COMP3121 Homework 1

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1 Question 1

The first statement is wrong. We can find two shortest paths between D and E, which are {D, B, A, E} and {D, C, F, E}. According to the definition of pivotal, node B is not pivotal for node pair of D - E.

The second statement is correct. The shortest path between A and C is {A, B, C} and the shortest path between A and D is {A, B, D}. According to the definition of pivotal, node B is pivotal for node pair of A - C and A - D.

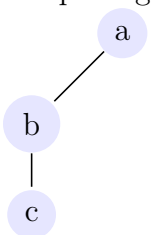
The third statement is correct. As mentioned before, shortest paths between D and E are {D, B, A, E} and {D, C, F, E}. According to the definition of pivotal, there is no pivotal for node pair of D - E.

The forth statement is correct. If two nodes are neighbors, there is no pivotal. We can check all non-neighbor node pairs and we can find that D is not pivotal for any of them. Therefore, the statement four is correct.

In conclusion, statement 2, 3, 4 are correct and statement 1 is wrong.

2 Question 2

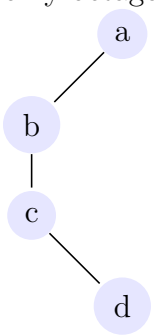
We can find two properties in these polygons. *1. every node in the graph has exactly two neighbors. 2. Every graph contains only one circle.* According to these two properties, let's say for any node b, a and c are its neighbors. If b is not the pivotal of a and c, then there is no pivotal in the graph, so among the four polygons, only square does not satisfy the property. For pentagon, hexagon, and octagon, every node is the pivotal for its two neighbors.



In conclusion, pentagon, hexagon, and octagon all satisfy that every node in this polygon is a pivotal of at least one node pair.

3 Question 3

Based on the properties in Question 2, let's say for any node b, a and c are its neighbors. and d is the neighbor of c. If b is a pivotal of at least two nodes pairs, the two pairs should be $a - c$ or $a - d$ (or in the other direction). Therefore, there should be at least 7 edges in the graph to ensure the statement could be right. For, pentagon, hexagon, and octagon, only octagon is satisfied. And we can easily find the two pairs in the upper a, b, c, d pattern.



In conclusion, only octagon satisfies that every node is a pivotal of at least two node pairs.

4 Question 4

The first statement is wrong. Before adding the edge, A and D are to local gatekeepers. After adding the edge, A and D are also local gatekeepers, but some other nodes also become local gatekeeper, such as B and E. Therefore, the statement is wrong.

The second statement is wrong. D is not a gatekeeper. We can check all the possible pairs of nodes and find that D does not satisfy the definition of gatekeeper.

The third statement is correct. For example, every path from B to E passes through A.

The forth statement is correct. A and D are the only 2 local gatekeepers in the graph. For A, B and E are its neighbors but are not connected by an edge. For D, B and C are its neighbors but are not connected by an edge.

In conclusion, statement 3, 4 are correct and statement 1, 2 are wrong.

5 Question 5

The first statement is correct. For example, every path from B to F passes through C.

The second statement is correct. A, B, C, E, and F are all local gatekeepers. Therefore, 5/6 of the nodes are local gatekeepers, which is over half.

The third statement is correct. If the edge between B and C is removed, the remained nodes will form a non-chordal circle. Therefore, all the nodes are local gatekeepers.

The forth statement is correct. If the edge between C and D is removed, every path from A to D passes through B. B will be gatekeeper.

In conclusion, all the statements are correct.