Assignment 1 - Fall 2017 Zaki G. Lindo

Assignment 1: Due Monday September 4, 2017, no later than 8pm.
$p.12 - 14$ # 16, 42, 47
$p.36 - 37$ # 7, 13

$p.12 - 14$

#16. For the statement about nonempty sets A, B, C, either prove that the statement is true or find a counterexample with nonempty sets that make it fail.

$$\text{If } A \subseteq (B \cap C) \text{ then } A \subseteq B \text{ and } A \subseteq C.$$

Proof: Let A, B, and C, be nonempty sets. Let $A \subseteq (B \cap C)$. We must show that $A \subseteq B$ and $A \subseteq C$. As $A \subseteq (B \cap C)$, for all $x \in A$, $x \in (B \cap C)$. By definition of *intersects*, for all $x \in B \cap C$, $x \in B$ and $x \in C$. As we showed $x \in B$, $x \in C$, and x was arbitrary, $A \subseteq B$ and $A \subseteq C$. Thus, If $A \subseteq (B \cap C)$ then $A \subseteq B$ and $A \subseteq C$.

#42. Determine if the function below is injective, surjective, or bijective. Prove it is true or give a counterexample for each property.

$$h : \mathbb{Z} \to \mathbb{Z} \text{ where } h(x) = 2x - 5.$$

(Injective) **Proof by Contradiction:** Let $h : \mathbb{Z} \to \mathbb{Z}$ where $h(x) = 2x-5$. For a contradiction, assume h is not injective. That is, for all $m, n \in \mathbb{Z}, m \neq n$ and $h(m) = h(n)$. As $h(m) = h(n), 2m - 5 = 2n - 5$. It follows that $2m = 2n$. Thus, $m = n$. As $m \neq n$ and $m = n$, we have a contradiction. Therefore, h is injective.

(Surjective) **Counter Example**: Let $h(x) = 2$. Solve for x.

$2 = 2x - 5$
$7 = 2x$
$7/2 = x$.

As $7/2 \notin \mathbb{Z}$ There exists $y \in \mathbb{Z}$ such that for all $x \in \mathbb{Z}, h(x) \neq y$. Therefore, h is not surjective.

(Bijective) As h is not surjective, h is not bijective.

#47. Carefully Prove the statement below.

If f and g are functions with $f : A \to B$, $g : B \to C$ and
$g \circ f$ is surjective then g is surjective.

Proof: Suppose f and g are functions with $f : A \to B$, $g : B \to C$ and $g \circ f$ is surjective. We want to show that g is surjective. We know $g \circ f(x) = g(f(x))$. Let $x \in A$. As $g(f(x))$ is surjective, there exists $y \in C$ such that $g(f(x)) = y$. Let $z \in B$ such that $f(x) = z$. Now we have that there exists $y \in C$ such that $g(z) = y$. As x was arbitrary, we can say that z was arbitrary. As z was arbitrary, for all $z \in B$, there exists $y \in C$ such that $g(z) = y$. Thus, g is surjective.

$p.36 - 37$

#7. Determine if the rule $x * y = xy$ is an operation on the set of negative integers.

This is not an operation on the set of negative integer. There exists $x, y \in -\mathbb{Z}$ such that $x * y \notin -\mathbb{Z}:$

Counter Example: Let $x = -1$. Let $y = -2$. This gives us $x * y = (-1)(-2) = 2 \notin -\mathbb{Z}$. As $2 \notin -\mathbb{Z}, x * y$ is not an operation on the set of negative integers.

#13. Determine if the operation defined below on the set $\mathbb{Z}$ is associative, commutative, has an identity, and if each element has an inverse. Either prove or give a counterexample for each property.

For $a, b \in \mathbb{Z}$, $a * b = a + 2b - 1$.

(Commutative): For all $a, b \in \mathbb{Z}$, $a * b = b * a$.

Let $a = 2$. Let $b = 4$.
$a * b = 2 + 2(4) - 1 = 9$.
$b * a = 4 + 2(2) - 1 = 7$.
$9 \neq 7$.
Thus, $*$ is not commutative on the set $\mathbb{Z}$.

(Associative): For all $a, b \in \mathbb{Z}$, $(a * b) * c = a * (b * c)$.

Let $a = 1$. Let $b = 2$. Let $c = -3$.
$(a * b) * c = (1 + 2(2) - 1) + (2)(-3) - 1 = 4 - 6 - 1 = -3$.
$a * (b * c) = 1 + 2(2 + 2(-3) - 1) - 1) - 1 = 1 + 2(-5) - 1 = -10$
$-10 \neq -3$.
Thus, $*$ is not associative on the set $\mathbb{Z}$.

(Identity): There exists $e \in \mathbb{Z}$ such that for all $a \in \mathbb{Z}, e * a = a$ and $a * e = a$.

Let $a = 1$.Let e be the identity for 1. Then $1 * e = 1$.
$1 * e = 1 + 2e - 1 = 1$,
$2e = 1$,
$e = 1/2$. But $e \notin \mathbb{Z}$. Therefore, 1 does not have an identity $e \in \mathbb{Z}$.

(Inverse):There exists $i \in \mathbb{Z}$ such that for all $a \in \mathbb{Z}, a * i = e$.

If we observe the counter example we used in our (Identity) presentation we will see $*$ does not have an inverse for that counter example, because there isn't an identity element $e \in \mathbb{Z}$ that we can say $a * i = e$.

Due to this, we can say not every element has an inverse under $*$.