

To start with, I will rewrite the inequality to this form:

$$\left| \sum_{k=1}^n O(k) - \sum_{k=1}^n E(k) \right| \leq n$$

The reason why I rewrote the inequality like this is because we can easily get the number of numbers from 1 to  $n$  divisible by  $k$  with formula  $\lfloor n/k \rfloor$ . As a result, the inequality is equivalent to:

$$\left| \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k-1} \right\rfloor - \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k} \right\rfloor \right| \leq n$$

Now we will solve this inequality for two cases – when the value in the vertical bars are positive and when it is negative.

When the value is positive, then:

$$\begin{aligned} \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k-1} \right\rfloor &\leq n + \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k} \right\rfloor \\ \left\lfloor \frac{n}{1} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \dots &\leq n + \left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + \dots \end{aligned}$$

Because  $n \leq n$ ,  $n/3 \leq n/2$ ,  $n/5 \leq n/4$  etc., this inequality has to hold in this case.

When the value is negative, then similarly:

$$\begin{aligned} \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k} \right\rfloor &\leq n + \sum_{k=1}^{\lfloor n/2 \rfloor} \left\lfloor \frac{n}{2k-1} \right\rfloor \\ \left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + \dots &\leq n + \left\lfloor \frac{n}{1} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{5} \right\rfloor + \dots \end{aligned}$$

Because  $n/2 \leq n$ ,  $n/4 \leq n$ ,  $n/6 \leq n/3$  etc., the inequality holds in both cases, so the proof is complete.