Let a, b, $c > 0$ be real numbers. From AG inequality we know that:

$$\frac{a+2b+3c}{6} \geq \sqrt[6]{ab^2c^3}$$

Now suppose that $x = a$, $y = 2b$, $z = 3c$ holds. Then we can get the value of the right side of the inequality:

$$xy^2z^3 = 108$$
$$a(2b)^2(3c)^3 = 108$$
$$ab^2c^3 = 1$$

So to get the smallest possible value of $x + y + z$, both sides of the inequality has to be equal. Hence, the numbers a, b, c has to be equal. As a result, $a = b = c = 1$ and the smallest possible value of $x + y + z$ is:

$$x + y + z = a + 2b + 3c = 1 + 2 + 3 = 6$$