

We know that inequality  $(a - 2b)^2 \geq 0$  has to hold. As a result, the inequality:

$$\begin{aligned}a^2 - 4ab + 4b^2 &\geq 0 \\5a^2 + 5b^2 &\geq 4a^2 + 4ab + b^2 \\5(a^2 + b^2) &\geq (2a + b)^2 \\5c^2 &\geq (2a + b)^2\end{aligned}$$

has to be true. And since the numbers  $a, b, c \geq 0$ , we can square root both sides:

$$\begin{aligned}\sqrt{5}c &\geq 2a + b \\ \frac{2a + b}{c} &\leq \sqrt{5}\end{aligned}$$

So the inequality has to hold. Q. E. D.