W1 HW A106230034 莫雅玄 數統系

例 6.4 $\hat{\theta}_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$, $\hat{\theta}_2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$

sol $E(x_i)=\mu$, $V(x_i)=\sigma^2=E(x_i^2)-\mu^2$

則 $E(\bar{x})=\mu$, $V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x}^2)-\mu^2$

$\Rightarrow E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$

$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\sigma^2$

$\Rightarrow E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$

$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$

$\therefore \hat{\theta}_2$ 為母體變異數 σ^2 之不偏估計量