Lab 2: Divide and Conquer (Continue) and Greedy Algorithm

1 Exercise

Counterfeit coin problem is based on the balancing of similar-looking coins, and the goal is to find the fake coin within a limited number of uses of the balance. Suppose that you are given *N* similar-looking coins, and there is exactly only one fake coin which is lighter than all other coins (which are real ones). In this problem, you are given a balance to compare the total weight of two groups of coins. The lighter side will rise and the heavier side will fall. Both sides balance when they have the same weight. (Note that you don't have any other device for measuring the accurate weight of a coin.) The running time is defined as the number of times of using the balance. Apply **divide-and-conquer** to **design** an algorithm for solving this problem.

Balance

Coins

2 Coin Changing Problem

Problem Description

Suppose you are given a coin set C consists of k different values $V = \{v_1, v_2 \dots v_k\}$.

- ✧ Each v_i $(1 \leq i \leq k)$ is a positive integer,
- ✧ $v_1 \leq v_2 \leq v_3 \leq \dots \leq v_k$
- ✧ $v_1 = 1$ dollar

The problem is to pick the minimum number of coins in C to make change for n dollars. That is to find the result set $A = \{a_1, a_2 \dots a_k\}$, such that

- ✧ $v_1 * a_1 + v_2 * a_2 + \dots + v_k * a_k = n$
- ✧ Let $P(n) = a_1 + a_2 + \dots + a_k$ be minimum number of coins for solving the problem.

Question

a) What is/are the difference(s) between this problem and the problem in the tutorial?

b) Can the greedy strategy in the tutorial apply to this problem? (If yes, state the reason(s). If no, provide one counter example)

3 Minimal Coverage Problem

Problem Description

Suppose that you are given a positive integer M and a set of line segments $S = \{ls_1, ls_2 \dots ls_n\}$.

For line segments in S:

- ✧ All line segments are along X axis with integer coordinates sets $L = \{L_1, L_2 \dots L_n\}$ and $R = \{R_1, R_2 \dots R_n\}$ where, $0 \le L_i \le R_i \le M \quad (1 \le i \le n)$
- ✧ An integer x is covered by segment $[L_i, R_i]$ when $L_i \le x \le R_i$

The problem is to choose the minimal amount of line segments, such that each integer in the interval $[0, M]$ is covered by some segment (L_i, R_i).

Question

a) Think of the following example. What are the possible segments that can cover the integer 0? Which is the best segment to be chosen?

b) Design a **greedy algorithm** for solving this problem. If $[0, M]$ cannot be covered by given line segments, your algorithm should return "NIL" (without quotes). Otherwise, your algorithm should return the result set C consists of all line segments used to cover $[0, M]$.

c) **Prove** that your greedy algorithm can achieve optimal solution.

Example

Example 1: given $S = \{[0, 2], [0, 3], [4, 6], [5, 7], [7, 10]\}$ and $M = 10$;

Result: $C = \{[0, 3], [4, 6], [7, 10]\}$;