# Lab 1: Algorithm Introduction

## 1 Problem

Let $F(x) = \sum_{k=0 \cdots n} a_k x^k$ be a polynomial equation, where $a_0 \cdots a_n$ are stored in an array $A[0 \cdots n]$. Given an input integer $x$, we can find the corresponding output value $F(x)$. For example, the coefficients of the polynomial $F(x) = 1 + x + 2x^2$ can be stored in an array $< 1, 1, 2 >$. When $x = 2$, $F(x)$ equals to 11.

The basic algorithm (illustrated in Algorithm 1) is designed for solving this problem. The idea is that, for every $k$ (line 2), compute $A[k] \cdot x^k$ (line 3-5) first and then sum up the results (line 7).

**Algorithm 1:** Basic Algorithm

```
  Input: coefficient array A[0···n] representing a_0···a_n
  Input: x
  Output: corresponding value F(x)
1 sum = 0
2 for k=0 to n do
3     temp = A[k]
4     for i = 1 to k do
5         temp = temp · x
6     sum = sum + temp
7 return sum
```

## 2 Time Complexity

Analyze the time complexity of Algorithm 1.

## 3 Incremental Algorithm

- Design an incremental algorithm for solving the problem.
  *hint*: $F(x) = \sum_{k=0 \cdots n} a_k x^k$ can be reorganized as $F(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + xa_n) \cdots))$
- Consider the polynomial equation $F(x) = 4 + 2x + 3x^2 + x^3$. Show the running steps of your algorithm when $x = 2$.
- Analyze the time complexity of your algorithm.