Logistic Regression

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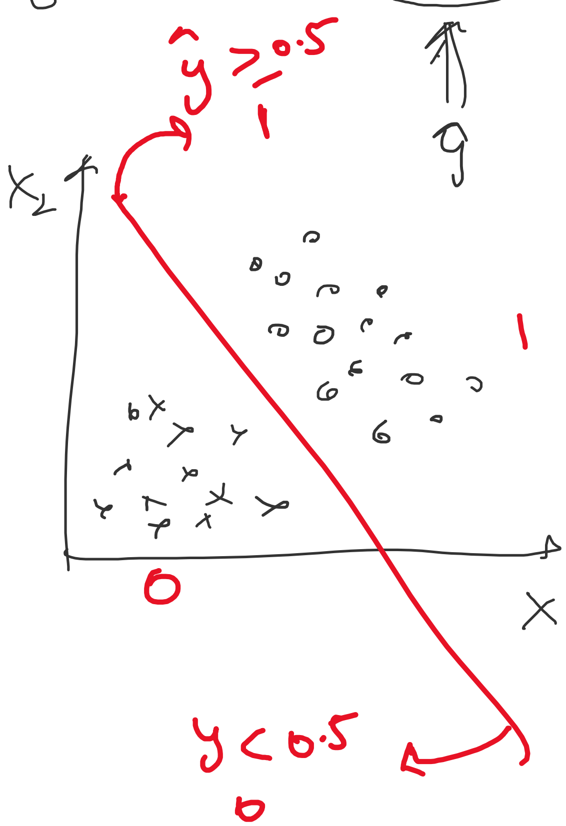
→ Classification

→ Regression → $\underset{Y}{(0-1)}$ → $\underline{y} = \begin{cases} 0 & \text{if } \hat{y} < 0. \\ 1 & \text{if } \hat{y} \geq 0. \end{cases}$

$\hat{y}$



Logistic Regression

↓

$\underline{\text{Sigmoid}} + \underline{\text{Linear}}$

⇓

$$g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{h(\omega) = \sum_{j=1}^{n} \omega_j x_j + \omega_0}$$

$$\boxed{\hat{y} \rightarrow g(h(\omega))}$$

Loss - f^n

$$P(y=1 \mid x; \omega) = \hat{y} \qquad y \in$$

$P(y=0 \mid x; \omega) = 1-\hat{y}$

$$\boxed{P(y \mid x; \omega) = (\hat{y})^{y}(1-\hat{y})^{(1-y)}} \rightarrow \text{Bernou}$$

$y=0 \rightarrow 1-y$

$y=1 \rightarrow y$

$$L(\theta) = p(y \mid X; \omega) = \prod_{i=1}^{m} P(y^{(i)} \mid x^{(i)};$$

$$= \prod_{i=1}^{m} (\hat{y}_{(i)})^{y^{(i)}} (1 - \hat{y}_{(i)}$$

log

$$\log(L(\theta)) = \sum_{i=1}^{m} \left[y^{(i)} \log(\hat{y}_{(i)}) + (1 -\right.$$

Maximize this

Minimization $= \Big| - \sum^{m} \left(y^{(i)} \log(\hat{y}^{(i)}) \right.$

Error Binary Cross

w_j

$$w_j = w_j - \alpha \frac{\partial J}{\partial w_j}$$

$$J = -\sum_{i=1}^{m} [y$$

$$\hat{y}_i = \frac{1}{1+e^{-z}}$$

$$\frac{\partial J}{\partial w_j} = \frac{\partial J}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial z} \times \frac{\partial z}{\partial w_j}$$

$$\frac{\partial J}{\partial \hat{y}} = -\sum_{i=1}^{m} \left[\frac{y_i}{\hat{y}_i} - \frac{(1-y_i)}{(1-\hat{y}_i)} \right] = -\sum_{i=1}^{m} [$$

$$\frac{\partial \hat{y}}{\partial z} = +\frac{1}{(1+e^{-z})^2} + e^{-z} = \frac{e^{-z}}{(1+e^{-z})^2} =$$

$$\frac{\partial z}{\partial w_j} = x_{ij}$$

$$\frac{\partial J}{\partial w_j} = \cancel{\frac{1}{m}} \boxed{\sum_{i=1}^{m} \left[(\hat{y}_i - y_i)\, x_{ij} \right]}$$

$$\boxed{w_j := w_j - \alpha \sum_{i=1}^{m} (\hat{y}_i - y_i)\, x_{ij}}$$