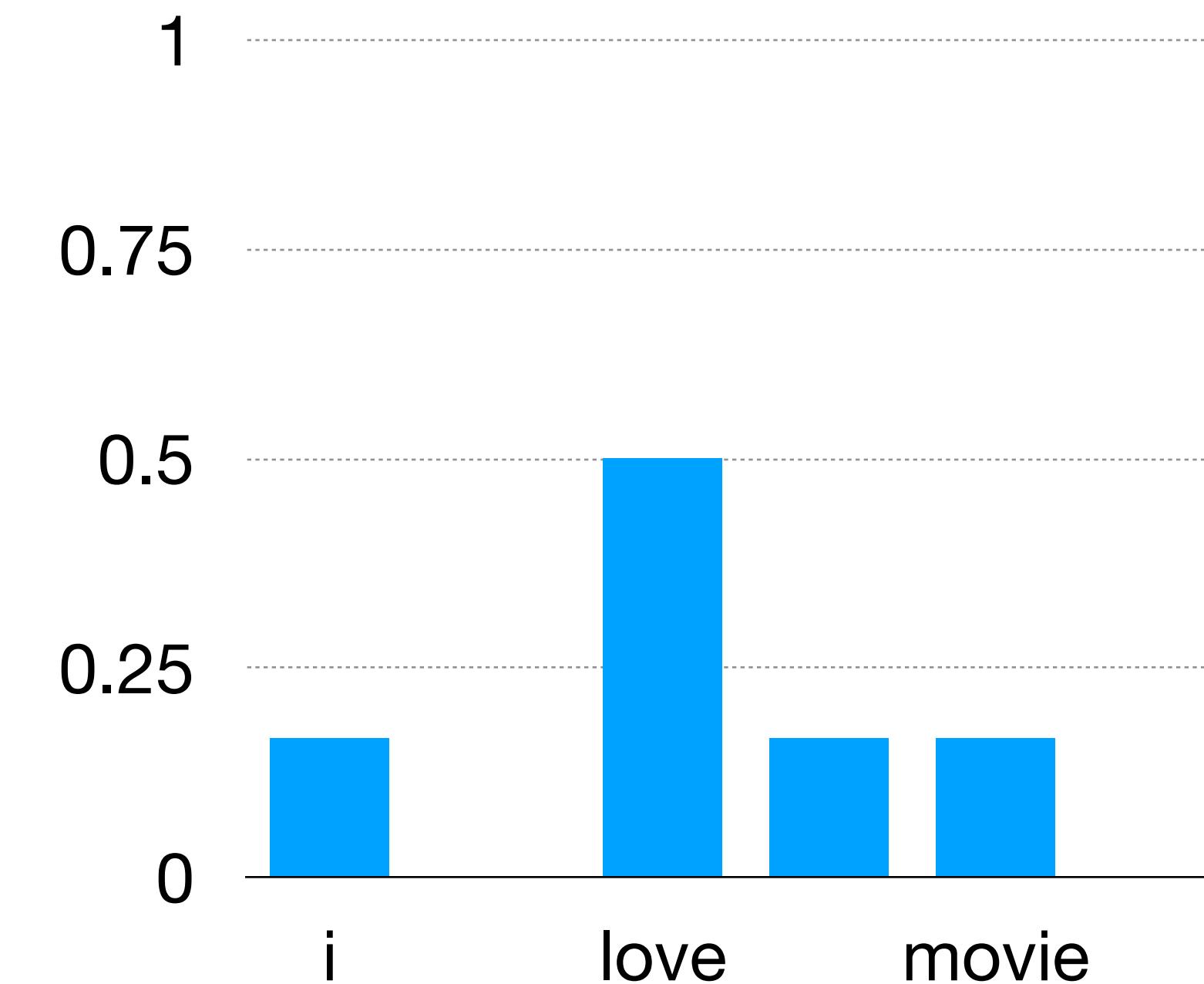
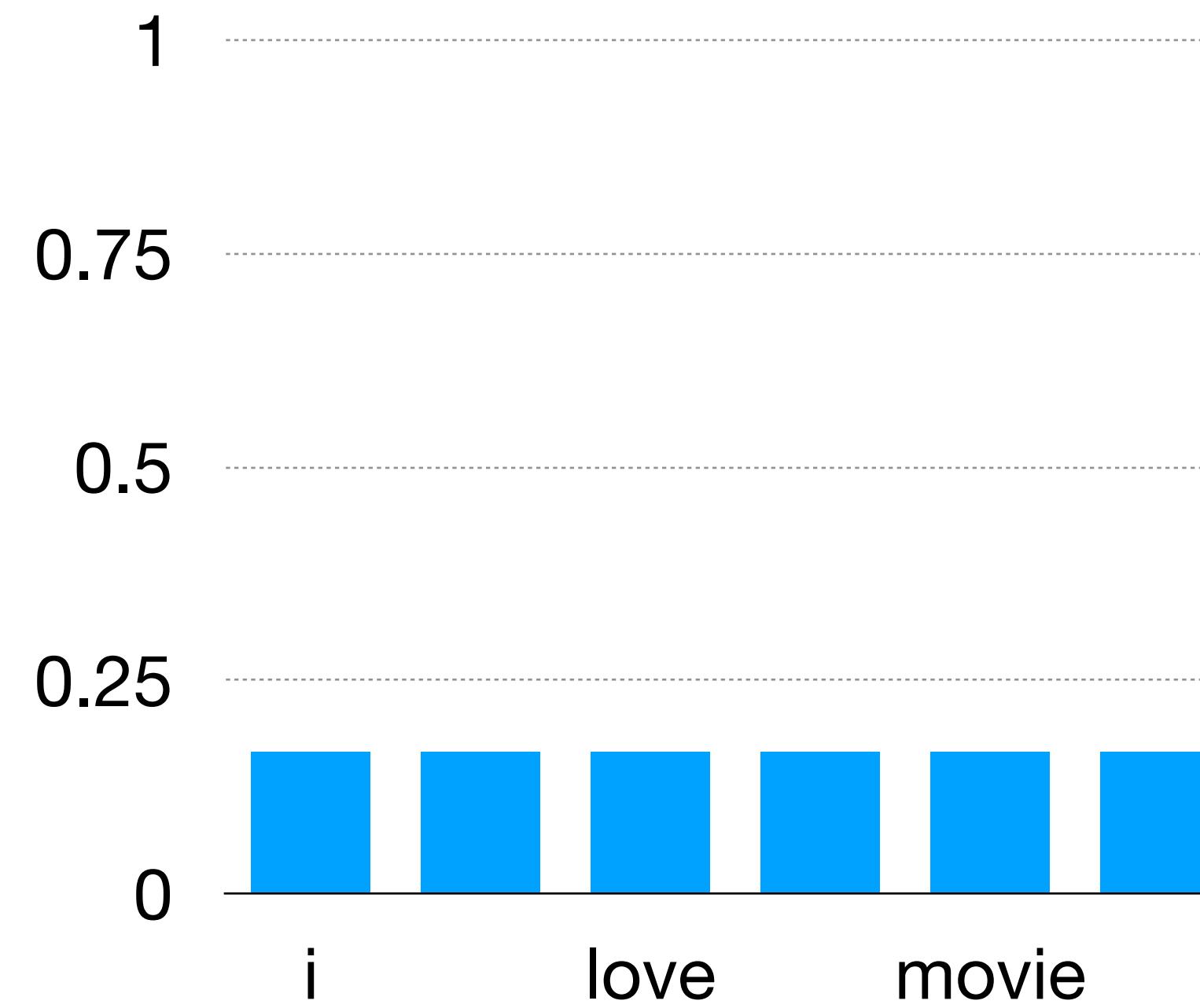
- vocabulary V: {i, hate, love, the, movie, actor}
- training data (movie reviews):
  - i hate the movie
  - i love the movie
  - i hate the actor
  - the movie i love
  - i love love love love love the movie
  - hate movie
  - i hate the actor i love the movie

labels:  
positive  
negative

# Naive Bayes, again

- Assumption: each word is independent of all other words, conditional on document label
- Given labeled data, we can use naive Bayes to estimate probabilities for unlabeled data
- Goal: infer probability distribution that generated the labeled data for each label

which of the below word distributions looks like one found in **positive reviews**?

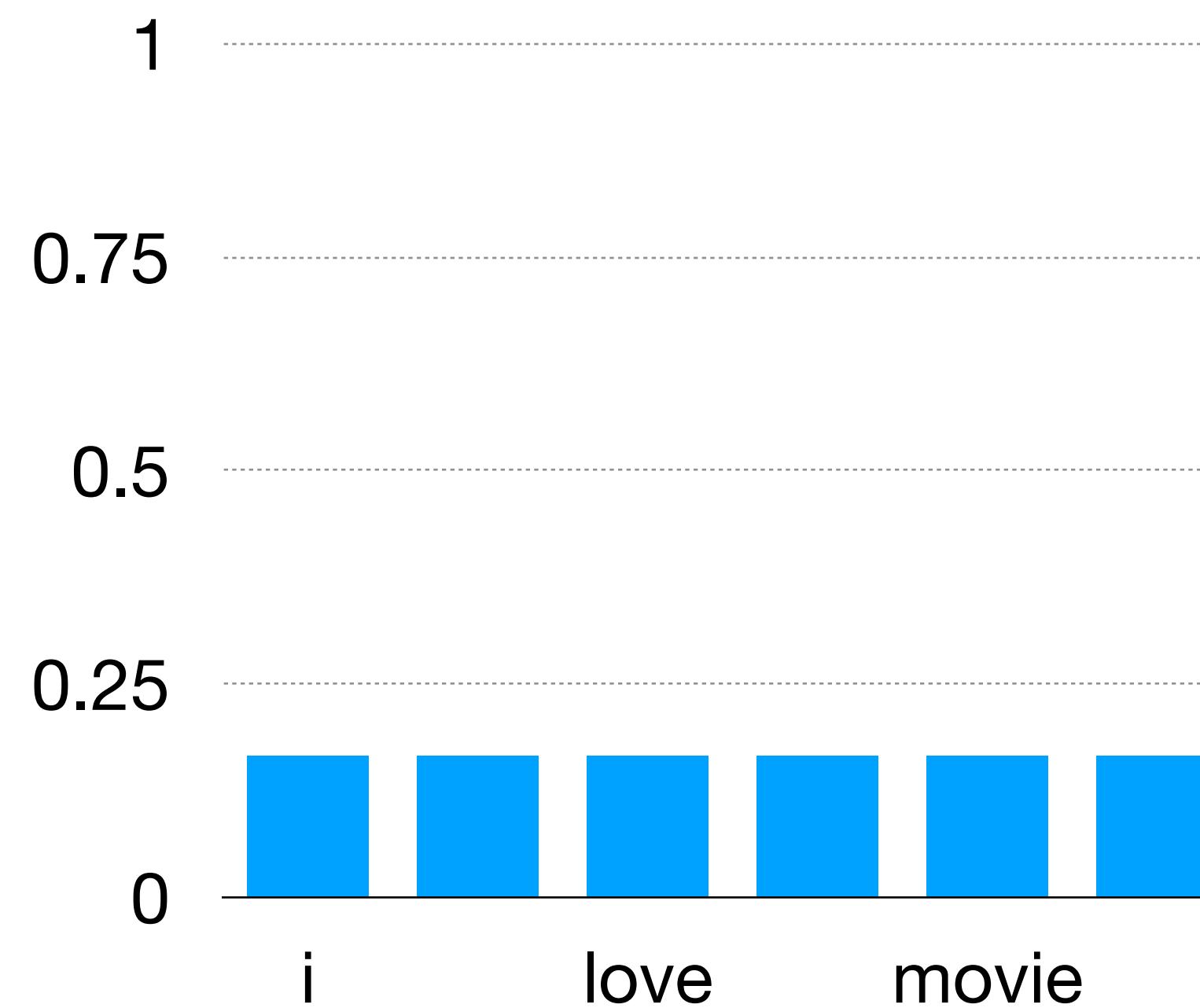


# ... back to our reviews

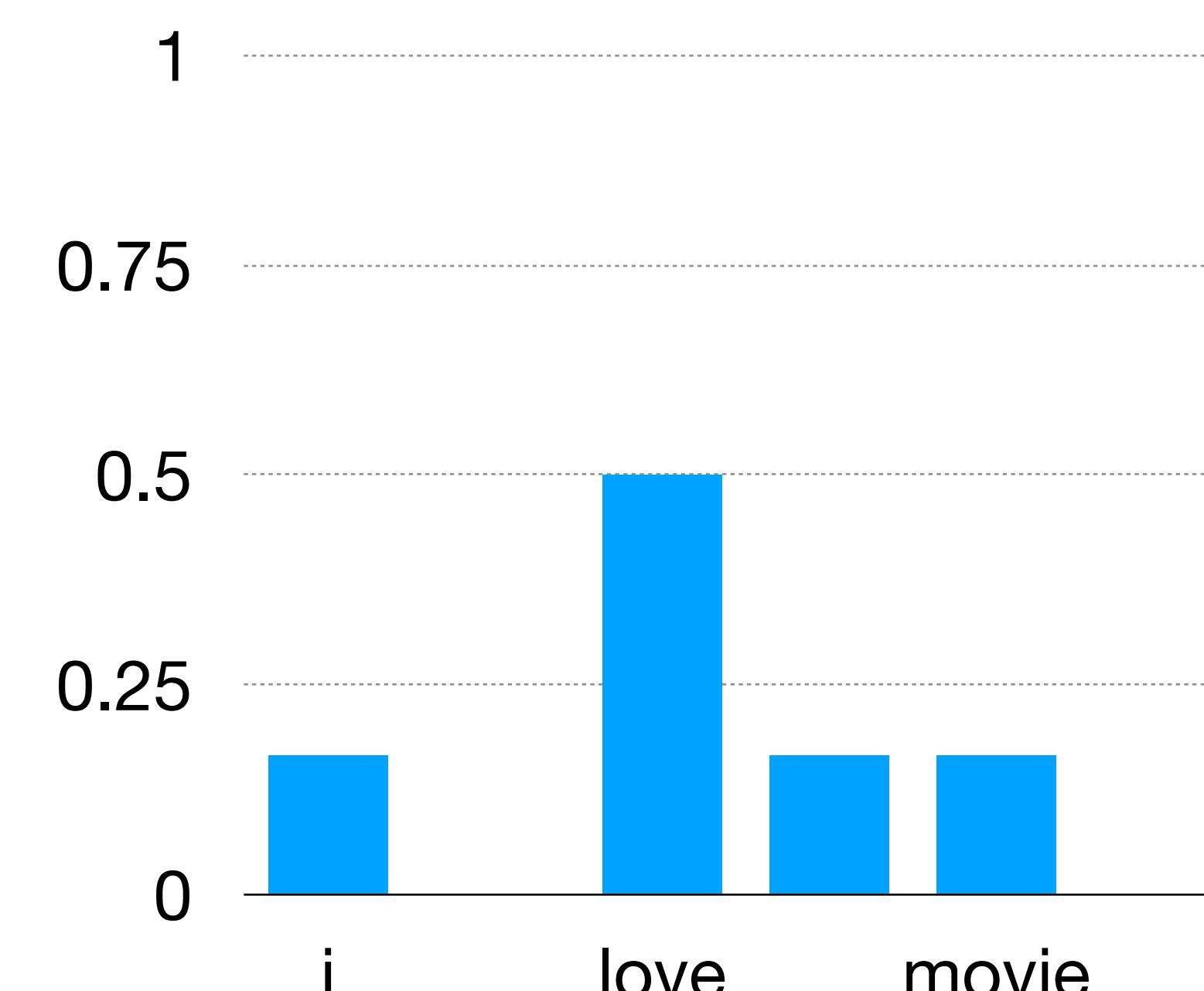
$$p(i \text{ love love love love love the movie})$$

$$= p(i) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie})$$

$$= 5.95374181\text{e-}7$$



$$= 1.4467592\text{e-}4$$



# Logarithms to avoid underflow

$$p(w_1) \cdot p(w_2) \cdot p(w_3) \dots \cdot p(w_n)$$

can get really small esp. with large  $n$

$$\log \prod p(w_i) = \sum \log p(w_i)$$

$$p(i) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie}) = 5.95374181e-7$$

$$\log p(i) + 5 \log p(\text{love}) + \log p(\text{the}) + \log p(\text{movie})$$

$$= -14.3340757538$$

*This implementation trick is very common in ML and NLP*

# Estimating word probabilities from counts

- Our goal: infer probability distribution that generated the labeled data for each label
- One way to estimate  $P(\text{word} \mid \text{pos})$  and  $P(\text{word} \mid \text{neg})$ : use counts in a text
- Probability estimated as relative frequency: “maximum likelihood estimation”, use the distribution that maximizes the likelihood of the training data

# Estimating word probabilities from counts

$p(X | y=\text{POS})$

word	count	$p(w   y)$
i	3	0.19
hate	0	0.00
love	7	0.44
the	3	0.19
movie	3	0.19
actor	0	0.00
<b>total</b>	<b>16</b>	

$p(X | y=\text{NEG})$

word	count	$p(w   y)$
i	4	0.22
hate	4	0.22
love	1	0.06
the	4	0.22
movie	3	0.17
actor	2	0.11
<b>total</b>	<b>18</b>	

$p(X | y=POS)$

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new review  $X_{\text{new}}$ : love love the movie

$$\log p(X_{\text{new}} | \text{POS}) = \sum_{w \in X_{\text{new}}} \log p(w | \text{POS}) = -4.96$$

$$\log p(X_{\text{new}} | \text{NEG}) = -8.91$$

# How likely are positive reviews in general?

- Reminder: We are estimating the probability of a label  $c$  (pos, neg) given a datapoint  $d$  as  $P(c | d) \sim P(d | c)P(c)$
- We approximated  $P(d|c)$  by the product of the probabilities of individual words
- $P(c)$  is the general probability of seeing a positive or negative review
- How do we estimate that probability?

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- We approximated  $P(d|c)$  by the product of the probabilities of individual words
- $P(c)$  is the general probability of seeing a positive or negative review
  - This lets us encode the inductive bias about the labels
- How do we estimate that probability?
  - The same was as for  $P(\text{word} | c)$ : through relative frequencies

# Computing the prior probability of “pos”, “neg”

- **i hate the movie**
- **i love the movie**
- **i hate the actor**
- **the movie i love**
- **i love love love love love the movie**
- **hate movie**
- **i hate the actor i love the movie**

This gives us the following counts and probability estimates:

label y	count	$p(Y=y)$	$\log(p(Y=y))$
POS	3	0.43	-0.84
NEG	4	0.57	-0.56

# Posterior probabilities for $X_{\text{new}}$

$$\begin{aligned}\log p(\text{POS} | X_{\text{new}}) &\propto \log P(\text{POS}) + \log p(X_{\text{new}} | \text{POS}) \\ &= -0.84 - 4.96 = -5.80\end{aligned}$$

$$\log p(\text{NEG} | X_{\text{new}}) \propto -0.56 - 8.91 = -9.47$$

What does NB predict?

# Naive Bayes summary

- Assumptions

# Naive Bayes summary

- Assumptions:
  - Independence between features, in our case between words in a text: A text is just a bag of words
  - For the test data, assume the probability distribution that makes the training data most likely. (Are there alternatives?)

# Naive Bayes summary

- Steps to use
  1. Training: learn  $p(c)$  and  $p(w|c)$  parameters for all classes and words, based on their counts in labeled training data
  2. Prediction: For a new document, use the learned parameters to predict the (non-normalized) posterior probability of class labels, choose the more likely one  
  
(Non-normalized: we dropped the denominator  $P(d)$ )

What if we see no positive training documents containing the word “awesome”?

$$p(\text{awesome} \mid \text{POS}) = 0$$

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$$p(\text{awesome} \mid \text{POS}) = 0$$

any review that contains “awesome” will have zero probability for the positive class!

# Add- $\alpha$ (pseudocount) smoothing

$$\text{unsmoothed } P(w_i | y) = \frac{\text{count}(w_i, y)}{\sum_{w \in V} \text{count}(w, y)}$$

$$\text{smoothed } P(w_i | y) = \frac{\text{count}(w_i, y) + \alpha}{\sum_{w \in V} \text{count}(w, y) + \alpha |V|}$$

what happens if we do  
add- $\alpha$  smoothing as  $\alpha$  increases?

# Example: Training

	Cat	Documents
Training	-	just plain boring
	-	entirely predictable and lacks energy
	-	no surprises and very few laughs
	+	very powerful
	+	the most fun film of the summer
Test	?	predictable with no fun

# Example: Prediction

Model Parameters

$P(+)$  =

$P(-)$  =

w	$P(w +)$	$P(w -)$
I	0.1	0.2
love	0.1	0.001
this	0.01	0.01
fun	0.05	0.005
film	0.1	0.1
...	...	...

New doc x =

# Other details

- Binarization
  - Issue: overcounting word repetitions
  - Solution:
- Negation handling
  - Issue: “not fun” as bag of words: we lose information of what is negated
  - Solution: