# Greedy Algorithm

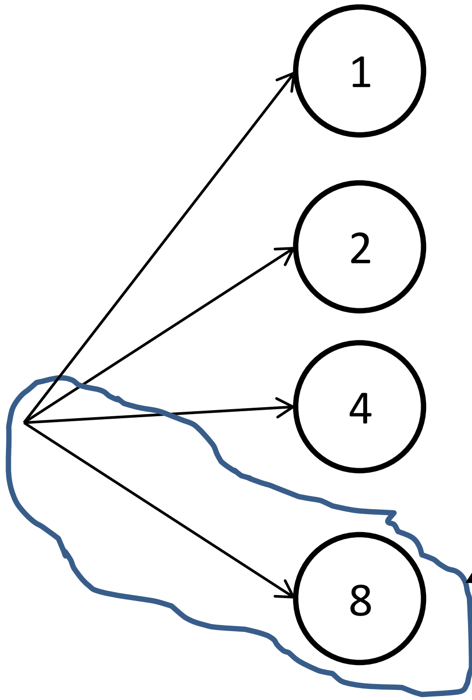
# Coins Changing Problem

- Input: an integer value N
  types of coins $\{v_1,v_2,\ldots,\}$ with $v_{i+1}=2v_i$ , $v_1 =1$
- Output: the minimum number of coins to change N

- Example:
  - N=15
  - $v_1=1$, $v_2=2$, $v_3=4$, $v_4=8$, ……
  - Optimal solution: 4 ($v_1$: 1, $v_2$: 1, $v_3$: 1, $v_4$: 1)

# Greedy Solution

- What is your greedy choice?
  - $v_G$ = Highest $v_i$ such that $\leq$ N--------(*)
  - Example (Previous): N=15

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**4 Choices**

# Greedy Choice is correct (Proof)

- Does this greedy choice guarantee the optimal solution?

- Proof:

  Let $\mathbf{S_{opt}}$ be the **set of coins** of optimal solution.

  Let $v_{opt}$ be the highest choice of $\mathbf{S_{opt}}$.

  Let $\mathbf{S_G}$ be the **set of coins** of greedy solution.

  Let $v_G$ be the highest choice of $\mathbf{S_G}$.

# Greedy Choice is correct (Proof)

- By (*), we have: $v_G \geq v_{opt}$.
- Case 1: if $v_G = v_{opt}$
  → the optimal solution contains the greedy choice
- Case 2: if $v_G > v_{opt}$

Optimal solution: $a_{opt} > 0, a_{opt-1}, \ldots, a_1 \geq 0$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Value of a coin | $v_G$ | …… | …… | $v_{opt}$ | $v_{opt-1}$ | …… | …… | $v_2$ | $v_1$ |
| Number of coins | 0 | …… | …… | $a_{opt}$ | $a_{opt-1}$ | …… | …… | $a_2$ | $a_1$ |

- **Claim**: for all i in 1..opt, $a_i$ must be either 0 or 1
  - Otherwise, if some $a_i > 1$, then we can reduce the number of coins in the optimal solution → **contradiction!**

# Greedy Choice is correct (Proof)

After the round-up procedure

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Value of a coin | $v_G$ | …… | …… | $v_{opt}$ | $v_{opt-1}$ | …… | …… | $v_2$ | $v_1$ |
| Number of coins | 0 | …… | …… | $a_{opt}$ | $a_{opt-1}$ | …… | …… | $a_2$ | $a_1$ |

- Case 2: $v_G > v_{opt}$ (continue ……)
- **Claim**: for all i in 1..opt, $a_i$ must be either 0 or 1
- Total value in optimal solution $\leq v_{opt} + v_{opt-1} + \ldots + 1$

  $= v_{opt+1} - 1 < v_G \leq N$

  → Total value in optimal solution $\leq N$ → **contradiction!**
- Thus, Case 2 is impossible
- The proof is complete