

Let  $x_i$  be the number on a clock at position of number  $i \in \{1, 2, \dots, 12\}$  on a ordinary clock,  $y_i = (x_i + x_{i+1}) \bmod 13$  for  $i \in \{1, 2, \dots, 11\}$  and  $y_{12} = (x_{12} + x_1) \bmod 13$  and  $s = y_1 + y_2 + \dots + y_{12}$ . Then we know that:

$$s \equiv y_1 + y_2 + \dots + y_{12} \equiv 2(x_1 + x_2 + \dots + x_{12}) \equiv 12 \cdot 13 \equiv 0 \pmod{13}$$

Consequently, if we show that there exists a solution when  $s = 13$  and that there is no solution for  $s = 0$ , the smallest possible sum is  $s = 13$ . For  $s = 13$ , the solution is:

$$(x_1, x_2, \dots, x_{12}) = (1, 12, 2, 11, 3, 10, 4, 9, 5, 8, 6, 7)$$

However, if  $s = 0$ , then every  $y_i = 0$ , thus:

$$y_i \equiv y_{i+1} \pmod{13}$$

$$x_i + x_{i+1} \equiv x_{i+1} + x_{i+2} \pmod{13}$$

$$x_i \equiv x_{i+2} \pmod{13}$$

This holds iff  $x_i = x_{i+2}$  which is a contradiction. Hence, the smallest value Sylva can find is  $s = 13$ .