I'm going to prove this inequality for two cases – when $m > n$ and $m < n$ (when $m = n$ it obviously holds). So suppose that $m > n$. Then inequality:

$$\frac{m!}{\left(m - \frac{m-n}{2}\right)!} \geq \frac{\left(\frac{m+n}{2}\right)!}{\left(\frac{m+n}{2} - \frac{m-n}{2}\right)!}$$

holds because a number of partial permutations of m items is bigger than a number of partial permutations of $\frac{m+n}{2}$ items due to the assumption. From this inequality we are able to get the desired inequality:

$$\frac{m!}{\left(\frac{m+n}{2}\right)!} \geq \frac{\left(\frac{m+n}{2}\right)!}{n!}$$

$$m! \cdot n! \geq \left(\left(\frac{m+n}{2}\right)!\right)^2$$

Now suppose $m < n$. Similarly:

$$\frac{n!}{\left(n - \frac{n-m}{2}\right)!} \geq \frac{\left(\frac{n+m}{2}\right)!}{\left(\frac{n+m}{2} - \frac{n-m}{2}\right)!}$$

holds. That implies:

$$\frac{n!}{\left(\frac{n+m}{2}\right)!} \geq \frac{\left(\frac{n+m}{2}\right)!}{m!}$$

$$m! \cdot n! \geq \left(\left(\frac{m+n}{2}\right)!\right)^2$$

So the inequality holds for all m, n. Q. E. D.