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1.

a.

First, we assume that  $S_k$  is the result at the end of the  $k$ th iteration.

$$S_k = x + (x+1) + \cdots + (x+k)$$

And if  $S_{k-1}$  is correct ( $2 \leq k \leq y-x$ ),  $S_k$  is correct, because of the two statement within the while() loop.

Because of the same reason, we can say that if  $S_1$  is correct,  $S_2$  is correct and  $S_k$  is correct.

Let us see whether  $S_1$  and  $S_2$  are correct:

In the first loop, nextNumber is  $(x+1)$ , result is  $x + (x+1)$ .  $\rightarrow S_1$  is correct.

In the second loop, nextNumber is  $(x+2)$ , result is  $x + (x+1) + (x+2) \rightarrow S_2$  is correct.

So, in the  $k$ th loop, nextNumber is  $(x+k)$ , result is  $x + (x+1) + \cdots + (x+k)$ .

**(loop invariance)**

We can know that  $S_1$  and  $S_2$  are correct. Therefore,  $S_k$  is correct.

b.

We already known that  $S_k = x + (x+1) + \cdots + (x+k)$ .

So we can know that  $S_n = x + (x+1) + \cdots + (x+n)$ .  $n=y-x$

We can know that the final result is correct because  $S_k$  is correct.

Thus, the function gives the correct result, which is  $\text{sum}(n) = x + (x+1) + \cdots + y$

2.

a.

	0	1
0	00	01
1	01	10

b.

PROC1(a, b, c)

Inputs: a, b, c, three binary digits.

Output: de, a two-digit number which is the result of  $a + b + c$ . If  $a + b + c < 10$ ,  $d=0$ .

$uv \leftarrow \text{PROC0}(a, b)$

$u_1v_1 \leftarrow \text{PROC0}(c, v)$

if  $u = u_1 = 0$ ,

$d \leftarrow 0$  and  $e \leftarrow v_1$

else

$d \leftarrow 1$  and  $e \leftarrow v_1$

return de

3.

```
def fun (n):  
    result=1  
    if n<2  
        return 1  
    else  
        for i in range(1, n +1):  
            result←result*i  
        return result  
a←0, e←0  
if a≤k do,  
    e←e + 1/fun(a)  
    a←a + 1  
return e
```

Explanation :

First, we should define a new function to give us the correct answer of natural numbers' factorial.

Next, a is 0 and e is 0. We enter the loop and do the loop until we add  $1/\text{fun}(k)$  as the last item.

Then we end the loop and get the value of e for  $n = 0, 1, 2, \dots, k$ , which is an estimate of ***e***.