PHYSICS 1100/1101
UNIVERSITY PHYSICS I & II

LABORATORY MANUAL

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Contents

Credits

This lab manual has been developed through a collaboration of numerous staff and faculty in the Department of Astronomy and Physics at Saint Mary's University. These include Dr. David Guenther who wrote the earliest edition of this manual, Dr. David Clarke who wrote and revised subsequent editions including the present one, Dr. Mike Dunlavy who has maintained the manual for most of the past several years and who contributed the original drafts of labs 1, 6, and 9, Drs. Malcolm Butler, David Turner, and Gary Welch who contributed original drafts of other labs (some no longer in the curriculum) and who contributed to the editing of the manual, and Ms. Shawna Mitchell, who prepared the original drafts of Appendix C and many of the figures. In addition, this manual has benefited from the numerous comments and critiques from lab demonstrators, instructors, and students enrolled in PHYS 1100/1101 and its predecessors (PHYS 1210/1211, PHY 210/211, PHY 205, and PHY 221). All comments and criticisms are gratefully received by the lab instructors who will channel them to those responsible for updating the manual.

DAC, August 2009

Introduction

1. Objectives of the Physics Laboratory

Most people, whether they have ever taken a University Physics course or not, develop an intuition for the physical world around them. So, for example, when the coyote (from the *Bugs Bunny and Roadrunner* series) falls only *after* he realises he has stepped out beyond the edge of the cliff, or when the anvil doesn't budge even after being rammed by a massive bull at top speed, or when Martians the size of ostriches are created from a small seed and a single drop of water, the audience is amused at the impossibility of the events. Yet how do we *know* these events are impossible? What laws of physics are being violated[1], and how do we know what these laws are?

More specifically, if a force acts on an object, we expect the object to move in the direction the force is applied. In class, our intuition on this matter is formalised by Newton's Second Law of Motion that states $F \propto a$, where F is the applied force and a is the acceleration of the object. The law appears to be a reasonable representation of what we expect. But how do we know the law of motion isn't, for example, $F \propto v$, where v is the velocity of the object? Can our everyday experiences tell us that this is not correct or must we test the correctness of this law of motion more carefully? In fact, history records that up until the time of Galileo Galilei (1564-1642), most scientists *did* believe $F \propto v$. Galileo was the first to conduct a series of experiments for which we have recorded evidence (similar to Experiment 3 in this manual), to deduce the correct form of the law (he got Newton's Second Law).

To many, this represents the goal of scientific experiment—to discover new laws of physics. But once the law is discovered, what is the point of doing the experiment over and over again? (Galileo's experiment has probably been carried out by nearly every first year science student at every university in the world for the past two centuries.)

Here, then, is at least a partial list of reasons why we ask physics students to do these labs, even though surely there is little more to discover in them:

1. to introduce students to the care and methodology required to do experimental science effectively;

2. to show students how the ideal theory taught in class or found in the textbook is applied to "real-world" situations in which friction, measuring errors, and uncertainties play a significant role;

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[1]The three examples violate Newton's Second Law, Conservation of Momentum, and Conservation of Mass, respectively.

3. to teach students the proper techniques of data and graphical analysis;

4. to introduce students to simple methods for estimating the uncertainties associated with a measurement or an experiment and to enable the student to assess the precision or quality of experimentally determined results;

5. to expose students to a variety of instruments and measuring techniques through hands-on use;

6. to instruct students on the proper way to keep a laboratory notebook and writing a formal scientific report; and

7. to instil upon students proper and safe laboratory conduct.

1.1 Preparation for the lab

Each lab section has assigned to it one lab *instructor* (a faculty member) and one lab *demonstrator* (normally a graduate or senior undergraduate student). These people are here to help you understand the objectives and methodology of the labs, and to ensure you are able to complete the lab during the three-hour lab period.

Before walking into the lab, you should have read and understood the material discussed in this *Introduction*. In addition, before arriving in the lab to perform an experiment, you should have carefully read the instructions for that experiment and completed all pre-lab preparations. At the beginning of each lab, the demonstrator will check your lab books to make sure you are, in fact, prepared for the experiment and make a note to this effect in your lab book. Your preparedness for the experiments will influence your final lab grade. If you do not understand what you are supposed to do in the lab after reading the instructions, you should prepare a list of questions to ask the lab instructor or demonstrator prior to the lab or during the instructor's office hours.

2. Laboratory Notebook

You are required to keep a lab notebook where you will record *everything* you do in connection with the experiments you perform. The notebook contains the only permanent record of what you did during the experiment. Hence, it should contain all of the details which are in any way relevant to the experiment. You, or any other scientist, should be able to reconstruct exactly what you did during the experiment from the notes recorded in your lab notebook.

The lab notebook used for PHYS 1100/1101 is the "**Blueline A-90 Physics Notebook**" sold at the University bookstore. This notebook contains bound pages with right-facing pages lined (for notes and calculations) and left-facing pages alternating between blank (for diagrams) and graph paper (for all graphs). The pages in the notebook are unnumbered, so *the first thing you need to do upon opening up your notebook is to number sequentially all the right-facing pages **in pen***, and leave a page at the beginning for a *Table of Contents* which should be kept up-to-date as you perform the experiments. A binder with loose-leaf

paper is not acceptable for a lab notebook, nor will you be permitted to use loose sheets of paper during the lab. *Everything* you do in the lab should be recorded in your lab notebook, however trite you may think the detail may be. These "tidbits" may well become critcal to your recollection of what you did in the lab when it comes time to write up a formal report on one of the experiments (§ 3).

You should always write in your lab notebook using pen. Errors are corrected by drawing a single neat line through the errant entry and entering the correction above or beside it in as neat a fashion as possible. If the entire page is spoiled, a simple cross through the entire page will suffice. *Under no circumstances should you tear out a page.* If you do, your numbered pages will give you away! Data should be recorded directly into the lab notebook in pen and *not*, for example, on a separate piece of paper, later to be copied into the notebook. Even scratch calculations should be written down in the notebook in pen. The idea is for you to keep all notes, errant or not, in your lab notebook. Often, one can learn as much from what didn't work as from what did, and so it is necessary to get into the habit of keeping your notebook in such a way that you can read your errors as well.

Summary:

- number all notebook pages and keep a Table of Contents;

- record everything in pen that happens during an experiment;

- use a pen to enter all data and observations into the notebook, and for doing all calculations;

- *never* remove any pages from your notebook.

2.1 Informal Lab Write-ups

One informal lab write-up is required with every experiment performed, and all informal write-ups are completed in-class. At the end of each experiment, the lab notebooks are collected by the lab demonstrator, whether or not you are finished, and graded by him or her to be returned the following week. Your in-class lab write-up need not be a detailed account of the experiment. Rather, it must contain all the information you would need should you be called upon to write the experiment up formally in the future (see § 3 on *Formal Reports*). *This is one of the primary criteria upon which your experiment will be graded.* While being forced to complete both the experiment and write-up in the same three hour period may make the lab seem rushed at first, it does force the student to learn how to record the salient facts effectively and efficiently. Of course, the other advantage is, other than the preparatory work for each experiment, there is no weekly lab homework! Examples of both an acceptable and an unacceptable informal lab report can be found in App. A, and the student should review these before coming to the first lab.

Before each lab, you should read the instructions in the manual carefully and understand them. In addition, you should enter the following in your lab notebook **before** coming to the lab:

1. the title and number of the experiment as it appears in the lab manual;

2. the purpose of the experiment (one or two sentences) *in your own words*;

3. a list (but not a derivation) of the important formulæ (as given in the manual) required to perform the data analysis with accompanying definitions for the symbols used;

4. answers to any and all of the boxed "Preparation questions" (but not the "Additional discussion questions") scattered throughout the *theory*, *procedure*, and *analysis* sections of the lab description.

During the lab, you should record the following:

1. the date, your name, and your partner's name;

2. if different from the manual, a list of the equipment used and/or a free-hand drawing of the experimental arrangement as assembled by you and your partner;

3. the procedures you followed that deviated from those in the lab manual (If your procedures were, in fact, identical to the instructions, just refer to the lab manual; there is no need to copy verbatim what already appears there.);

4. an accurate and effective record of your measurements (Often the best way to record data[2] is in a table, and you will be expected to do so whenever possible.);

5. uncertainty estimates given for *every* measurement, as well as how this uncertainty was determined ("by eye", "half-the-range rule", "scatter in the measurements", *etc.*; see § 5);

6. comments about difficulties or anomalies you encountered;

7. all calculations connected with the experiment, including uncertainty propagation (for repeated calculations, a written example of one will suffice); and finally

8. your conclusions in which you state your final results with an estimate of their accuracy, and whether your results agree with "accepted values" (if any) to within experimental uncertainty. If your results disagree with accepted values, you should list some possible sources of error that were not accounted for in the lab that may account for the observed differences.

The "Additional discussion questions" are intended for the formal report (§ 3) and need not be answered in the informal report. However, time permitting, attempting to answer these questions in the informal report could lead to "bonus points"...

Performing all these tasks in the three-hour session may, at first, seem daunting. By the end of the year, however, your lab experience should seem much more "relaxed" as you become

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[2]Note that the word *data* is plural, and takes the plural form of the verb. Thus, one writes "The data *were* recorded in Table 1.", and not "The data *was* recorded in Table 1." The singular form of *data* is *datum*.

more efficient in what you record, and how you record it. If you are running out of time, concentrate on getting the raw data, even if that means skipping some of the analysis. A formal report can be done without your analysis, but not without the data!

2.2 Grading

The informal lab write-ups are graded on a scale of 0 to 10 using the following criteria:

1. your performance in the lab, including pre-lab preparation, efficient use of the three-hour period, *etc.*;

2. quality of writing and legibility;

3. proper calculations and graphs;

4. proper evaluation of your results including uncertainty propagation and conclusions; and

5. completeness.

The last criterion is determined from the answer to the following question: "At some point in the future, could the student generate a formal write-up from what appears in the informal report?".

Labs are marked by the lab demonstrator, and returned to you the following week, giving you time to prepare for the next lab one week hence. Read the grader's comments carefully; they are there to provide direction on improving future reports, as well as to ensure your lab is complete enough should you be required to convert the informal report into a formal one (§ 3).

If you miss a lab altogether, you will get a zero for that lab. If your absence is excused (*e.g.*, for a medical reason), you will get a chance later in the semester to make up the lab "for credit" during "make-up week".

2.3 Make-up week

If worse comes to worst and you do not get all your data during the regular lab session, you should plan to make use of "make-up week", scheduled the week before formal labs are assigned. Here, you can gather any data you may have missed or gathered incorrectly on as many labs as you may need. Except for excused absences (*e.g.*, medical), make-up week is *not* to redo labs for credit. Rather, they are there to allow students to gather the data they may have missed in the regular lab, so that they can do whatever formal report is assigned, which is worth much more to the final lab grade than a single informal report.

3. Formal Reports

You are required to write one formal report per semester on one of the experiments performed in the lab. These will generally be graded by your lab instructor. If your lab notebook has been kept as described in the previous section, the effort should amount to little more than reformatting and/or transcribing what already appears in your notebook. You will have two weeks to complete each report.

The formal report is *not* written in your lab notebook, but on standard sized $(8\frac{1}{2}'' \times 11'')$ paper (white blank for computer-generated, white lined for hand-written). If graphs are hand-drawn, they should be drawn on separate sheets of graph paper (and not *quadrille* paper; if you don't know the difference, find out before doing your report!) using, as always, rulers for straight lines. All pages can be stapled together at the top left corner; duo-tangs or other covers are unnecessary but acceptable if you feel the need.

Adherence to standard English grammar is mandatory. In this world, if you cannot communicate your thoughts intelligibly, the chances of gaining employment are severely limited, regardless of what other assets you might possess. Therefore, the report must be written using full sentences and in paragraph form and not, for example, in point form (although "bulleted paragraphs" are acceptable when it improves the clarity and organisation). Clearly, texting abbreviations are absolutely *verboten*! An example of a formal report for which most instructors would give a high grade appears in App. B.

Your formal report should contain the following:

1. the date the experiment was performed, your name, and your partner's name;

2. the date the formal report was completed;

3. Purpose—in your own words, state the objectives of the experiment using full sentences;

4. Theory—this should be a self-contained summary of all the equations used in the data analysis. If the derivations of the equations are simple, they should be included here too. However, if the derivations are lengthy, you may cite references (*e.g.*, a text book, this manual) where the expressions are derived in all their gory detail. As a guide, try to keep this section to less than one or two pages. **Under no circumstances, should you plagiarise this lab manual, or any other document!**.

   Note: It is *not* necessary to answer the "preparation questions" in your formal report. These should already have been answered and graded in your informal report;

5. Equipment—list all equipment used and include a good-sized diagram (at least half a page) showing how the experiment was assembled;

6. Procedure—this should be a self-contained summary of what you did in the lab to carry out the stated objectives. Do not refer to the lab instructions here, and certainly don't plagiarise them! This should be your own account of what you did, and written in the *past-passive* tense (*e.g.*, "The mass of the sample was measured..."). Avoid using the imperative tense (*e.g.*, "Measure the mass of the sample...") and avoid using personal pronouns (*e.g.*, "I measured the mass of the sample...");

7. Raw Data—in this section, you should record all data gathered during the lab along with an estimate of the uncertainty for each measurement. Without an estimate of the uncertainty, the data entry is nearly worthless. See § 5 on *Experimental Uncertainty*. Where ever possible, record the data in a table with the table properly titled;

8. Data Analysis—all data manipulation (using the equations listed in the Theory section) should be done here, including all uncertainty propagation. Where ever possible, generate graphs of the data in which the independent variable appears on the x-axis, and the dependent variable appears on the y-axis. See § 6 on *Graphical Analysis*;

9. Discussion—here, you should indicate the accuracy of your results, recount any anomalies or difficulties you experienced in the lab, propose possible modifications to the lab to avoid these problems, and answer all questions in the *Additional discussion questions* section. These should be answered as part of the text, and not in "point form";

10. Conclusions—here, you should simply state what you found. A few sentences will suffice and should include what your experimental values were (including the uncertainties) and whether they agreed with any accepted values. **IF YOUR RESULTS DISAGREE WITH THE ACCEPTED VALUES, DON'T SAY THAT THEY DO AGREE!!**

As a guide, a formal report will probably be no less than 6 pages, and no more than 12. Don't think volume is necessarily a plus. Obvious padding of the report with excessive verbiage will go against you. If your writing is legible, you may write your report by hand. However and in general, typed (*e.g.*, word processor) reports are usually preferred. Diagrams should be done by machine *only* if such diagrams are better than carefully hand-drawn ones. Similarly with equations; if your word-procesor cannot handle equations effectively (*e.g.*, see the equations in App. B), then you should leave enough space in your document to insert the equations neatly by hand afterward.

Finally, the formal report is your report. Thus, these should be done individually. Even if your lab partner is asked to write up the same lab, you must not hand in carbon copies of the same report. **Copied reports will get zero.** This applies to both the one who copies, and the one who allowed their report to be copied.

4. Attendance

Attendance to all laboratory sessions is *mandatory*. If you must miss a session for any unavoidable reason (*e.g.*, medical), please discuss this with your instructor previous to the session you must miss whenever possible. The laboratory sessions are regularly scheduled parts of the course, and you should no more schedule work or other obligations during this time than you would during your lectures. If you do have a problem attending regularly scheduled laboratory sessions, please talk to your lab instructor. Often, reasonable requests can be accommodated.

5. Experimental Uncertainty

No measurement made in the laboratory can be 100% precise; there is *always* some degree of *uncertainty* associated with every measured value. If q represents the quantity being measured (*e.g.*, a mass, length, time, *etc.*), then the associated uncertainty is written as Δq, and we write the measurement as:

$$q \pm \Delta q.$$

The Greek letter Δ (capital letter "delta", their "D") is often used to indicate "a change in". You are probably already familiar with its use in *rises* and *runs*, in which the rise, Δy, is the "change in y", and the run, Δx, is the "change in x". The $\pm$ symbol (read "plus or minus") is taken to mean that the quantity, while measured to be q, could indeed be anywhere between $q - \Delta q$ and $q + \Delta q$; we simply cannot state with any degree of certainty where in this range q might be given the measuring devices available to us.

Science is an objective pursuit and it is the analysis of the uncertainties that makes it objective. The statement that a measured quantity is "pretty close" is *never* an acceptable conclusion in a lab report. You need to know *how close* your result is to existing values so that you can determine whether or not your results agree or disagree with what may already be known. The *only* way to do this is to calculate the effect your measured uncertainties have on your final results, a process known as *propagation of uncertainties*.

In propagating your uncertainties from the measurements to the final results, you will learn to apply a set mathematical rules which follow from simple arithmetic and, on occasion, elementary calculus. In fact, the method you will learn in this lab is a simplified version of the "full-blown" analysis and gives quick "rules-of-thumb" for propagating uncertainties. As simplified as these rules may be, you will still find that the propagation of uncertainties will consume much of your time preparing your informal reports. The sooner you learn to do these extensive arithmetic operations accurately and quickly, the better will be your performance in this, or any other science or engineering lab.

5.1 Errors and Uncertainties

Contrary to popular usage (even by seasoned scientists!), the terms *error* and *uncertainty* are not synonymous. Let us begin, therefore, by making the distinction between these two important concepts.

Definition: **Error** is the difference between an experimental result and the "accepted" value. The smaller your *error*, the more *accurate* your results.

Definition: **Uncertainty** is a measure of how precisely an instrument may make a measurement. The smaller your *uncertainty*, the more *precise* your readings.

As an example of an experimental error, suppose you have determined that the acceleration of gravity is 9.9 m s^{-2}, while the "accepted" value is 9.8 m s^{-2}. In this case, the "error" is $9.9 - 9.8 = 0.1 \text{ m s}^{-2}$. When an "error" is made, it is up to the experimentalist to indicate in his or her report what the possible sources of error might be. Perhaps friction played an unquantified role in the experiment, or perhaps the system didn't start exactly at rest as

assumed by the theory. Or, perhaps the local acceleration of gravity *really* is 9.9 m s^{-2} and not 9.8 m s^{-2}, in which case the experimentalist must show some independent evidence to support this "alternate" view. (Before doing so, however, you should check to confirm that you have not simply made a blunder in your calculations!)

There are numerous examples of experimental uncertainty in the lab. In fact *every* measurement taken during an experiment has an associated uncertainty. If you are measuring the length of a steel rod with a ruler whose smallest divisions are millimetres (mm), there is no way you could report the length of the rod to be 47.23931 mm. The best you could probably do is to report 47.2 mm. And while 47.2 mm may be the best value you can determine, you probably wouldn't be able to *swear* that the measurement wasn't 47.1 mm or 47.3 mm. However, with a good degree of confidence, you could probably state that the length is *greater than* 47.0 mm (you can see that the tip of the rod is beyond the 47th mm mark on the ruler) and *less than* 47.4 mm (you can see that the tip of the rod is clearly less than half way between the 47th and 48th mm markings). Therefore, you would state the length of the rod is somewhere in the range of 47.1 and 47.3, thus 47.2 ± 0.1 mm. The ± 0.1 bit is the uncertainty. Note this is *not* an error. No *error* was committed because you were unable to measure the length any better than to within a fraction of a mm. The uncertainty is simply a statement of the inescapable fact that nothing can be measured with infinite precision.

> **First Law of Experimental Science**: All measured quantities must be accompanied by an estimate of the uncertainty.

Some experiments require making measurements with metre sticks or 2-metre sticks. In such cases the best accuracy one can hope for is typically ± 1 mm (in large part, the parallax caused by the thickness of the wood may prevent accurate intra-mm measurements), in which case a measurement given as 57.6 cm would normally be recorded as "0.576 ± 0.001 m." Note that, while the measurement may have been made in centimetres, one might record it in metres to maintain work in the SI (mks) system of units. It is normal practice to include a statement like "(estimated reading uncertainty)" after the measurement in order to indicate how the uncertainty was established.

The "half-the-range rule"

In some cases, it is not convenient to "read off" the uncertainty from the measuring device itself, as with the metre-stick in the example above. In these cases, another way to estimate the uncertainty is to take the same reading several times and then use what we will call the "half-the-range rule". For example, in experiment 3, you are to measure the time for an object to slide down the air-track five times. If these readings were 2.12, 2.14, 2.15, 2.16, and 2.18 seconds, then the average value of these readings is 2.15 and the range is $2.18 - 2.12 = 0.06$. So half the range is 0.03, and you would quote your experimental reading as 2.15 ± 0.03 s.

Note: In some labs, you may be instructed to use *Gaussian statistics* (*i.e.*, "standard

deviations") for uncertainty estimates. In fact, many calculators have "standard deviation" and "mean" buttons on them which can be used blindly to extract averages and uncertainties from your data. However, these techniques apply to data bases which contain a large number of entries. In this lab, our data bases will usually comprise of 5, maybe 10 readings which hardly qualifies for large-number statistical methods such as standard deviations. In this case, the "half-the-range rule" used in this lab is actually *preferable* to full-blown statistical analysis, and much easier to use.

5.2 Expressing Uncertainties

In the opening paragraph of this section, we introduced the notation $q \pm \Delta q$ as a way to express a measured quantity q with its uncertainty. Δq is called the *absolute* uncertainty, and has the same units as q itself. Thus, if $q = 4.23$ m and $\Delta q = 0.03$ m, we would write this as:

$$4.23 \pm 0.03 \text{ m}, \hspace{3cm} \text{(absolute uncertainty)}$$

and not, for example, "4.23 m $\pm$ 0.03 m" which isn't necessarily incorrect, just awkward.

Alternately, one may wish to express an uncertainty as a *fraction* of the measured value. Thus, we may write our uncertain measurement as:

$$q \pm \frac{\Delta q}{q},$$

where the *fractional uncertainty* (also known as the *relative uncertainty*), $\Delta q/q$, is *always* unitless. Thus, in our example above, we would write:

$$4.23 \text{ m } \pm \ 0.0071 \text{ (frac. unc.)}, \hspace{2cm} \text{(fractional uncertainty)}$$

(since $0.03/4.23 = 0.0071$) where the designation "(frac. unc.)" is optional, and needed only if you think there is any possibility the reader will confuse the fractional uncertainty for an absolute uncertainty. If you are careful with your placement of the units (in this example, m) *and there are units to place*, this won't be an issue. For an absolute uncertainty, the units are placed after the *uncertainty* (4.23 ± 0.03 m), whereas for a fractional uncertainty, the units are placed after the *reading* (4.23 m $\pm$ 0.0071). This is sufficient to distinguish between the two. Note that this convention is easy to remember, as it follows the normal rules of spoken English. Thus, one would say "4.23 plus or minus 0.03 metres", and not "4.23 metres plus or minus 0.03" if one wanted to be certain both numbers were understood as metres. Similarly, one would say "4.23 metres plus or minus 0.0071 fractional uncertainty", and not "4.23 plus or minus 0.0071 fractional uncertainty metres", which doesn't really make sense.

Converting between absolute and fractional uncertainties, as one has to do frequently in propagating uncertainties, is easy. Let the fractional uncertainty of the quantity q be f_q. Then we have:

$$f_q \ = \ \frac{\Delta q}{q} \hspace{1cm} \text{(converting from absolute to fractional uncertainty)};$$

$$\Delta q \ = \ f_q \, q \hspace{1cm} \text{(converting from fractional to absolute uncertainty)}.$$

A *percentage uncertainty* is just the fractional uncertainty multiplied by 100. Thus, 4.23 m $\pm$ 0.0071 could be expressed as 4.23 m $\pm$ 0.71%; it is completely a matter of taste. This is entirely analogous to whether you express the money in your pocket in terms of dollars (*e.g.*, \$22.43) or in cents (*e.g.*, 2,243¢). The value of the money in your pocket is the same regardless of the units in which you express it. Between the two, this manual normally uses fractional uncertainties though, on occasion, percentage uncertainties are used when convenient.

By and large, final results should always be expressed with an absolute uncertainty. However and as the examples in § 5.4 show, one needs to convert back and forth between absolute and fractional (percentage) uncertainties frequently while propagating uncertainties, and thus you will need to become at ease with these conversions.

If the datum is expressed in scientific notation, it and the absolute uncertainty should be expressed as follows:

$$(2.21 \pm 0.05) \times 10^{-6} \text{ kg}$$

and *not*

$$2.21 \times 10^{-6} \pm 5.0 \times 10^{-8} \text{ kg}$$

which is much more cumbersome. Similarly, a fractional uncertainty should be expressed as:

$$2.21 \times 10^{-6} \text{ kg } \pm 0.023$$

and *not*

$$(2.21 \text{ kg } \pm 0.023) \times 10^{-6}$$

which, in fact, is not the equivalent statement.

5.3 Significant Figures

A former staff member of the Department of Astronomy and Physics (who shall remain nameless) left a sign in the Burke-Gaffney Observatory (the dome on top of the Loyola Residence) with the remarkably precise coordinates for the Observatory of 44°37′45″2145 N, 63°44′49″4671 W. Surely the person was just trying to be helpful, but unfortunately displayed no sense whatever of *significant figures*. Quoting a precision to the nearest *ten thousandth* of an arcsecond (corresponding to the nearest 3 mm on the surface of the Earth) begs the question: "To which tuft of carpet do these coordinates refer?". Obviously for every measurement taken, there is an appropriate number of significant figures one can quote reasonably, and this number is intimately tied to the uncertainty of the measurement.

Uncertainties may be expressed with one or two significant figures, but no more. The last significant figure in the experimental quantity should correspond to the last significant figure in the absolute uncertainty. For example:

4.2316 ± 0.03	has too many significant figures;
4.2316 ± 0.0312	has too many significant figures in the uncertainty;
4.2 ± 0.03	has too few significant figures;
4.23 ± 0.03	is just right;
4.232 ± 0.031	is OK too.

While the final results should be expressed with the appropriate number of significant figures, *intermediate steps should be retained with all the precision your calculator permits*. Rounding off each and every step of the calculation can lead to significant "round-off errors" which could grow significantly larger than the uncertainty itself.

5.4 Propagating Uncertainties

Invariably, one is asked to convert the raw data with their associated uncertainties into a result attained by "plugging" the data into specified equations. In order to express the final result with an associated uncertainty, one has to propagate the uncertainties through the relevant equations. For this, there are two primary rules we will follow in this lab:

Rule 1: When *adding* or *subtracting* uncertain quantities,
 add the ABSOLUTE uncertainties.

Rule 2: When *multiplying* or *dividing* uncertain quantities,
 add the FRACTIONAL (or PERCENTAGE) uncertainties.

Let's look at how these two rules are used mathematically. Suppose we have two uncertain measurements: $q \pm \Delta q$ and $r \pm \Delta r$. According to rule 1:

$$\Delta(q+r) \; = \; \Delta q + \Delta r; \qquad \Delta(q-r) \; = \; \Delta q + \Delta r. \tag{I.1}$$

Notice the propagated absolute uncertainty is the same regardless of whether we are taking a sum or a difference. *In particular, notice that $\Delta(q-r) \neq \Delta q - \Delta r$!*

Now introduce a third uncertain quantity, $s \pm \Delta s$. It follows from rule 1 that:

$$\Delta(q+r+s) \; = \; \Delta q + \Delta r + \Delta s; \qquad \Delta(q-r-s) \; = \; \Delta q + \Delta r + \Delta s,$$

etc. You can see how things would go if we had four or more terms: just *add* all the absolute uncertainties regardless of whether the term is being added or subtracted.

Next, according to rule 2, the *fractional* uncertainty in qr and q/r are given by:

$$f_{qr} \; = \; \frac{\Delta(qr)}{qr} \; = \; \frac{\Delta q}{q} + \frac{\Delta r}{r}; \qquad f_{q/r} \; = \; \frac{\Delta(q/r)}{q/r} \; = \; \frac{\Delta q}{q} + \frac{\Delta r}{r}, \tag{I.2}$$

Notice the fractional uncertainties are added regardless of whether the factors are multiplied or divided. For products/quotients of three uncertain quantities, we have:

$$\frac{\Delta(qrs)}{qrs} \; = \; \frac{\Delta q}{q} + \frac{\Delta r}{r} + \frac{\Delta s}{s}; \qquad \frac{\Delta[q/(rs)]}{q/(rs)} \; = \; \frac{\Delta q}{q} + \frac{\Delta r}{r} + \frac{\Delta s}{s},$$

etc. Again, the generalisation to four or more factors is clear: just add up all the fractional uncertainties.

Powers of uncertain quantities, such as q^n, can be handled just like products. For $n = 2$, we have $q^2 = qq$ and thus:

$$\frac{\Delta(q^2)}{q^2} \;=\; \frac{\Delta(qq)}{qq} \;=\; \frac{\Delta q}{q} + \frac{\Delta q}{q} \;=\; 2\frac{\Delta q}{q},$$

Similarly, for $n = 3$, $q^3 = qqq$ and:

$$\frac{\Delta(q^3)}{q^3} \;=\; \frac{\Delta q}{q} + \frac{\Delta q}{q} + \frac{\Delta q}{q} \;=\; 3\frac{\Delta q}{q},$$

and so on. Thus, in general, the uncertainty in the quantity q^n is given by:

$$\frac{\Delta(q^n)}{q^n} \;=\; n\frac{\Delta q}{q}, \tag{I.3}$$

which can be extended to apply even for non-integer values of n. Thus, for $n = \frac{1}{2}$, we have:

$$\frac{\Delta(\sqrt{q})}{\sqrt{q}} \;=\; \frac{1}{2}\frac{\Delta q}{q}. \tag{I.4}$$

Occasionally, the two basic rules aren't enough. For example, what is the uncertainty of an arbitrary function such as the sine, cosine, or even log of an uncertain quantity? Our answer comes from the calculus.

The first derivative of a function, $f(x)$, is written:

$$f'(x) \;=\; \frac{df(x)}{dx}.$$

Now, "$df(x)$" is the infinitesimal change in $f(x)$ for the corresponding infinitesimal change in x, namely "dx". Let us replace the infinitesimal changes with their "macroscopic" counterparts, namely the rise, $\Delta f(x)$, and the run, Δx. Thus, write:

$$f'(x) \;\approx\; \frac{\Delta f(x)}{\Delta x},$$

and solve for $\Delta f(x)$, the uncertainty in $f(x)$:

$$\Delta f(x) \;\approx\; f'(x)\,\Delta x. \tag{I.5}$$

For example, suppose $\theta = 32 \pm 1°$, and we want to know what $\cos\theta$ is with an uncertainty. Since the derivative of a cosine is a sine[3], we have from equation (I.5):

$$\Delta(\cos\theta) \;=\; \sin\theta\,\Delta\theta.$$

---

[3]Of course, the derivative of the cosine is actually minus the sine, but we are only interested in the absolute value of the differences when determining the uncertainties.

For $\theta = 32 \pm 1°$, $\sin\theta = 0.5299$, $\Delta\theta = 0.0175$ rad (angles outside trig functions are *always* expressed in radians, *never* degrees!), and thus $\Delta(\cos\theta) = 0.0093$. Since $\cos\theta = 0.8480$, we report:

$$\cos(32 \pm 1°) = 0.8480 \pm 0.0093 \approx 0.848 \pm 0.009.$$

Example 1: Suppose $m = 3.21 \pm 0.02$ kg, $d = 14.7 \pm 0.2$ m, $t = 29.5 \pm 0.3$ s, $E = 0.90 \pm 0.03$ J, and $r = 3.95 \pm 0.05$ m. Evaluate the following expression propagating all uncertainties:

$$F = \frac{md}{t^2} + \frac{E}{r} \tag{I.6}$$

Solution: There are at least two ways to tackle this problem. The first way, and what is often followed in this manual, is to develop an algebraic expression for the final uncertainty before any numbers are used. To this end, we see that the right hand side of equation (I.6) has two terms, and so we start by letting:

$$A = \frac{md}{t^2}; \quad B = \frac{E}{r}.$$

Then equation (I.6) becomes $F = A + B$ and from equation (I.1), we have:

$$\Delta F = \Delta A + \Delta B. \tag{I.7}$$

Now, ΔB is a quotient of two factors, and thus from rule 2 [equation (I.2)]:

$$\frac{\Delta B}{B} = \frac{\Delta E}{E} + \frac{\Delta r}{r} \quad \Rightarrow \quad \Delta B = \frac{E}{r}\left(\frac{\Delta E}{E} + \frac{\Delta r}{r}\right), \tag{I.8}$$

while ΔA has three factors, and thus:

$$\frac{\Delta A}{A} = \frac{\Delta m}{m} + \frac{\Delta d}{d} + \frac{\Delta t^2}{t^2} \quad \Rightarrow \quad \Delta A = \frac{md}{t^2}\left(\frac{\Delta m}{m} + \frac{\Delta d}{d} + 2\frac{\Delta t}{t}\right), \tag{I.9}$$

using equation (I.3) for the fractional uncertainty of t^2. Substituting equations (I.9) and (I.8) into (I.7), we get:

$$\Delta F = \frac{md}{t^2}\left(\frac{\Delta m}{m} + \frac{\Delta d}{d} + 2\frac{\Delta t}{t}\right) + \frac{E}{r}\left(\frac{\Delta E}{E} + \frac{\Delta r}{r}\right). \tag{I.10}$$

Equation (I.10) looks a bit nasty but take heart; it's a "worst-case-scenario". All expressions in the theory sections of these labs are no worse and usually simpler to deal with than equation (I.6).

To complete the problem, use equation (I.6) to evaluate F and equation (I.10) to evaluate the propagated uncertainty, ΔF. Thus,

$$F = \frac{(3.21\,\text{kg})(14.7\,\text{m})}{(29.5\,\text{s})^2} + \frac{(0.90\,\text{J})}{(3.95\,\text{m})} = 0.2820\,\text{N}$$

$$\Delta F = \frac{(3.21\,\text{kg})(14.7\,\text{m})}{(29.5\,\text{s})^2}\left(\frac{0.02}{3.21} + \frac{0.2}{14.7} + 2\frac{0.3}{29.5}\right) + \frac{0.90\,\text{J}}{3.95\,\text{m}}\left(\frac{0.03}{0.90} + \frac{0.05}{3.95}\right) = 0.0127\,\text{N},$$

and we'd report $F = 0.282 \pm 0.013$ N (or $F = 0.28 \pm 0.01$ N).

The second way to propagate uncertainties is to substitute the uncertain numbers directly into the original expression [e.g., equation (I.6)], and manipulate the uncertainties along with the numbers according to the two rules of uncertainty propagation. This method is advised only after you have become good at handling the numbers efficiently and accurately. Its disadvantage is that it is easy to make mistakes often requiring a whole slew of arithmetic operations to be repeated. The advantage is it avoids algebraic derivations such as equation (I.10). Thus:

$$
\begin{aligned}
F &= \frac{(3.21 \pm 0.02\,\text{kg})(14.7 \pm 0.2\,\text{m})}{(29.5 \pm 0.3\,\text{s})^2} + \frac{(0.90 \pm 0.03\,\text{J})}{(3.95 \pm 0.05\,\text{m})} \\[2mm]
&= \frac{(3.21\,\text{kg} \pm 0.0062)(14.7\,\text{m} \pm 0.0136)}{(29.5\,\text{s} \pm 0.0102)(29.5\,\text{s} \pm 0.0102)} + \frac{(0.90\,\text{J} \pm 0.0333)}{(3.95\,\text{m} \pm 0.0127)} \\[2mm]
&= (0.0542\,\text{N} \pm 0.0402) + (0.2278\,\text{N} \pm 0.0460) \\[2mm]
&= (0.0542 \pm 0.0022\,\text{N}) + (0.2278 \pm 0.0105\,\text{N}) \\[2mm]
&= 0.2820 \pm 0.0127\,\text{N} \\[2mm]
&= 0.282 \pm 0.013\,\text{N},
\end{aligned}
$$

as before. In the second line, absolute uncertainties are converted to fractional uncertainties, and are distinguishable from absolute uncertainties only by the positioning of the units. In the third line, fractional uncertainties are added together and then converted back to absolute uncertainties in the fourth line. Finally, the absolute uncertainties are added in the fifth line and rounded off appropriately in the sixth and final line.

Example 2: Suppose $g = 9.81 \pm 0.01\,\text{m}\,\text{s}^{-2}$, $S = 1.05 \pm 0.02\,\text{m}$, and $\theta = 12.0 \pm 0.5°$. Evaluate the following expression propagating all uncertainties:

$$v = \sqrt{gS\sin\theta}$$

Solution: First, determine the uncertainty in $\sin\theta$ using equation (I.5):

$$\sin(12.0 \pm 0.5°) = \sin 12.0° \pm \cos 12.0°\frac{\pi}{180}0.5° = 0.2079 \pm 0.0085 = 0.2079 \pm 4.11\%,$$

where the factor $\pi/180$ converts $0.5°$ to radians. Note that $\sin\theta$ *has no units*, and thus we cannot use the positioning of units to distinguish between fractional and absolute uncertainties. Instead, we can use either the (frac. unc.) designation introduced in § 5.2, or percentage uncertainties which are distinguishable from absolute uncertainties by the % sign. Here, we choose the latter. Thus,

$$
\begin{aligned}
v &= [(9.81 \pm 0.01\,\mathrm{m\,s^{-2}})(1.05 \pm 0.02\,\mathrm{m})(0.2079 \pm 0.0085)]^{1/2} \\
&= [(9.81\,\mathrm{m\,s^{-2}} \pm 0.10\%)(1.05\,\mathrm{m} \pm 1.90\%)(0.2079 \pm 4.11\%)]^{1/2} \\
&= [2.141\,\mathrm{m^2 s^{-2}} \pm 6.11\%]^{1/2} \\
&= 1.463\,\mathrm{m\,s^{-1}} \pm 3.06\% \\
&= 1.463 \pm 0.0448\,\mathrm{m\,s^{-1}} \\
&= 1.46 \pm 0.04\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

In the second line, absolute uncertainties are converted to percentage uncertainties (fractional uncertainties times 100), and these are then combined in the third line. The square root is performed in the fourth line using equation (I.4) and the percentage uncertainty is converted to an absolute uncertainty in the fifth line. The answer is rounded off to an appropriate number of significant figures in the sixth and final line.

5.5 Comparing Uncertainty with Error

The whole point of propagating uncertainties is to interpret your data. If, for example, you determine the acceleration of gravity to be $9.83\,\mathrm{m\,s^{-2}}$ and the "accepted" value is $9.81\,\mathrm{m\,s^{-2}}$, was the experiment a success or were your results inaccurate? Or could the difference of $0.02\,\mathrm{m\,s^{-2}}$ you found be significant? Without an estimate of your uncertainty, you cannot answer these questions, and thus the value of your experiment is substantially reduced.

By propagating your uncertainties, you can address all these questions. Suppose your propagated uncertainty for your estimated value of g were $0.04\,\mathrm{m^{-2}}$. Thus, you report the acceleration of gravity to be $9.83 \pm 0.04\,\mathrm{m\,s^{-2}}$. Since the error (*i.e.*, $9.83 - 9.81 = 0.02$) is *less* than the uncertainty (*i.e.*, 0.04), then the difference between your value and the accepted value is *insignificant* and your value agrees with the accepted value *to within experimental uncertainty*. On the other hand, if your uncertainty were 0.01, then the error is *greater* than the uncertainty and the error is *significant*. Thus, you report a real difference between your value and the accepted value. In this case, it is the responsibility of the experimentalist to determine what, if any, errors might have been committed during the lab that might have caused the discrepancy, and to follow up on these possibilities. If no errors were found, it may be possible that the experimentalist has observed a *real* effect, in which case the scientific knowledge base has been expanded. These are the results practising scientists hope for.

In general, if your measured value is $q_{\mathrm{exp}} \pm \Delta q_{\mathrm{exp}}$, and the accepted value is $q_{\mathrm{acc}} \pm \Delta q_{\mathrm{acc}}$[4], then the final test you make of your experiment is the following:

---

[4]In this lab, the uncertainty of the "accepted value" is often taken as zero since, in general, it will usually be true that $\Delta q_{\mathrm{acc}} \ll \Delta q_{\mathrm{exp}}$

1. Determine the experimental error: $\epsilon = |q_{\mathrm{exp}} - q_{\mathrm{acc}}|$

2. Compute the total uncertainty: $\Delta q = \Delta q_{\mathrm{exp}} + \Delta q_{\mathrm{acc}}$.

3. a) If $\epsilon < \Delta q$, you declare the following:

   The results confirm the accepted value to within experimental uncertainty.

   b) If $\epsilon > \Delta q$, you declare the following:

   The results do not *confirm the accepted value to within experimental uncertainty.*

If you find a significant difference in your experimental results (your "error" is greater than your uncertainty), *don't conclude that your results confirmed or were "pretty close to" the accepted value!* Instead, declare the discrepancy and look for possible reasons for this discrepancy. **You will not be graded low because your results didn't agree with the so-called accepted value, but you will be graded low if you make false conclusions!**

The Prime Directive of Experimental Science:

NEVER CONCLUDE
WHAT YOU DO NOT FIND!

5.6 Exercises

Answers to the following exercises are found in App. D.

1. Convert the following absolute uncertainties to fractional uncertainties.

   a) 43.2 ± 0.1 m.
   b) $(2.0613 \pm .0011) \times 10^{-6}$ kg.
   c) -5.639 ± 0.031 s.

2. Convert the following percentage uncertainties to absolute uncertainties.

   a) $2063.$ N $\pm 4.3\%$
   b) 6.07214×10^{-15} J $\pm 0.031\%$
   c) $-19.3°$C $\pm 12\%$

3. Express the following with an appropriate number of significant figures.

   a) 17.3 ± 0.02 m
   b) 6.15392 ± 0.03419 s
   c) 57.31 K ± 0.05

d) $20\,\mathrm{N} \pm 0.03\%$

4. Propagate the following uncertainties:

a) Let $m = 4.32 \pm 0.01\,\mathrm{kg}$, $d = 63.25 \pm 0.2\,\mathrm{m}$, $t = 17.2 \pm 0.1\,\mathrm{s}$, $E = 1.1 \pm 0.1\,\mathrm{J}$, and $r = 4.21 \pm 0.01\,\mathrm{m}$. Evaluate the following expression propagating all uncertainties:

$$F = \frac{md}{t^2} + \frac{E}{r}$$

b) Let $d_1 = 6.31 \pm 0.01\,\mathrm{m}$, $d_2 = 6.42\,\mathrm{m} \pm 0.01$, $d_3 = 3.15\,\mathrm{m} \pm 0.02$, $t_f = 14.2 \pm .1\,\mathrm{s}$, and $t_i = 3.6\,\mathrm{s} \pm 1\%$. Evaluate the following expression propagating all uncertainties:

$$v = \frac{d_1 + d_2 + d_3}{t_f - t_i}$$

c) Let $F = 3.62 \pm 0.01\,\mathrm{N}$, $x = 1.55 \pm 0.05\,\mathrm{m}$, and $\theta = 44 \pm 1°$. Evaluate the following expression propagating all uncertainties:

$$W = Fx\cos\theta$$

5. Compare the following values of theoretical *vs.* experimental results, and state whether each experimental result agrees or disagrees with the theoretical value.

theory	experiment	agree or disagree?
$9.81\ \mathrm{m\,s^{-2}}$	$9.79 \pm 0.01\ \mathrm{m\,s^{-2}}$	
$331.5\ \mathrm{m\,s^{-1}}$	$351.4\ \mathrm{m\,s^{-1}} \pm 0.074$	
$1.616 \times 10^{-25}\ \mathrm{\AA}$	$1.36 \times 10^{-25} \pm 0.03 \times 10^{-24}\ \mathrm{\AA}$	
$2.0\ \mathrm{kg}$	$2.0 \pm 1.0\ \mathrm{g}$	

6. Graphical Analysis

6.1 Drawing a graph

Graphical analysis techniques are used to identify trends in your data, to suggest relationships between variables, and to identify sources of error. You should take great care in presenting your data in graphical form so the reader can understand your experimental results at a glance. The purpose of this section is to indicate an acceptable format for graphs in the lab notebook, to practise generating graphs, and to perform some rudimentary graphical analysis on some sample data.

In an experiment to measure the spring constant of a spring, we hang various masses from a vertical spring and measure the distance the spring stretches in each case. Force balance requires that $kx = Mg$, and thus,

$$x = \frac{g}{k}M,$$

is the linear relationship we are testing. On an x *vs.* M plot, we expect our data to follow a straight line with slope $m = g/k$ and pass through the origin. (Note we are using lower-case m for the slope, and upper case M for the masses.) Suppose the data gathered in this mock-experiment are given in the following table:

M (kg)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x ($\pm$ 0.002 m)	0.012	0.022	0.035	0.040	0.063	0.074	0.086	0.097

Figure I.1 displays these tabulated data in a perfectly acceptable graph, which will be referred to throughout this section. The essentials of an acceptable graph include:

- a useful title;

- all axes labelled and annotated;

- all straight lines drawn with a ruler;

- as is the case for everything in the lab notebook, all graphs are drawn in ink;

- graphs should be drawn on proper graph paper (and not, for example, blank, lined, or quadrille paper);

- a graph should use as much of the page as practical; and

- data points should be plotted with "error bars" (see below) where they are larger than the symbol being used.

An example of a useless title is "Position *vs.* mass". This is useless, since presumably this can be gleaned from the labels on the axes. An example of a useful title for the same plot is "Spring distortion as a function of mass load", as given in Fig. I.1. Useful titles tend to be longer than useless titles, but they should still fit on one line.

Axes are labelled with the variable name and its associated units. Annotations (numbers and tick marks) should be chosen sensibly. Typically, major tick marks should be separated by multiples of 1, 2, or 5 (multiplied by some power of ten if necessary) in the units of the variable. Never use multiples of 3, 6, 7, or 9, and certainly never use fractional values (*e.g.*, 2.5, 4.327, *etc.*), as such choices make the job of interpolating between tick marks by eye much more difficult. Multiples of 4 and 8 are rarely used, but occasionally may be justified.

As emphasised in the previous section, measured data are *always* accompanied by uncertainties. Uncertainties on a graph can be represented by a symbol (*e.g.*, the capital letter
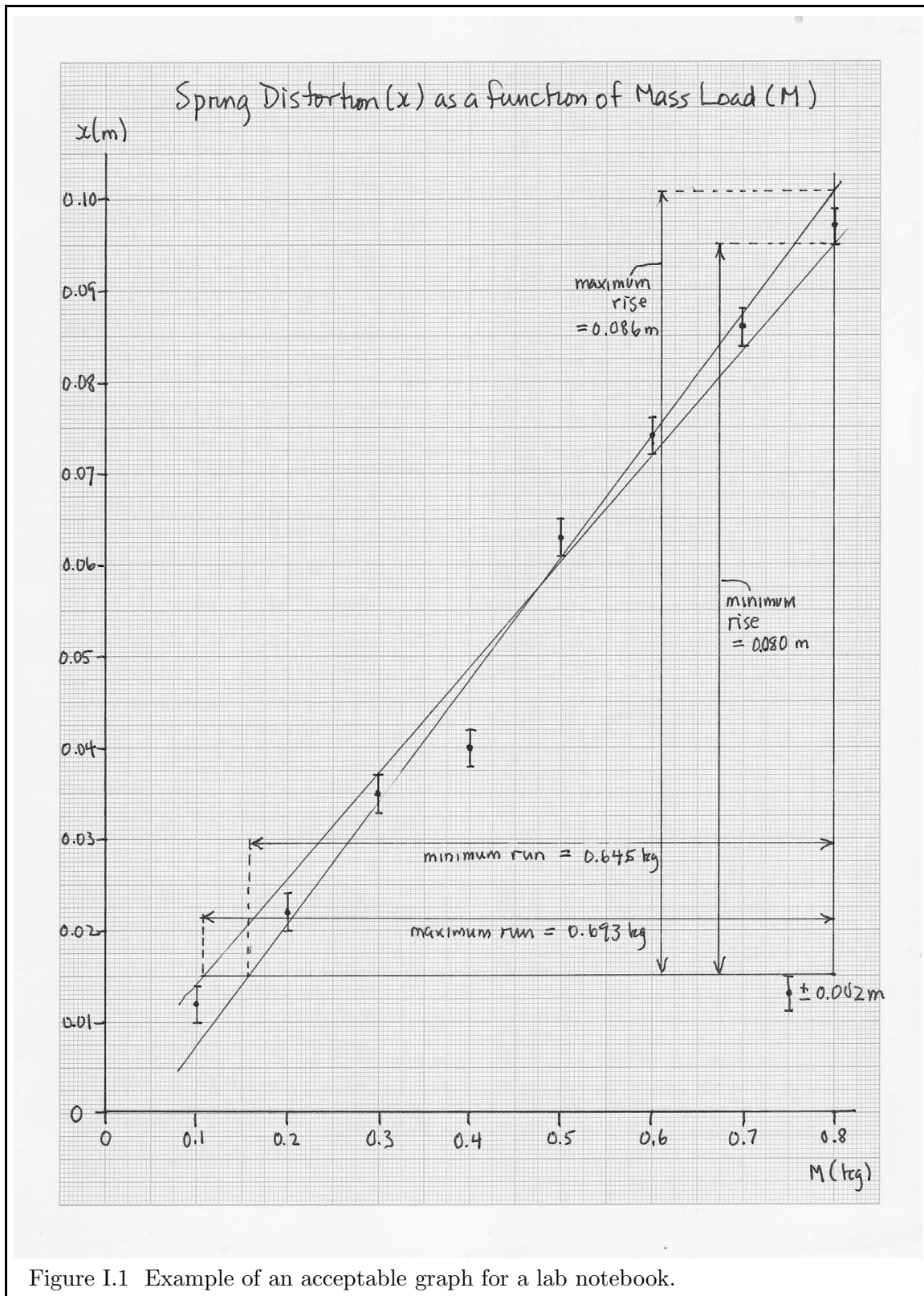
Figure I.1 Example of an acceptable graph for a lab notebook.

"I") centred on the data-point, whose span covers the limits of uncertainty, as shown in Fig. I.1. These are often called "error bars", but as they are really indicative of the uncertainty, they should probably be called "uncertainty bars". Common usage does, however, refer to them as "error bars", and so we shall (grudgingly) follow this convention.

One vital use of a graph is it allows the experimentalist to see instantly any possible errant data. The point at $m = 0.4$ kg is clearly "off", and we should go back to our experiment and remeasure that point. Perhaps the reading was supposed to have been 5.2 cm, and we errantly wrote down 4.2 cm instead. For the purpose of this example, we'll just ignore this data point from now on.

6.2 Determining the slope of a graph

Often, the "analysis" part of "Graphical Analysis" amounts to looking for linear relationships between the dependent variable (or some function of it) and the independent variable (or some function of it). When fitting a straight line through data points on a graph, there are three general guidelines to follow:

- If the data clearly do not lie on a straight line, don't force one!

- There is usually no reason to force the best-fit line to pass through the origin; it need not be anchored at the "(0,0)" point. In fact, the actual intercept may have some physical significance or may be indicative of uncertainties or errors in your experiment.

- When there is strong evidence the data are linear, draw the best fit (by eye) *straight* line through as many of the error bars as possible (not necessarily through the data points themselves). In particular, do not draw a hand-drawn wavy curve nor a connect-the-dots jagged line through the data. Best-fit straight lines have data points distributed evenly on both sides, and are most easily drawn using a transparent ruler.

Slopes are determined by measuring the largest rise and run on your best fit line, and dividing the former by the latter (slope is "rise over run"). Note that you cannot necessarily use the differences of the co-ordinates of the two extreme points, since these may or may not lie on your best fit line. Slopes are determined directly from the graph.

Determining the uncertainty of the slope depends on how large the uncertainties are and whether they can be seen on the graph. If the error bars are large enough to be drawn effectively on the graph as they are on Fig. I.1, draw two straight lines through the data. The first line is drawn with the greatest slope consistent with the data (m_{max}) and the second line is drawn with the smallest slope consistent with the data (m_{min}). While least squares techniques *can* be used, they rarely generate better answers than when an experienced person simply fits the minimum and maximum slopes on a graph by eye, measures the two slopes directly, and then reports the slope as:

$$m = \frac{m_{max} + m_{min}}{2} \pm \frac{m_{max} - m_{min}}{2}. \tag{I.11}$$

This should remind you of the "half-the-range rule" (§ 5.1).

Suppose now the error bars are too small to be drawn on the graph. If the data are supposed to lie on a straight line and the experiment went as expected, then the data will appear to lie on a straight line to within the accuracy of the graph. In this case, the slope of the data (and the uncertainty in the slope) can be computed directly from the tabulated data, with the graph serving only to verify the linear quality of the data.

The final possibility is that the error bars are too small to be plotted on the graph, and the data are supposed to follow a straight line but clearly do not. In this case, errors in the theory, gathering the data, analysing the data, or even generating the graph must be sought for, found, and corrected if possible.

Back now to the example in Fig. I.1. Minimum and maximum rises and runs are indicated on the plot (note they span most of the graph), from which minimum and maximum slopes are computed. Note that the "errant" data point has not been used in fitting the lines and lies well outside their bounds. Thus, we find:

$$m_{\text{min}} = \frac{\text{rise}_{\text{min}}}{\text{run}_{\text{max}}} = \frac{0.080}{0.693} = 0.115 \text{ m kg}^{-1}; \quad \text{and}$$

$$m_{\text{max}} = \frac{\text{rise}_{\text{max}}}{\text{run}_{\text{min}}} = \frac{0.086}{0.645} = 0.133 \text{ m kg}^{-1}.$$

Note that $m_{\text{min}} = \text{rise}_{\text{min}}/\text{run}_{\text{max}}$ and *not* $\text{rise}_{\text{min}}/\text{run}_{\text{min}}$ (think about why this should be so). Therefore, from equation (I.11) we get a slope of:

$$m = 0.124 \pm 0.009 \text{ m kg}^{-1} = 0.124 \text{ m kg}^{-1} \pm 7.3\%.$$

Since the slope is not actually k but g/k, we write $k = g/m$ and find the spring constant from m propagating all uncertainties. Taking the uncertainty of $g = 9.81 \text{ m s}^{-2}$ to be zero, we find $k = 79.1 \text{ N m}^{-1} \pm 7.3\% = 79.1 \pm 5.7 \text{ N m}^{-1}$. You should confirm all these calculations yourself, including measuring m_{min} and m_{max} from the graph, to make sure you can do this kind of analysis properly.

6.3 Exercises

Answers to the following exercises are found in App. D.

1. Consider Experiment 3, in which one is supposed to measure the acceleration of gravity. The independent coordinate is the distance (S) over which the air track rider travels, and the dependent coordinate is the time (t) it takes for the rider to travel that distance. Theoretically, one expects the two variables to be related according to:

$$S = \frac{g \sin \theta}{2} t^2,$$

$$\text{or} \quad t^2 = \frac{2}{g \sin \theta} S, \tag{I.12}$$

where g is the acceleration of gravity and θ is the angle of inclination of the air track rider (see Experiment 3 if you want more details). Suppose, in performing this experiment, the data in the following table are gathered.

S (m)	0.2	0.4	0.6	0.8	1.0	1.2	1.4
t (± 0.01 s)	0.48	0.69	0.88	0.97	1.08	1.19	1.27

On a piece of graph paper (not "quadrille" paper), plot these data, including the error bars, with the independent variable (S) on the horizontal axis and the dependent variable (t) on the vertical axis. Having graphed the data, can you spot any potentially errant data? Are you justified in throwing out these errant data? Can your data be fit to a straight line?

2. Plot a second graph for which the dependent variable is t^2. Now can you spot any errant data? Do your data points now follow a straight line? According to equation (I.12), t^2 plotted against S should follow a straight line with slope $2/(g\sin\theta)$. Thus, measure the slope on your graph along with an uncertainty and, assuming $\sin\theta = 0.174 \pm 0.003$, determine a value and an uncertainty for g.

3. An interesting exercise might be to measure the discrepancies among the students' results in the class. Everyone started off with the same data, but it is very unlikely that everyone came up with identical estimates for g. Compare your values with your neighbour's. Is the difference between your and your neighbours' values significant? *i.e.*, Is the difference larger than your estimate of the uncertainty? If so, one or both of you are in error and you should try to identify and correct the error(s) made.

Experiment 1
Measuring Density

Purpose

Ostensibly, the purpose of this experiment is to determine the density of a sample of wood, and thus identify its species. The real purpose of this experiment, however, is to demonstrate some of the basic skills an experimental physicist and engineer must master, such as:

- preparing and using a lab notebook;

- using some basic measuring devices;

- measuring and recording uncertain data;

- interpreting and analysing experimental results.

Apparatus

1. one large and one small wooden block,

2. metre stick,

3. Vernier caliper,

4. micrometer,

5. mass balance.

Theory

The density of a uniform object (represented by the symbol ρ, the Greek letter "rho") is given by:

$$\rho = \frac{m}{V},\tag{1.1}$$

where m is the mass of the object and V its volume. The SI units for density are $\mathrm{kg\,m^{-3}}$.

species	density ($kg\,m^{-3}$)
red cedar	380
willow	420
Canadian spruce	450
European redwood	510
Oregon pine	535
sycamore	590
white ash	670
maple	755
water (at 4°C)	1,000

Table 1.1 Densities of various species of wood, with water included for comparison. Values are taken from `http:// www.simetric.co.uk`.

The volume of a rectangular wooden block is given by multiplying together its dimensions, namely its length, l, width, w, and height, h. Thus, equation (1.1) becomes

$$\rho \;=\; \frac{m}{lwh}. \tag{1.2}$$

While it is commonly known that wood floats (although ebony can be as dense as 1,120 $kg\,m^{-3}$ and would thus *sink*), the range in densities for wood may surprise some. Balsa wood, with a density of 170 $kg\,m^{-3}$, is 1/8 the density of the densest wood, Lignum Vitae, with a density of 1,370 $kg\,m^{-3}$. Table 1.1 gives the densities of a sample of common wood species, and you will use this to identify the variety of wood in your blocks.

Given that m, l, w, and h are all uncertain quantities, ρ must also be an uncertain quantity and we should look at how the uncertainties in the measured quantities propagate to give us $\Delta\rho$. Rule 2 in § 5.4 of the *Introduction* tells us to *add* the fractional uncertainties of all quantities being multiplied or divided in a given term. Since the right hand side of equation (1.2) has just one term with four factors, we can immediately write down:

$$\frac{\Delta\rho}{\rho} \;=\; \frac{\Delta m}{m} + \frac{\Delta l}{l} + \frac{\Delta w}{w} + \frac{\Delta h}{h}. \tag{1.3}$$

Procedure

This lab requires the use of both the Vernier caliper and the micrometer; devices for measuring lengths with a fair degree of precision. You will be given instruction on their use during the lab, which will follow the discussion in App. C.

1. Measure the masses of both blocks of wood using the mass balance, being sure to record an uncertainty with each measurement.

What do I use for the uncertainty? At first glance, one might record half or even less of the smallest gradation on the balance as the uncertainty. Thus, if the smallest gradation is 1 g, a "first-guess" of the uncertainty might be $\pm\,0.5$ g or even $\pm\,0.2$ g depending on how well you think you can interpolate between the finest gradations. However, there are other factors to consider as well. How well did the scale balance? How much can you change the mass reading and still have the scale "look" balanced? Does shifting the mass in the tray slightly affect the balance? These tests and others might result in you recording a greater uncertainty than just half the smallest gradation. However you decide what to use as the uncertainty, be sure to make some note of this with your reading.

2. Measure the dimensions of the large block twice: once with the metre stick, once with the Vernier caliper. Be sure to record all uncertainties with the measurements.

3. Measure the dimensions of the small block twice: once with the Vernier caliper, once with the micrometer. Be sure to record all uncertainties with the measurements.

Analysis

1. Using equations (1.2) and (1.3), compute the density of the larger block for each set of dimension measurements, propagating all uncertainties.

2. Using equations (1.2) and (1.3), compute the density of the smaller block for each set of dimension measurements, propagating all uncertainties.

Conclusions

1. Do the two densities measured for each block agree to within experimental uncertainty? If not, check the obvious possibilities, such as bad arithmetic, slipped data entries, *etc.* Time permitting, you might even go back and double-check some of your measurements. Failing this, can you identify any possible source of error or uncertainty you may not have considered as you were doing the measurements? ("Human error" is *never* an acceptable answer to a question such as this.)

2. Using Table (1.1), what is your best guess of the species of wood in each block? Are you able to identify the species of wood uniquely by all your density measurements, or was the uncertainty of some measurements large enough to make more than one identification possible? If so, which ones?

Additional discussion question

1. What are the advantages and limitations of each of the measuring devices used?

Experiment 2
Equilibrium—Adding Force Vectors in Two Dimensions

Purpose

The purpose of this experiment is to verify Newton's Second Law of Motion, $\sum \vec{F} = m\vec{a}$, when the net acceleration of the system is zero.

Apparatus

1. force table,

2. several masses and mass holders,

3. mass balance,

4. cardboard square,

5. protractor.

Theory

Force is a vector quantity and thus has magnitude and direction. If several forces, $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, *etc.*, act simultaneously on a mass m, the resultant force $\vec{F}_R$ is equal to the vector sum of the individual forces and accelerates the mass according to Newton's Second Law of Motion, $\vec{F}_R = m\vec{a}$. In the particular case of an object in equilibrium, $\vec{a} = 0$, and hence $\vec{F}_R = 0$. That is, when an object is in equilibrium, the vector sum of all forces acting on it is identically zero.

Figure 2.1 shows three forces acting on a central body (a ring). The resultant force can be depicted graphically by drawing the vectors head to tail (the order in which the vectors are added is not important). The resultant force is represented by the vector whose tail coincides with the tail of the first vector drawn, and whose head coincides with the head of the final vector drawn, as shown in Fig. 2.2.

Figure 2.1 Three forces acting on a central object (ring).
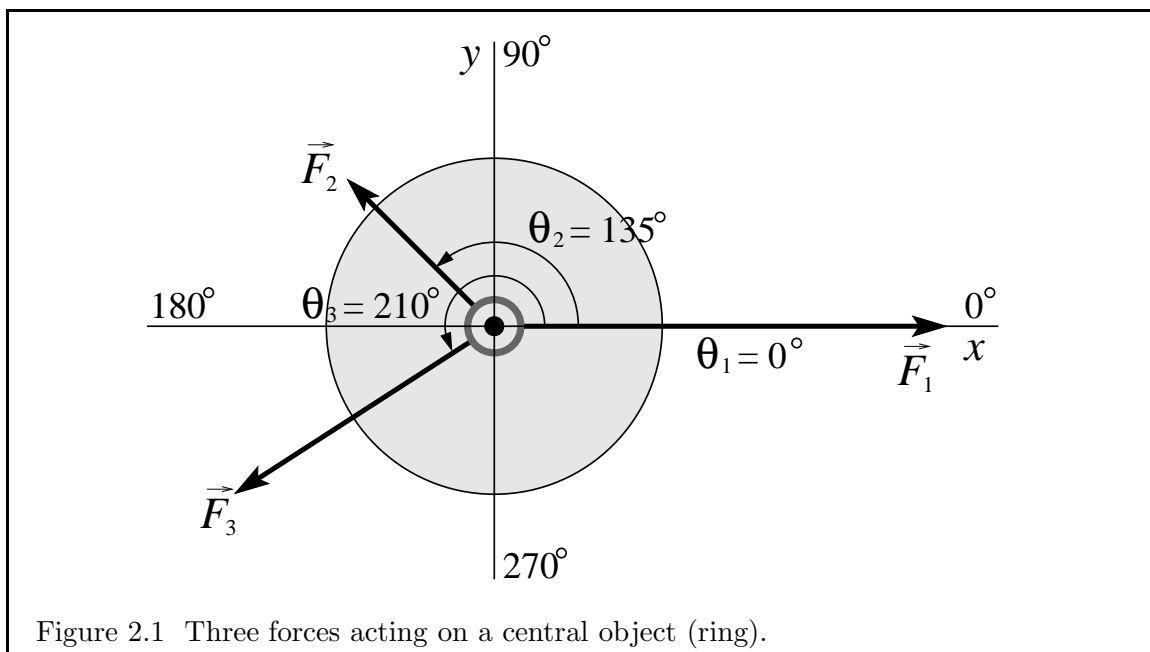
Forces, like any vectors, can be added together by resolving them into components and then adding the x-components together and the y-components together to obtain the x- and y-components of the resultant vector. For example, suppose the three forces in Fig. 2.1 are: $\vec{F_1} = 1.12 \pm 0.01$ N directed at $\theta_1 = 0 \pm 1°$, $\vec{F_2} = 0.52 \pm 0.01$ N at $\theta_2 = 135 \pm 1°$, and $\vec{F_3} = 0.83 \pm 0.01$ N at $\theta_3 = 210 \pm 1°$. We would like to resolve these forces onto an x-y coordinate system (with the x-axis directed at $0°$) and propagate their uncertainties. Let us start by examining the x-component of $\vec{F_1}$.

$$
\begin{aligned}
F_{1x} = F_1 \cos\theta &= (1.12 \pm 0.01 \text{ N}) \cos(0 \pm 1°) \\
&= (1.12 \pm 0.01 \text{ N})(\cos 0° \pm 0.0175 \sin 0°) \\
&= (1.12 \pm 0.01 \text{ N})(1 \pm 0) = 1.120 \pm 0.010 \text{ N},
\end{aligned}
$$

where we have used equation (I.5) to propagate the uncertainty in the angle ($\pm 1°$ con-

Figure 2.2 Graphical depiction of the forces in Fig. 2.1.
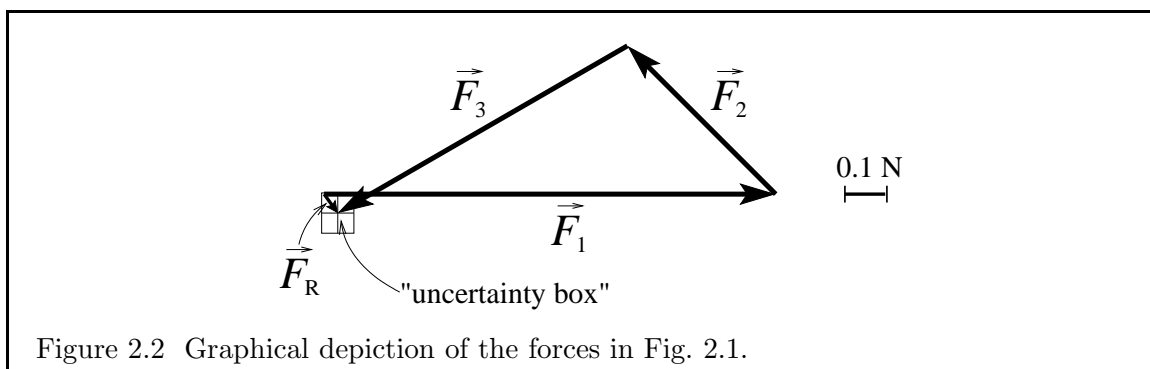
verted to $\pm 0.0175\,\text{rad}$) to an uncertainty in the cosine. Now according to equation (I.5), the uncertainty in $\cos\theta$ is proportional to $\sin\theta$ which, when $\theta = 0$, is zero! Surely the uncertainty in $\cos\theta$ can't be *zero*! In fact, equation (I.5) is an approximation that can give suspicious results when θ is any multiple of 90°, including zero. In truth, $\cos(0 \pm 1°)$ should lie *somewhere* between $\cos 0° = 1$ and $\cos \pm 1° = 0.99985$. Thus, we might report $\cos\theta = 0.99992 \pm 0.00008 = 0.99992 \pm 0.008\%$ using the "half-the-range rule". This is indeed a tiny percentage uncertainty—much smaller than the other uncertainties we will encounter—and thus we are justified in using the approximation that equation (I.5) gave us, namely $\cos\theta = 1 \pm 0$.

Now what of the y-component of $\vec{F}_1$?

$$
\begin{aligned}
F_{1y} &= F_1 \sin\theta = (1.12 \pm 0.01 \text{ N}) \sin(0 \pm 1°) \\
&= (1.12 \pm 0.01 \text{ N})(\sin 0° \pm 0.0175 \cos 0°) \\
&= (1.12 \pm 0.01 \text{ N})(0 \pm 0.0175)
\end{aligned}
$$

To proceed, we need to convert the absolute uncertainties into fractional uncertainties, which poses an immediate problem: How does one convert 0 ± 0.0175 into a fractional uncertainty? Formally, $0.0175/0 = \infty$, which makes no sense. What has gone wrong?

We have to be mindful of the assumptions that went into the expressions using fractional uncertainties. Rule 2 on page 12 assumes $\Delta q \ll q$, which clearly is not the case when $q = 0$! In physics, we can rarely just blindly plug-and-chug into formulæ; we always have to *think* about what we are doing. In this case, because we have violated the assumption that $\Delta q \ll q$, we have run into trouble when we blindly use the results of that assumption.

Instead, let us determine the *maximum* and *minimum* values of the y-component consistent with these data. The most negative our y-component can be is $(1.12 + 0.01 \text{ N})(0 - 0.0175) = -0.020 \text{ N}$ and the most positive is $(1.12 + 0.01 \text{ N})(0 + 0.0175) = 0.020 \text{ N}$. Thus, we should quote our y-component as:

$$
F_{1y} = 0 \pm 0.020 \text{ N}.
$$

Calculating the components of $\vec{F}_2$ and $\vec{F}_3$ is more straight forward since none of the angles are a multiple of 90°. These are given below:

$$
\begin{aligned}
F_{2x} &= (0.52 \pm 0.01 \text{ N}) \cos(135 \pm 1°) = -0.368 \pm 0.014 \text{ N} \\
F_{2y} &= (0.52 \pm 0.01 \text{ N}) \sin(135 \pm 1°) = 0.368 \pm 0.014 \text{ N} \\
F_{3x} &= (0.83 \pm 0.01 \text{ N}) \cos(210 \pm 1°) = -0.718 \pm 0.016 \text{ N} \\
F_{3y} &= (0.83 \pm 0.01 \text{ N}) \sin(210 \pm 1°) = -0.415 \pm 0.018 \text{ N}
\end{aligned}
$$

Note that an extra significant figure has been carried in all components as these are intermediate results, and we wish to minimise the effect of round-off errors on the final results.

Preparation question 1: Verify that the components of $\vec{F}_2$ and $\vec{F}_3$ are given as above.

The components of the resultant force are obtained by adding together the components of the individual forces. Thus:

$$\begin{aligned}
F_{Rx} &= F_{1x} + F_{2x} + F_{3x} &= 0.034 \pm 0.040 \text{ N} \\
F_{Ry} &= F_{1y} + F_{2y} + F_{3y} &= -0.047 \pm 0.052 \text{ N}.
\end{aligned} \tag{2.1}$$

Since both components are consistent with 0, we would conclude that to within experimental uncertainty, the force vectors summed to zero (as expected for a system in equilibrium). Note that if even *one* component were not consistent with zero, we would have to claim that our forces did *not* sum to zero to within experimental uncertainty, and then possibly search for reasons why they didn't.

In Fig. 2.2, the propagated uncertainties are depicted by an "uncertainty box" located at the tip of $\vec{F}_3$ with a width of 0.08 N (to represent the uncertainty in F_{Rx}, namely ± 0.040 N) and a height of 0.10 N (to represent the uncertainty in F_{Ry}, namely ± 0.052 N). If we construct our force diagram carefully enough, the vector $\vec{F}_R$ drawn from the tail of $\vec{F}_1$ to the tip of $\vec{F}_3$ should have the components given by equation (2.1). Further, if to within experimental uncertainty we found our forces added to zero, then all of $\vec{F}_R$ should lie within the uncertainty box. Conversely, if we found our forces did not add to zero to within experimental uncertainty, the tail of $\vec{F}_R$ should lie *outside* the uncertainty box.

Procedure

On the apparatus shown in Fig. 2.3, forces $\vec{F}_1$, $\vec{F}_2$, *etc.*, are applied to a small ring by strings which pass over a pulley and to which masses are hung. The magnitude of each force is obtained by calculating the weight of the total mass hanging from the string, while the direction of each force is determined from an angular scale on the "force table". In this experiment you apply several forces to the ring and adjust the directions of the forces until the ring remains stationary and centred around a central post.

1. Examine the force table and note how both the magnitude and direction of the forces can be adjusted. The central pin serves as a reference for centring the ring and also prevents the masses from falling off in grossly unbalanced situations. The total weight on a string is the weight of the hanger plus the weight of the added mass, *which you will have to weigh using the mass balance, since the numbers written on the masses are only good to within a few grams.*

2. Begin by estimating the precision with which forces can be declared balanced. Load *two* mass hangers with equal masses (~ 100 g) and position the arms precisely at $0°$ and $180°$ using the measuring device provided (piece of "notched" cardboard) for accuracy. The masses on the hangers *including the hangers* should be as identical as possible, using the 1 and 2 gram masses as needed. The central ring should be free of the central pin and, even when the force table is tapped briskly, the central ring should not move. Now find by experiment the largest increment in mass, Δm, which, when added to one of the mass hangers, just causes the ring to drift when tapping the force table. Record

Figure 2.3 The force table with three of the four hangers in place.
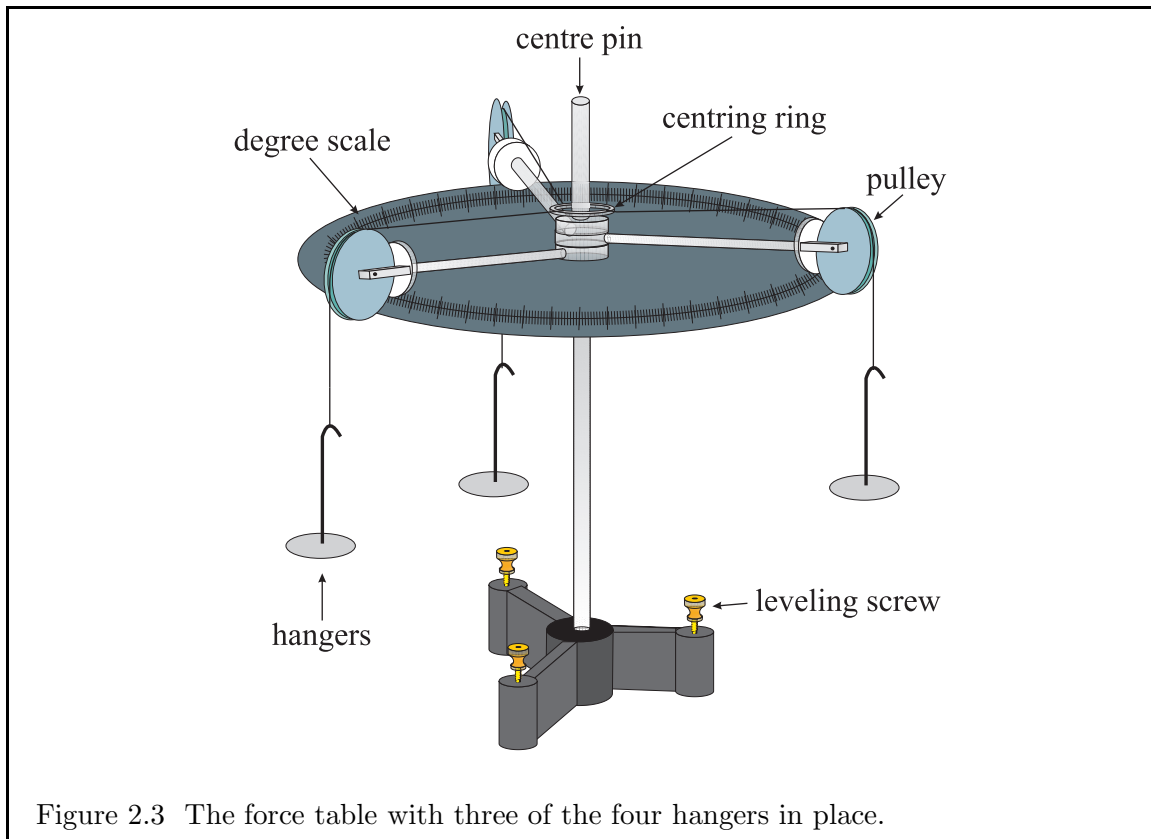
the value $\Delta w = \frac{1}{2}\Delta mg$, which is the uncertainty for all weights used in the rest of this experiment.

Preparation question 2: Why do you suppose we use $\frac{1}{2}\Delta mg$, and not just Δmg as the uncertainty in the weights?

3. Next, estimate the precision with which angles can be determined at force balance. Load *three* mass hangers with equal masses ($\sim 100\,\mathrm{g}$) and position the arms precisely at 0°, 120°, and 240°. Make certain that the strings are aimed directly at the centre and, if they are not, slide the knots around the ring until they are. The central ring should be free of the central pin and, even when the force table is tapped briskly, the central ring should not move. Leaving two of the arms fixed, nudge the third arm clockwise until tapping the force table causes the ring to drift. Record the angular position of the arm. Return the arm to its equilibrium position, then nudge it counterclockwise until tapping the force table once again causes the ring to drift. Record this second angular position of the arm. Half of the difference between the two positions is the uncertainty for all angular measures in the rest of this experiment.

PART I. Three-force experiment

4. Load three mass hangers with three *unequal* masses (*e.g.*, 100, 150, and 200 g), making sure the greatest mass is less than the sum of the other two.

Preparation question 3: Why must the greatest mass be less than the sum of the other two?

---

5. Adjust the arm directions very precisely until the ring is free of the central pin and remains centred even while tapping briskly on the force table. Record the total weight, w, hanging from each string (including the hanger!) as measured by the mass balance. Record the angular position of each arm, using the "notched" cardboard square for accuracy.

PART II. Four-force experiment

6. Add the fourth mass hanger to the force table.

7. Load all four mass hangers with unequal masses (between 50 g and 250 g) making sure the greatest mass is significantly less than the sum of the other three.

8. Adjust the arm directions until the ring remains centred and free of the central pin even while briskly tapping on the force table. Record all four masses and their positions.

Analysis

1. Resolve each force in Part I into their x- and y-components, propagating the uncertainties in both the magnitude and direction of the forces as done in the Theory section.

2. Calculate the x- and y-components of the resultant force and their uncertainties [*e.g.*, equation (2.1) in the Theory section)]. To within experimental uncertainty, is your resultant vector consistent with zero?

3. In the manner of Fig. 2.2, draw to scale your measured force vectors (magnitude and direction) without worrying about the uncertainties. Treat this diagram as you would a graph, and use a *full* sheet of graph paper taking care to represent the vectors as accurately and as large as possible. Be sure to indicate the scale used for your diagram (*e.g.*, Fig. 2.2). If the measurements and your drawing are absolutely accurate, then a closed triangle should result. However, because of experimental uncertainties, your triangle will probably be slightly open.

4. In the manner of Fig. 2.2, draw the resultant vector. If your diagram is done accurately (and big) enough, the components you computed in analysis step 2 should correspond nicely to the resultant vector you just drew.

5. Again in the manner of Fig. 2.2, draw an "uncertainty box" around the tip of the resultant vector. If, in analysis step 2, you found that the resultant vector was consistent with zero, your resultant vector should lie completely within the uncertainty box you just drew. Otherwise, not.

6. Repeat analysis steps 1–5 for the four forces in Part II.

Conclusions

1. Did analysis step 2 show that the forces were balanced in each of Parts I and II to within experimental precision? Why or why not?

2. Do your force diagrams in analysis step 5 confirm that the forces were balanced in each of Parts I and II to within experimental precision? Why or why not?

Additional discussion question

1. What, if anything, could cause the answers to the above questions to be different? In the event analysis steps 2 and 5 arrive at different conclusions (*i.e.*, one confirms force balance, the other does not), which of the two do you believe and why?

Experiment 3
Determining the Acceleration of Gravity

Purpose

The purpose of this experiment is to determine the acceleration of gravity using a linear air track.

Apparatus

1. linear air-track,

2. linear air-track rider with attached metal "flag",

3. one photogate,

4. one accessory photogate,

5. ruler,

6. Vernier caliper,

7. ten risers.

Theory

An object subject to a constant acceleration, a, will travel a distance, S, in a time, t, according to the kinematical equation of motion,

$$S \; = \; v_0 t + \frac{1}{2} a t^2, \tag{3.1}$$

where v_0 is the initial velocity. If the object starts from rest ($v_0 = 0$) on a frictionless plane inclined at an angle θ, then it will accelerate under the influence of gravity alone down the

Figure 3.1 The Inclined Plane.
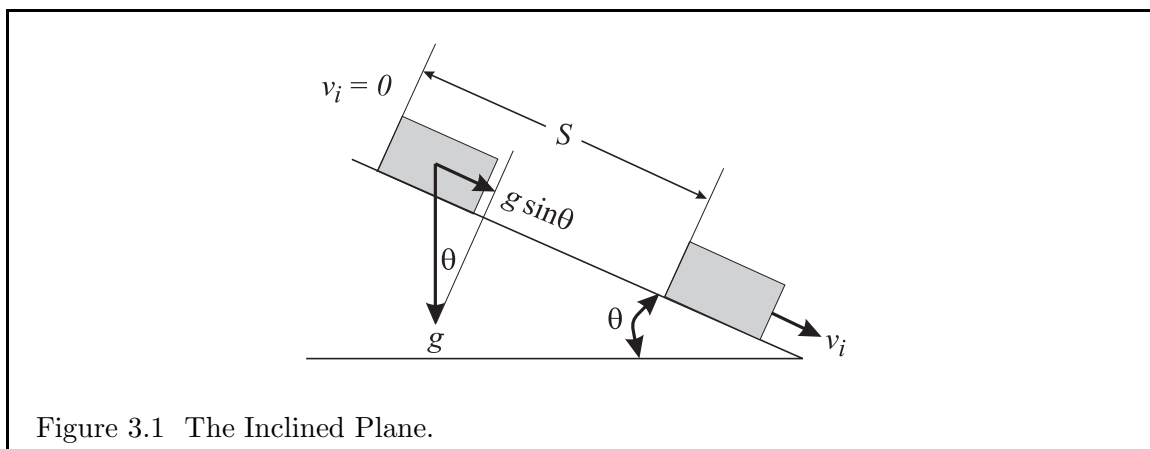
plane. The acceleration along the incline is $a = g \sin \theta$, where g is the acceleration of gravity at the Earth's surface (see Figure 3.1). Thus, equation (3.1) becomes:

$$S = \frac{g \sin \theta}{2} t^2,$$

or, rearranging to isolate t,

$$t^2 = \frac{2}{g \sin \theta} S. \tag{3.2}$$

Therefore, a plot of t^2 (not t!) vs. S should yield a straight line with a slope, m, given by:

$$m = \frac{2}{g \sin \theta}.$$

Thus, if we measure the slope from a t^2 vs. S graph, the experimentally determined acceleration of gravity, $g_{\exp}$, is given by:

$$g_{\exp} = \frac{2}{m \sin \theta}. \tag{3.3}$$

Note that the slope is not equal to g directly. Rather, the slope has to be substituted into equation (3.3) in order to obtain g.

Finally, note that the uncertainty in $g_{\exp}$, $\Delta g_{\exp}$, is given by:

$$\frac{\Delta g_{\exp}}{g_{\exp}} = \frac{\Delta m}{m} + \frac{\Delta \sin \theta}{\sin \theta}, \tag{3.4}$$

where Δm is the experimental uncertainty in the slope, and $\Delta \sin \theta$ is the experimental uncertainty in $\sin \theta$.

> **Preparation question 1:** Derive equation (3.4). To do this, you may wish to review § 5.4 of the *Introduction*. This isn't meant to be difficult; it's a two- or three-liner at most.

Procedure

1. The linear air-track apparatus should already be set up for you as depicted in Fig. 3.2. Set the airflow to maximum so that the air-track rider slides smoothly along the air-track.

Figure 3.2 The inclined linear air-track with the air-track rider in its starting position.
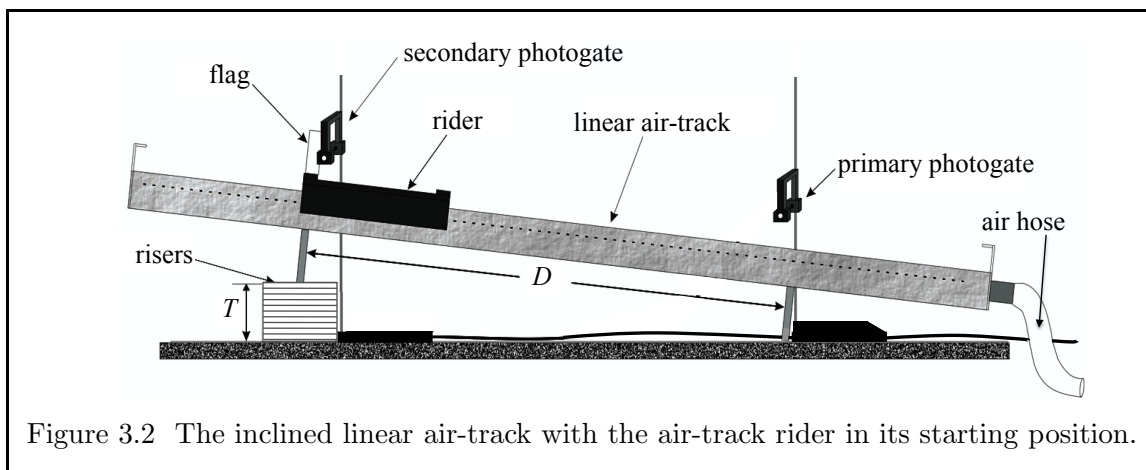
2. Release the rider near the centre of your track from rest. If the rider starts to move, the track is not level and you will need to adjust the screw on the leg with the single rubber foot. *Turning the screw clockwise raises the track.* Once the track is levelled so that the rider does not move, place the rider at another location and make sure the rider doesn't move from rest there either. If it does, your track may be slightly bent or warped, in which case you will need to select the best metre or so of track on which the rider moves the least from rest.

3. Measure the horizontal distance, D (Fig. 3.2), between the legs of the air-track where they come in contact with the table. Record this value along with its uncertainty.

4. Measure the thickness of the ten risers together at five different places using the Vernier caliper provided. Take the average of these five values as the thickness, T (Fig. 3.2), and use the "half-the-range" rule for the uncertainty. If you need to be reminded how to use a Vernier caliper, see App. C or ask your demonstrator.

5. Place the ten risers underneath the leg of the air-track with the single foot, as shown in Fig. 3.2.

6. **Using only the felt-tipped pen provided**, carefully mark eight vertical "tick-marks" *along the straightest 1.4 m of the air-track*, each precisely 0.2 m apart. These tick marks should be placed so that the rider glides over them as it slides down the track. **Do not mark the metal surface of the air track with anything other than the pens provided, as pencils and ball-point pens will damage the surface and/or mark it permanently.** The highest tick mark is the "zero-point",

followed by the 0.2, 0.4, 0.6, ..., 1.4 m positions, the latter being closest to the bottom of the track.

7. Create a table with six columns in your lab book with the headings: S, t_1, t_2, t_3, t_4, and t_5. Including the headings, this table will require 16 lines assuming the data are entered double-spaced.

8. Set the primary photogate (with the largest base) to *pulse mode*. This is the setting that causes the timer to start when the flag of the air-track rider cuts the beam of one of the photogates, and stop when the flag cuts the beam of the other.

9. With the flag on the downhill side of the rider, hold the rider so that its downhill edge is precisely at the zero-point tick mark. Then, carefully slide the secondary photogate (with the smallest base) toward the flag until either the little red light flashes on, or the timer on the primary photogate starts. The idea is that with the rider precisely placed at the zero-point, the flag cuts the photogate beam the instant the rider is released, and the timer is started. **Once the secondary photogate is positioned satisfactorily, do not move it for the rest of the experiment.**

10. Using the rider, position the primary photogate at the 0.2 m mark in the same manner as the secondary photogate was positioned at the zero-point in the previous step.

11. Return the rider precisely to the zero-point and release it from rest. The timer should then record the time, t, for the rider to slide 0.2 m. Repeat this step five times, and record each time in your data table under the columns t_1, t_2, *etc.* If any of the five times are clearly "off", repeat this step until you have five consistent (but not necessarily identical) values.

12. Repeat step 11, with the primary photogate positioned at each of the 0.4, 0.6, 0.8, ..., 1.4 m positions, and record your readings in your data table. This will give you seven data points for plotting.

Analysis

1. Calculate the sine of the angle of inclination using $\sin\theta = T/D$, and its associated uncertainty, $\Delta\sin\theta$, using equation (3.6) below. Note there is no point in calculating θ itself, since θ is never actually used, only $\sin\theta$ is used.

Preparation question 2: Show that

$$\frac{\Delta\sin\theta}{\sin\theta} = \frac{\Delta T}{T} + \frac{\Delta D}{D},\qquad(3.6)$$

where ΔT and ΔD are the measured absolute uncertainties in T and D respectively from procedure steps 3 and 4.

---

2. Create another table with five columns in your lab book with the headings: S, t, Δt, t^2, and $\Delta(t^2)$. t is the average of your five readings in your raw data table, Δt is the uncertainty in t given by the "half-the-range rule", and $\Delta(t^2)$ is the uncertainty in t^2 given by:

$$\Delta(t^2) \;=\; 2t\,\Delta t. \tag{3.7}$$

Complete this table by performing all the necessary calculations.

Preparation question 3: Derive equation (3.7).

---

3. On a page of graph paper, plot t^2 *vs.* S and include the uncertainty bars $\Delta(t^2)$.

4. Evaluate the slope, m, of the graph along with an estimate of its uncertainty (see § 6 on *Graphical Analysis* in the *Introduction*).

5. From equations (3.3) and (3.4), calculate your experimentally determined value for g, along with its associated uncertainty.

6. After completing all the analysis and when you are certain you will not need to take any further measurements, use a damp cloth to clean off all the tick marks you put on the air track with the felt marker.

Conclusions

1. Does your experimental value of g agree with the accepted local value of g ($9.81 \mathrm{~m\,s^{-2}}$) to within experimental uncertainty? If not, what sources of experimental error might account for the difference that you haven't already accounted for? (Hint: glance at the first two discussion questions.)

Additional discussion questions

1. If there were friction in the system, would it increase or decrease your experimental value of g? Explain.

2. If the rider were released *before* it cut the first photogate beam, would this increase or decrease your experimental value of g? Explain.

3. In addition to the slope of the t^2 *vs.* S plot, you might also determine the t^2-intercept. What ought this intercept be? If you found that to within experimental uncertainty, your t^2-intercept was greater than zero, what effect, if any, could this have on your experimentally determined value of g? How might a positive t^2-intercept be interpreted physically?

4. An alternate method of using an air-track to measure g consists of setting up one of the photogates in *gate mode*, where the timer starts as soon as the beam is cut and stops as soon as the same beam is restored. Thus, in this mode, the photogate measures the time it takes for the flag to pass through the beam. In such a configuration, the timer can be used to measure the speed of the air-track rider at the bottom of the incline. Demonstrate how that information could be used to measure g experimentally. Would you expect the uncertainties using this method to be greater or less than the uncertainties you obtained in this experiment?

Experiment 4
Ballistic Pendulum and Projectile Motion

Purpose

The purpose of this experiment is to verify the law of conservation of momentum in an inelastic collision using the ballistic pendulum.

Apparatus

1. ballistic pendulum (see Fig. 4.1),

2. balance and masses,

3. metre stick,

4. carbon paper,

5. masking tape.

Theory

This experiment studies the principle of conservation of momentum through a combined study of projectile motion and the motion of a simple pendulum.

Consider a projectile of mass M_1, fired horizontally from a known height y above the ground, where y is measured from the *bottom* of the projectile. According to the laws of kinematics for constant acceleration, the projectile falls to the ground in a time t given by:

$$t = \sqrt{\frac{2y}{g}}. \tag{4.1}$$

During that time, it travels a horizontal distance x, as shown in Fig. 4.2, and thus the initial horizontal velocity v_1 is given by:

$$v_1 = \frac{x}{t}. \tag{4.2}$$

Figure 4.1   The ballistic pendulum apparatus.
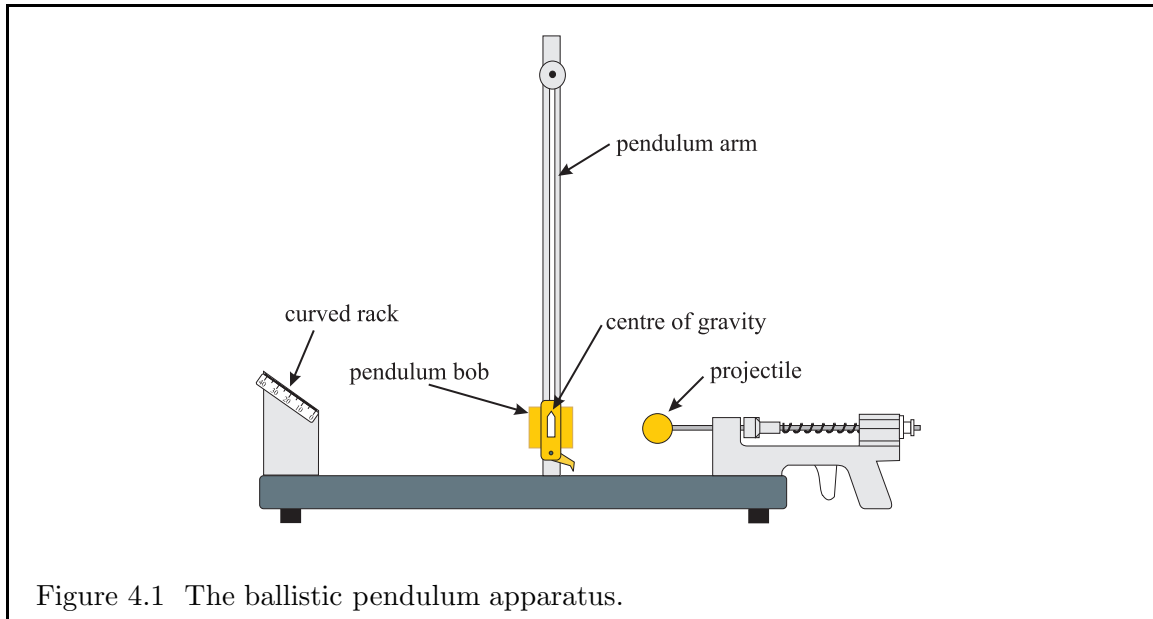
Preparation question 1: Can you think of any reason(s) why equation (4.2) might underestimate v_1, even if the measurements of x and t are accurate?

Figure 4.2   Determining the initial velocity of the projectile from its trajectory.
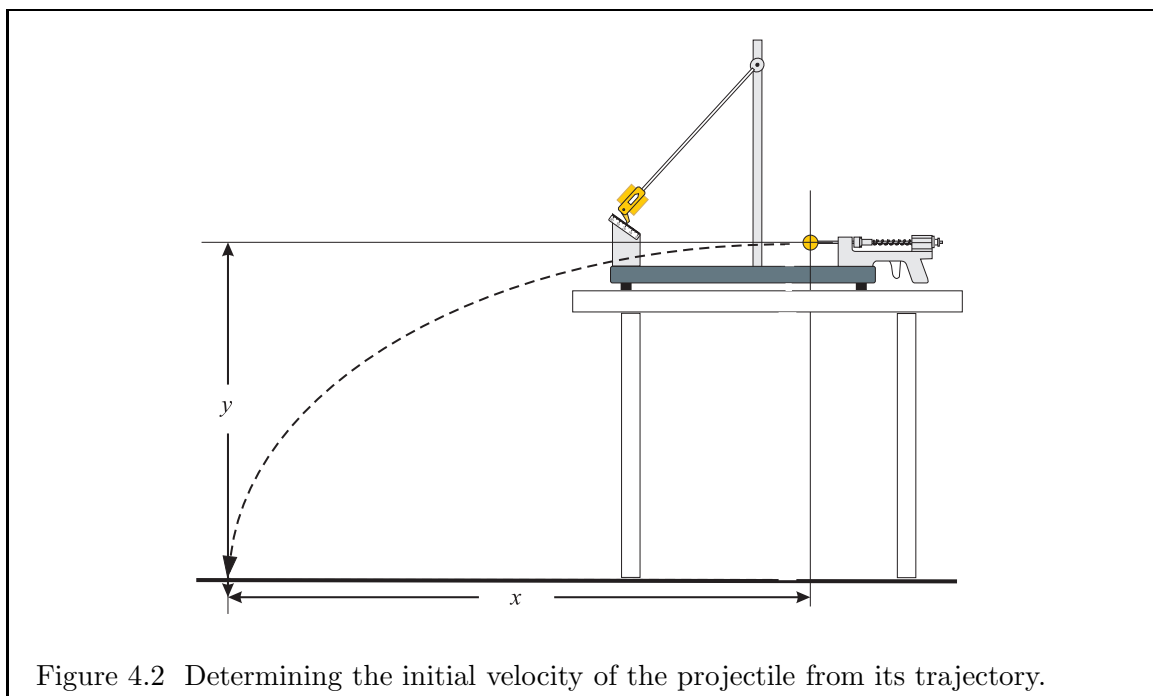
Combining equations (4.1) and (4.2) gives us:

$$v_1 = x\sqrt{\frac{g}{2y}}. \tag{4.3}$$

Preparation question 2: If $g = 9.81$ is taken to be exact, show that the uncertainty in v_1 is given by:

$$\frac{\Delta v_1}{v_1} = \frac{\Delta x}{x} + \frac{1}{2}\frac{\Delta y}{y}, \tag{4.4}$$

where Δx and Δy are respectively the absolute uncertainties in x and y.

Suppose that instead of being allowed to travel until it hits the ground, the projectile strikes and becomes attached to a pendulum bob of mass M_2 initially at rest. Immediately after this *inelastic collision*, the combined system of the bob and projectile has a mass $M_1 + M_2$, moves off with a horizontal velocity v_{12}, and begins its trajectory upward. At the bottom of the trajectory, the kinetic energy, K_i, is given by:

$$K_i = \frac{1}{2}(M_1 + M_2)v_{12}{}^2, \tag{4.5}$$

while its potential energy, U_i, is zero if we associate the bottom of the trajectory with a height of zero.

The pendulum now swings under the constraint of the arm, and comes to rest a height h above its starting position where it is held in place by a "tooth and claw" mechanism as shown in Fig. 4.3. Here, the kinetic energy, K_f, is zero (because it has come to rest), and the potential energy, U_f, is given by:
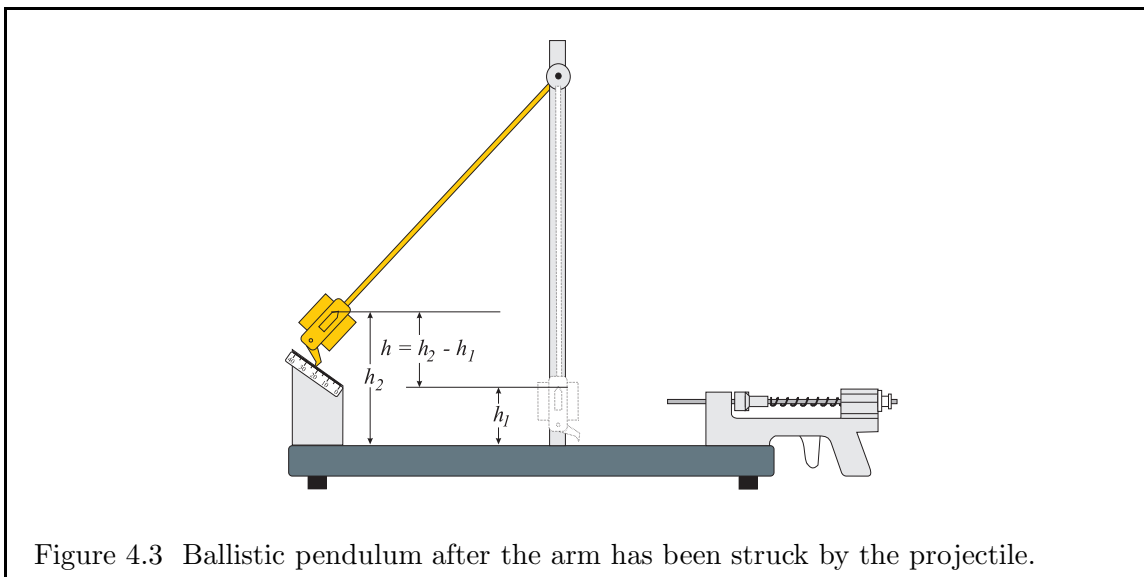
$$U_f = (M_1 + M_2)gh. \tag{4.6}$$

Figure 4.3 Ballistic pendulum after the arm has been struck by the projectile.

According to the principle of conservation of mechanical energy, $U_i + K_i = U_f + K_f$ and thus, from equations (4.5) and (4.6), we get:

$$0 + \frac{1}{2}(M_1 + M_2){v_{12}}^2 \;=\; (M_1 + M_2)gh + 0$$

$$\Rightarrow \quad v_{12} \;=\; \sqrt{2gh}. \tag{4.7}$$

Preparation question 3: If g is taken to be exact, show that the uncertainty in v_{12} is given by:

$$\Delta v_{12} \;=\; \sqrt{\frac{g}{2h}}\, \Delta h, \tag{4.8}$$

where Δh is the absolute uncertainty in h.

In this lab, we are interested in verifying whether momentum is conserved in an inelastic collision. Here, M_1 with speed v_1 [equation (4.3)] immediately before the collision collides and sticks to M_2 and acquires a velocity v_{12} [equation (4.7)] immediately after the collision. According to the principle of conservation of momentum, the momenta of the system immediately before and after the collision should be the same, thus:

$$M_1 v_1 \;=\; (M_1 + M_2)v_{12}$$

This is most easily tested experimentally by testing the validity of the equation:

$$M_1 v_1 - (M_1 + M_2)v_{12} \;=\; 0, \tag{4.9}$$

where the absolute uncertainty of the left hand side is given by:

$$M_1 \Delta v_1 + v_1 \Delta M_1 + (M_1 + M_2)\Delta v_{12} + v_{12}(\Delta M_1 + \Delta M_2). \tag{4.10}$$

This can be easily verified using the rules of uncertainty propagation in §5.4.

Procedure

PART I. Determining the before-collision velocity v_1

1. Measure the mass of the projectile, M_1, using the balance provided. Record this value in your lab book.

2. Move the pendulum bob out of the path of the projectile. Do not remove the pendulum arm, but instead rest it in the curved rack.

3. Place the entire apparatus (see Fig. 4.2) near the edge of the table so that when the projectile is fired, it lands on the floor (but please don't shoot your neighbours!). Use masking tape to outline where on the table the apparatus is to sit for the duration of the experiment.

4. Measure the distance y between the bottom of the projectile as it rests in the launcher, and the floor. Estimate and record the uncertainty, Δy.

5. Fire two or three test shots, and see where they land on the floor. Place the carbon paper at that point (with white paper layer touching the floor) and tape it down.

6. Keep the position of the ballistic pendulum apparatus fixed. By firing the projectile, determine the range, x, of the projectile's motion (Fig. 4.2) using the marks that are made when the projectile lands on the carbon paper. Repeat this procedure ten times and record your measurements in a table. Find the average range ($\bar{x}$) with an appropriate uncertainty (*e.g.*, the sum of the uncertainty in the distance between where the projectile is launched and the leading edge of the carbon paper taped to the floor, and the uncertainty in the position of the dots from the carbon paper as determined from the "half-the-range rule" described in § 5 of the *Introduction*).

PART II. Determining the after-collision velocity v_{12}

7. Record the effective mass of the pendulum bob, M_2, printed on the base of the gun assembly. Use an uncertainty of ± 1 in the last digit quoted.

8. Let the pendulum bob hang freely so that when you fire the projectile (mass M_1), it is caught and trapped by the bob (mass M_2). You may have to adjust the orientation of the bob slightly to that the projectile is caught and trapped by the bob nearly every time the projectile is fired.

9. With the bob hanging freely, measure the height h_1 of the centre of mass of the bob (indicated by a pointer attached to the bob; Fig. 4.3) above the base.

10. With the bob hanging freely and at rest, fire the projectile so that the bob is caught by the curved rack. Measure the height h_2 of the centre of mass of the bob above the base.

    HINT: Each grove is 1 mm higher than the previous. Thus, to get h_2 in mm, you can simply add the groove number to the height of the centre of mass of the bob above the base when it is trapped in "groove 0" (as numbered on the side of the curved track).

11. Repeat step 10 nine more times to obtain ten measurements for h_2 altogether, recording your data in a table. From these data, find the average height, $\bar{h}_2$, and determine the associated uncertainty, Δh_2, from the "half-the-range rule".

12. Finally, determine $h = \bar{h}_2 - h_1$ (Fig. 4.3), the height that the bob rises with the projectile, where $\Delta h = \Delta h_2 + \Delta h_1$.

Analysis

1. From equations (4.3) and (4.4), determine v_1 and its uncertainty.

2. From equations (4.7) and (4.8), determine v_{12} and its uncertainty.

3. Substitute your measured values for M_1, v_1, and v_{12} and the recorded value of the effective mass of the bob (M_2), into equation (4.9), propagating all uncertainties [expression (4.10)].

Conclusions

1. To within experimental uncertainty, did you find momentum to be conserved? If not, what sources of experimental error might account for the difference that haven't already been accounted for? (Hint: Glance at the first discussion question.)

Additional discussion questions

1. There are other sources of error that have not been accounted for in this lab. For example, what would the effect of air resistance be in measuring the range of the projectile? How would the loss of energy and momentum caused by the interaction of the curved rack with the bob affect the results? Does the fact that the grooves on the curved rack are discrete instead of continuous have any influence on the results?

2. Does it matter to the conservation of momentum whether or not energy is conserved? That is, can momentum be conserved even if kinetic energy is not? Explain.

Experiment 5
Shear Modulus

See Appendices A and B for examples of informal and formal reports for this experiment.

Purpose

The purpose of this experiment is to measure the shear modulus (*a.k.a.* modulus of rigidity) for two metal rods, and then identify the metal in each rod from tabulated values.

Apparatus

1. torsion apparatus (Fig. 5.1),

2. assorted masses and a mass hanger,

3. mass balance,

4. micrometer,

5. metre stick,

6. two rods of unknown metal.

Theory

Under the influence of an external force, all solids deform to some extent. Materials that restore their original shape once the external force has been removed are described as *elastic*; examples include springs, diving boards, rubber bands, *etc.* If the *elastic limit* of a material is exceeded, permanent deformation of the object results. *Inelastic* materials such as bread dough, metal solder, *etc.*, are materials for which the elastic limit is relatively small.

The *stress* on a body (Σ) associated with an applied external force (F) is defined as $\Sigma = F/A$ where A is the area over which the force is *distributed* (and not necessarily the area over which it is *applied*). The deformation of the body is called the *strain* (S) and is

Figure 5.1 Schematic diagram of the torsion apparatus.
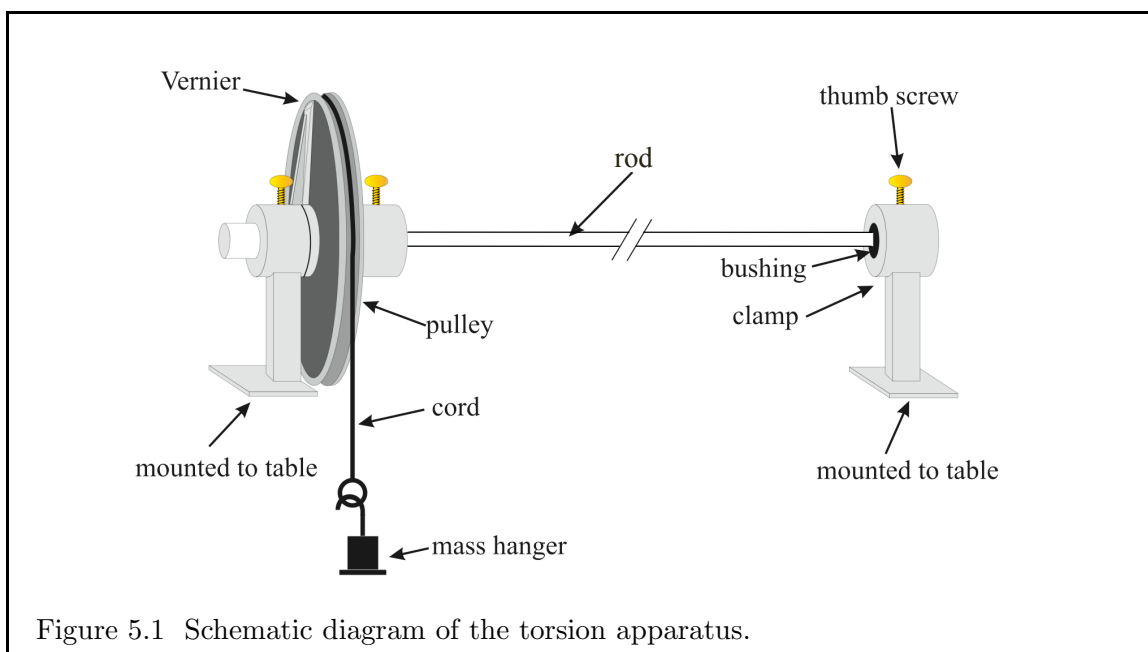
usually expressed as a ratio of the distortion of the body (x) to some characteristic length (L). Thus, $S = x/L$. For an elastic body and small strains, the stress is linearly proportional to the strain ($\Sigma \propto S$), and the constant of proportionality is known as the *elastic modulus*.

There are basically two types of stresses: *compressional* (also known as *tensile*) stress in which the force is applied normal to the surface; and *shearing* stress, in which the force is applied tangential to the surface. Compressing or stretching a spring is an example of a compressional stress. Let the cross-sectional area of the spring be A, let the applied force be F, and let the spring of relaxed length L be stretched by a distance x when the force is applied. The stress is then given by F/A, and the strain is given by x/L. Thus, we write :

$$\frac{F}{A} = -Y\frac{x}{L}, \tag{5.1}$$

where Y is referred to as *Young's Modulus*, and is the specific name given to the elastic modulus for a compressional stress. Equation (5.1) may be rewritten as:

$$F = -\frac{AY}{L}x \equiv -kx, \tag{5.2}$$

where, in this example, $k = AY/L$ is the spring constant. Note that the spring constant depends as much upon the geometry of the spring (its length and cross-sectional area) as it does upon the material with which the spring is made. On the other hand, Y depends only upon the type of material used to make the spring. Thus, there is a particular value of Y for springs made with copper, another value for steel, *etc.*, irrespective of the dimensions of the spring.

Figure 5.2 A sheared rectangular box.
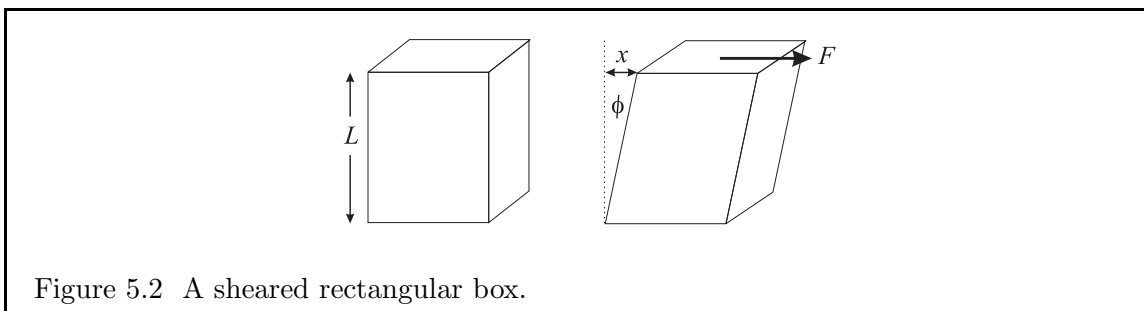
Preparation question 1: Using equation (5.2), show that if a spring is cut in half, its spring constant doubles.

For a sheared system, we again define the strain as a unitless quantity, namely the distortion x divided by the length of the object, L, as depicted in Fig. 5.2. In this case, we write:

$$\Sigma \;=\; \frac{F}{A} \;=\; GS \;=\; G\frac{x}{L} \;\equiv\; G\tan\phi, \tag{5.3}$$

where G is the *Shear Modulus*, also known as the *Modulus of Rigidity*, and ϕ is the shearing angle.

A long rod fixed at one end and twisted at the other is an example of a sheared system. To compute the shear modulus, G, of a solid rod with radius R and length L, divide it into a series of concentric hollow cylinders. The total stress on the rod is then the sum of all the stresses on the concentric cylinders from which we can compute G.

In Fig. 5.3, a hollow cylinder of length L, radius r, thickness dr, and cross-sectional area $dA = 2\pi r\, dr$, is twisted by a torque $d\tau = r\,dF$, where dF is the infinitesimal force applied to the surface of the thin cylindrical shell. Thus, the stress on the cylinder is given by[5]:

$$\Sigma \;=\; \frac{dF}{dA} \;=\; \frac{1}{2\pi r^2}\frac{d\tau}{dr}. \tag{5.4}$$

Now, the strain on the cylinder, as depicted in Fig. 5.3 is given by

$$S \;=\; \tan\phi \;=\; \frac{x}{L} \;=\; \frac{r\theta}{L}. \tag{5.5}$$

Substituting equations (5.4) and (5.5) into $\Sigma = GS$ gives us:

$$\frac{1}{2\pi r^2}\frac{d\tau}{dr} \;=\; G\frac{r\theta}{L}.$$

Solving for $d\tau$ we get:

$$d\tau \;=\; \frac{2\pi G\theta}{L}r^3 dr,$$

---

[5]Note that the stress is distributed over the cross-sectional area of the rod, and not over its surface area.
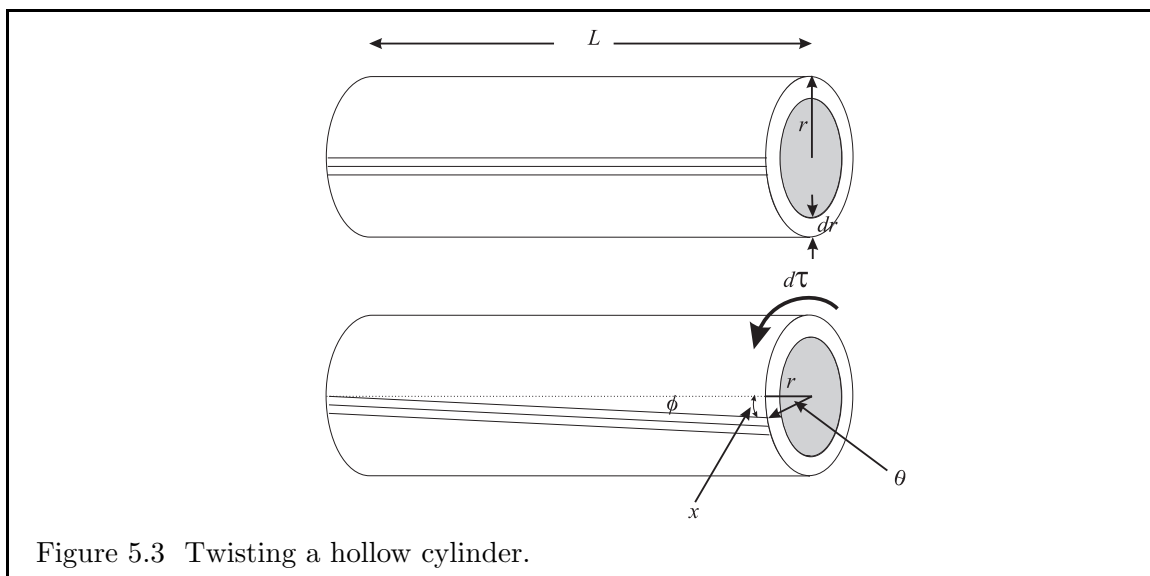
Figure 5.3 Twisting a hollow cylinder.

and then integrating $d\tau$ over the entire radius of the rod, we get:

$$\tau \;=\; \int_0^R \frac{2\pi G\theta}{L} r^3 dr \;=\; \frac{2\pi G\theta}{L} \int_0^R r^3 dr \;=\; \frac{\pi G\theta R^4}{2L}. \tag{5.6}$$

In this experiment, the torque is applied to the rod by hanging a mass, M, from one side of a pulley of diameter D as shown in Fig. 5.1. Thus, the torque is given by $\tau = MgD/2$ and comparing this to equation (5.6) gives an expression for the twist angle, θ:

$$\theta \;=\; \frac{gDL}{G\pi R^4} M,$$

where θ is expressed in radians. If θ is measured in degrees, we have instead:

$$\theta \;=\; \frac{180\, gDL}{G\pi^2 R^4} M. \tag{5.7}$$

Equation (5.7) has the form $\theta \propto M$, with the slope, m, given by:

$$m \;=\; \frac{180\, gDL}{G\pi^2 R^4},$$

which, when solved for G, yields:

$$G \;=\; \frac{180\, gDL}{m\pi^2 R^4}. \tag{5.8}$$

Therefore, if one measures the twist angle as a function of the load mass, the slope of the θ vs. M plot, m, can be used in equation (5.8) to give an estimate of the Shear Modulus, G.

> **Preparation question 2:** If $g = 9.81$ is taken to be exact, show that the
> uncertainty in G is given by:
>
> $$\frac{\Delta G}{G} = \frac{\Delta D}{D} + \frac{\Delta L}{L} + \frac{\Delta m}{m} + 4\frac{\Delta R}{R}. \qquad (5.9)$$
>
> Note the factor of 4 in front of the fractional uncertainty of R. Thus, R will
> have to be measured particularly carefully to minimise its disproportionate
> contribution to the fractional uncertainty of G.

Procedure

1. The masses indicated on the weights are not entirely accurate, so we need to deter-
   mine their uncertainty. Using the balance provided, weigh five of the "50 g" masses
   individually, and then take the mean $\pm$ half the range of the values as your individual
   mass value. If you find the masses are identical, use the reading uncertainty on the
   mass balance as the uncertainty instead. Repeat for the "100 g" masses.

2. Wrap the piece of cord provided around the wheel, and then measure (using the metre
   stick) how much of the cord was needed to wrap around the wheel exactly once. Note
   that because of the thick axle, it is difficult to measure the diameter of the wheel
   directly, and so you shouldn't attempt to do so. The diameter of the wheel, D, is the
   circumference divided by π. Record both D and an estimate of its uncertainty.

3. Insert the rod with a "B" stamped on one of the ends into the sockets of the table
   clamps (Fig. 5.1). Be certain that the thumbscrews are tightened into the notches of
   the bushings, otherwise you may damage the rod during the experiment.

4. Twist the wheel by hand by about $10°$ and release. The wheel should wiggle back and
   forth a few times as it returns to its original position. Record the position that the
   wheel returns to. Now twist the wheel by the same amount in the opposite direction,
   and release. The wheel should return to the same position that you recorded. *If it
   doesn't return to the same position to within* $0.5/deg$, *tell your instructor; your torsion
   wheel may have to be replaced with another before you continue.*

5. Use the metre stick to measure the length of the thin portion of the rod between the two
   metal bushings at either end. Record this length L and an estimate of its uncertainty.

6. Use the micrometer to measure the diameter of the rod in at least five different loca-
   tions. **Do not over-tighten the micrometer caliper.** If you are still unfamiliar
   with this device, see App. C or ask your demonstrator. From these measurements, de-
   termine the average diameter and an associated uncertainty (using the "half-the-range
   rule"). The radius of the rod, R, is half the average diameter, and ΔR is half the
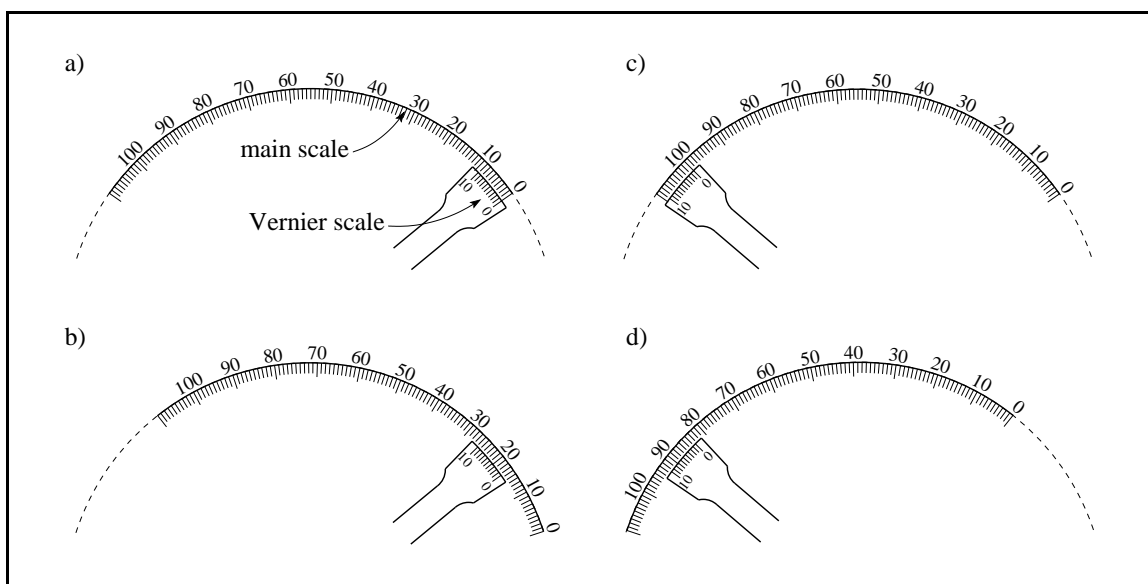   estimated uncertainty of the diameter.

Figure 5.4   a) To "zero" the angle for clockwise twists (procedure step 8), line up the '0' on the Vernier scale with the '0' on the main scale. b) In this example of a clockwise twist angle (procedure step 9), the reading is 16.7°. c) To "zero" the angle for *counter*clockwise twists (procedure step 12), line up the '0' on the Vernier scale with the '100' on the main scale. b) In this example of a *counter*clockwise twist angle (procedure step 13), the reading is 83.4° and thus $\theta = 100° - 83.4° = 16.6°$.

7. Create a table with seven columns and at least 22 lines, assuming the data are entered double-spaced and one line is left for the headings. The first column is to record the total mass hung on the hanger and its uncertainty, columns 2–5 are to record the four twist angles you will measure, column 6 is for the average twist angle, and column 7 is for the "half-the-range" uncertainty in the twist angle.

8. Wrap the cord or strap around the wheel in a clockwise fashion. Set the Vernier scale accurately to 0° with the mass hanger being the only weight on the cord. *Refer to Fig. 5.4 a) and b) to be certain you are using the Vernier angular scale correctly.*

9. Apply masses to the mass hanger in "50 gram" increments (actual increment determined in step 1) starting with one mass and ending with ten masses (maximum capacity of the hanger). Record in your table the masses applied in column 1 and the resulting twist angles (θ) in column 2. Remember that the uncertainty of the mass applied is the sum of the absolute uncertainties of each mass. What is your estimate of the reading uncertainty in θ?

10. With the maximum mass on the hanger, jiggle the mass hanger gently. Take the reading again, and record this possibly new value of the twist angle at the bottom of the third column of your data table.

11. Complete the third column by removing the "50 g" masses one at a time, recording each twist angle in your data table as you do. Your angle measurements should read almost the same as they did in step 9, although they will probably not be *exactly* the same.

12. Once all the masses have been removed, wrap the cord (or strap) around the wheel in a counter-clockwise fashion and, with the mass hanger being the only weight on the cord, set the Vernier scale accurately to 100°. *Refer to Fig. 5.4 c) and d) to be certain you are using the Vernier angular scale correctly. Particularly for the 100° side, many students get this confused.*

13. Repeat steps 9, 10, and 11 recording your twist angles in columns 4 and 5 of your data table. Note that this time, the twist angle $\theta = 100° -$ the reading.

    Important: The four angles for a given mass should not differ by much more than 0.5°. If they do, consult your instructor as there may be something wrong with your equipment or with how you are reading the angles.

14. Time permitting, replace rod "B" with the rod stamped with an "S" on one of its ends and repeat steps 5–13 using the "100 g" masses instead of the "50 g" masses.

Analysis

1. For rod "B", compute the average of the four twist angles, $\bar{\theta}$, for each mass and record this in the sixth column of your table. In the seventh and final column, record the uncertainty in θ, $\Delta\theta$, using the "half-the-range rule" or the reading uncertainty, whichever is greater.

2. Generate a plot of $\bar{\theta}$ *vs.* M, including error bars for both $\Delta\bar{\theta}$ and M as appropriate. According to equation (5.7), the data should follow a straight line.

3. Find the maximum and minimum slopes consistent with the data, and determine an average slope with an associated uncertainty [equation (I.11) in the *Introduction*].

4. From equations (5.8) and (5.9), determine the shear modulus, G, of the "B"-rod and its uncertainty.

5. Time permitting, repeat analysis steps 1–4 for the "S"-rod.

Conclusions

1. Use Table 5.1 to identify the type of metal in rods "B" and "S". Is it possible to identify the metal uniquely?

metal	G (10^{10} N m^{-2})
lead	1.31
aluminum	2.70
brass	3.45
cast iron	4.10
bronze	4.48
iron (malleable)	6.40
carbon steel	7.70
steel	8.50
molybdenum	11.80

Table 5.1 Table of shear moduli (G) for various metals. Values are taken from
http://www.engineeringtoolbox.com.

Additional discussion questions

1. If the uncertainty in the measured value of G is too large, it will be impossible to make
a unique identification of the metal in the rod. Assuming this to be the case, what is
the single most important modification you could make to this experiment that would
improve the precision of the results enough to make a unique identification?

2. We never actually measured the mass of the mass hanger. Yet, done properly, your
data should have fit a straight line passing through the origin. What did we do that
accounted for the mass of the mass hanger?

Experiment 6
Determining g Using a Simple Pendulum

Purpose

The purpose of this experiment is to determine a value for the gravitational acceleration g at the Earth's surface using a simple pendulum.

Apparatus

1. metal pendulum "bob",

2. metal frame and thread for the pendulum bob,

3. metre stick,

4. stopwatch.

Theory

The period of oscillation, T, for a simple pendulum oscillating with a small amplitude (a few degrees or less) is given by:

$$T \;=\; 2\pi\sqrt{\frac{L}{g}}, \tag{6.1}$$

where L is the length of the string as measured from the pivot point to the centre of mass of the bob, and g is the gravitational acceleration at the Earth's surface.

> **Preparation question 1:** Starting with a free-body diagram for a pendulum, derive equation (6.1) using your textbook for "inspiration" if you like.
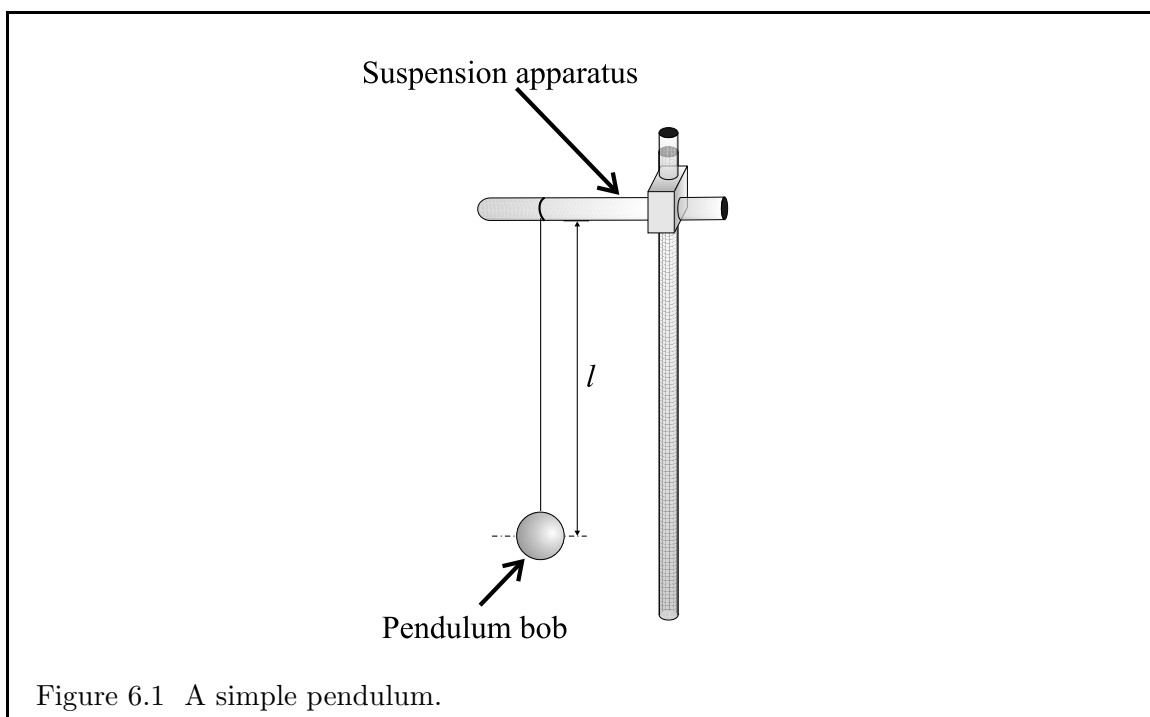
Figure 6.1 A simple pendulum.

Notice that the period of oscillation doesn't depend on either the mass of the pendulum bob, nor the amplitude of oscillation *so long as the amplitude is **small***.

Squaring both sides of equation (6.1), we get:

$$T^2 = \frac{4\pi^2}{g} L, \tag{6.2}$$

Therefore, a plot of the square of the swing period, T^2, *vs.* the string length, L, should yield a straight line with slope:

$$m = \frac{4\pi^2}{g},$$

which, when solved for g, gives:

$$g = \frac{4\pi^2}{m}. \tag{6.3}$$

In this lab, we will use equation (6.3) as a simple way to measure the acceleration of gravity.

Preparation question 2: What is the uncertainty in g, Δg, in terms of the uncertainty in the slope, Δm?

Procedure

1. Attach the string to the pivot bar of the stand so that the length of string between the pivot point (the underside of the horizontal bar in the metal frame) to the centre of

mass for the pendulum bob as close to 90 cm as practical. Take care that the thread does not come loose during the experiment (Fig. 6.1).

2. Make a table in your lab book with eight columns and at least 16 lines (assuming you'll enter your data double-spaced).

3. Using the metre stick, measure the actual length, L, of the string from the pivot point to the centre of mass of the bob, and record it and an estimate of the uncertainty in the first column of your table.

4. Pull the bob back so that the angle the string makes with the vertical is no more than $5°$ (use a protractor, if necessary, to get an idea of what "less than $5°$" means!), and release. Use the stopwatch to measure the elapsed time for *ten* complete swings of the bob (as measured from the *top* of the swing) in seconds. It is not a good idea to start counting right when the pendulum is released; wait for one swing to complete before starting. NOTE: one swing is completed when the pendulum returns to its starting position. Record this time (call it t_1) and an estimate of the uncertainty (reading, reaction delay, *etc.*), in column 2.

> **Preparation question 3:** Why is it better to measure the time between moments when the bob is at the *top* of its swing than when it is at the *bottom* of its swing?

5. Repeat step 4 four additional times, recording the times for ten swings in columns 3–6 headed with, say, t_2, t_3, t_4, and t_5.

6. Repeat steps 3, 4, and 5 for several other lengths (*e.g.*, 0.8, 0.7, ..., 0.3 m).

Analysis

1. For each string length, compute the average time, $\bar{t}$, for ten swings and a half-the-range uncertainty. Divide this by 10 to get your measurement of the period, T, and record this value along with its uncertainty in column 7 of your table. Note that when you divide $\bar{t}$ by 10 to get T, you should also divide its uncertainty by 10 to get ΔT.

2. For each length, compute T^2 and its uncertainty, $\Delta(T^2)$, [*e.g.*, equation (I.3) in the *Introduction*], and record these values in the eighth and final column of your table. Provide in your lab book a sample calculation for how you got *one* value of T^2 and $\Delta(T^2)$ from your raw measurements t_1, t_2, *etc.* The rest you can just record in your table without writing out the derivations in detail.

3. Generate a graph with T^2 (the dependent variable) on the vertical axis and L (the independent variable) on the horizontal axis, including uncertainty bars for both variables if visible. If the uncertainty bars are not large enough to plot, make a note of that on your graph. Review § 6 of the *Introduction* if you have forgotten what constitutes a good graph and how to measure the slope from a graph.

4. By drawing two straight lines through your data, one with the maximum slope consistent with your data, the other with the minimum slope consistent with your data, determine the slope of the best fit line, m, along with an uncertainty, Δm [equation (I.11) in the *Introduction*].

5. Use equation (6.3) to compute g with an associated uncertainty.

Conclusions

1. Does your experimentally determined value of g agree with the local accepted value of $9.81 \ \mathrm{m \, s}^{-2}$? If not, what are the possible sources of experimental error?

Additional discussion questions

1. Do your maximum- and minimum-slope best-fit lines encompass the origin? If not, can you explain why they might not?

2. Compare your experimentally determined value for g from this lab with that determined in Lab 3. Which value was more *accurate*? Which value was more *precise*? Comment, then, on which methodology is better for determining g.

3. Recall that equation (6.3) is derived assuming that the swing amplitude of the pendulum is relatively small. Time permitting and with the pendulum string length of say 0.6 m, measure T (one measurement for ten swings is sufficient here) for an amplitude of (approximately) $10°$, $20°$, and $40°$. Including your previous measurement for $5°$, this gives you four data points. Is there a trend? If you were to have used an amplitude that was too large, how would this have affected your experimentally determined value of g?

Experiment 7
Standing Waves

Purpose

The purpose of this experiment is to investigate standing waves on a stretched string. As a consequence, the *linear density* (in units of kg m^{-1}) of the string may be determined.

Apparatus

1. oscillator,

2. frequency counter,

3. transformer,

4. electrical leads,

5. standing wave driver,

6. pulley,

7. string,

8. UV lamp,

9. several masses and a mass hanger,

10. mass balance,

11. metre stick.

Theory

The velocity of propagation, v, of a transverse wave on a taut string is given by:

$$v = \sqrt{\frac{T}{\mu}},$$

where T is the tension on the string (N) and μ is the linear density of the string (kg m^{-1}). In the case of wave motion, we also have,

$$v \; = \; f\lambda,$$

where f is the frequency of vibration (Hz = s^{-1}) and λ is the wavelength of the wave (m). Thus,

$$f \; = \; \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}. \tag{7.1}$$

If L is the length of the string and n is the number of loops in the standing wave (see Fig. 7.1), then

$$\lambda \; = \; \frac{2L}{n}.$$

Preparation question 1: Why do standing waves exist only when the wavelength is given by $\lambda = 2L/n$, where n is an integer? Why, for example, could a standing wave not be created with $\lambda = 4L$ (and thus $n = 1/2$)?

Further, if the tension is supplied by a mass, M, hanging from the string, $T = Mg$. Substituting for λ and T in equation (7.1), we get our principle result to be checked experimentally:

$$f \; = \; \frac{n}{2L}\sqrt{\frac{Mg}{\mu}}. \tag{7.2}$$

Note that there are *three* independent variables that can be varied: n, M, and L. We can rearrange equation (7.2) in three different ways to "expose" the dependence f has on

Figure 7.1 Examples of standing waves.
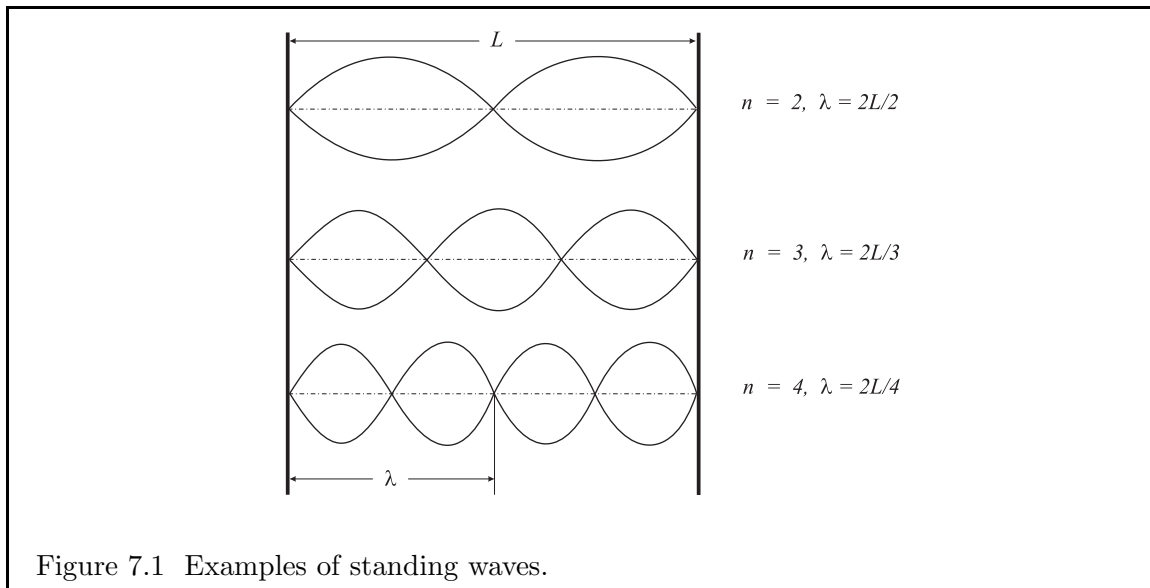
each of these variables. Thus:

$$f = m_1 n, \qquad \text{where} \quad m_1 = \frac{1}{2L}\sqrt{\frac{Mg}{\mu}}; \tag{7.3}$$

$$f = m_2\sqrt{M}, \quad \text{where} \quad m_2 = \frac{n}{2L}\sqrt{\frac{g}{\mu}}; \tag{7.4}$$

$$f = m_3\frac{1}{L}, \qquad \text{where} \quad m_3 = \frac{n}{2}\sqrt{\frac{Mg}{\mu}}. \tag{7.5}$$

By plotting f against n (holding each of L and M constant), f against $\sqrt{M}$ (holding each of n and L constant), and f against $1/L$ (holding each of n and M constant), we should get three straight lines with measurable slopes, and thus three ways of estimating the linear density of the string, μ. Thus, for example, from equation (7.3) we have:

$$\mu = \frac{Mg}{4L^2 m_1^2} \quad \text{and} \quad \frac{\Delta\mu}{\mu} = \frac{\Delta M}{M} + 2\frac{\Delta L}{L} + 2\frac{\Delta m_1}{m_1} \tag{7.6}$$

In this lab, we will find that the fractional uncertainty in the masses, $\Delta M/M$, is negligible compared to the other sources of uncertainty, and thus we will ignore its contribution.

Preparation question 2: Derive similar expressions to equations (7.6) for μ and its fractional uncertainty in terms of m_2 and m_3 (assuming the uncertainties for both g and n are zero).

Procedure

Figure 7.2 is a schematic diagram illustrating the apparatus used in this experiment. The driver (a two-cone speaker) is used to vibrate the string which is fixed on a pulley at the other end. The frequency and amplitude of vibration are controlled by the oscillator. The desired frequency is that which excites resonant waves in the string (*i.e.*, provides the maximum observed amplitude in the string vibrations).

WARNING: The UV lamp illuminates the phosphorescent string and makes it easier to see. UNDER NO CIRCUMSTANCES SHOULD YOU LOOK DI-RECTLY AT THE UV BULB WHILE IT IS ON! Prolonged exposure of the unprotected eye to the emission from the UV bulb can cause severe and permanent eye damage.

1. With the oscillator off, become familiar with the oscillator controls. The large dial adjusts the oscillator frequency and has a logarithmic scale from which you can get a quick read of the output frequency. Touching the right side of the large dial is a "fine-adjust" knob for fine-tuning the frequency. In addition, the "range" knob is a frequency "multiplier" (*e.g.*, set at 10, the frequency output is ten times what the large dial is set to) and the "amplitude" knob controls the power output.
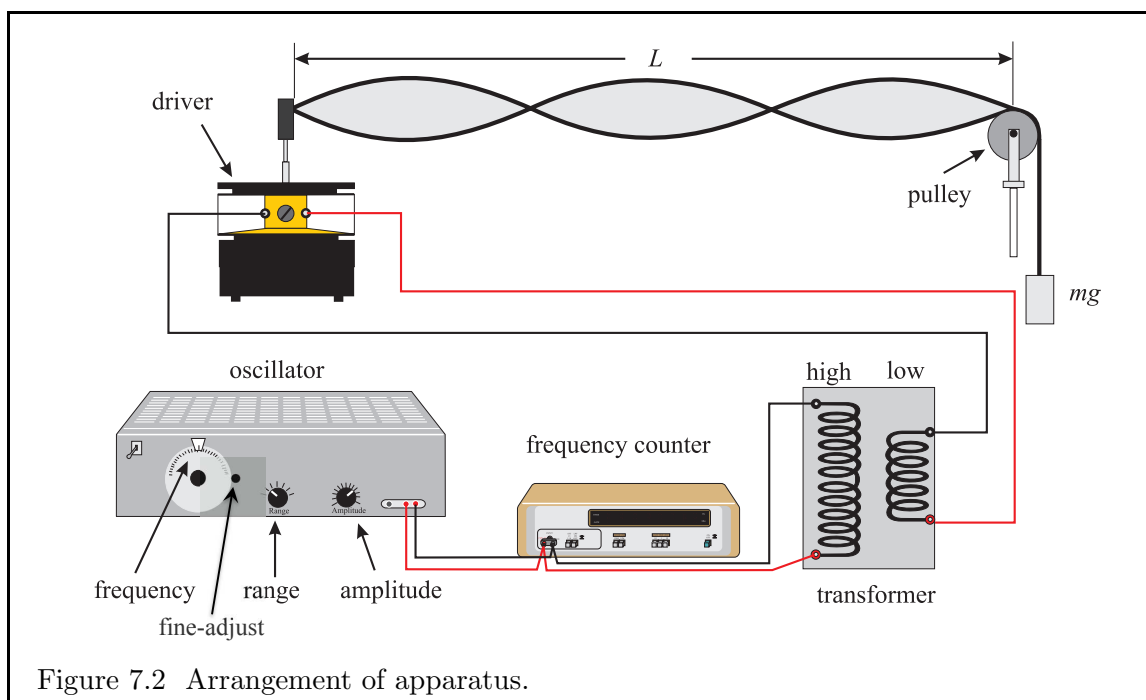
Figure 7.2 Arrangement of apparatus.

2. While the large dial has a logarithmic scale of the output frequency, the uncertainty of this value is high. Thus, we use an electronic frequency counter to measure the output frequency. For highest accuracy, it is necessary to have the "10 s" selector pushed in, which means that the device takes only one reading every ten seconds. *Thus it is critical that after adjusting the oscillator frequency, you let the value of the readout settle for several seconds before recording the value.*

3. Turn the oscillator and frequency counter on. As a trial run, place a 20 gram mass on the hanger and set the length of the string to about 1 metre. Note that at the pulley end, the string is measured from the top of the pulley. Adjust the frequency until the resonance frequency for a one-loop standing wave (f_1) is found. Since $f \propto n$, if you happen to find the frequency f_n of an n-loop standing wave first, adjust the frequency to $f_1 = f_n/n$ to find the one-loop resonance. Note also it is possible to hit a weak $n = 1$ resonance at a frequency $f_1/2$ (analogous to pushing a child on a swing every other oscillation). Thus, once you think you have found f_1, try doubling the frequency to make sure you do get two loops ($n = 2$).

   At first, the amplitude knob on the oscillator can be set quite high, but it may be necessary to reduce the amplitude once resonance is found, especially for low values of n. Note also that the frequency must be adjusted very gradually as one approaches resonance.

4. There will be a lot of frequencies to measure in this lab, and so rather than measuring each frequency five times to apply the half-the-range-rule to each, we'll do it carefully

for one resonant frequency, and apply that uncertainty to all frequencies measured in the rest of the lab.

a) With the 20 g mass still on the mass hanger, find the two-loop ($n = 2$) resonance of the ~ 1 m length string. To do this, watch the string as you carefully adjust the frequency until the central node is as sharp as it can be and the two loops are at their maximum amplitude. Remember to wait several seconds before recording the frequency from the frequency counter. There is no need to record a reading uncertainty here.

b) Next, turn the frequency dial well away from resonance, and repeat the previous step until you have ten measures of the $n = 2$ resonant frequency. Delete any questionable values (those clearly too high or too low) and from the rest, determine the average frequency, $\bar{f}$, and its uncertainty, Δf, from the half-the-range rule.

c) Compute the fractional uncertainty, $\Delta f / \bar{f}$. This is the fractional uncertainty you will use for all resonant frequencies you measure in the rest of the lab.

5. Fix the length of the string ($L \sim 1$ metre) and the mass on the hanger ($M \sim 25$ grams). Record the actual length of the string and measure the mass hanging from the string (including the mass hanger) using the balance provided. Find the resonant frequencies for as many resonances (*i.e.*, $n = 1, 2, 3, \ldots$) as you can, recording your data in a two-row table (top row for n, bottom row for f.)

6. Fix and record the length of the string ($L \sim 1$ metre) and use $n = 2$. Find the two-loop resonant frequencies for masses 20, 25, 30, 35, ..., 60 g hanging on the hanger. Use the mass balance to determine the masses (including the mass hanger) rather than relying on the values stamped on the masses themselves. Record your data in a two-row table (top row for M, bottom row for f).

7. Fix and record the mass ($M \sim 25$ g) on the hanger and use $n = 2$. Find the two-loop resonant frequencies for varying lengths (*e.g.*, $L = 0.5, 0.6, 0.7, \ldots, 1.2$ metres) and record your data in a two-row table (top row for L, bottom row for f).

Analysis

1. From your first data table, plot f *vs.* n (§ 6.1, *Introduction*) and measure the slope m_1 directly from your graph (§ 6.2, *Introduction*). Use equations (7.6) to determine the linear density of the string, μ, and its uncertainty. Check to make sure that $\Delta M / M$ (ΔM is the reading uncertainty of the mass balance) is much less than the other uncertainties (*e.g.*, $\Delta L / L$). If so, it can be ignored in the uncertainty propagation. If not, it must be carried forward.

2. Do what you need to do to determine μ and its uncertainty from your second data table (f and M). With f plotted, as usual, on the vertical axis, think carefully about

what should be plotted along the horizontal axis so that the data follow a straight line [*e.g.*, glance back at equation (7.4)].

3. Do what you need to do to determine μ and its uncertainty from your third data table (f and L). Again, think carefully about what should be plotted along the horizontal axis to obtain a linear plot.

Conclusions

1. Are the three experimentally determined values for μ consistent to within your experimental uncertainty?

2. Which measurement is the most *precise*?

Additional discussion questions

1. What are the primary sources of uncertainty in this experiment? How might the accuracy and effectiveness of this experiment be improved?

2. Why is a guitar designed to have strings with different thicknesses? Given that the strings are mounted with similar tensions (why might this be desirable?), how do the differing diameters affect the resonant frequencies (and thus the pitch) of the different strings?

3. The microphone (pickup) on an electric guitar is not placed at the middle (along the length) of the guitar string where you might expect the maximum amplitude of vibration to be. Why not? [Hint: when you pluck a string you excite a variety of different modes (values of n) or resonances simultaneously, and not just the $n = 1$ (single loop) mode.]

Experiment 8
Specific Heat and Latent Heat of Fusion

PART I

Purpose

The purpose of the first part of this experiment is to find the specific heat of brass.

Apparatus

1. heated brass samples,

2. water,

3. calorimeter (Figure 8.1),

4. electronic thermometer,

5. balance.

Theory

By definition, heat is the thermal energy transferred between a system and its surroundings as a result of temperature differences. When a hot piece of metal is immersed in cool water, heat "flows" from the metal into the water. The heat flow continues until both the metal and the water are at the same temperature. A schematic diagram illustrating the apparatus (called a *calorimeter*) used in this heat transfer experiment is given in Fig. 8.1.

According to the principle of conservation of energy, the net heat gained or lost by a closed system such as the metal and water in a calorimeter is zero. Thus,

$$Q_w + Q_m \; = \; 0 \quad \Rightarrow \quad Q_w \; = \; -Q_m. \tag{8.1}$$

Figure 8.1 Schematic diagram of the calorimeter.
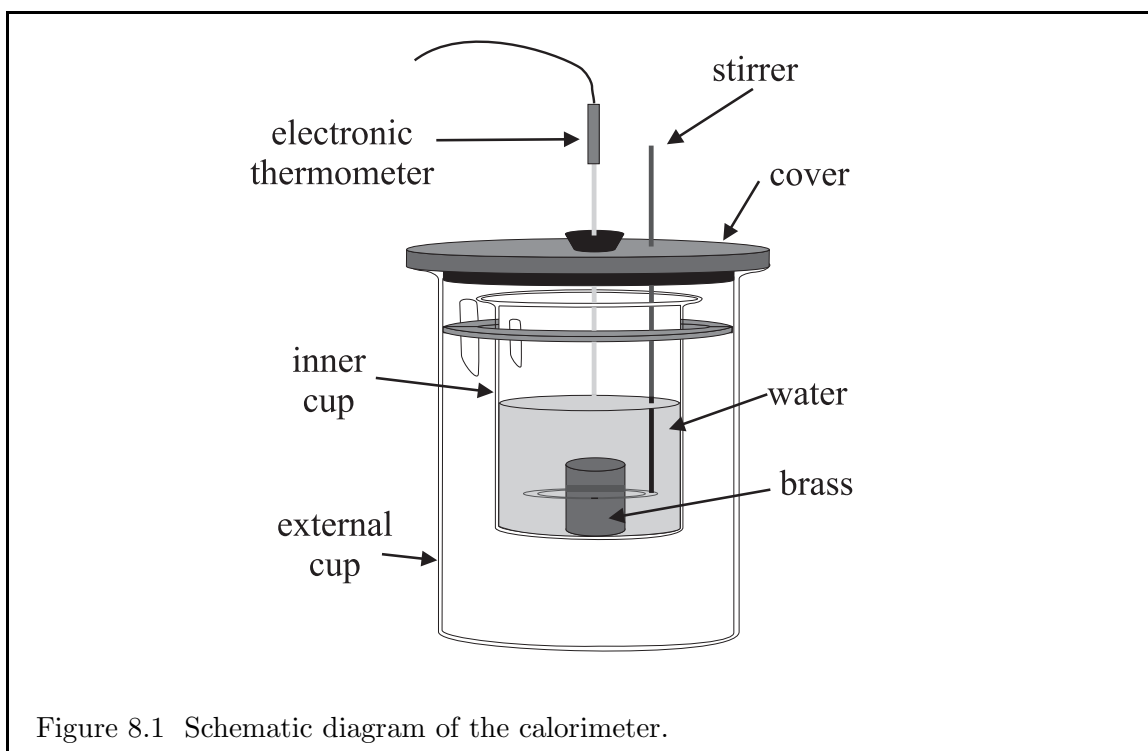
Now, for a given mass M of material with specific heat c, the heat gained or lost as its temperature changes from T_i to T_f is:

$$Q = Mc(T_f - T_i).$$

Thus, if the initial temperature of the metal is T_m and its final temperature is $T < T_m$, then,

$$Q_m = M_m c_m(T - T_m) < 0.$$

Further, if the initial temperature of the water is T_w and its final temperature is $T > T_w$ (*i.e.*, the metal and water attain the same final temperature), then,

$$Q_w = M_w c_w(T - T_w) > 0.$$

Substituting the expressions for Q_m and Q_w into equation (8.1), we get:

$$M_w c_w(T - T_w) = -M_m c_m(T - T_m) = M_m c_m(T_m - T)$$

and therefore:

$$c_m = c_w \frac{M_w}{M_m} \frac{T - T_w}{T_m - T}. \tag{8.2}$$

> **Preparation question 1:** In this experiment, the heat transfer to the calorimeter is negligible and can be ignored. If this weren't true, how would equation (8.2) be modified to take the calorimeter into account? Assume that the inner cup of the calorimeter has a mass M_c, a specific heat c_c, and is initially at temperature T_w.

From equation (8.2), c_m can be calculated from the given value of c_w (4,186 J kg^{-1} °C^{-1}) and all the measured temperatures and masses. Further, the fractional uncertainty in c_m is given by:

$$\frac{\Delta c_m}{c_m} = \frac{\Delta M_w}{M_w} + \frac{\Delta M_m}{M_m} + \frac{2\Delta T}{T - T_w} + \frac{2\Delta T}{T_m - T}. \tag{8.3}$$

> **Preparation question 2:** Derive equation (8.3).

Procedure and Analysis

1. Measure and record the mass of the calorimeter and its uncertainty.

2. Partially fill the calorimeter (about 2/3) with cool (say $\sim$ 15° C) tap water. Measure and record the mass of the water and the calorimeter together. Calculate M_w, propagating its uncertainty properly.

Be prepared to perform steps 3–5 in rapid succession. As always, all measurements are to be recorded with their uncertainties.

3. Record the initial temperature of the water (T_w).

4. Record the temperature T_m of the hot large sample of brass (same as that of the hot water bath). Check that the tip of the thermometer is fully immersed in the hot water and not above the surface nor touching the bottom, either one of which could give you a reading very different from the temperature you seek.

5. Select a large sample of brass, transfer it into the water in the calorimeter, and quickly replace the top. **For this experiment to be successful, the transfer must be made within 10 s.** Record the number that appears on the attached string both in your lab book and on the chalkboard (the latter so that no one else accidentally takes your sample after you have returned it to the hot water bath).

6. Stir the water gently for at least 30 seconds while monitoring its temperature. Record the highest temperature attained by the water (T).

7. Measure the mass of the brass sample (M_m) and return it to the hot water bath. Discard the water in the calorimeter.

8. Calculate c_m and its uncertainty using equations (8.2) and (8.3).

9. Repeat steps 2 through 8 for a small sample of brass.

10. *After performing Part II*, repeat steps 2 through 8 for the same large sample of brass again. You do this after Part II to give your sample time to heat up again.

Conclusions

1. Are your three measurements of c_m consistent to within experimental uncertainty?

2. "Standard values" of c_m for brass differ, according to the source. In part, this is because brass, not being an element, is made with slightly varying compositions. Searching textbooks and on-line, an average value for c_m seems to be $373. \pm 7.$ J kg^{-1} °C^{-1}. Is your value consistent with this range?

Additional discussion question

1. Are your results more precise for the large sample or small sample of brass? Explain.

PART II

Purpose

The purpose of the second part of this experiment is to find the latent heat of ice.

Apparatus

1. ice,

2. water,

3. calorimeter (Fig. 8.1),

4. electronic thermometer,

5. balance.

Theory

As in Part I, Part II demonstrates that heat is transferred from a hot source to a cold source, and that heat transfer may also cause a phase change in a substance, *i.e.* solid to liquid. Consider the case of a block of ice with mass M_i and temperature 0°C immersed in a warm water bath of mass M_w and temperature T_w. Some of the heat transferred from the warm water bath to the ice breaks the intermolecular bonds of the solid ice crystals to form liquid water. The heat, Q_i, required to convert 0°C ice of mass M_i to water of the same temperature and mass is given by:

$$Q_i \;=\; M_i L_i,$$

where L_i is the latent heat of fusion of ice.

The remaining heat transferred from the warm water bath increases the temperature of the water that was once ice from 0°C to its final, equilibrium temperature T. This heat (Q_w) is given by:

$$Q_w \;=\; M_i c_w (T - 0°C) \;=\; M_i c_w T,$$

where T must now be expressed in Celcius. When equilibrium is reached, the sum of Q_i and Q_w must be equal to the heat loss by the warm water bath. Thus,

$$M_i L_i + M_i c_w T \;=\; -M_w c_w (T - T_w) \;=\; M_w c_w (T_w - T)$$

and so,

$$L_i \;=\; c_w \frac{M_w}{M_i}(T_w - T) - c_w T. \tag{8.4}$$

From measurements of the masses and temperatures, one can determine the heat of fusion of ice, L_i. Further, the absolute uncertainty in L_i is given by the rather nasty looking equation:

$$\Delta L_i \;=\; c_w \left[\frac{\Delta M_w}{M_i}(T_w - T) + \frac{M_w \Delta M_i}{M_i^2}(T_w - T) + \frac{2\Delta T M_w}{M_i} + \Delta T \right]. \tag{8.5}$$

We'll resist the temptation to ask you to derive this! (Though you could; it really is just straight-forward algebra.)

Procedure and Analysis

1. Partially fill the calorimeter (about 2/3) with lukewarm (say $\sim 35°$ C, or near body temperature) tap water. Measure and record the mass of the water and the calorimeter together. Calculate M_w, propagating its uncertainty properly.

Be prepared to perform steps 2 and 3 in rapid succession. Once again, all uncertainties must be recorded with the measurements.

2. Record the initial temperature of the water (T_w) just before adding the ice.

3. Crush three ice cubes, and add the solid portion only (no melt water!) to the water in the calorimeter. Quickly replace the top.

4. Stir the water gently until all the ice has melted while monitoring its temperature. Record the lowest temperature attained by the water (T).

5. Measure the mass of the calorimeter + water + melted ice, and calculate M_i. Discard the water in the calorimeter.

6. Calculate L_i and its uncertainty from equations (8.4) and (8.5).

7. Repeat steps 1 through 6 two more times using at first two crushed ice cubes, then one crushed ice cube.

Conclusions

1. Are your three measurements of L_i consistent to within experimental uncertainty?

2. What is your best estimate of L_i? Does it agree with the accepted value of $L_i = 3.33 \times 10^5$ J kg^{-1} to within experimental uncertainty?

Additional discussion questions

1. Does the amount of ice affect the latent heat of fusion? Does the amount of ice affect the experimental uncertainty? If so, in what way and why?

2. When a substance changes from liquid to vapour, is the work done by the substance (W) positive or negative, and does its internal energy (E) increase or decrease? Explain.

3. Which cools a cup of hot coffee more: 40 g of water at $0°$ C or 20 g of ice at $0°$ C? Work it out!

Experiment 9
Equipotential Surface Mapping

Purpose

The purpose of this lab is to map equipotential contours and electric field lines about two pairs of charged electrodes.

Apparatus

1. DC power supply,

2. multimeter (used as a voltmeter),

3. parallel and circular electrode sheets,

4. wire leads and probe,

5. ruler.

Theory

If a charge, Q, exerts an electrical force, $\vec{F}_E$, on another charge, q, the *electric field*, $\vec{E}_Q$, emanating from Q is defined to be:

$$\vec{E}_Q = \frac{\vec{F}_E}{q} \, \text{N C}^{-1}.$$

The concept of an "electric field" is an important one in physics. It postulates that a charged object, Q, sets up a "field" in all space about it such that should another charged object, q, enter that field, an electrical force is exerted upon q. We say that the force is mediated by the field. The field from Q exists whether or not q is there to "feel" it. (A falling tree in the woods does indeed make a sound even if no one is there to hear it!) The force, however, exists only when the two charges are present.

> **Preparation question 1:** Write down Coulomb's Law for two spherical charges, Q and q. What is the electrical field emanating from Q?

> **Preparation question 2:** Write down Newton's Law of gravitation for two spherical masses, M and m. Since this is similar in form to Coulomb's Law, we might expect there to be a gravitational field analogous to the electrical field. What is the gravitational field emanating from M? What are its units? What is the gravitational field near the surface of the Earth?

The electric force is conservative, and thus we can define a potential energy whose incremental change, dU_E, is minus the work done by the electrical force over an incremental displacement, $d\vec{r}$:

$$dU_E = -\vec{F}_E \cdot d\vec{r}.$$

The *potential*, V_E, is to the potential energy, U_E, what the electric field, $\vec{E}$, is to the electrical force, $\vec{F}_E$. Thus,

$$dV_E = \frac{dU_E}{q} = -\frac{\vec{F}_E}{q} \cdot d\vec{r} = -\vec{E} \cdot d\vec{r}. \tag{9.1}$$

The units for potential, V, are $\mathrm{J\,C^{-1}}$, or volt (V). Equivalently, we can write:

$$\vec{E} = -\frac{dV_E}{dr}\,\hat{r}, \tag{9.2}$$

where $\hat{r}$ is a unit vector pointing in the opposite direction of $\vec{E}$. Thus the units for electric field may also be written as $\mathrm{V\,m^{-1}}$. (Indeed, these are more commonly used than $\mathrm{N\,C^{-1}}$.)

Because the potential is the dot product of two vectors, it is a scalar quantity (like temperature) and can be visualised in 2-D using *contours*. If you have ever looked at a weather map (*e.g.*, Fig. 9.1), you are already familiar with contours. Contours are lines that join up points with equal values of the scalar being mapped. In Fig. 9.1 in which temperature contours are shown, the 20° contours, for example, join up all the points on the map where the temperature is expected to reach 20°. If your city lies between the 20° and 25° contour lines, you can expect a high somewhere between these temperatures.

Contours of V_E are known as *equipotential contours*, or just *equipotentials* for short[6]. If two points lie on the same equipotential, $dV_E = 0$, and no electrical work is done on a charge that may move between these two points. The electrical force, therefore, is not inclined to move the charged object from one of these points to the other.

A gravitational analogy may be helpful. A level surface is a surface that lies along the same *gravitational* equipotential, and an object placed on this surface can be expected to stay put. Thus, shelves are built level and not slanted. On a slanted shelf, one end is at a higher potential than the other, and an object placed at the higher end can be expected

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[6]In 3-D, the locus of points with the same potential form *surfaces*, and are thus known as *equipotential surfaces*.

Figure 9.1 A typical weather map with temperature contours.
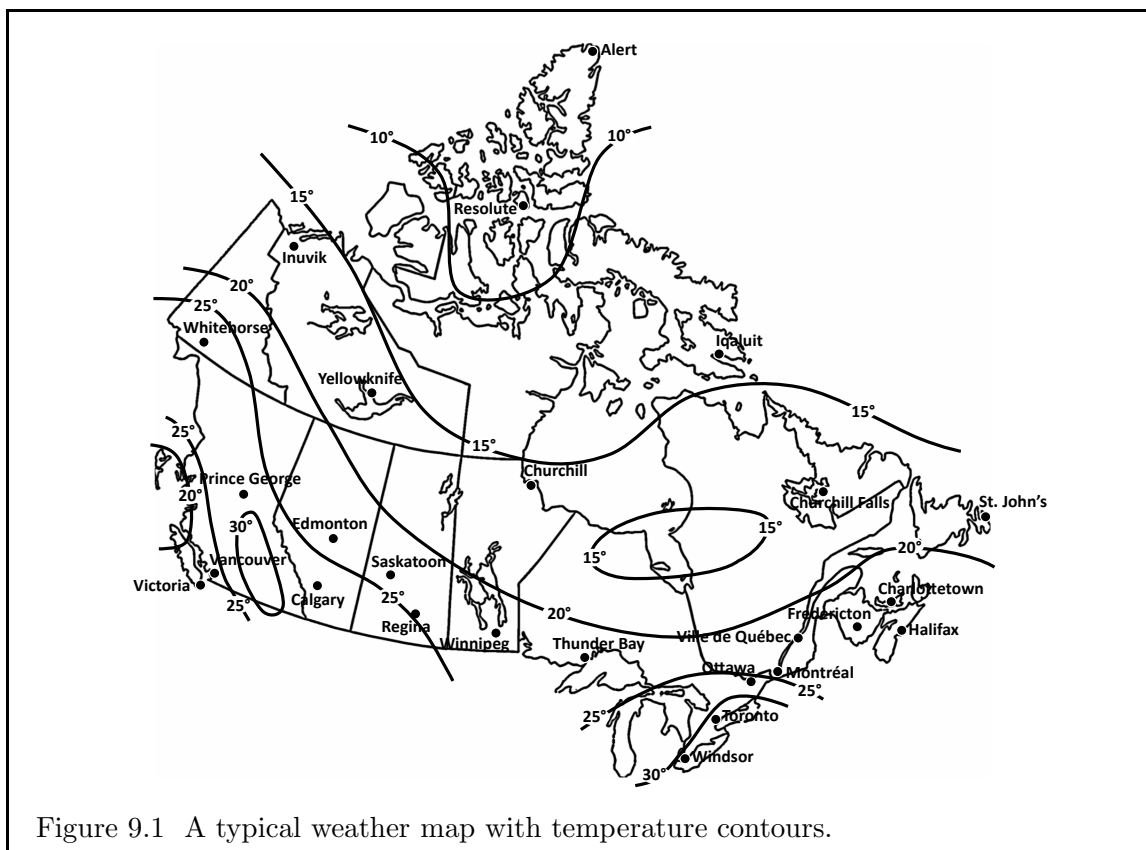
to slide down the shelf. On a level shelf, the gravitational force is still there, but it doesn't cause the object to move. So it is with electrical potential and the electrical force.

If $d\vec{r}$ lies along a V_E contour, the change in V_E along $d\vec{r}$, dV_E, is zero by definition. Thus, equation (9.1) requires that $\vec{E} \cdot d\vec{r} = 0$, and $\vec{E}$ is perpendicular to $d\vec{r}$. Since $d\vec{r}$ lies along the equipotential, the electric field, $\vec{E}$, must be everywhere perpendicular to equipotentials. We will use this fact to map electric field "lines" from equipotential contours mapped directly from measurements taken with an electrical probe placed around charged electrodes.

Procedure

In this experiment, we use an apparatus in which *electrodes* (essentially metal objects connected to opposite ends of a "DC source" or battery) are fixed to a special low-conductivity surface which permits a very low current to move through it. Thus, an ordinary voltmeter (which needs *some* current to pass through it for a reading) can be used to measure potential differences between two arbitrary points on the surface. Finding points with the same potential, then, allows us to map out equipotential contours, and thus electric field lines.

Unlike all the other labs in this manual which are *quantitative*, this is a *qualitative* lab and thus you may consider today to be an "uncertainty-free holiday". No uncertainty
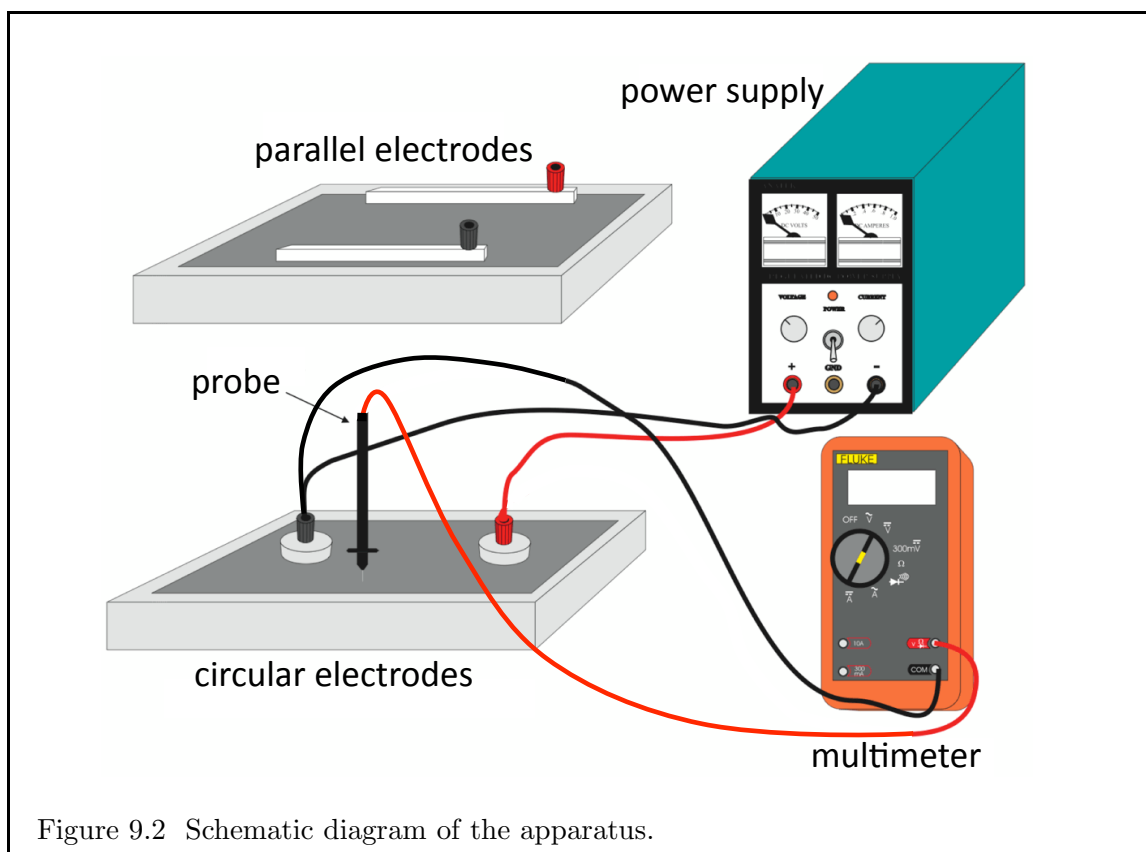
Figure 9.2 Schematic diagram of the apparatus.

propagation or analysis is expected in this write-up.

NOTE: For your safety and the safety of the equipment, do not turn on the power supply until your lab instructor or a demonstrator verifies that you have connected everything according to instructions and that the proper instrument settings are being used.

PART I. Parallel electrodes: a "capacitor"

1. **Make sure your power supply is turned off.** Set the multimeter dial to the DC-voltage location, typically labelled with a "V" that has both a straight line and a dashed line above it so that it acts as a voltmeter.

2. With the leads provided, connect the positive parallel electrode (red) to the positive terminal (red) on the power supply and the negative parallel electrode (black) to the negative terminal (black) on the power supply (see Fig 9.2, which actually shows the "circular electrodes" being used; you should use the "parallel electrodes" for Part I).

3. Connect the negative terminal of the multimeter (black, labelled "COM") to the negative parallel electrode. The positive terminal of the multimeter (red "V") has a probe attached to it which you will use to measure the potential (relative to the negative electrode) at various points between the electrodes.

4. With the power supply still switched off, turn the current knob to full and the voltage knob to zero.

Have your lab instructor or demonstrator check your connections and settings before proceeding.

5. Turn on the power supply and place the probe on the positive (red) parallel electrode. Increase the voltage slowly until the potential measured by the multimeter between the two parallel electrodes reads +20 V. (Once you have set the voltage on the power supply, do not adjust it until this part of the experiment is finished.)

6. Holding the probe *vertically*, press its tip firmly—but not too hard—at some arbitrary point between the parallel electrodes, and take a reading from the multimeter. Leaving the tip of the probe in place but tilting the probe substantially toward one or the other electrode, does the reading on the multimeter change? Now let off on the pressure of the probe somewhat, and determine how firmly the probe must be held so that the reading remains stable. What can you say about the importance of how the probe is held?

7. Along a perpendicular line that bisects the parallel electrodes, place the probe at 3.0, 6.0, 9.0, and 12.0 cm from the *positive* (red) electrode, and for each point record the voltage. On the scaled drawing of the parallel plate electrodes (page 79), draw (for this lab only, in pencil if you like) a dot at each of these four points and include the voltage reading (V). If you make too many errors on the sheet and want to start over, go to page 80.

8. Starting at the 3.0 cm point, place the probe at as many other points (each roughly 2 cm apart) as needed to map out the equipotential contour (*i.e.*, points with the *same* value of V) essentially from one side of the apparatus to the other. Mark each point of the equipotential contour on the scaled drawing, then draw a smooth line between the points.

9. Repeat step 8 for each of the other three points in step 7 to give you a total of four equipotentials.

10. Now draw at least six representative electric field lines separated evenly across the width of the drawing, remembering that field lines must everywhere be perpendicular to both the surface of the electrode and equipotential contours.

## PART II. Circular electrodes: a "dipole"

11. Repeat steps 1–5 in Part I for the circular electrodes.

12. Using the probe, map the equipotential contour for $V_E = 3.0$ volts, and record these positions on the scale diagram provided on page 81. Each position should be roughly 2.0 cm apart. Probe as many places as needed to trace the equipotential contour either off the page, or back to its beginning (a minimum of 10 locations, possibly more).

13. Repeat step 11 for each of $V_E =$ 6.0, 9.0, 12.0, 15.0, and 18.0 Volts, for a total of six equipotential contours.

14. On your scale drawing, draw at least six electric field lines that connect the two electrodes. Remember that electric field lines are perpendicular to the surface of the electrodes, and to every equipotential contour they cross.

15. Turn off the power supply and set its voltage and current knobs to zero. Remove all the electrical leads (wires) and turn off the multimeter.

Analysis, Discussion, and Conclusions

1. Include your two field-line maps with your lab write-up. This is the only time you'll be adding pages to your lab notebook.

2. Plot the data in procedural step 7 on a V_E *vs.* d graph (where d is the distance from the positive electrode), including the points $V_E = 20$ volts at $d = 0$ cm and $V_E = 0$ volts at $d = 15$ cm (and thus touching the negative electrode). This will give you six points that should lie on a straight line. What is the slope of your graph? What does this value represent? (Hint: what are its units?[7])

3. In general, static electric field lines, such as the ones drawn in this lab, must begin on a positive point charge and end on a negative point charge. In this lab, this is manifest by (most of) the electric field lines starting and ending on one of the electrodes (where the point charges reside). Do all your electric field lines behave this way? If any of the field lines you drew do not start and/or end on an electrode, where do you suppose their "charge anchors" are?

4. Describe the shape of the electric field lines for both electrode setups. What do you think influences the shapes of the equipotentials and thus the field lines?

5. A couple of times in the lab procedures, it was mentioned that the electric field lines must be perpendicular to the surfaces of the electrodes as well. Thus, the surfaces of the electrodes must themselves be equipotentials. Explain why this would be so.

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[7]Need another hint? Glance back at equation (9.2)
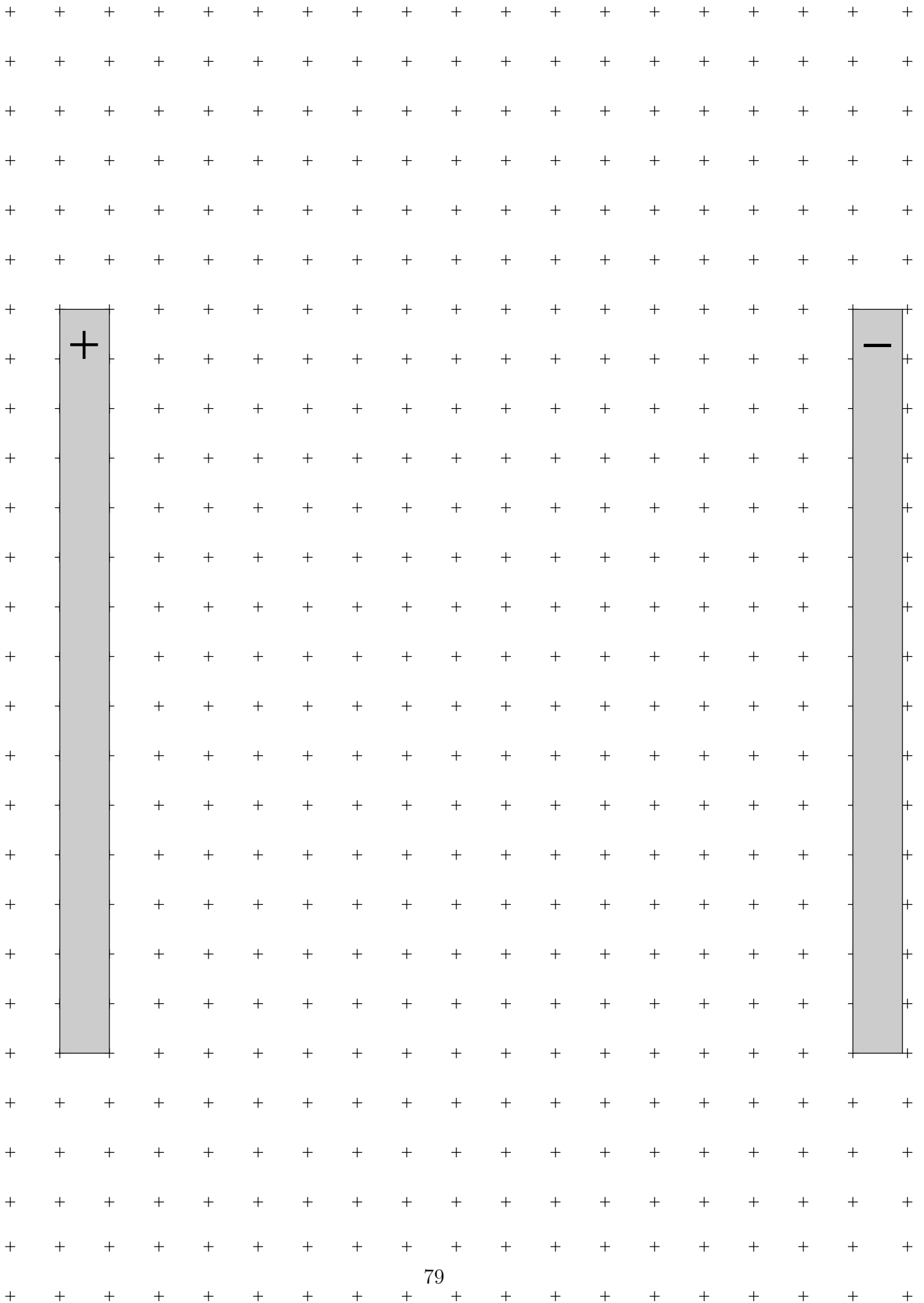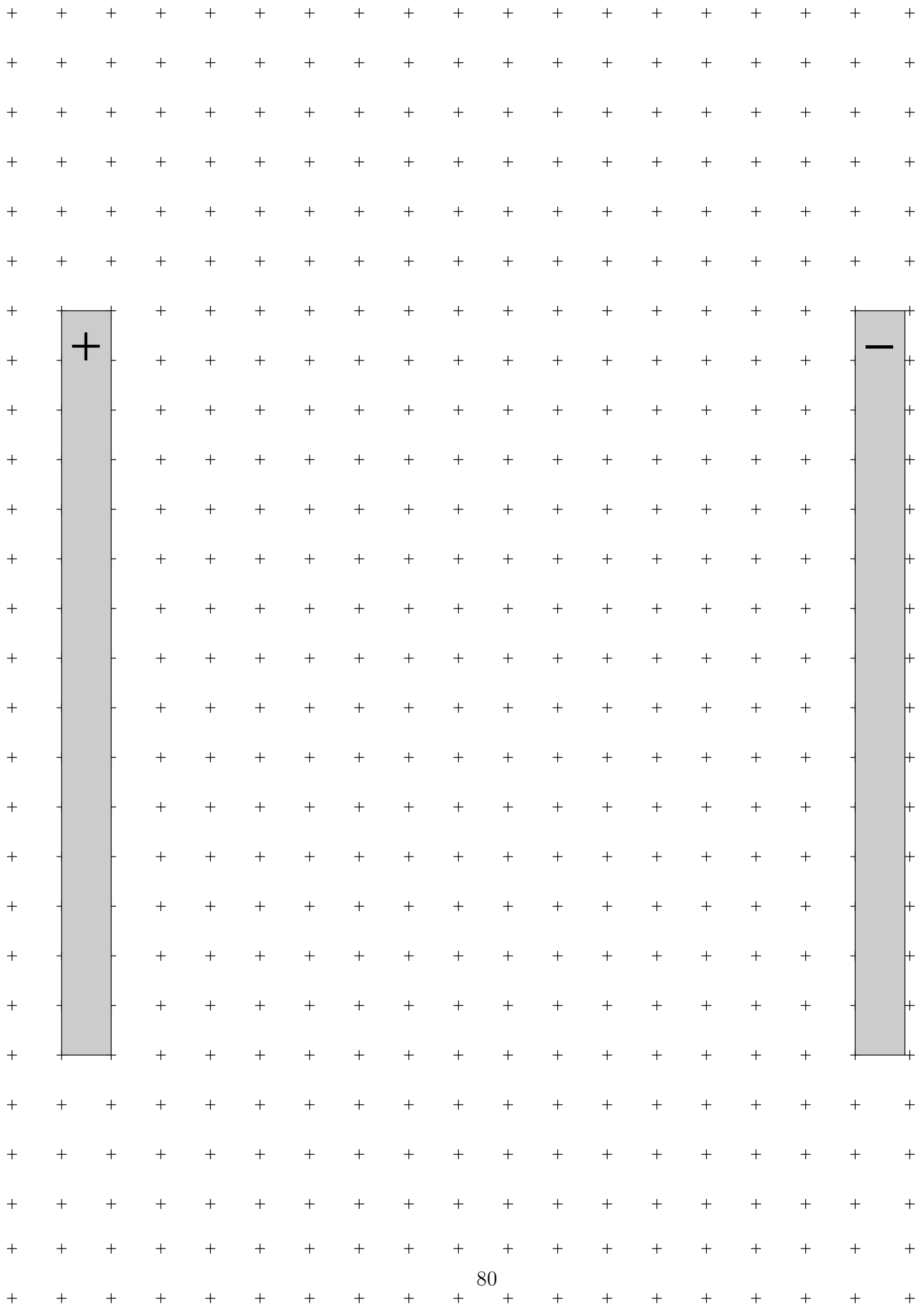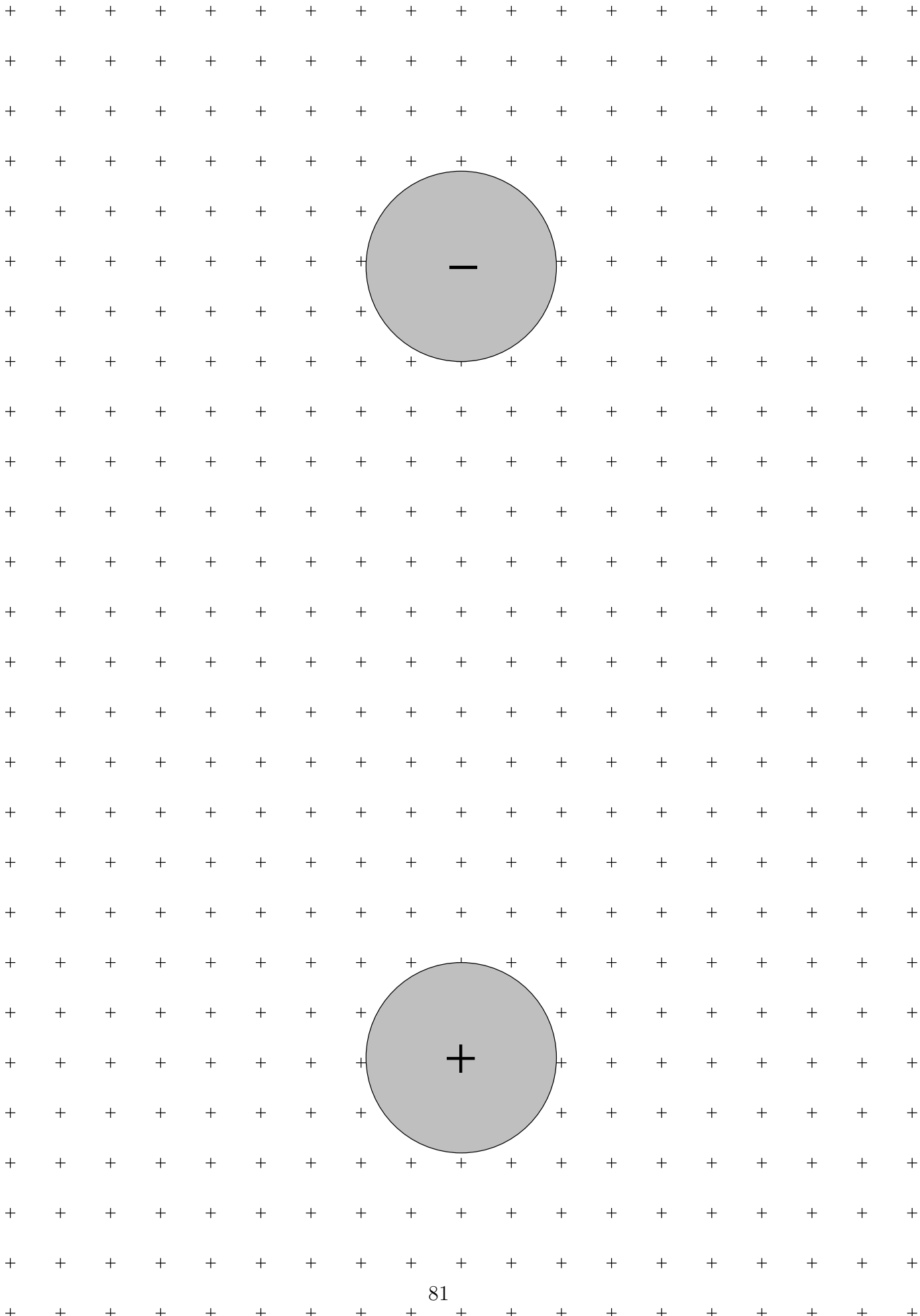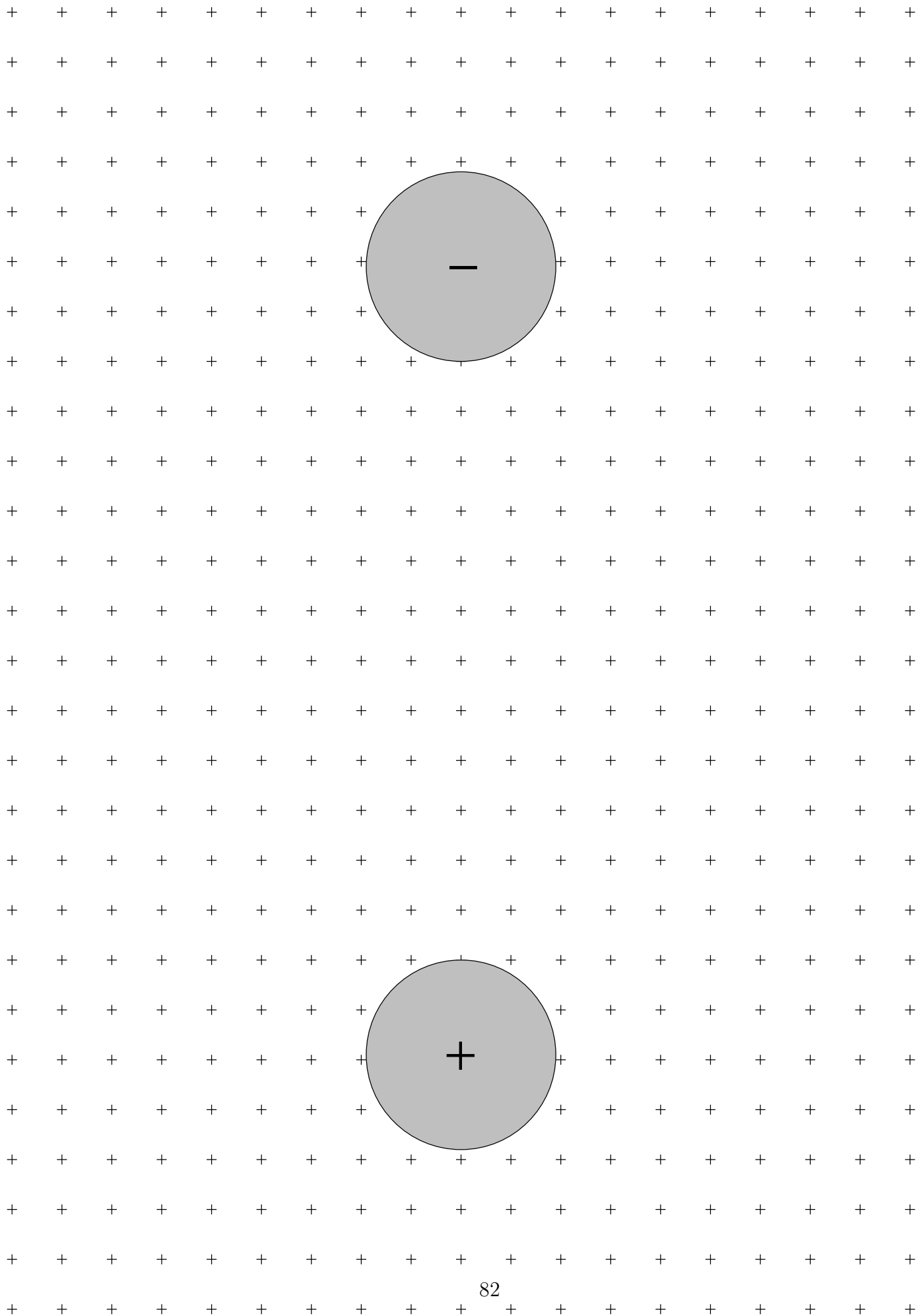
Experiment 10
Direct Current I-R Circuits

Purpose

The purpose of this experiment is to verify Kirchhoff's rules in simple DC *I-R* circuits.

Apparatus

1. DC voltage source,

2. four digital multimeters (acting as voltmeters, ammeters, and ohmmeters),

3. resistors,

4. electrical leads.

Theory

The current, I, driven through a DC (direct current) circuit depends upon the effective resistance, R_{eff}, in the circuit and the potential difference, V, applied across the resistance. According to the definition of resistance,

$$ V \;=\; IR_{\text{eff}} \quad \Rightarrow \quad I \;=\; \frac{V}{R_{\text{eff}}}, \tag{10.1} $$

where the unit of resistance is "ohm" (Ω). Contrary to what is often understood from high school and even from some college-level electronics texts, this is *not* "Ohm's Law"; nothing in equation (10.1) says that R_{eff} has to be constant. Ohm's law states that for a certain class of resistors known as *Ohmic resistors* (*e.g.*, ceramics, most metals), R_{eff} *is* a constant regardless of the potential difference, V, applied. Thus, for resistors obeying Ohm's Law, I plotted against V yields a straight line with slope $1/R_{\text{eff}}$. For *non-Ohmic resistors* (those that don't obey Ohm's Law), an I *vs.* V plot does not yield a straight line.

An example of a non-Ohmic resistor is a semi-conducting diode. While one can always find the effective resistance of such a resistor by measuring V and I and then using equation (10.1), the value of R_{eff} depends not only on the potential difference across it, but also on

Figure 10.1 A DC *I-R* circuit with two resistors in series.
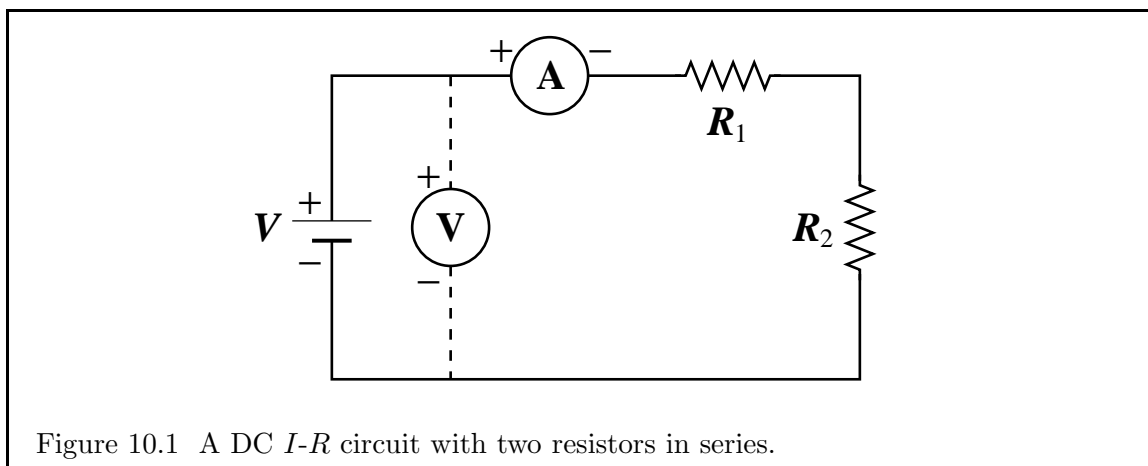
the direction of the current through it. It is this property that is at the base of the entire computer-chip industry.

Kirchhoff's two rules for circuit analysis (the ones to be checked indirectly in this lab) are:

1. *Junction rule:* The sum of all currents into and out of any point (or junction) is zero: $\sum I = 0$.

2. *Loop rule:* The sum of all voltages around any closed loop is zero: $\sum V = 0$.

In the circuit depicted in Fig. 10.1, a DC voltage supply (battery) is placed in "series" with two resistors, R_1 and R_2, and an ammeter, A, to measure the current. The ammeter has its own "internal resistance" which is generally small and usually negligible; we will ignore this resistance in this lab. A voltmeter, V[8], is also shown in Fig. 10.1 placed in "parallel" with the resistors using dotted lines. The dotted lines indicate that the voltmeter is not actually connected to the circuit as the ammeter is, but can be introduced into the circuit simply by "touching" its two leads to either side of a circuit element whose potential drop is being measured. As indicated in Fig. 10.1, the voltmeter is measuring the potential gain across the battery or, equivalently, the potential drop across the two resistors. Voltmeters also have their own internal resistance which, by design, is very large (typically several million Ω) so that the voltmeter diverts as little current from the main circuit as possible. In principle, this drain does affect the current passing through the resistors, though we shall ignore this effect in this lab.

For the circuit depicted in Fig. 10.1, we use the second of Kirchhoff's rules to find:

$$V - IR_1 - IR_2 \; = \; 0 \quad \Rightarrow \quad V \; = \; I(R_1 + R_2)$$

---

[8]The symbol "V" is probably the most overused symbol in circuit analysis. It is used for potential, the symbol for Volt, to label the power supply, and to label the voltmeter. One generally has to go on context to determine what usage is being made of the "V" and, at times, things can look a little crazy. For example, to say the voltage is ten volts, one would write: $V = 10\,\mathrm{V}$ where, if one didn't notice the use of italics, one might be tempted to "cancel the Vs" and write nonsense such as $1 = 10$.

Figure 10.2 A DC *I-R* circuit with three resistors in series.
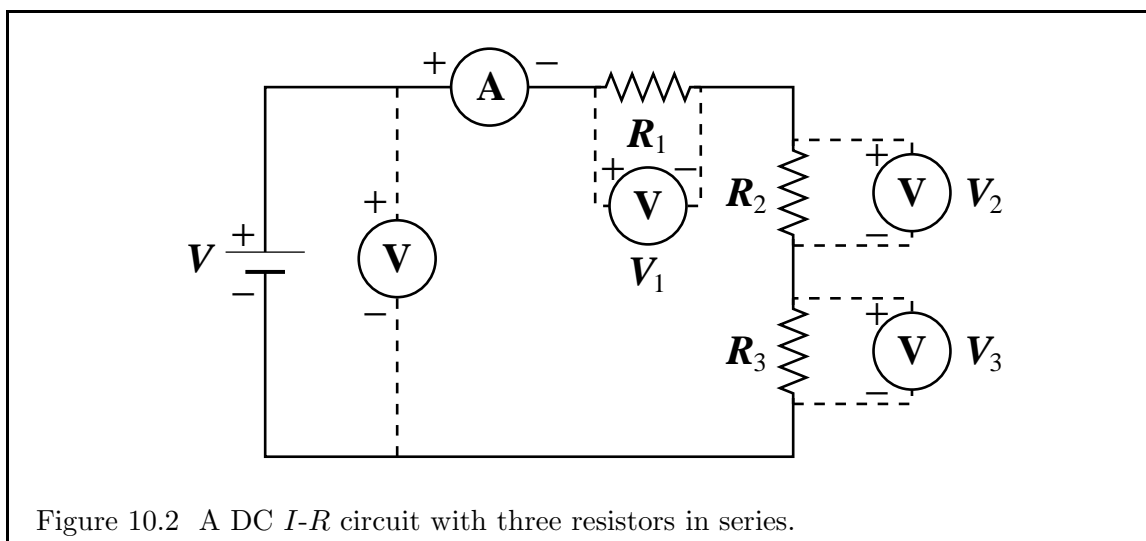
$$\Rightarrow \quad I = \frac{V}{R_1 + R_2}. \tag{10.2}$$

Given that V, R_1, and R_2 are uncertain quantities, it is easy to show that the uncertainty in I is given by:

$$\frac{\Delta I}{I} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2} + \frac{\Delta V}{V}. \tag{10.3}$$

Comparing equations (10.1) and (10.2), we find for this circuit that:

$$R_{\text{eff}} = R_1 + R_2.$$

Thus, resistors in series add.

The circuit depicted in Fig. 10.2 shows *three* resistors in series. Applying the second of Kirchhoff's rules once again, we find:

$$V - IR_1 - IR_2 - IR_3 = 0 \quad \Rightarrow \quad V = I(R_1 + R_2 + R_3)$$

$$\Rightarrow \quad I = \frac{V}{R_1 + R_2 + R_3}, \tag{10.4}$$

with the uncertainty in I given by:

$$\frac{\Delta I}{I} = \frac{\Delta R_1 + \Delta R_2 + \Delta R_3}{R_1 + R_2 + R_3} + \frac{\Delta V}{V}. \tag{10.5}$$

Comparing equations (10.1) and (10.4), we find for this circuit that:

$$R_{\text{eff}} = R_1 + R_2 + R_3.$$

For the circuit in Fig. 10.3 which shows two resistors in *parallel* across the power supply, we need to use both of Kirchhoff's rules to determine the effective resistance in the circuit.
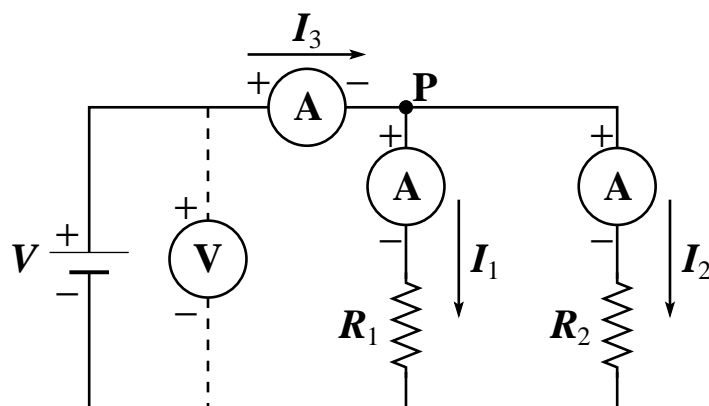
Figure 10.3 A DC I-R circuit with two resistors in parallel.

As illustrated, the current I_3 goes toward point P while currents I_1 and I_2 go away from point P. Thus:

$$I_3 - I_1 - I_2 \; = \; 0 \;\; \Rightarrow \;\; I_3 \; = \; I_1 + I_2. \tag{10.6}$$

Applying the second rule to the two loops containing the voltage supply, we get:

$$V - I_1 R_1 \; = \; 0 \;\; \Rightarrow \;\; I_1 \; = \; V/R_1, \tag{10.7}$$

$$V - I_2 R_2 \; = \; 0 \;\; \Rightarrow \;\; I_2 \; = \; V/R_2. \tag{10.8}$$

Substituting equations (10.7) and (10.8) into equation (10.6) yields:

$$I_3 \; = \; V \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \tag{10.9}$$

Comparing equations (10.1) and (10.9), we find for this circuit that:

$$\frac{1}{R_{\text{eff}}} \; = \; \frac{1}{R_1} + \frac{1}{R_2} \;\; \Rightarrow \;\; R_{\text{eff}} \; = \; \frac{R_1 R_2}{R_1 + R_2}$$

and we say "resistors in parallel add in reciprocal".

Finally, the uncertainties in the currents are given by:

$$\frac{\Delta I_1}{I_1} \; = \; \frac{\Delta R_1}{R_1} + \frac{\Delta V}{V}; \qquad \frac{\Delta I_2}{I_2} \; = \; \frac{\Delta R_2}{R_2} + \frac{\Delta V}{V};$$
$$\frac{\Delta I_3}{I_3} \; = \; \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \right) \frac{R_1 R_2}{R_1 + R_2} + \frac{\Delta V}{V}. \tag{10.10}$$

Preparation question 1: Derive equations (10.3), (10.5), and all three of equations (10.10).

Procedure and Analysis

> **WARNING:** Never short out the terminals of the power supply. That is, do not connect the power supply to a circuit consisting only of a multimeter and no other resistors, as this may damage the power supply and/or the multimeter.

Use of the Fluke multimeter: A multimeter is a device that can be used as an ohmeter, voltmeter, or ammeter, simply by setting a switch. The sensitivity of the device can also be controlled by an appropriate setting.

For an ohmmeter, turn the multimeter dial to the ohmmeter location, typically marked with an "Ω". **With the resistor isolated from the rest of the circuit**, connect one lead from the black input on the meter labelled "COM" to one end of the resistor, and connect another lead from the red input on the meter labelled "Ω" to the other end of the resistor. This provides you with a resistance readout in ohms (Ω). The uncertainty of the resistance reading is $\pm 0.25\% + 2$ in the last digit. Thus, for example, if the reading is $0.704\,\mathrm{k\Omega}$, the uncertainty is $0.0025 \times 0.704 + 0.002 = 0.0038\,\mathrm{k\Omega}$, and you would quote $0.704 \pm 0.004\,\mathrm{k\Omega}$.

For an ammeter, turn the multimeter dial to the DC-current location, typically labelled with an "A" that has both a straight line and a dashed line above it, and place the meter in *series* with the device whose current is being measured. Thus, connect one lead from the black input on the meter labelled "COM" to the "low voltage" side (labelled "$-$" in the figures) and connect another lead from the red input on the meter labelled "10 A" to the "high voltage" side (labelled "$+$" in the figures). This will give you a current readout between 0 and 10 A. If the current is low enough (less than 0.3 A), move the lead from the "10 A" input to the "300 mA" input. This will give a current readout between 0 and 300 mA, **and should be used for low currents only**. The uncertainty of the DC current reading is $\pm 1\% + 3$ in the last digit.

For a voltmeter, turn the multimeter dial to the DC-voltage location, typically labelled with a "V" that has both a straight line and a dashed line above it, and place the meter in *parallel* across the device whose voltage is to be measured. Thus, connect one lead from the black input on the meter labelled "COM" to the "low voltage" side ("$-$") and connect another lead from the red input on the meter labelled "V" to the "high voltage" side of the *same* device ("$+$"). This will give you a potential drop readout between 0 and 600 volts. The uncertainty of the DC voltage reading is $\pm 0.25\% + 2$ in the last digit[9].

PART I: Two resistors in series

1. Create a data table in your lab notebook with three columns: one for the measured applied voltage (V_m), one for the measured current (I_m), and one for the computed current (I_c).

---

[9]Uncertainties are from Fluke's web site: `http://ca.mouser.com/fluke/`.

2. Choose two resistors for use as R_1 and R_2. Measure and record their resistances using the multimeter as an ohmmeter and assemble the circuit as shown in Fig. 10.1.

3. For three different values of applied voltage V, record measured values for V_m and I_m. Use equations (10.2) and (10.3) to determine the expected current (I_c) and its uncertainty based on Kirchhoff's rules, and record these in column 3.

PART II: Three resistors in series

4. Create a data table in your lab notebook with nine columns: one for the measured applied voltage (V_m), one for the measured current (I_m), one for the computed current (I_c), three for the measured voltages across each of the resistors ($V_{1,m}$, $V_{2,m}$, and $V_{3,m}$), and three for the computed voltages across each of the resistors ($V_{1,c}$, $V_{2,c}$, and $V_{3,c}$).

5. Assemble the circuit as shown in Fig. 10.2. Using the multimeter as an ohmmeter, measure and record the resistances of the three resistors.

6. For three different values of applied voltage V, record measured values for V_m, I_m, $V_{1,m}$, $V_{2,m}$, and $V_{3,m}$. Use equations (10.4) and (10.5) to compute the expected current (I_c) and its uncertainty. Compute the expected voltages across each of the resistors ($V_{1,c}$, $V_{2,c}$, and $V_{3,c}$) using $V_{i,c} = I_c R_i$, where $i = 1, 2, 3$, and where the uncertainties follow the usual rules of uncertainty propagation. Record each of the computed values in the appropriate columns of your table.

PART III: Two resistors in parallel

7. Create a data table in your lab notebook with seven columns: one for the measured applied voltage (V_m), three for measured currents ($I_{1,m}$, $I_{2,m}$, and $I_{3,m}$), and three for computed currents ($I_{1,c}$, $I_{2,c}$, and $I_{3,c}$).

8. Choose two resistors for use as R_1 and R_2. Measure and record their resistances using the multimeter as an ohmmeter and assemble the circuit as shown in Fig. 10.3.

9. For three different values of applied voltage V, record measured values for the voltage across the power supply, V_m, and the three currents, $I_{1,m}$, $I_{2,m}$, and $I_{3,m}$, as indicated in Fig. 10.3. Using equations (10.7) – (10.10), calculate the expected currents and their uncertainties and record these values in the columns $I_{1,c}$, $I_{2,c}$, and $I_{3,c}$.

Conclusions and discussion

1. Is there a significant difference between any of your measured and calculated values in any of the three tables? If so, what sources of error do you suspect may have crept into your experiment?

2. Which of your results, if any, confirm Kirchhoff's first rule? Second rule? How?

3. Kirchhoff's rules are statements on the conservation of energy and the conservation of charge. Which is which?

4. Have you been able to verify Ohm's Law? If so, what data from your experiment has verified this? If not, how might you modify this lab so that you could?

Appendix A
Examples of Informal Reports

A.1 An Example of a Good Informal Report

In the following pages is an example of an informal report for Experiment 5, written by I. M. A. Keener, that most instructors and graders would consider excellent. Everything up to and including the "Preparation questions" was presumably done before Keener came to the lab. The purpose is stated with complete sentences, the theory section just states the relevant equations with the symbols defined, and all "Preparation questions" are correctly answered.

Referring to the lab manual for the apparatus and procedures is fine so long as there were no deviations from these during the lab. If you had to use some different equipment or deviate from the stated procedures, you should record those deviations in your report.

Complicated figures need not be reproduced in the report; a reference to the relevant figure in the manual is acceptable. However, simple diagrams and/or those you feel are needed but not in the manual should be drawn with pen and ruler on one of the blank pages in your lab book, and should fill as much of the page as practical.

In the *Data* section, the data should be clearly identified, tabulated where possible, and written *in ink*. Errors will happen, and thus you should write small enough or leave blank lines between entries so if you do make an error, you can neatly cross out the error and write the correct value above it. Note you can still read the errant entries in this example.

The *Data Analysis* section is distinct from the *Data* section, and you should keep them separate. All calculations are performed here, *in ink*. Graphs are done according to the instructions in § 6 of the *Introduction*. Thus, you should draw your graph with pen and ruler, and it should fill as much of the graph page as practical. The title should be descriptive but not overly long, and annotations should be chosen sensibly. Axes are labelled with the variable names and units, and error bars are included where practical. If the data are being fit to a straight line, lines with the minimum and maximum slopes consistent with the data and not forced through the origin should be drawn, and the slopes should be measured directly from these.

If there is insufficient time to finish the experiment, do what you can giving the highest priority to gathering the raw data. After all, your ability to generate a formal report for the experiment in the future will depend more on whether you have all the data than on whether you complete all the analysis. If you are unable to get all the data during the

regularly scheduled lab, you should plan on gathering the missing data during "make-up week" (§ 2.3 of the *Introduction*).

The *Discussion* and *Conclusions* sections may be merged together in the informal report, if you wish. Here, we expect you only to state the facts as you determined them. If you have time, extend your discussion by answering the questions at the end of the experiment, and suggest realistic ways the lab may be improved where appropriate.

Finally, note that this sample report includes only the data and analysis for one of the rods, not both, as this is sufficient to demonstrate what a good lab report should look like. In practise, it is expected that students will be able to analyse *both* rods for a complete report.

Most graders would give this lab a 7 or 8 out of 10, losing 2 points for not including the second rod. A completed report like this would earn the full ten points.

Experiment 5: Shear Modulus

Performed Nov. 17, 2008 by I.M.A. Keener

Lab Partner : R.U. Really

Purpose :   The purpose of this lab is to demonstrate how a metal can be
identified by measuring its shear modulus.

Apparatus :   ~~tort~~ torsion apparatus (see Fig. 5.1 of the lab manual)
assorted masses and mass hanger
mass balance
micrometer
metre-stick.
rod of unknown metal ($\sim$ 1 m in length)

Theory :   See the theory section for Experiment 5 in the lab manual
for additional details and derivations.

$$\theta = \frac{180\,g\,D\,L}{G\,\pi^2\,R^4}\,M \qquad\qquad (1)$$

where θ = the twist angle of the rod
g = 9.81 ms^{-2} (exact)
D = diameter of torsion wheel
L = length of rod
G = shear modulus of metal in rod
R = radius of rod
M = mass hanging from the torsion wheel that provides the
twist (torque).

Many of these quantities are illustrated in Fig 5.3 of the manual.

When plotted on a Θ vs. M graph, the data should follow a straight line with a slope

$$m = \frac{180\, g\, D\, L}{G\, \pi^2\, R^4} \qquad\qquad (2)$$

which can be solved for G to get

$$G = \frac{180\, g\, D\, L}{m\, \pi^2\, R^4} \qquad (N\, m^{-2}) \qquad\qquad (3)$$

Preparation questions:

1. A Hooke's Law force, in terms of the Young Modulus, Y, is given by

$$F = -\frac{AY}{L}\, x$$

where A is the cross-sectional area of the spring and L its length. Thus, the spring constant, k, is given by

$$k = \frac{AY}{L}$$

Now, cut the spring in half, to the new length, $L' = L/2$. Then the new spring constant, k', is given by

$$k' = \frac{AY}{L/2} = 2\frac{AY}{L} = 2k$$

and the spring constant is doubled, as required.

2. From equation (3), G is made up of products and quotients of the uncertain quantities D, L, m, and R. Thus, from repeated application of equation I.2 in the introduction to the lab manual,

$$\frac{\Delta G}{G} = \frac{\Delta\left(\frac{180g}{\pi^2}\right)^0}{\frac{180g}{\pi^2}} + \frac{\Delta D}{D} + \frac{\Delta L}{L} + \frac{\Delta m}{m} + \frac{\Delta R^4}{R^4}$$

$$= \frac{\Delta D}{D} + \frac{\Delta L}{L} + \frac{\Delta m}{m} + 4\frac{\Delta R}{R} \qquad (4)$$

as required, and by using equation I.3 in the lab manual.

Procedure : See experiment 5 of the lab manual for details. Only the "B"-rod was examined, as this is sufficient to demonstrate a good lab report.

Data : The enumeration refers to the procedure number in the manual.

1. Mass measurements

m = 50.5g , 49.7g , 50.5g , 50.4g , 49.9g

$\Rightarrow$ $\bar{m}$ = 50.20 g ; Δm = 0.40g (half the range)

Since Δm > reading uncertainty of the mass balance, we shall use

m = 50.2 ± 0.4g

as the mass of the individual weights.

2. Circumference of the torsion wheel, $C = 49.3 \pm 0.3$ cm

The uncertainty is larger than the reading uncertainty of the metre stick because the circumference had to be measured by wrapping the cord about the wheel, and then measuring the length of cord. This introduces uncertainty in judging where the ends of the cord meet, kinks in the cord, etc., and we estimate this effect to be about triple the simple reading uncertainty of ± 0.1 cm.

Thus, $D = C/\pi = 15.7 \pm 0.1$ cm

4. Length of rod "B" : $L = 100.0 \pm 0.1$ cm (reading error of metre stick)

5. Radius of rod "B" :

We measured the diameter with a micrometer five times, and found :

$d = 3.924$ mm, 3.930 mm, 3.926 mm, 3.927 mm, 3.928 mm

all with a reading uncertainty (on the micrometer) of ± 0.002 mm

Thus $\bar{d} = 3.927$ mm, $\Delta d = 0.003$ mm (half the range)

Since the half the range uncertainty > reading uncertainty, we take

$d = 3.927 \pm 0.003$ mm

$\Rightarrow R = d/2 = 1.9635 \pm 0.0015$ mm.

6-12 Data table for mass loads and their associated twist angles. for rod "B".

M (g)	$\theta_1{}^\circ$	$\theta_2{}^\circ$	$\theta_3{}^\circ$	$\theta_4{}^\circ$	$\bar{\theta}^\circ$	$\Delta\theta^\circ$
50.2 ± 0.4	2.0	2.1	1.9	2.2	2.05	0.15
100.4 ± 0.8	4.1	4.2 ~~4.1~~	4.0	4.2	4.13	0.10
150.6 ± 1.2 ~~100.6 ± 0.1~~	5.9	5.9	6.1	6.0	5.98	0.10
200.8 ± 1.6	8.5 ~~6.5~~	8.4	8.6	8.6	8.52	0.10
251.0 ± 2.0	10.3	10.4	10.4	10.7	10.43	0.15
301.2 ± 2.4	11.8	12.0	12.1	12.1	11.95	0.15
351.4 ± 2.8	14.2	14.8	14.5	14.6 ~~14.1~~	14.52	0.30
401.6 ± 3.2	16.3	16.4	16.1	16.5 ~~16.3~~	16.33	0.20
451.8 ± 3.6	18.0	18.0	18.0	17.9	17.97	0.05
502.0 ± 4.0	19.9	20.6	19.8	20.3	20.15	0.40

estimated reading uncertainty for twist angle: ± 0.05° (Vernier scale)

θ_1: increasing mass, clockwise twist

θ_2: decreasing mass, clockwise twist

θ_3: increasing mass, counter-clockwise twist

θ_4: decreasing mass, counter-clockwise twist.

Analysis:

1. See data table

2. See plot. Uncertainty bars for masses < 200 g are too small to plot.

3. To find the slope,

$$m_{max} = \frac{max\ rise}{min\ run} = \frac{20.85 - 2.00}{500 - 53} = 0.04217\ \%/g$$

$$m_{min} = \frac{min\ rise}{max\ run} = \frac{19.85 - 2.00}{500 - 35} = 0.03839\ \%/g$$

Thus $m = \frac{m_{max} + m_{min}}{2} \pm \frac{m_{max} - m_{min}}{2} = 0.4\ 0.04028 \pm 0.00189\ \%/g$

$$= \underline{40.3 \pm 1.9\ \%/kg}$$

4. Thus, from equation (3), we have

$$G = \frac{(180)(9.81)(0.157)(1.000)}{(40.3)\ \pi^2 (0.0019635)^4} = 4.689 \times 10^{10}\ N\,m^{-2}$$

From equation (4),

$$\frac{\Delta D}{D} = 0.00637 \qquad \frac{\Delta L}{L} = 0.001 \qquad \frac{\Delta R}{R} = 0.00076$$

and $\frac{\Delta m}{m} = 0.0469 \qquad \Rightarrow \frac{\Delta G}{G} = 0.0573$

Dependence of twist angle (θ) on load mass (M) on the torsion apparatus. Rod "B"
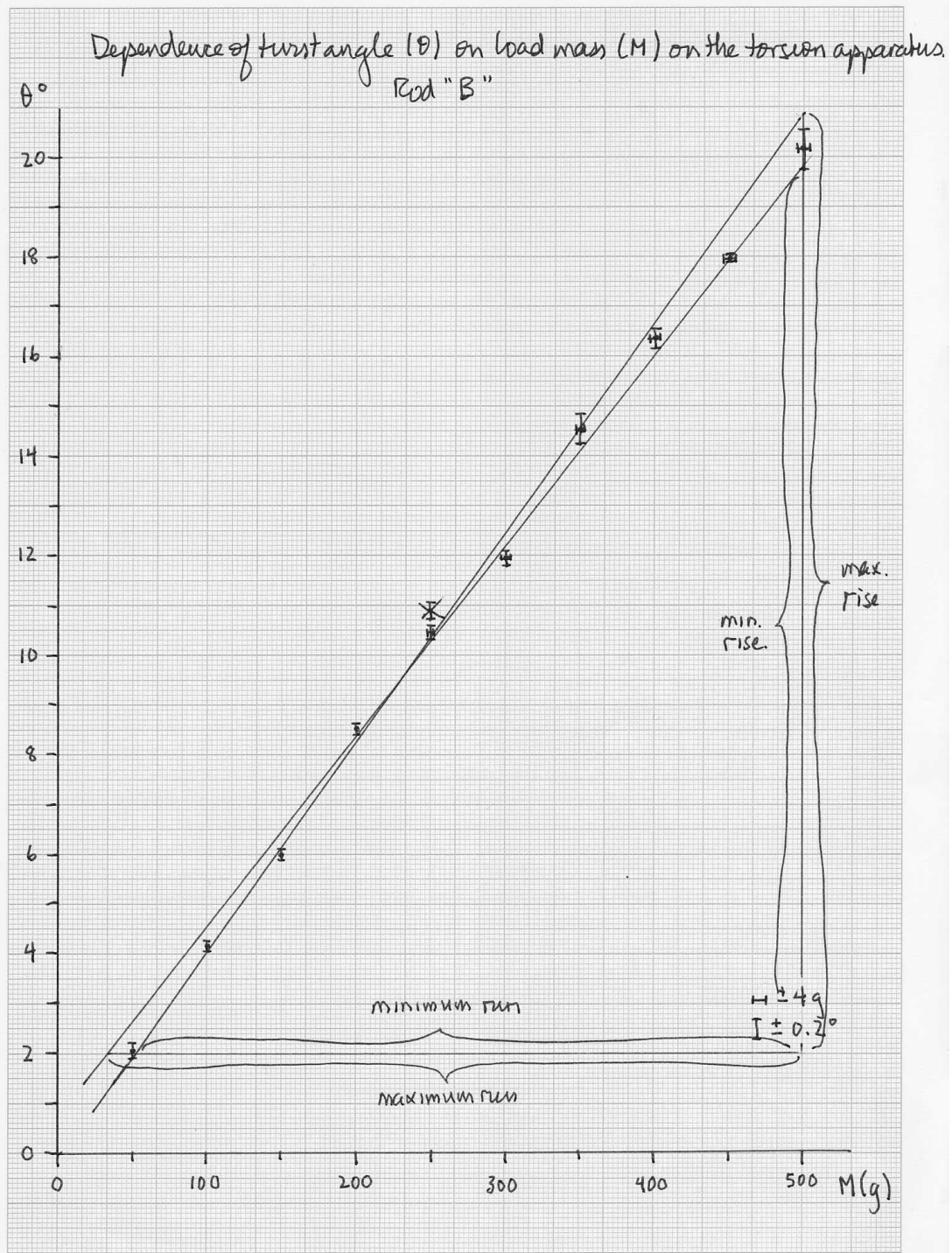
$$\Rightarrow \Delta G = 0.27 \times 10^{10} \, Nm^{-2}$$

Thus, I find that $G = (4.69 \pm 0.27) \times 10^{10} \, Nm^{-2}$ for rod "B".

Discussion

1. I have found G, the shear modulus, to be in the range:

$$4.42 \times 10^{10} \leq G \leq 4.96 \times 10^{10} \, Nm^{-2} \quad (rod \text{ "B"})$$

According to the table of shear moduli given in the manual, this is consistent with bronze $(4.48 \times 10^{10} \, Nm^{-2})$.

2. By far the biggest source of uncertainty is the determination of the slope and thus, ultimately, measuring the twist angles. Thus, to improve the lab, one would have to review how the angles are determined and improve on this substantially.

A.2 An Example of a Poor Informal Report

In the following pages is an example of an informal report for Experiment 5, written by R. U. Really, that most instructors would consider very poor. Since the *Purpose* and *Theory* sections are to be completed *before* the lab, Really has no excuse for this brevity or for not answering the "Preparation questions". The *Purpose* is stated using an incomplete sentence, and the *Theory* section should at least contain the equations to be used and a list of defined symbols.

There is no division between the *Data* and *Data Analysis* sections. Errant entries are obliterated and cannot be read. Symbols (*e.g.*, D, L, *etc.*) are introduced without being defined. Some data are recorded with their uncertainties (*e.g.*, D, L, and R), so that's good, but some data are not recorded with their uncertainties (*e.g.*, M), and other data are missing altogether (*e.g.*, the five measurements that went into determining R).

The graph is inadequate. It could have been drawn much bigger and the lines were drawn without a ruler. The title is useless and the axes are not labelled. Error bars are not drawn to scale, the minimum and maximum sloped lines are forced through the origin when they should not be, and the rises and runs are chosen so small that reading error from the graph now dominates the uncertainty.

What is worse, Really has completely forgotten to reconcile units. He has used the slope as directly measured from the graph (0.039) into his equation for G without first converting to °/kg, and has used the measurements of D, R, and L directly without first converting them into metres. No wonder his shear modulus was *five orders of magnitude* off!

Really's carelessness has rendered this lab report virtually useless. Evidently his partner, I. M. A. Keener, even alerted him to the potential problem with the units, yet there is no evidence Really bothered following this up.

Unlike Keener's report, Really's report shows little organisation, little preparedness, and little attention to proper analysis procedure. It is difficult to imagine how Really would be able to prepare a formal report from this two months down the road, and even more difficult to imagine how anyone else would be able to follow what was done in this experiment. Virtually any grader would give this report a severe failing grade (could be zero!).

#5 Shear Modulus

by R.V. Really
and I.M.A. Keener

Purpose — to measure the shear modulus of a rod

Theory, apparatus, procedures — see lab manual

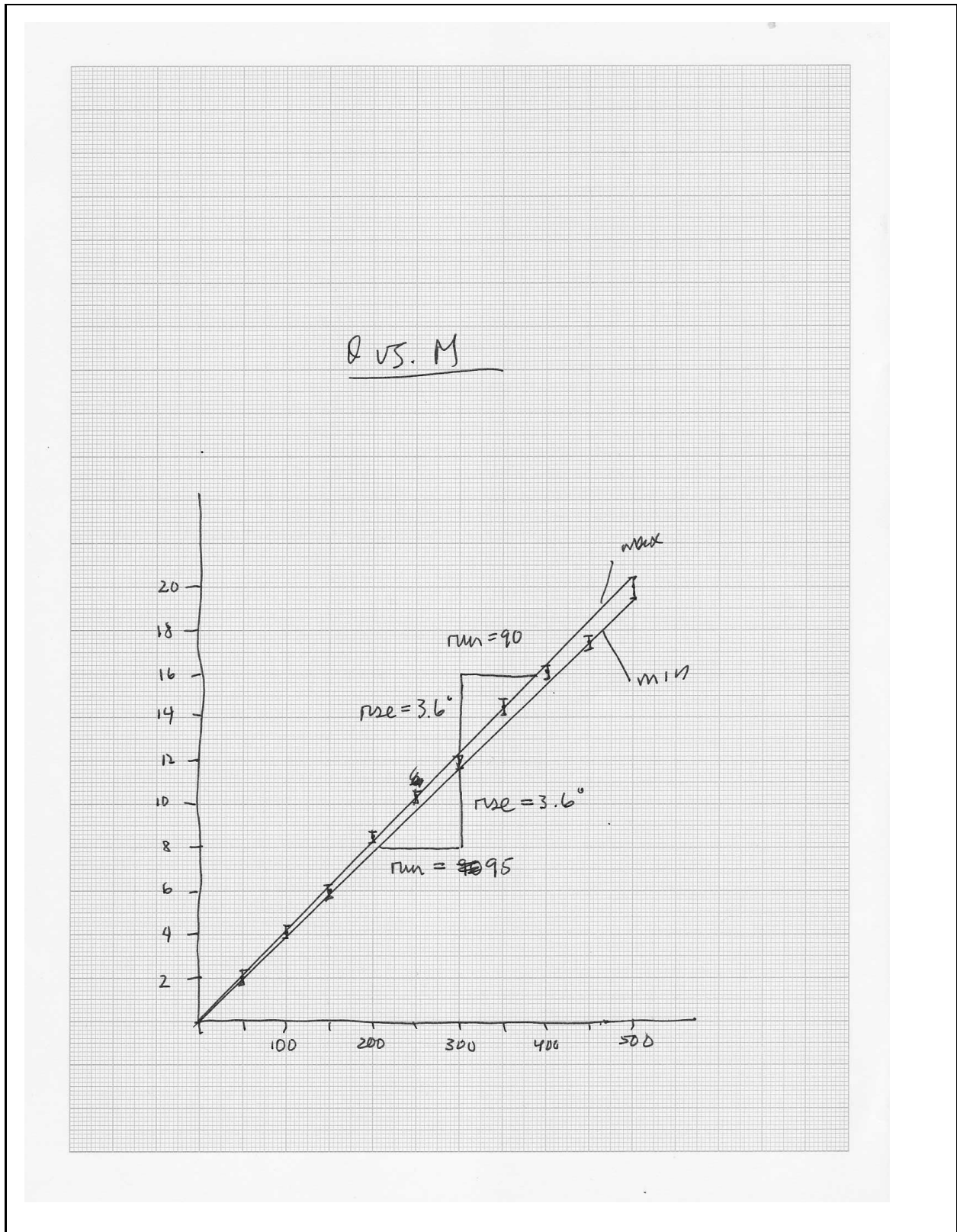Data M = 50.5 49.7 50.5 50.4 49.9

$\overline{m}$ = 50.2 ± .4 g.

D = 15.7 ± 0.1 cm
L = 100.0 ± 0.1 cm
R = 1.9635 ± 0.0015 mm

M	θ_1	θ_2	θ_3	θ_4	$\overline{\theta}$	$\Delta\theta$
50.2	2	2.1	1.9	2.2	2.05	0.15
100.4	4.1	4.2	4.0	4.2	4.13	0.10
150.6	5.9	5.9	6.1	6.0	5.98	0.10
200.8	8.5 8.4	8.4	8.6	8.6	8.52	0.10
251	10.3	10.4	10.4	10.7	10.43	0.15
301.2	11.8	12.0	12.1	12.1	11.95	0.15
351.4	14.2	14.8	14.5	14.6	14.52	0.3
401.6	16.3	16.4	16.1	16.5	16.33	0.2
451.8	18.0	18.0	18.0	17.9	17.57	0.05
502.0	19.9	20.6	19.8	20.3	20.15	0.4

plot

Q vs. M

max

min

run = 90

rise = 3.6°

rise = 3.6°

run = 95

20
18
16
14
12
10
8
6
4
2

100 200 300 400 500

slope m_{max} = $\dfrac{3.6}{90}$ = 0.04

m_{min} = $\dfrac{3.6}{95}$ = 0.038

$m = 0.039 \pm 0.001$

$G = \dfrac{180 \, g \, DL}{m \pi^2 R^4}$ = 4.8457×10^5

Conclusions

G is nowhere near any of the tabulated numbers (all about 10^5 bigger!), so I can't tell what the metal was.

My lab partner thinks I got the units screwed up.

Biggest source of error is human error.

Appendix B
Example of a Formal Report

In the following pages is an example of a formal report for Experiment 5 for which most instructors would give high marks (90% or better). Notice for the most part, it is little more than a transcription of the example of a good informal report in App. A, with more discussion filled in here and there. This is possible *only* if your lab notebook is kept complete and in good order, like Keener's. If you maintain your lab notebook more like Really does, your task of writing up a *good* formal report will be significantly more difficult.

As per § 3 of the *Introduction* ("Theory" item #4), answers to the "preparation questions" are *not* reproduced here. These were to help prepare the student for the experiment before coming to the lab, have already been answered in the informal report, and have served their purpose. However, note that *all* the "Discussion questions" are answered in this report, which were not all necessarily answered in the informal report.

Notice how well the word processor used to generate this report handles equations. If your word processor cannot do equations well, you are advised to leave spaces for each equation, then write them in by hand after the report is printed out. Good handwriting is *always* better than bad word processing.

As with the sample informal report in § A.1, this sample formal report includes only the data and analysis for one of the rods, not both, as this is sufficient to demonstrate what a good formal lab report should look like. In practise, students would be expected to include the analysis of both rods in a formal report, having used the "make-up week" (§ 2.3) to obtain any data that were not collected during the actual lab.

Use this example as a guide only. Your instructor may give you specific instructions which supersede some of the styles illustrated here.

Formal Lab Report

Experiment 5:   Shear Modulus

Performed by: I. M. A. Keener
Partner: R. U. Really
Lab Performed: November 17, 2008
Report Submitted: March 23, 2009

Purpose

The purpose of this experiment is to demonstrate how a metal can be
identified by measuring its shear modulus.

Apparatus

1. torsion apparatus (see Fig. 1),

2. assorted masses and a mass hanger,

3. mass balance,

4. metre stick,

5. micrometer,

6. rod of unknown metal.

See Fig. 1 for a schematic diagram of the arrangement and use of the
apparatus.

Theory

When a rigid object is acted upon by an external force, some shear will
result (Fig. 2). Experimentally, it is known that the "stress", Σ, exerted
on the object (force divided by the area over which the force acts) is
proportional to the "strain" (shearing distortion), x/L, and we write:

$$\Sigma \;=\; G\,\frac{x}{L}, \tag{1}$$

where x is the distortion, L is the scale length of the object (e.g., see
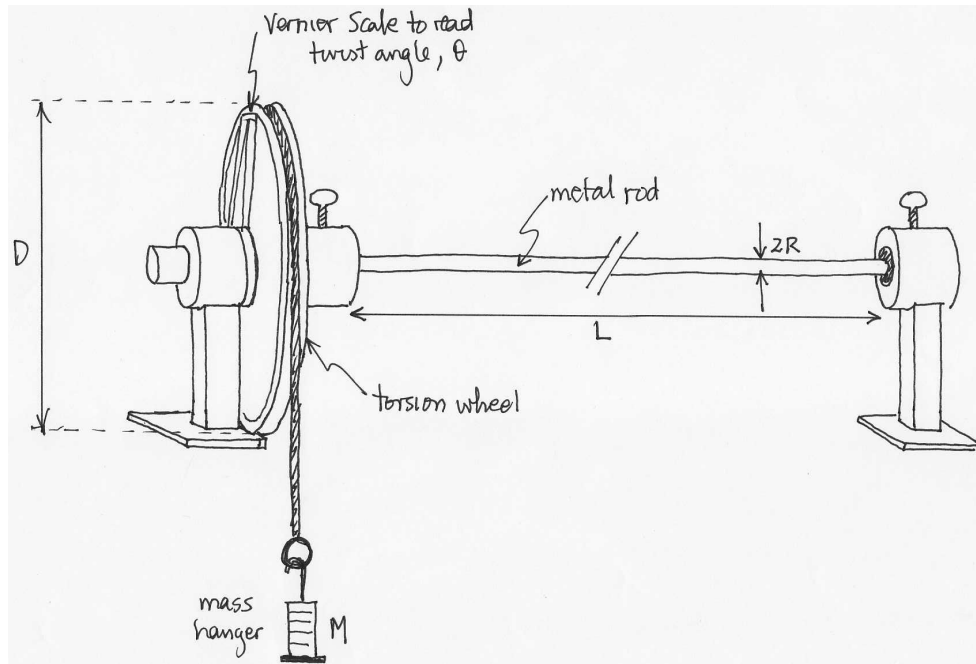Fig. 2), and G is the shear modulus on the material.

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Figure 1. Apparatus for the Shear Modulus lab, with the important quantities labelled. (Reproduced from the lab manual.)
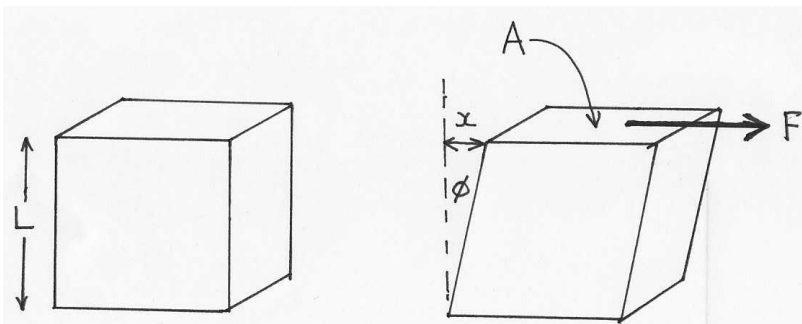

Figure 2. When a force, F, is applied to a solid object, the strain (x/L) is proportional to the stress (F/A) for small distortions. (Reproduced from the lab manual.)

In this lab, we twist a long metal rod. In this case, the stress comes from an applied torque rather than an applied force, and the strain is a twist angle, not a shearing distance. If, as in Fig. 1, the torque is supplied by a mass, M, hanging from a wheel of diameter D, which in turn twists the metal rod of radius R and length L, the twist angle, θ, is related to the mass load, M, according to:

$$\theta \;=\; \frac{180\text{gDL}}{\text{G}\pi^2\text{R}^4}\,\text{M,} \tag{2}$$

where θ is measured in degrees. See equation 5.7 and its derivation in the "PHYS 1100/1101 Lab manual", Edition 4.1 (2009; henceforth "the manual"). Thus, a plot of θ vs. M should yield a straight line whose slope, m, is given by:

$$\text{m} \;=\; \frac{180\text{gDL}}{\text{G}\pi^2\text{R}^4}. \tag{3}$$

By measuring this slope, we can then find an experimentally determined value for the shear modulus:

$$\text{G} \;=\; \frac{180\text{gDL}}{\text{m}\pi^2\text{R}^4}. \tag{4}$$

The uncertainty in G follows from rule 2 in § 5.4 in the manual, as well as equation (I.3). Thus,

$$\frac{\Delta\text{G}}{\text{G}} \;=\; \frac{\Delta\text{D}}{\text{D}} + \frac{\Delta\text{L}}{\text{L}} + \frac{\Delta\text{m}}{\text{m}} + 4\frac{\Delta\text{R}}{\text{R}}. \tag{5}$$

Procedure

1. Five of the "50 g" masses were weighed using the mass balance. The average of these five values and a half-the-range uncertainty were recorded.

2. The circumference of the torsion wheel (Fig. 1) was measured by measuring the length of the cord needed to wrap tightly around the wheel, then dividing by π. An uncertainty of ± 3 mm was quoted for the circumference---three times the reading uncertainty---because of the additional uncertainty introduced by this method of measuring.

3. The rod stamped with a "B" was inserted into the torsion apparatus. Its length, L, was measured using a metre stick and its radius was measured five times with a micrometer, with its value taken as the mean of the five readings with a half-the-range uncertainty recorded.

4. The cord was wrapped clockwise about the torsion wheel and, with only the mass hanger hanging from the cord, the Vernier angular reading was

set precisely to zero. Masses were then applied in 50 g increments up
to 500 g, and the twist angle for each mass was recorded in column θ_1
of Table 1.

5. With the maximum mass on the hanger, the hanger was jiggled gently
   from below, and the twist angle was measured again and recorded at the
   bottom of column θ_2 in Table 1. Masses were removed in 50 g
   increments with each twist angle reading recorded in column θ_2.

6. With the mass hanger empty, the cord was wrapped around the wheel in
   the counter-clockewise direction, and steps 4 and 5 were repeated,
   filling columns θ_3 and θ_4 in Table 1. Column $\bar{\theta}$ contains the means of
   the values in columns θ_1--θ_4, while column $\Delta\theta$ is the half-the-range
   uncertainty.

7. The graph in Fig. 3 is $\bar{\theta}$ plotted against M, with the uncertainty bars
   drawn where visible. All subsequent analysis was performed from this
   graph.

Raw Data

1. The masses of the five "50 g" masses are:

   50.5 g, 49.7 g, 50.5 g, 50.4 g, 49.9 g

   Thus, $\bar{m} = 50.20$ g; $\Delta m = 0.40$ g.

   As the masses are added/removed from the hanger, the total mass is
   adjusted in units of 50.2 ± 0.4 g.

2. circumference of wheel = C = 49.3 ± 0.3 cm. Thus, D = 15.7 ± 0.1 cm.

3. L = 100.0 ± 0.1 cm

   The five measured rod diameters are:

   3.927 mm, 3.930 mm, 3.926 mm, 3.927 mm, 3.928 mm.

   Thus, $\bar{d}$ = 3.927 mm and Δd = 0.003 mm (half the range). Since the
   half-the-range uncertainty is greater than the reading uncertainty,
   the former is retained as the uncertainty in d.

   Thus, R = $\bar{d}/2$ = 1.9635 ± 0.0015 mm.

4. The data from procedure items 4--7 are tabulated in Table 1.

   The reading uncertainty for the Vernier scale in θ is $\pm\ 0.05°$ which,

M (g)	θ_1 (°)	θ_2 (°)	θ_3 (°)	θ_4 (°)	$\bar{\theta}$ (°)	$\Delta\theta$ (°)
50.2 ± 0.4	2.0	2.1	1.9	2.2	2.05	0.15
100.4 ± 0.8	4.1	4.2	4.0	4.2	4.13	0.10
150.6 ± 1.2	5.9	5.9	6.1	6.0	5.98	0.10
200.8 ± 1.6	8.5	8.4	8.6	8.6	8.52	0.10
251.0 ± 2.0	10.3	10.4	10.4	10.7	10.43	0.15
301.2 ± 2.4	11.8	12.0	12.1	12.1	11.95	0.15
351.4 ± 2.8	14.2	14.8	14.5	14.6	14.52	0.30
401.6 ± 3.2	16.3	16.4	16.1	16.5	16.33	0.20
451.8 ± 3.6	18.0	18.0	18.0	17.9	17.97	0.05
502.0 ± 4.0	19.9	20.6	19.8	20.3	20.15	0.40

Table 1. Mass load and twist angles gathered in this lab.
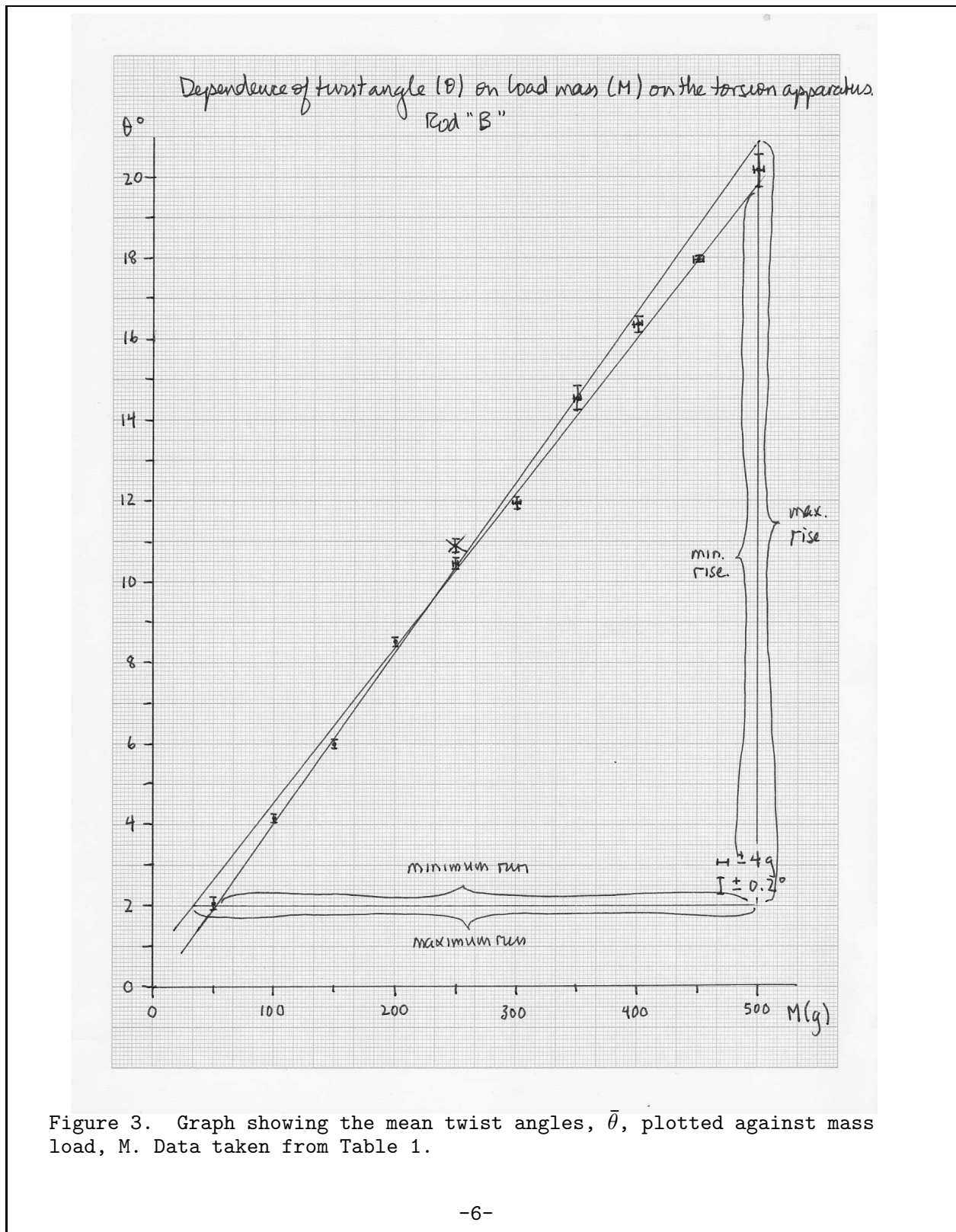Reading uncertainty for all twist angles is ± 0.05°.

in all cases, is less than or equal to the half-the-range uncertainty.
Thus, the latter is retained for the uncertainty for all mean twist
angles.

Data Analysis

Figure 3 includes the fit-by-eye best-fit lines with the maximum and
minimum slopes, and consistent with most of the data (passes through most
of the error bars). Also indicated on the graph are the rises and runs
used to measure these slopes. Thus, we find:

$$m_{max} \ = \ \frac{\text{max rise}}{\text{min run}} \ = \ \frac{20.85 - 2.00}{500. - 53.} \ = \ 0.04217 \ °/g;$$

$$m_{min} \ = \ \frac{\text{min rise}}{\text{max run}} \ = \ \frac{19.85 - 2.00}{500. - 35.} \ = \ 0.03839 \ °/g;$$

$$\Rightarrow \quad m \ = \ \frac{m_{max} + m_{min}}{2} \pm \frac{m_{max} - m_{min}}{2} \ = \ 0.04028 \pm 0.00189 \ °/g$$
$$= \ 40.3 \pm 1.9°/\text{kg}.$$

Thus, from equation (4), we get:

Figure 3.   Graph showing the mean twist angles, $\bar{\theta}$, plotted against mass load, M. Data taken from Table 1.

$$G \;=\; \frac{(180)(9.81)(0.157)(1.000)}{(40.3)\pi^2(0.0019635)^4} \;=\; 4.689 \times 10^{10} \; \mathrm{Nm}^{-2}.$$

To find ΔG using equation (5), we first evaluate all the relevant fractional uncertainties. Thus,

$$\frac{\Delta D}{D} = 0.00637; \qquad \frac{\Delta L}{L} = 0.001; \qquad \frac{\Delta R}{R} = 0.00076; \qquad \frac{\Delta m}{m} = 0.0469.$$

Substituting these values into equation (5) yields:

$$\frac{\Delta G}{G} \;=\; 0.0573 \quad \Rightarrow \quad \Delta G \;=\; 0.27 \times 10^{10} \; \mathrm{Nm}^{-2}.$$

Therefore, the shear modulus for the metal rod stamped "B" is:

$$G \;=\; (4.69 \pm 0.27) \times 10^{10} \; \mathrm{Nm}^{-2}.$$

and thus $4.42 \times 10^{10} \, \mathrm{Nm}^{-2} \leq G \leq 4.96 \times 10^{10} \, \mathrm{Nm}^{-2}$. According to Table 5.1 in the manual, this is consistent with bronze ($G = 4.48 \times 10^{10} \, \mathrm{Nm}^{-2}$), and no other metals. However, it is noted that while not tabulated in Table 5.1, the shear modulus of copper, a metal closely related to bronze, is $G = 4.50 \times 10^{10} \, \mathrm{Nm}^{-2}$. Thus, the uniqueness of the identification is based, in part, on the exclusivity of Table 5.1.

Discussion

1. While it was possible to identify the metal uniquely (given the separation of values in Table 5.1), there is still much one could do to improve the precision of this lab. By far the largest source of uncertainty in this lab is the determination of the slope of the θ vs. M graph, whose fractional uncertainty is 7 times higher than the next largest source (measurement of D). Thus, one would obtain the greatest improvement by working on how the angular twists are measured.
   The imprecision of the angular measurements in the current lab isn't just reading error; Table 1 shows that the "spread" in θ to be typically much greater. There may be some "hystereses" in the rod (in which the rod doesn't completely "untwist" right after the torque is relieved), and thus the readings obtained while adding the masses and removing them may be systematically different. It may be that there is some slipping of the rod in the clamps or bushings at the $0.2°$ level which this lab wouldn't necessarily detect. Thus, to obtain an order of magnitude increase in the precision and accuracy of the twist angle would require a substantial redesign of this experiment.

2. The mass of the hanger was accounted for by step 4 of the Procedure, in

which the Vernier scale was set precisely to zero with just the mass hanger hanging from the cord. This effectively set the origin of the θ vs. M graph to be (0,0).

Conclusions

The shear modulus for the metal rod stamped "B" is $G = (4.69 \pm 0.27) \times 10^{10}$ Nm^{-2}. To within experimental uncertainty, this is consistent with the rod being made of bronze, whose shear modulus (from Table 5.1 in the manual) is $G = 4.48 \times 10^{10} \, \text{Nm}^{-2}$.

Appendix C
Using Vernier Scales

C.1 Vernier Scales

Throughout your scientific career, you will encounter instruments that use Vernier scales to make precise measurements. These measurements are accomplished by the combined use of two scales: the first is the main scale which is used to make the coarse measurement; the second is the Vernier scale which can measure accurately one tenth to one hundredth the finest division on the main scale. The final value reported for a measurement using an instrument with a Vernier scale is the sum of the readings from the main scale and the Vernier scale.
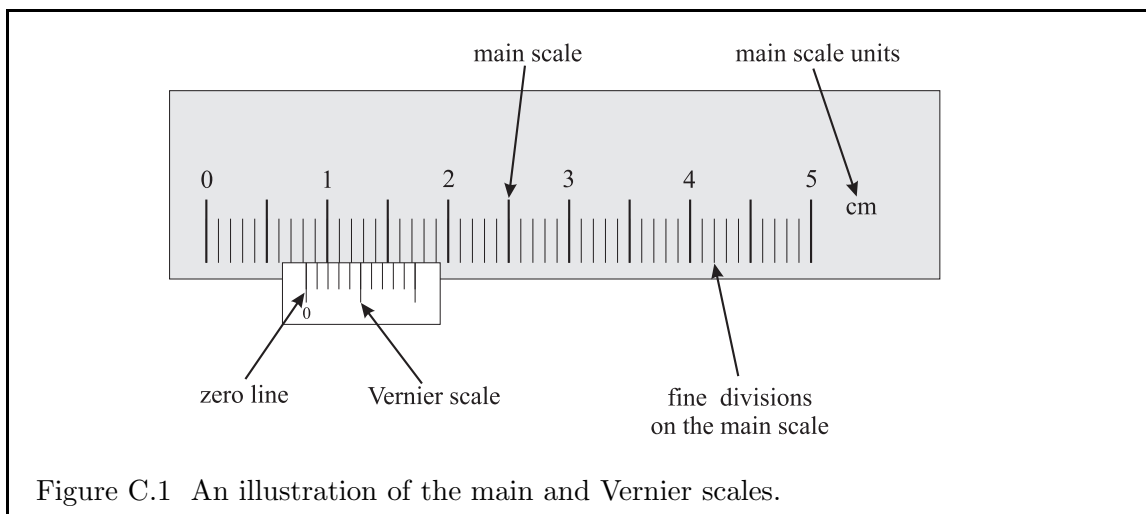
Figure C.1 An illustration of the main and Vernier scales.

     Figure C.1 illustrates three important components of a typical Vernier device: the "main scale", the "Vernier scale", and the "zero line". On some instruments, the zero line may not be identified as such, and so one takes as the zero line the line at the end of the Vernier scale closest to the *smallest* main scale reading (Fig. C.2).

     To read the Vernier device, first determine between which two divisions on the main scale the zero line falls. The *lesser* of these two main scale divisions is the main scale reading. The Vernier scale reading is then determined by identifying which of the divisions on the
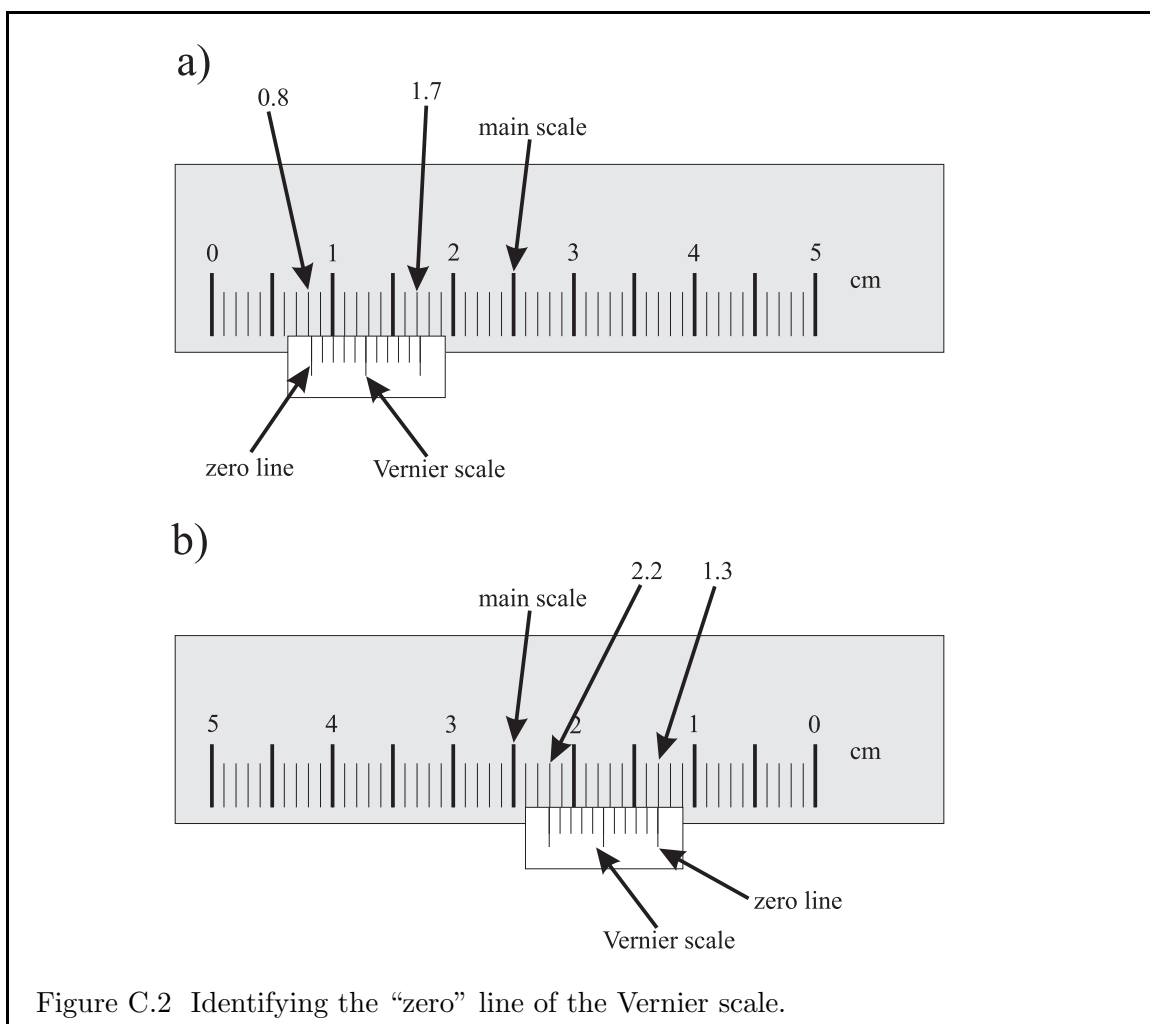
Figure C.2 Identifying the "zero" line of the Vernier scale.

Vernier scale lines up best with a main scale division.

Example 1. In Fig. C.2a, the zero line falls between 0.8 and 0.9 cm. Thus, the main scale reading is 0.8 cm. The Vernier division aligned with a main scale division is the third division right of the zero line (which happens to line up with the 1.1 division on the main scale, but note that the '1.1' is irrelevant), and since there are ten divisions on the Vernier scale, the Vernier reading is 3/10 of a main scale division. Thus, the Vernier reading is $0.1 \times 3/10 = 0.03$ cm and the final reading is $0.8 + 0.03 = 0.83$ cm.

As for the uncertainty, you can see that the third Vernier scale division is the *only* one to line up with a main scale division, and therefore the reading cannot be 0.82 nor 0.84 (neither the second nor fourth divisions line up!). Thus, an uncertainty of ± 0.01 is too high. Therefore, quote your result as 0.830 ± 0.005, where the extra '0' is added after the 0.83 in order to make the significant figures of the measurement match that of the uncertainty.

Example 2. In Fig. C.2b, the main scale increases from right to left and the zero line falls between 1.3 (just on the right) and 1.4 (on the left). Thus, the main scale reading is 1.3 cm.
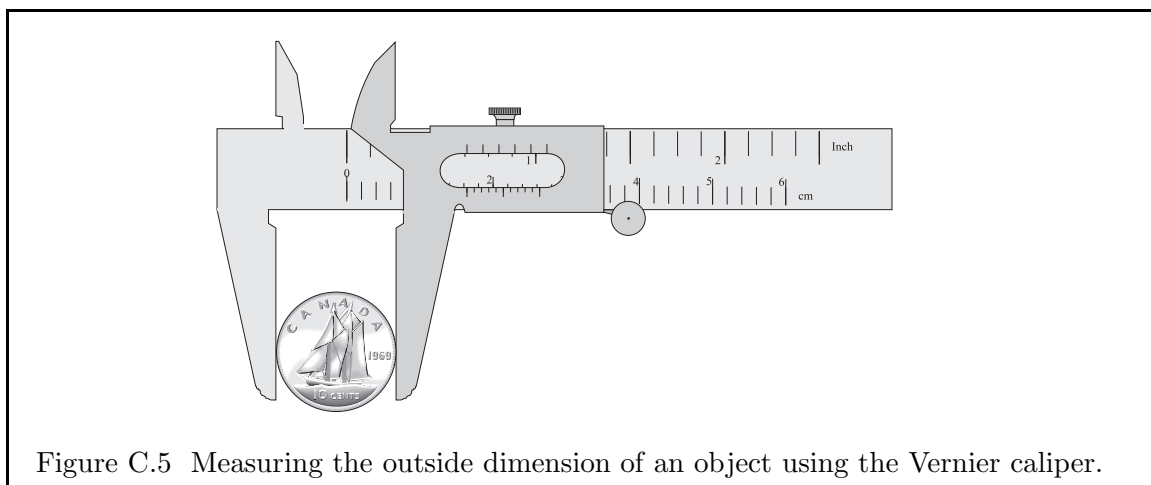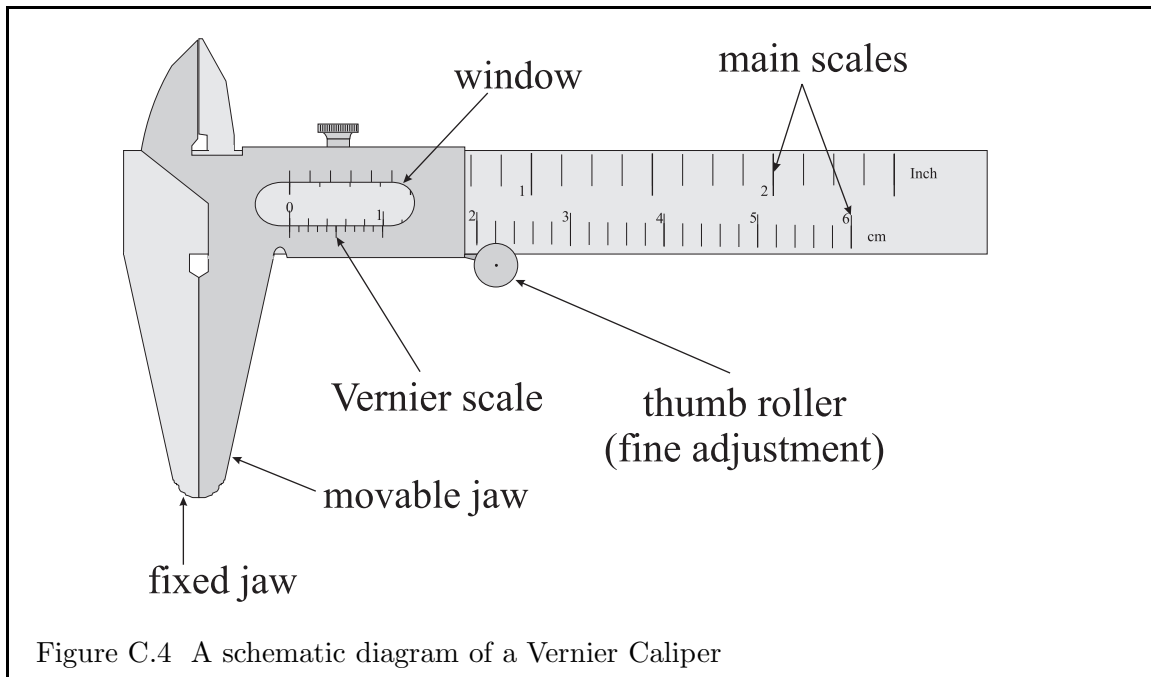
In this case, both the zeroth and the first Vernier divisions line up with main scale divisions equally well, and thus take the Vernier reading as half way in between 0.00 and 0.01. Thus, your final reading should be $1.305 \pm 0.005\,\mathrm{cm}$.

Figure C.3 gives six other examples of Vernier readings. For each, identify and label the main scale, the Vernier scale, and the zero line, and write down the reading along with an estimate of your uncertainty. (see App. D for answers.)

Figure C.3 Six settings of a Vernier scale.
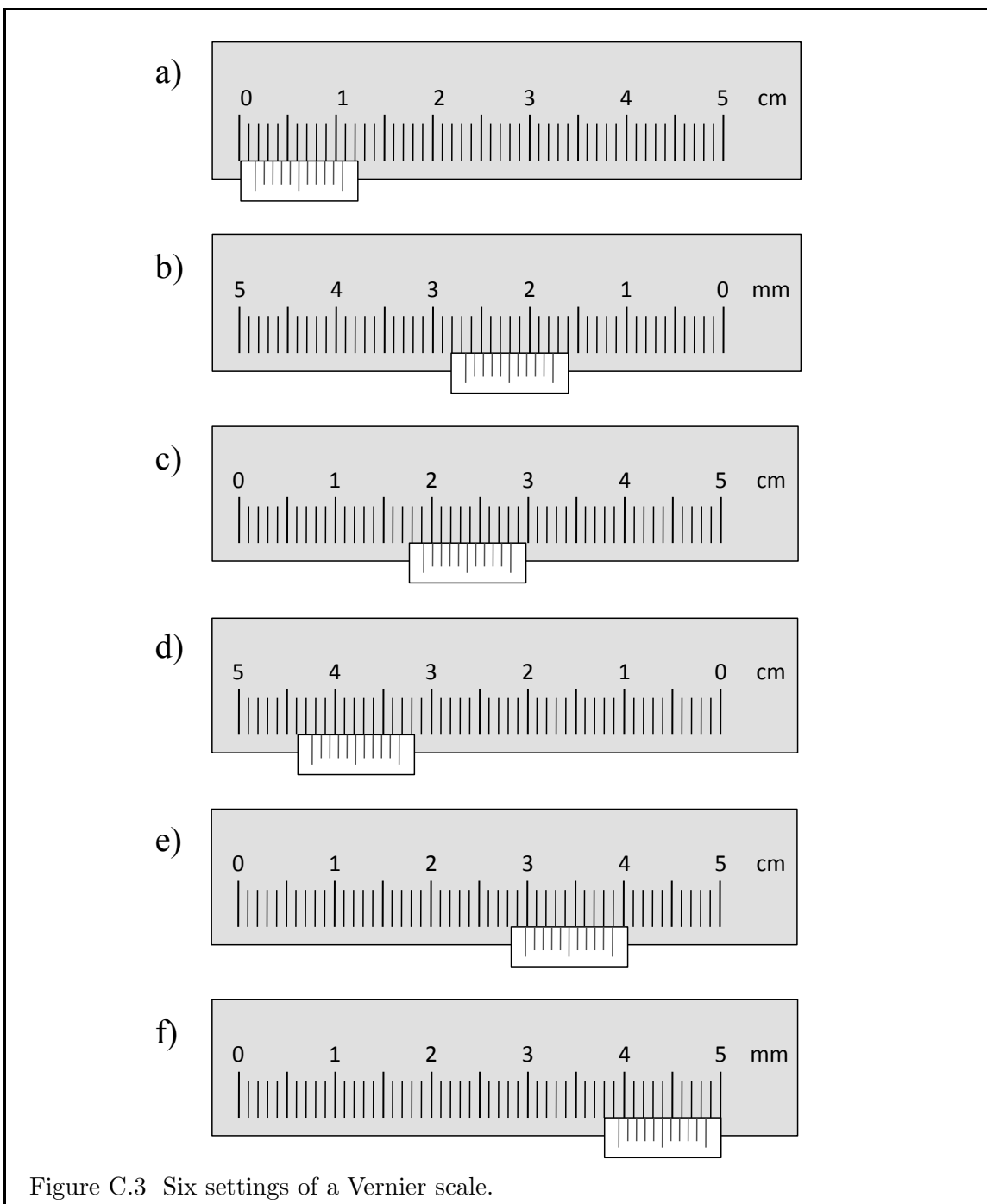
C.2 The Vernier Caliper

One of the most frequently encountered Vernier devices is the Vernier caliper, used to measure the outside or inside dimensions of objects with a high degree of precision. Most common calipers can measure to a precision of $\pm\,0.05\,\mathrm{mm}$. Calipers consist of a handle with the main scale running down its length and a jaw permanently fixed to one end. A second jaw containing a window with the Vernier scale along the window's edge slides up and down the handle and has a thumb roller for making fine adjustments (Fig. C.4).

Figure C.4 A schematic diagram of a Vernier Caliper

Figure C.5 Measuring the outside dimension of an object using the Vernier caliper.

To use a Vernier caliper to measure the outside dimensions of an object, you first open

the jaws of the caliper and place the object between the opened jaws (Fig. C.5). Slowly close the caliper on the object using the thumb roller until the jaws hold the object snugly. **Do not close the caliper jaws on the object without using the thumb roller**, otherwise, you could damage or deform the object you are measuring or damage the caliper itself. When you have closed the caliper jaws, you can read the measurement from the scales.
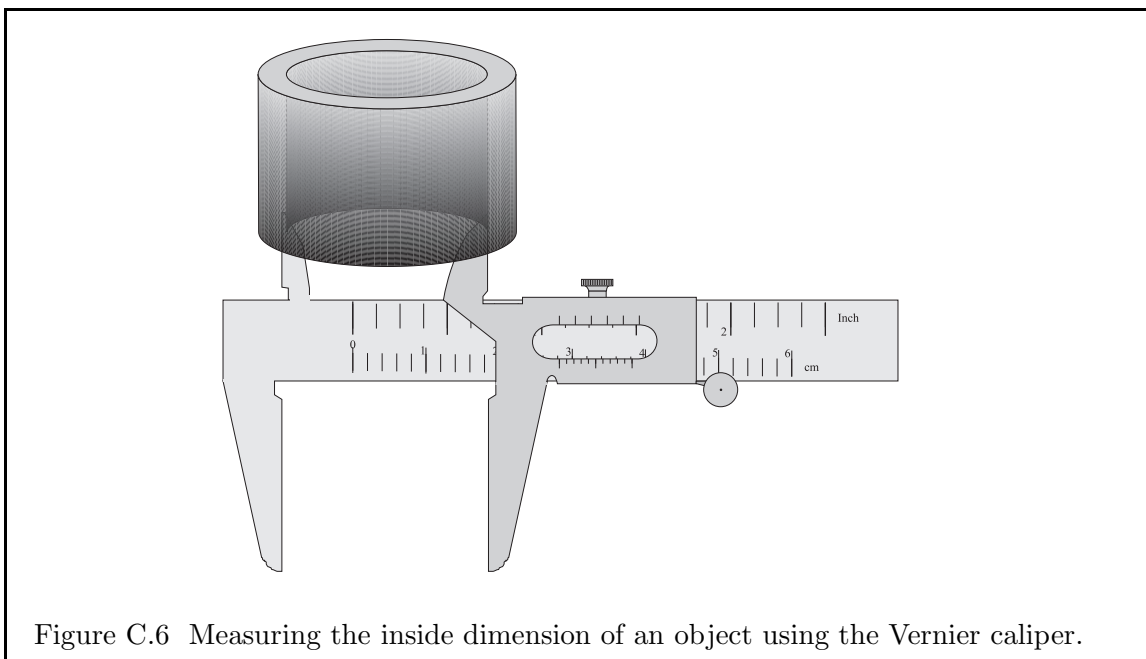

Figure C.6 Measuring the inside dimension of an object using the Vernier caliper.

Measuring the inside dimension of an object with a Vernier caliper is essentially the same as measuring the outside except that you use the small curved jaws opposite the main jaws (Fig. C.6). These jaws measure distances by the separation of the straight edges of the two small jaws. For example, to measure the inside of a pipe, place the small jaws inside the pipe and slowly open the calipers (using the thumb roller) until the edges of the jaws touch the inside of the pipe on opposite sides, then read your measurement from the scales.

Points to remember:

- Before using the caliper, determine its "zero" value by closing the jaws together with nothing in between them. Record this reading, and subtract it from all subsequent measurements made. Ideally, this value should be 0.00 mm. If your actual "zero" is outside the range ± 0.1 mm, please inform the lab demonstrator.

- Make sure you know what the units are for the scales you are using. Some calipers have both inch and centimetre scales on the handle.

- Do not drop the calipers.

- Do not close the calipers with excessive force.

- Do not bend the handle or jaws.

- Do not write on the scales with pens or pencils.

- Always open the jaws before removing an object.

- It is always good practice to take several measurements and take the average to get your final measurement (with the uncertainty being half-the-range if it is greater than the reading uncertainty).

C.3 The Micrometer

The second most frequently encountered Vernier device is the micrometer, used to measure the outside dimensions of objects with a very high degree of precision. Used carefully, the micrometers that you will use in this lab can measure with a precision of $\pm\, 0.002\,\mathrm{mm} = 2\,\mu\mathrm{m}$ (micrometre), whence its name.

Figure C.7 A schematic diagram of the micrometer.
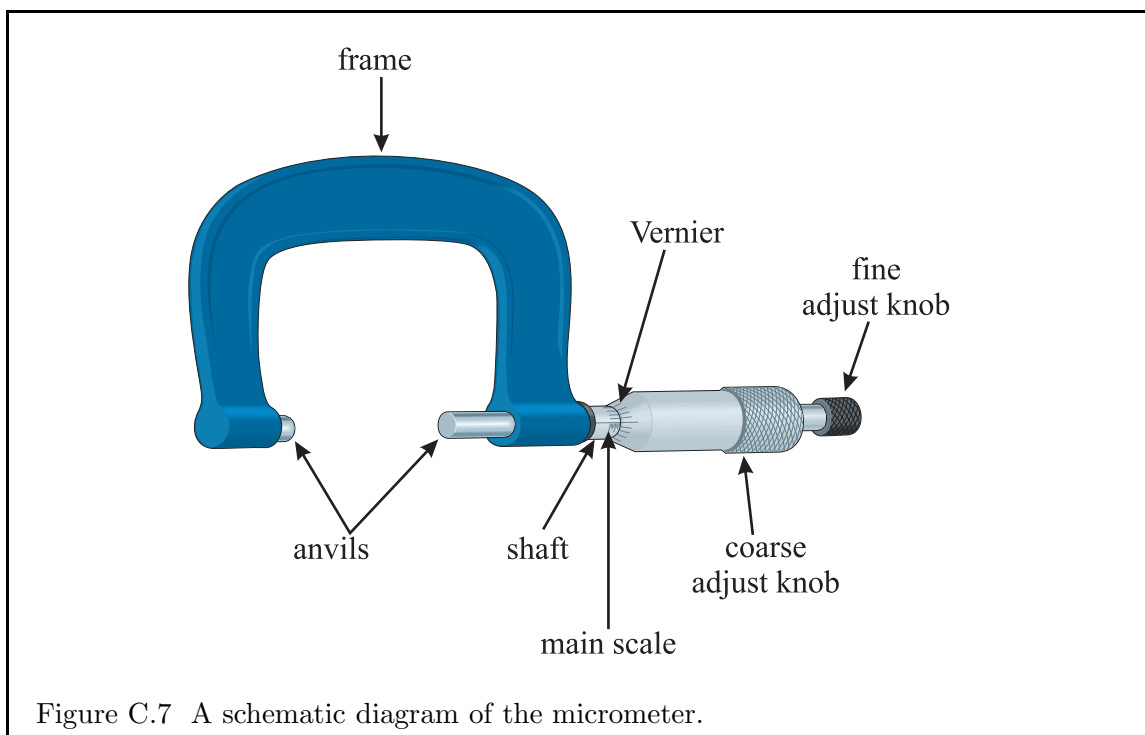
   Micrometers look like a "u" (the frame) with a handle attached to one end (Fig. C.7). Attached to the inside of the frame are two anvils which are closed snugly onto the object being measured. On the inside of the handle is the main scale printed right on the shaft, with the Vernier scale printed on the inside of the barrel. Normally, the main scale is divided into half-millimetres, and the Vernier scale is divided into 50 divisions. At the end of the handle are two knobs for opening and closing the anvils. The micrometer is designed so that

for every full revolution of the barrel, the anvils spread by exactly one division on the main scale, that is by 0.5 mm.

To use a micrometer, place the object to be measured between the opened anvils. Close the anvils on the object using the fine adjust knob until the anvils are holding the object snugly. The clutch built into the fine adjust knob will prevent the anvils from closing too tightly around the object. **Therefore never use the coarse adjust knob to close the anvils on an object.** Failure to use the fine adjust knob may deform or damage the object, the micrometer, or both, and will certainly distort your measurement.

To read the measurement on the main scale, find the last division uncovered by the edge of the barrel. In Fig. C.8, for example, the last division uncovered is 16 (just). Because the main scale is divided into *half*-millimetres, be aware that your main scale reading could be a full integral number (*e.g.*, 16.0 mm), or a half-integral number (*e.g.*, 16.5 mm).
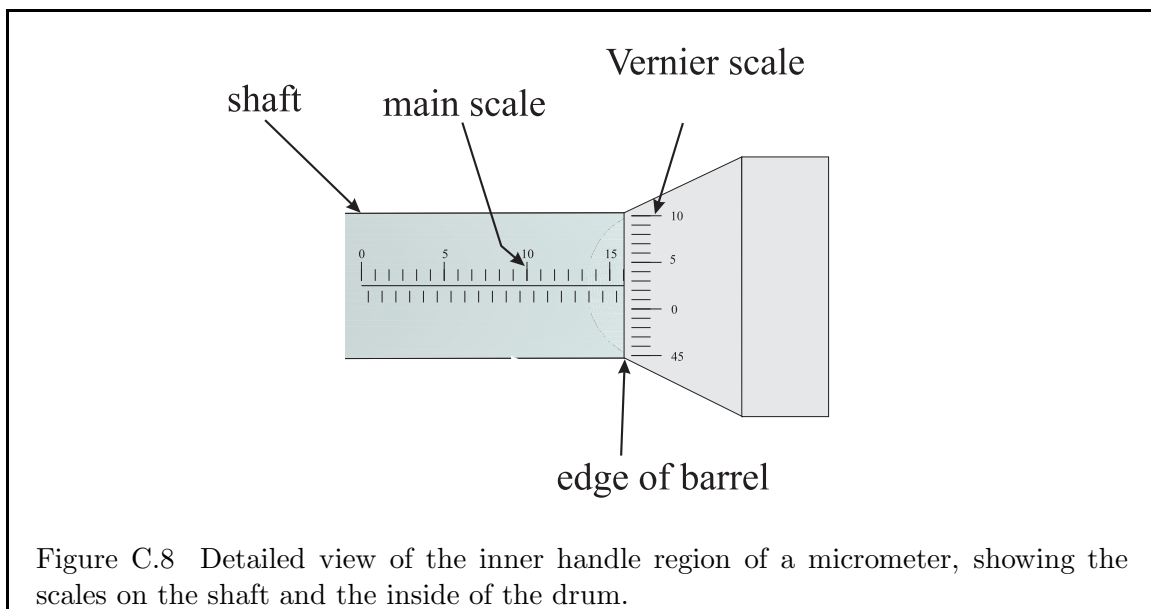

Figure C.8   Detailed view of the inner handle region of a micrometer, showing the scales on the shaft and the inside of the drum.

Since the Vernier scale has 50 divisions and since the anvils spread by 0.5 mm per revolution of the barrel, the Vernier scale divides each half millimetre into 50 equal parts. Thus, the units of the divisions on the barrel are $0.5/50 = 0.01$ mm. To obtain the Vernier reading, pick off the number on the Vernier scale aligned with the central line running along the length of the shaft (and perpendicular to the main scale divisions). In general, two divisions on the Vernier scale will straddle this central line, and you should do the best you can to interpolate the Vernier reading. In Fig. C.8, for example, the central line points to about 2.6 on the Vernier scale. Since the units of the Vernier scale are 0.01 mm, we must add to the main scale reading (16.000 mm) the Vernier scale reading (0.026 mm) to get a net reading of 16.026 mm. As for the uncertainty, it is generally safe to use ± 0.002 mm, although you may feel confident you can read to within ± 0.001 mm.

Points to remember:

- Before using the micrometer, determine its "zero" value by closing the anvils together with the fine adjust knob. Record this reading, and subtract it from all subsequent measurements made. Ideally, this value should be 0.000 mm. If your actual "zero" is outside the range ±0.01 mm, please inform the lab demonstrator.

- Make sure you know what the units are for the scales you are using. Some micrometers have different units than mm used in the examples here.

- Do not drop the micrometers.

- Always use the fine adjustment knob to close the anvils.

- Do not close the anvils with excessive force.

- Do not write on the scales with pens or pencils.

- Always open the anvils before removing an object.

- It is always good practice to take several measurements and take the average to get your final measurement (with the uncertainty being half-the-range if it is greater than the reading uncertainty).