

Firstly, we need to prove that if each animal on the farm gets same amount of food, the animals cannot be equal. Suppose that animals are equal when they reach similar level of satisfaction after eating given amount of food. However, pigs have marginally bigger minimum food consumption than chicken. Therefore, the animals can't be equal when everyone gets the same amount of food.

Let c_c, c_p be amount of food chickens and pigs get. If, on average, an animal in government gets the same amount of food as an animal outside of government, the equation:

$$\frac{o_c c_c + o_p c_p}{o_c + o_p} = \frac{2g_c c_c + 2g_p c_p}{g_c + g_p}$$

where o_c, o_p is the number of chickens and pigs outside of government and g_c, g_p is the number of chickens and pigs in government, has to hold.

So the only thing remaining is to show a case when this equation holds. For $c_c = 1, c_p = 10, o_c : o_p = 1 : k$ and $g_c : g_p = k : 1$ we can show that there exist a real number k which satisfies the relation:

$$\begin{aligned}\frac{o_c + 10ko_c}{o_c + ko_c} &= 2 \cdot \frac{kg_p + 10g_p}{kg_p + g_p} \\ \frac{1 + 10k}{1 + k} &= 2 \cdot \frac{k + 10}{k + 1} \\ 10k + 1 &= 2k + 20 \\ 8k &= 19 \\ k &= \frac{19}{8}\end{aligned}$$

So in this case, on average, an animal in government gets the same amount of food as an animal outside of government, when $o_c : o_p = 8 : 19$ and $g_c : g_p = 19 : 8$.

As this is the only thing we have to show, the proof is complete. Q. E. D.