Gravitational Wave Signatures of Preheating in Higgs–R^2 Inflation

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Overview

1. **Higgs–R^2 inflation**

2. **Preheating and tachyonic instability**

3. **Gravitational Wave**

Higgs Inflation and R^2 Inflation

R^2 model and Higgs model:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\mathrm{P}}^2}{2}R + \alpha R^2\right) \qquad S = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{P}}^2}{2}R + \frac{\xi}{2}\phi^2 R - \frac{\lambda}{4}(\phi^2 - v^2)^2 + ..\right]$$

Spectral index and the tensor-to-scalar ratio:

$$n_s \simeq 1 - \frac{2}{N_{\mathrm{CMB}}} - \frac{9}{2N_{\mathrm{CMB}}^2}\,,$$

$$r \simeq \frac{12}{N_{\mathrm{CMB}}^2}\,.$$

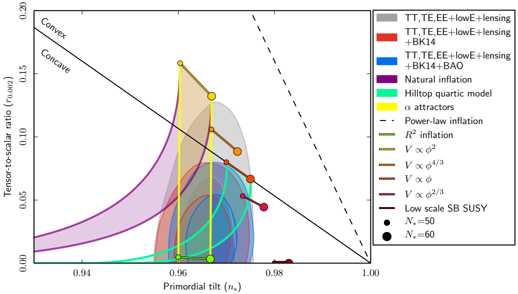

Figure: n_s and r observational constraints from Planck2018 results[Akrami et al., 2020]

Higgs–R^2 inflation

Action in Jordan frame:

$$S = \int d^4x \sqrt{-g_{\rm J}} \left[\frac{M_{\rm P}^2}{2} R_{\rm J} + \alpha R_{\rm J}^2 + \frac{1}{2}\xi\phi^2 R_{\rm J} - \frac{1}{2} g_{\rm J}^{\mu\nu} \partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4 \right],$$

Transfer to Einstein frame and define χ:

$$\Omega^2 = 1 + 4\alpha\frac{\psi}{M_{\rm P}^2} + \xi\frac{\phi^2}{M_{\rm P}^2}, \qquad \chi = \frac{\sqrt{6}}{2} M_{\rm P} \ln\left(1 + 4\alpha\frac{\psi}{M_{\rm P}^2} + \xi\frac{\phi^2}{M_{\rm P}^2}\right),$$

We get:

$$S = \int d^4x \sqrt{-g_{\rm E}} \left[\frac{M_{\rm P}^2}{2} R_{\rm E} - \frac{1}{2} g_{\rm E}^{\mu\nu} \partial_\mu\chi\partial_\nu\chi - \frac{1}{2} g_{\rm E}^{\mu\nu} e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_{\rm P}}} \partial_\mu\phi\partial_\nu\phi - V_{\rm E}(\phi,\chi) \right],$$

$$V_{\rm E}(\phi,\chi) = \frac{M_{\rm P}^4}{16\alpha} e^{-2\sqrt{\frac{2}{3}}\frac{\chi}{M_{\rm P}}} \left[4\lambda\alpha\frac{\phi^4}{M_{\rm P}^4} + \left(e^{\sqrt{\frac{2}{3}}\frac{\chi}{M_{\rm P}}} - 1 - \frac{\xi\phi^2}{M_{\rm P}^2} \right)^2 \right].$$

Higgs–R^2 inflation

$$V(\chi, \phi_{\min}) = \frac{M_{\mathrm{P}}^4}{4\,(4\alpha + \xi^2/\lambda)} \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_{\mathrm{P}}}}\right)^2$$

- $4\alpha \gg \xi^2/\lambda$ R^2-like inflation
- $\xi^2/\lambda \gg 4\alpha$ Higgs-like inflation

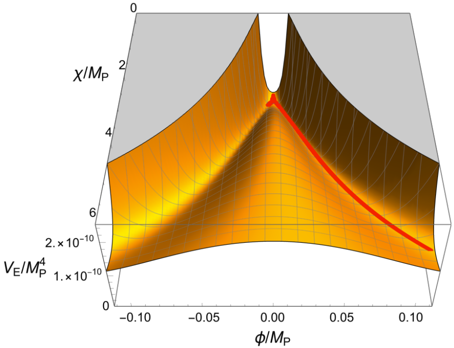

Figure: An example of potential shape.

Preheating——parameter selections

- The magnitude of the curvature power spectrum:

$$A_s \simeq \frac{V}{24\pi^2 M_{\rm P}^4 \epsilon_V} \sim 2.1 \times 10^{-9}\,,$$

$$\frac{\xi^2}{\lambda} + 4\alpha \approx 2.4 \times 10^9\,.$$

- Fix $\lambda = 0.01$. Only one free parameter which is ξ
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$$BP1 \rightarrow BP7$$
$$R^2 \rightarrow \mathrm{Higgs}$$

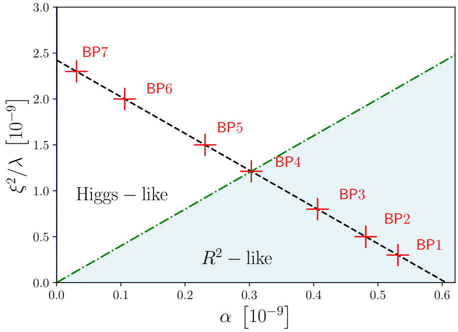

Figure: Seven benchmark points(BPs)

Preheating——lattice simulations

- EOM+Friedmann equations

$$\rho_{\rm pot} = \langle V(\chi,\phi)\rangle \ ,$$
$$\rho_{\rm kin} = \left\langle \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}f(\chi)\dot{\phi}^2 \right\rangle \ ,$$
$$\rho_{\rm grad} = \left\langle \frac{1}{2a^2}\left(\nabla\chi\right)^2 + \frac{1}{2a^2}f(\chi)\left(\nabla\phi\right)^2 \right\rangle \ .$$

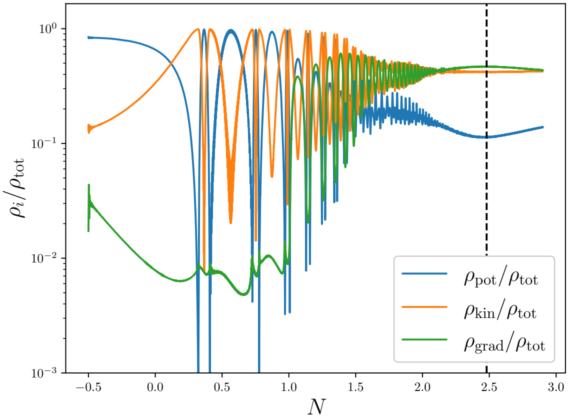

Figure: Energy density of BP7

Preheating——tachyonic instability

Tachyonic instability:

$$\delta\ddot{\varphi}_{\mathbf{k}} + 3H\delta\dot{\varphi}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + m^2_{\varphi,\mathrm{eff}}\right)\delta\varphi_{\mathbf{k}} \approx 0\,.$$

$$m^2_{\chi,\mathrm{eff}} \approx \frac{M_\mathrm{P}^2}{12\alpha} + \frac{\xi}{4\alpha}\phi^2\,,$$
$$m^2_{\phi,\mathrm{eff}} \approx 3\left(\lambda + \frac{\xi^2}{4\alpha}\right)\phi^2 - \frac{\xi M_\mathrm{P}}{2\sqrt{6}\alpha}\chi\,,$$

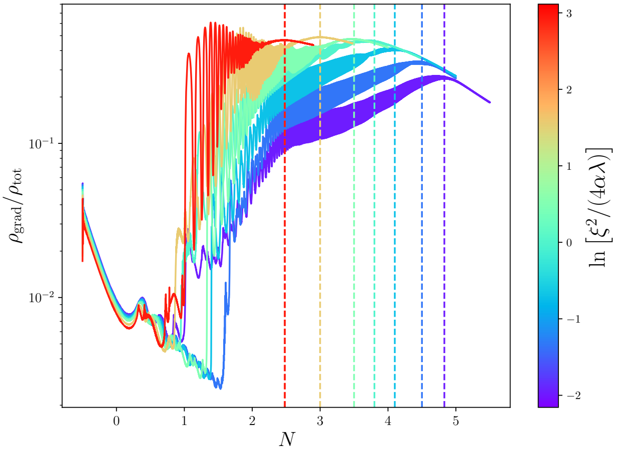

Figure: Gradient energy from BP1 to BP7

Gravational waves

$$\bar{h}_{ij}'' + \left(k^2 - \frac{a''}{a}\right)\bar{h}_{ij} = 2a\frac{T_{ij}^{\mathrm{TT}}}{M_{\mathrm{P}}^2}$$

$$T_{ij}^{\mathrm{TT}} = \left[\partial_i\chi\partial_j\chi + f(\chi)\partial_i\phi\partial_j\phi\right]^{\mathrm{TT}} ,$$

GW spectrum[Dufaux et al., 2007]:

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln k} .$$

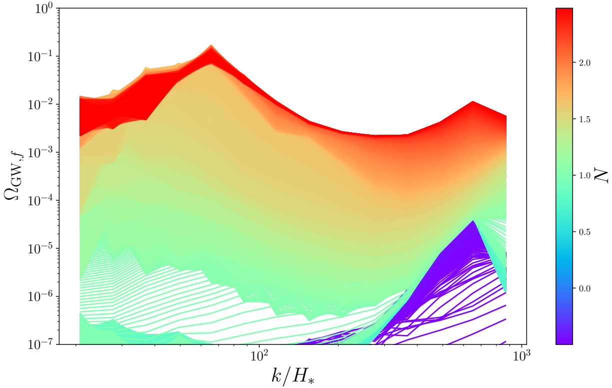

Figure: Evolution of the GW spectrum for BP7

Gravitational Wave

GW spectrum today[Dufaux et al., 2007]:

$$\Omega_{\mathrm{GW}} h^2 \approx 9.3 \times 10^{-6} \frac{1}{\rho_{c,f}} \frac{d\rho_{\mathrm{GW}}}{d\ln k}\bigg|_{\tau=\tau_f},$$

$$f = 4 \times 10^{10} \left(\frac{k}{a_f \rho_{c,f}^{1/4}} \right) \mathrm{Hz}\,.$$

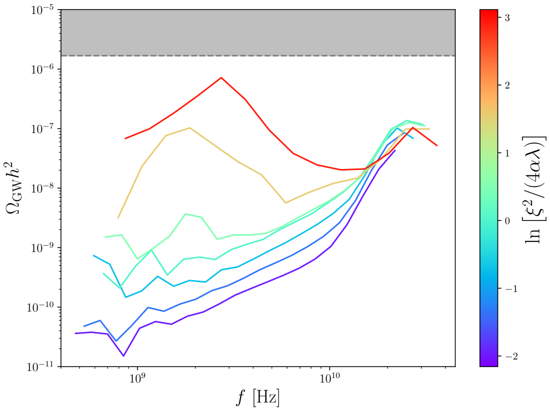

Figure: Present-day GW spectrum $\Omega_{\mathrm{GW}} h^2$ for all the BPs

Summary

- We consider Higgs–R^2 model:

$$S = \int d^4x \sqrt{-g_{\rm E}} \left[\frac{M_{\rm P}^2}{2} R_{\rm E} - \frac{1}{2} g_{\rm E}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} g_{\rm E}^{\mu\nu} e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_{\rm P}}} \partial_\mu \phi \partial_\nu \phi - V_{\rm E}(\phi, \chi) \right],$$

- During the preheating, inhomogeneities increases exponentially because of the tachyonic instability. A large ξ bring more severe instability.

$$m_{\phi,\rm eff}^2 \approx 3\left(\lambda + \frac{\xi^2}{4\alpha}\right)\phi^2 - \frac{\xi M_{\rm P}}{2\sqrt{6}\alpha}\chi\,,$$

- We observe that the GW spectrum grows in accordance with the enhancement of the inhomogeneities in the fields during preheating.

References

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Thank you