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260. Single Number III [\(/problems/single-number-iii/\)](/problems/single-number-iii/)

Aug. 25, 2019 | 2.9K views

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Given an array of numbers `nums`, in which exactly two elements appear only once and all the other elements appear exactly twice. Find the two elements that appear only once.

Example:

Input: [1,2,1,3,2,5]
Output: [3,5]

Note:

1. The order of the result is not important. So in the above example, [5, 3] is also correct.
2. Your algorithm should run in linear runtime complexity. Could you implement it using only constant space complexity?

Solution

Overview

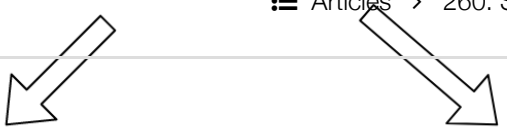
The problem could be solved in $\mathcal{O}(N)$ time and $\mathcal{O}(N)$ space by using hashmap.

To solve the problem in a constant space is a bit tricky but could be done with the help of two bitmasks.

Find single number

Find single number

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Use two bitmasks,
 $O(N)$ time, $O(1)$ space

Use hashmap,
 $O(N)$ time, $O(N)$ space

Approach 1: Hashmap

Build a hashmap : element \rightarrow its frequency. Return only the elements with the frequency equal to 1.

Implementation

Java

Python

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```
1 from collections import Counter
2 class Solution:
3     def singleNumber(self, nums: List[int]) -> List[int]:
4         hashmap = Counter(nums)
5         return [x for x in hashmap if hashmap[x] == 1]
```

Complexity Analysis

- Time complexity : $O(N)$ to iterate over the input array.
- Space complexity : $O(N)$ to keep the hashmap of N elements.

Approach 2: Two bitmasks

Prerequisites

This article will use two bitwise tricks, discussed in details last week :

- If one builds an array bitmask with the help of XOR operator following $\text{bitmask} \oplus= x$ strategy, the bitmask would keep only the bits which appear odd number of times. That was discussed in details in the article Single Number II (<https://leetcode.com/articles/single-number-ii/>).

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

$x = 2$

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

$\text{bitmask} \oplus x$,
the first appearance of x

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

$\text{bitmask} \oplus x \oplus x$,
the second appearance of x

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

- $x \& (-x)$ is a way to isolate the rightmost 1-bit, i.e. to keep the rightmost 1-bit and to set all the others bits to zero. Please refer to the article Power of Two (<https://leetcode.com/articles/power-of-two/>) for the detailed explanation.

$x \& (-x)$
keeps the rightmost 1-bit
and sets all the other bits to 0

$x = 7$

0	0	0	0	0	1	1	1
---	---	---	---	---	---	---	---

$-x = \sim x + 1$

1	1	1	1	1	0	0	1
---	---	---	---	---	---	---	---

$x \& (-x)$

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

$x = 6$

0	0	0	0	0	1	1	0
---	---	---	---	---	---	---	---

$-x = \sim x + 1$

1	1	1	1	1	0	1	0
---	---	---	---	---	---	---	---

$x \& (-x)$

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

Intuition

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An interview tip. Imagine, you have a problem to identify an array element (or elements), which appears exactly given number of times. Probably, the key is to build first an array bitmask using XOR operator. Examples: In-Place Swap (leetcode.com/articles/single-number-ii/356460/Single-Number-II/324042), Single Number (<https://leetcode.com/articles/single-number/>), Single Number II (leetcode.com/articles/single-number-ii/356460/Single-Number-II/324042).

So let's create an array bitmask : $\text{bitmask} \hat{=} x$. This bitmask will *not* keep any number which appears twice because XOR of two equal bits results in a zero bit $a \hat{=} a = 0$.

Instead, the bitmask would keep only the difference between two numbers (let's call them x and y) which appear just once. The difference here it's the bits which are different for x and y .

$\text{bitmask} \hat{=} x \hat{=} y \hat{=} a \hat{=} a$
 $=$
 difference
 between x and y

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

$x = 1$

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

$y = 2$

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

$a = 3$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

$a = 3$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

$\text{bitmask} \hat{=} x \hat{=} y \hat{=} a \hat{=} a$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

Could we extract x and y directly from this bitmask? No. Though we could use this bitmask as a marker to separate x and y .

Let's do $\text{bitmask} \& (-\text{bitmask})$ to isolate the rightmost 1-bit, which is different between x and y . Let's say this is 1-bit for x , and 0-bit for y .

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

$x = 1$

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

isolate rightmost 1-bit which is different for x and y	$y = 2$	<div>Articles > 260. Single Number III</div> <div>0 0 0 0 0 0 1 0</div>
	$a = 3$	0 0 0 0 0 0 1 1
	$a = 3$	0 0 0 0 0 0 1 1
	$bitmask =$ $bitmask \oplus x \oplus y \oplus a \oplus a$	0 0 0 0 0 0 1 1
	$diff =$ $bitmask \& (-bitmask)$	0 0 0 0 0 0 0 1

Now let's use XOR as before, but for the new bitmask $x_bitmask$, which will contain only the numbers which have 1-bit in the position of $bitmask \& (-bitmask)$. This way, this new bitmask will contain only number x $x_bitmask = x$, because of two reasons:

- y has 0-bit in the position $bitmask \& (-bitmask)$ and hence will not enter this new bitmask.
- All numbers but x will not be visible in this new bitmask because they appear two times.

$x_bitmask$	0 0 0 0 0 0 0 0
$x = 1$	0 0 0 0 0 0 0 1
$y = 2$	0 0 0 0 0 0 1 0
$a = 3$	0 0 0 0 0 0 1 1
$a = 3$	0 0 0 0 0 0 1 1
$x_bitmask =$ $x_bitmask \oplus x \oplus a \oplus a$	0 0 0 0 0 0 0 1
$diff =$ $bitmask \& (-bitmask)$	0 0 0 0 0 0 0 1

Voila, x is identified. Now to identify y is simple: $y = bitmask \oplus x$.

Implementation

Java

Python

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```
1 class Solution:
2     def singleNumber(self, nums: List[int]) -> List[int]:
3         # difference between two numbers (x and y) which were seen only once
4         bitmask = 0
5         for num in nums:
6             bitmask ^= num
7
8         # rightmost 1-bit diff between x and y
9         diff = bitmask & (-bitmask)
10
11        x = 0
12        for num in nums:
13            # bitmask which will contain only x
14            if num & diff:
15                x ^= num
16
17        return [x, bitmask^x]
```

Complexity Analysis

- Time complexity : $\mathcal{O}(N)$ to iterate over the input array.
- Space complexity : $\mathcal{O}(1)$, it's a constant space solution.

Analysis written by @liaison (<https://leetcode.com/liaison/>) and @andvary (<https://leetcode.com/andvary/>)

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pixelbookcoder (pixelbookcoder) ★ 2 ⓘ August 28, 2019 6:58 PM
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```
nums = [1,2,1,3,2,5]
seen = []
for num in nums:
    if num in seen:
```

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Fnaf (fnaf) ★ 2 ⓘ August 29, 2019 9:47 PM
@andvary can you do a series on segment tree?

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