EBERHARD KARLS
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Faculty of Economics and Social Sciences

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TAKE-HOME ASSIGNMENT - Summer Term 2024

B401: Continuous-time Derivatives Pricing

Organization

This take-home assignment aims at making you familiar with the basics in continuous-time derivatives pricing and practical applications of the contents encountered during the lecture. The assignment is due by **Monday, July 22, 2024 (1:00 PM)**. Please submit your assignment by uploading your files (poster, code, data) to Ilias. The presentation of your assignment will take place shortly after the submission (details tba).

The calculations can be done with an advanced software package (R, Python, SAS, MatLab), or you can use Excel as well. No matter which program you use, please **insert comments** describing your computational steps and showing all results and needed answers. Most importantly, make sure that all your **calculations are traceable**, results are highlighted, and sources are labeled. This implies that your code is smoothly running, and your results are displayed after a **rational amount of time**. Additionally, you have to hand in a 1-page A3-format presentation poster **(minimum font size: 11)**, which contains your name and student ID in the top left corner of your poster. Furthermore, the poster must be a **self-explanatory** solution to the below stated problems. This means that it contains all the necessary plots, tables, interpretations, and justifications.

We offer support within teaching sessions/practical classes. The date of the first session will be announced during the lecture. In this session, we will have a detailed look at the tasks of the assignment. Of course, you are always welcome to ask questions after the lecture.

The final grade is mainly based on the complexity of the product chosen and your poster. However, the technical implementation counts as well. A proper application of the valuation methods, a reasonable parameter choice, and your economic reasoning are the crucial drivers of your final grade.

Some important notes on **individual** solutions: You may form groups to discuss the general contents discussed in the lecture, but you have to name your student fellows with whom you talked to (also in the top left corner of your poster). However, the implementation steps and economic reasoning have to be done **individually**, as this is **not** a group assignment.

In case you consider the PhD track, we offer the option to get a special assignment on demand. Therefore, please send an email to benjamin.harsch@uni-tuebingen.de until the first teaching session to discuss the way your assignment can be raised to a PhD-level.

Problems

Part 1: Designing certificates

I. **Product design:**
Design a marketable structured product consisting of at least two components. Explain the intuition behind your product and describe the components briefly. *Note:* When designing your product, you should already think about how the certificate can create value for potential buyers.

II. **Product payoff:**
Illustrate the payoff profile and the market situation(s) under which the product does (and does not) outperform its underlying in a reasonable graphic.

III. **Target group:**
Define the target buyer group of your product. Explain the characteristics of potential buyers (e.g., understanding of risk, income orientation, regulatory conditions) and focus on the needs and advantages potential buyers seek, which your product provides.

IV. **Volume:**
Justify a realistic estimate of the total volume of your product that you can sell (e.g., through market research of similar products and market share potential). Explain how the volume changes depending on the market condition and the investor's demand.

Part 2: Valuation of certificates

Choose a traded product based on your underlying in Part 1 for which at least one year daily data of historic prices is available and which cannot be evaluated with closed-form Black-Scholes pricing functions.

V. **Volatility of the underlying:**
Estimate the 100-trading day backward-looking volatility of the return of your underlying for each day within a reasonably chosen time window. Implement a pricing approach for your product. Determine the forward-looking volatility of the underlying implied in the observed market price of your product for each day of your chosen time window. Comment on potential deviations between the forward- and backward-looking volatilities.

VI. **Volatility of the product:**
Estimate the 100-trading day backward-looking volatility of the return of your product for each day within your chosen time window. Compare this volatility for each day of your time window to the pricing model implied volatility of your product, which might be derived using a one-period binomial tree model (or alternatively, from a binomial tree evaluated until maturity).
Check whether the following instantaneous property derived from Ito's Lemma is valid for your numerical values:

$$\sigma_{P,t} = \sigma_{S,t} \cdot \left| \frac{\partial P_t}{\partial S_t} \right| \cdot \frac{S_t}{P_t}$$

Note: $\sigma_{S,t}$ and $\sigma_{P,t}$ are the standard deviation of your underlying S and product P respectively. Further, S_t and P_t denote the price of the underlying and product. For your product assume that the value equals the price of your product.

VII. **Replicating Portfolio:**
Determine the replicating portfolio for your product for each day within your observation window. Additionally, illustrate the equity fraction as a function of the price of the underlying at a specific point in time. Interpret your results in terms of the characteristics of your product in falling markets.

Part 3: Portfolio insurance strategy

For this task, assume a fictive investor with an investment horizon of $T^* = 1$ *year, planning to invest 10,000 euros. Assume that the investor has the possibility to invest in a stock or index, and put options for portfolio insurance purposes.*

VIII. **Performance analysis (without risk management):**
Use a Monte-Carlo simulation to derive the payoff/return distribution at the end of the investment horizon T^* of an investment in the stock/index. Calculate meaningful performance and risk measures and explain.

IX. **Performance analysis (with risk management):**
Implement a portfolio insurance strategy by including put options to hedge the portfolio against downside risks. Consider fictitious puts, which prices are based on the historical volatility of the underlying, and analyze the impact of

i) the fraction of initial wealth invested in put options, and

ii) the strike K of the put options

on the performance and risk measures of the resulting portfolio. Illustrate and discuss your observations.

X. **Stress scenario analysis**
For a specific portfolio allocation from Problem IX, consider that the volatility for pricing purposes of the options equals the historical volatility of the underlying ± 5 percentage points and/or a sudden drop in the stock/index price by 20% after half a year. Compare the results and explain.