**Assignment 1 - Fall 2017 Zaki G. Lindo**

Assignment 1: Due Monday September 4, 2017, no later than 8pm.
$p.12 - 14$ # 16, 42, 47
$p.36 - 37$ # 7, 13

$p.12 - 14$

#16. For the statement about nonempty sets A, B, C, either prove that the statement is true or find a counterexample with nonempty sets that make it fail.

$$\text{If } A \subseteq (B \cap C) \text{ then } A \subseteq B \text{ and } A \subseteq C.$$

**Proof:** Let $A$, $B$, and $C$, be nonempty sets. Let $A \subseteq (B \cap C)$. We must show that $A \subseteq B$ and $A \subseteq C$. As $A \subseteq (B \cap C)$, for all $x \in A$, $x \in (B \cap C)$. By definition of *intersects*, for all $x \in B \cap C$, $x \in B$ and $x \in C$. As we showed $x \in B$, $x \in C$, and $x$ was arbitrary, $A \subseteq B$ and $A \subseteq C$. Thus, If $A \subseteq (B \cap C)$ then $A \subseteq B$ and $A \subseteq C$.

While your reasoning is fine, your writing technique needs work. We don't usually use "Let" when assuming the hypothesis of an if..then statement, we use something such as assume or suppose. Then you totally forgot to use "Let" when you absolutely needed it. You do not keep discussing "for all $x$" throughout things and then say $x$ was arbitrary since you never "let $x \in A$ to choose an arbitrary one to work with.That should have happened immediately after assuming $A \subseteq (B \cap C)$. Wording "By definition of intersects" is not quite grammatically how we say it, you can just say that since $x \in B \cap C$ then $x \in B$.

#42. Determine if the function below is injective, surjective, or bijective. Prove it is true or give a counterexample for each property.

$$h : \mathbb{Z} \to \mathbb{Z} \text{ where } h(x) = 2x - 5.$$

(Injective) **Proof by Contradiction:** Let $h : \mathbb{Z} \to \mathbb{Z}$ where $h(x) = 2x-5$. For a contradiction, assume $h$ is not injective. That is, for all $m, n \in \mathbb{Z}, m \neq n$ and $h(m) = h(n)$. As $h(m) = h(n), 2m - 5 = 2n - 5$. It follows that $2m = 2n$. Thus, $m = n$. As $m \neq n$ and $m = n$, we have a contradiction. Therefore, $h$ is injective.

You are using "proof by contradiction" incorrectly here. The negation of "For all $m, n \in \mathbb{Z}$, if $h(m) = h(n)$ then $m = n$" is NOT "for all $m, n \in \mathbb{Z}, m \neq n$ and $h(m) = h(n)$". Your quantifier is wrong! Also why are you doing a proof by contradiction when you proved inside of this that if $h(x) = h(y)$ then $x = y$?? It makes the proof double the length so gives you more places to go wrong.

(Surjective) **Counter Example**: Let $h(x) = 2$. Solve for $x$.

$2 = 2x - 5$
$7 = 2x$
$7/2 = x$.

As $7/2 \notin \mathbb{Z}$ There exists $y \in \mathbb{Z}$ such that for all $x \in \mathbb{Z}, h(x) \neq y$. Therefore, $h$ is not surjective.

(Bijective) As $h$ is not surjective, $h$ is not bijective.

This is fine, but why start with "Let $h(x) = 2$"? Why not just assume $y = 2$ and suppose there was an $x$ with $h(x) = y$.

#47. Carefully Prove the statement below.

If $f$ and $g$ are functions with $f : A \to B$, $g : B \to C$ and $g \circ f$ is surjective then $g$ is surjective.

**Proof:** Suppose $f$ and $g$ are functions with $f : A \to B$, $g : B \to C$ and $g \circ f$ is surjective. We want to show that $g$ is surjective. We know $g \circ f(x) = g(f(x))$. Let $x \in A$. As $g(f(x))$ is surjective, there exists $y \in C$ such that $g(f(x)) = y$. Let $z \in B$ such that $f(x) = z$. Now we have that there exists $y \in C$ such that $g(z) = y$. As $x$ was arbitrary, we can say that $z$ was arbitrary. As $z$ was arbitrary, for all $z \in B$, there exists $y \in C$ such that $g(z) = y$. Thus, g is surjective.

You need to learn the definition of surjective, there is nothing in that definition that would tell you to start by assuming you have $x \in A$. Also we never say that $g(f(x))$ is surjective, it is not a function, it is an element of C.

$p.36 - 37$

#7. Determine if the rule $x * y = xy$ is an operation on the set of negative integers.

This is not an operation on the set of negative integer. There exists $x, y \in -\mathbb{Z}$ such that $x * y \notin -\mathbb{Z}$ :

**Counter Example**: Let $x = -1$. Let $y = -2$. This gives us $x * y = (-1)(-2) = 2 \notin -\mathbb{Z}$. As $2 \notin -\mathbb{Z}, x * y$ is not an operation on the set of negative integers.

Well done. We don't usually use $-\mathbb{Z}$ to mean the negative integers, it is more common to see $\mathbb{Z}^-$.

#13. Determine if the operation defined below on the set $\mathbb{Z}$ is associative, commutative, has an identity, and if each element has an inverse. Either prove or give a counterexample for each property.

$$\text{For } a, b \in \mathbb{Z},\ a * b = a + 2b - 1.$$

(Commutative): For all $a, b \in \mathbb{Z}$, $a * b = b * a$.

Let $a = 2$. Let $b = 4$.
$a * b = 2 + 2(4) - 1 = 9$.
$b * a = 4 + 2(2) - 1 = 7$.
$9 \neq 7$.
Thus, $*$ is not commutative on the set $\mathbb{Z}$.

(Associative): For all $a, b \in \mathbb{Z}$, $(a * b) * c = a * (b * c)$.

Let $a = 1$. Let $b = 2$. Let $c = -3$.
$(a * b) * c = (1 + 2(2) - 1) + (2)(-3) - 1 = 4 - 6 - 1 = -3$.
$a * (b * c) = 1 + 2(2 + 2(-3) - 1) - 1) - 1 = 1 + 2(-5) - 1 = -10$
$-10 \neq -3$.
Thus, $*$ is not associative on the set $\mathbb{Z}$.

(Identity): There exists $e \in \mathbb{Z}$ such that for all $a \in \mathbb{Z}, e * a = a$ and $a * e = a$.

Let $a = 1$.Let $e$ be the identity for 1. Then $1 * e = 1$.
$1 * e = 1 + 2e - 1 = 1$,
$2e = 1$,

$e = 1/2$. But $e \notin \mathbb{Z}$. Therefore, 1 does not have an identity $e \in \mathbb{Z}$.

(Inverse):There exists $i \in \mathbb{Z}$ such that for all $a \in \mathbb{Z}, a * i = e$.

If we observe the counter example we used in our (Identity) presentation we will see $*$ does not have an inverse for that counter example, because there isn't an identity element $e \in \mathbb{Z}$ that we can say $a * i = e$.

Due to this, we can say not every element has an inverse under $*$.

You did well, but in associative there is an extra ")-1" in one of the equations, careful. Also for the identity we don't say "an identity for 1", just an identity since the same $e$ must work for every element of the set.

Total Score: 30.5 out 35