



137. Single Number II [↗ \(/problems/single-number-ii/\)](/problems/single-number-ii/)

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Given a **non-empty** array of integers, every element appears *three* times except for one, which appears exactly once. Find that single one.

Note:

Your algorithm should have a linear runtime complexity. Could you implement it without using extra memory?

Example 1:

Input: [2,2,3,2]
Output: 3

Example 2:

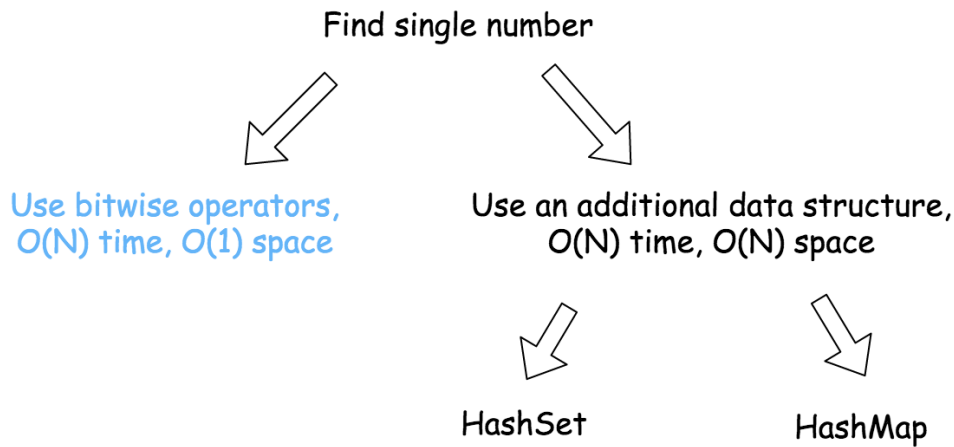
Input: [0,1,0,1,0,1,99]
Output: 99

Solution

Overview

The problem seems to be quite simple and one could solve it in $\mathcal{O}(N)$ time and $\mathcal{O}(N)$ space by using an additional data structure like set or hashmap.

The real game starts at the moment when Google interviewer (the problem is quite popular at Google the last six months) asks you to solve the problem in a constant space, testing if you are OK with bitwise operators.



Approach 1: HashSet

The idea is to convert an input array into hashset and then to compare the tripled sum of the set with the array sum

$$3 \times (a + b + c) - (a + a + a + b + b + b + c) = 2c$$

Implementation

Java

Python

Copy

```
1 class Solution:
2     def singleNumber(self, nums):
3         return (3 * sum(set(nums)) - sum(nums)) // 2
```

Complexity Analysis

- Time complexity : $\mathcal{O}(N)$ to iterate over the input array.

- Space complexity : $\mathcal{O}(N)$ to keep the set of $N/3$ elements.


Approach 2: HashMap

Let's iterate over the input array to count the frequency of each number, and then return an element with a frequency 1.

Implementation

Java

Python

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```

1 from collections import Counter
2 class Solution:
3     def singleNumber(self, nums):
4         hashmap = Counter(nums)
5
6         for k in hashmap.keys():
7             if hashmap[k] == 1:
8                 return k

```

Complexity Analysis

- Time complexity : $\mathcal{O}(N)$ to iterate over the input array.
- Space complexity : $\mathcal{O}(N)$ to keep the hashmap of $N/3$ elements.

Approach 3: Bitwise Operators : NOT, AND and XOR

Intuition

Now let's discuss $\mathcal{O}(1)$ space solution by using three bitwise operators (<https://wiki.python.org/moin/BitwiseOperators>)

$\sim x$ that means bitwise NOT

$x \& y$ that means bitwise AND

$x \oplus y$ that means bitwise XOR

XOR

Let's start from XOR operator which could be used to detect the bit which appears odd number of

times: 1, 3, 5, etc.

XOR of zero and a bit results in that bit

$$0 \oplus x = x$$

XOR of two equal bits (even if they are zeros) results in a zero

$$x \oplus x = 0$$

and so on and so forth, i.e. one could see the bit in a bitmask only if it appears odd number of times.

Bitmap	0	0	0	0	0	0	0
$x = 2$	0	0	0	0	0	0	1 0

Bitmap $\wedge x$, the first appearance of x	0	0	0	0	0	0	1 0
--	---	---	---	---	---	---	-----

Bitmap $\wedge x \wedge x$, the second appearance of x	0	0	0	0	0	0	0 0
--	---	---	---	---	---	---	-----

Bitmap $\wedge x \wedge x \wedge x$, the third appearance of x	0	0	0	0	0	0	1 0
--	---	---	---	---	---	---	-----

That's already great, so one could detect the bit which appears once, and the bit which appears three times. The problem is to distinguish between these two situations.

AND and NOT

To separate number that appears once from a number that appears three times let's use two bitmasks instead of one: `seen_once` and `seen_twice`.

The idea is to

- change `seen_once` only if `seen_twice` is unchanged
- change `seen_twice` only if `seen_once` is unchanged

$x = 2$	0	0	0	0	0	0	1 0
---------	---	---	---	---	---	---	-----

1st	$seen_once = \sim seen_twice \& (seen_once \wedge x)$	0 0 0 0 0 0 1 0
	$seen_twice = \sim seen_once \& (seen_twice \wedge x)$	0 0 0 0 0 0 0 0
2nd	$seen_once = \sim seen_twice \& (seen_once \wedge x)$	0 0 0 0 0 0 0 0
	$seen_twice = \sim seen_once \& (seen_twice \wedge x)$	0 0 0 0 0 0 1 0
3d	$seen_once = \sim seen_twice \& (seen_once \wedge x)$	0 0 0 0 0 0 0 0
	$seen_twice = \sim seen_once \& (seen_twice \wedge x)$	0 0 0 0 0 0 0 0

This way bitmask `seen_once` will keep only the number which appears once and not the numbers which appear three times.

Implementation

Java

Python

Copy

```

1 class Solution:
2     def singleNumber(self, nums: List[int]) -> int:
3         seen_once = seen_twice = 0
4
5         for num in nums:
6             # first appearance:
7             # add num to seen_once
8             # don't add to seen_twice because of presence in seen_once
9
10            # second appearance:
11            # remove num from seen_once
12            # add num to seen_twice
13
14            # third appearance:
15            # don't add to seen_once because of presence in seen_twice
16            # remove num from seen_twice
17            seen_once = ~seen_twice & (seen_once ^ num)
18            seen_twice = ~seen_once & (seen_twice ^ num)
19
20        return seen_once

```

Complexity Analysis

- Time complexity : $\mathcal{O}(N)$ to iterate over the input array.

- Space complexity : $O(1)$ since no additional data structures are allocated.

Analysis written by @liaison (<https://leetcode.com/liaison/>) and @andvary (<https://leetcode.com/andvary/>)

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(/kenanlv)

kenanlv (kenanlv) ★ 17 ⌚ August 13, 2019 2:05 PM

How could you come up with solution 3 during an interview???

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(/casd82)

casd82 (casd82) ★ 29 ⌚ August 16, 2019 5:57 PM

This is art.

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(/nitiz)

nitiz (nitiz) ★ 13 ⌚ August 12, 2019 3:51 PM

Wow! Just amazing. Thanks for the clear explanation.

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(/kkzeng)

kkzeng (kkzeng) ★ 63 ⌚ August 19, 2019 10:03 AM

Really elegant solution in the last approach. It's easy to see why this would work in a case like [1, 1, 1, 2, 2, 2, 3] and I know that order doesn't affect this solution i.e. [1, 2, 2, 1, 3, 1, 2] but I'm not sure I understand why it doesn't affect the solution?

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(/auto676)

auto676 (auto676) ★2 🕒 August 16, 2019 11:21 AM

[mention:(display:andvary)(type:username)(id:andvary)] Can you please explain how you got the expressions in the bit manipulation approach?

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(/szipan)

szipan (szipan) ★1 🕒 August 15, 2019 12:56 AM

For the bitwise algorithm, I think the commutative property of the two operators (seenOnce and seenTwice) should be proved first, i.e, why applying them to a number sequence in two different orders generates the same result. Or else, I think an interviewee only solves the problem by chance.

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(/user4722i)

user4722i (user4722i) ★5 🕒 August 26, 2019 2:29 AM

Simply sort the array. Compare neighbors when you find the one which doesn't match its neighbors stop.

Complexity of sort would be $O(n\log n)$

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(/byheddy)

byheddy (byheddy) ★0 🕒 a day ago

yo boy ♂ next door

have a look at my binary search which is, certainly, $O(n\lg n)$

```
class Solution {
    public int singleNumber(int[] nums) {
        Read More
```

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(/dushuangli0835)

dushuangli0835 (dushuangli0835) ★4 🕒 September 13, 2019 10:33 PM

First two didn't match the requirement. they use $O(n/3)$ extra space for hashmap. Sorting also doesn't match, because $O(n\log n)$ larger than $O(n)$.

this question have to use part of the quicksort. every time find the position of pivot. use the pivot final position to check whether the single number on its left or right.

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