Question 1

Which of the following statement(s) is correct?

1. As every player chooses a dominant strategy in a game of two players, these strategies achieve a social optimal.
2. Both players in a game may not have dominant strategies but must have at least one best response to the other player.
3. Mixed strategy equilibrium is achieved when, the strategy of each player is selected under their respective probability distribution; no player is capable of improving his own payoff by unilateral change.
4. The spirit of mixed strategy equilibrium is to make the behavior of the player unpredictable by the other player through randomizing the probability in strategy selection. As such, one can only choose one probability distribution between 2 pure strategies so that there is no difference in payoff.

1\. Incorrect. “Prisoners’ dilemma”, all prisoners have a dominant strategy, which is to confess. The social optimal strategy is all of them choose not confess.
2\. Correct. Best response to one player’s choice always exists.
3\. Correct. This is the premise of mixed strategy.
4\. Correct. Mixed strategy equilibrium makes the expectation of adversary’s payoff equal.

Question 2

What is the Nash equilibrium for the pure strategy under the following payoff matrix?

		Player B	
		L	R
Player A	U	$3, 5$	$4, 3$
	D	$2, 1$	$1, 6$

1. (U,L)
2. (U,R)
3. (D,L)
4. (D,R)

We’ll use “A=U” to represent “A chooses U”.
For A:
When B=L, A=U
When B=R, A=U
For B:
When A=U, B=L
When A=D, B=R
Therefore, (U,L) is one Nash equilibrium.
1 is correct.

Question 3

Select all Nash equilibrium under the following payoff matrix

		Player B	
		L	R
Player A	U	$1, 1$	$4, 2$
	D	$3, 3$	$2, 2$

1. (D,L)
2. (1/3, 2/3)
3. (1/2, 1/2)
4. (1/4, 3/4)
5. (U,R)
6. (D,R)

1, 3 and 5 are correct.
For pure strategy:
When B chooses L, A will choose D;
When B chooses R, A will choose U.
When A chooses U, B will choose R;
When A chooses D, B will choose L.
For mixed strategy, suppose A chooses U with probability of p, and B chooses L with probability of q.
For A,
A chooses U will get 1*q+4(1-q)=4-3q;
A chooses D will get 3q+2(1-q)=2+q. 2+q=4-3q, and q=1/2.
For B,
B chooses L will get 1*p+3(1-p)=3-2p,
B chooses R will get 2p+2(1-p)=2. 2=3-2p, and p=1/2.
The Nash equilibrium for mixed strategy is (1/2, 1/2).
Therefore, the Nash equilibria are (D, L), (U, R) and (1/2, 1/2).

Question 4

Consider the following 2-player game. Select the statement(s) which is/are correct.

		Player B	
		L	R
Player A	U	$3, 3$	$1, 2$
	D	$2, 1$	$3, 0$

1. The pure strategy equilibrium is (U, L)
2. The game has no pure strategy equilibrium.
3. If the payoff of player B in (U, L) is changed (positive payoff), there will be (can be) no Nash equilibrium.

4. If the payoff of player A in (U, L) is changed (positive payoff), there will be (can be) no Nash equilibrium.

For pure strategy:
For A:
  When B=L, A=U
  When B=R, A=L
For B:
  When A=U, B=L
  When A=D, B=L
Therefore, (U,L) is one Nash equilibrium.
1 is correct and 2 is incorrect.

		B	
		L	**R**
A	**U**	3, y	1, 2
	D	2, 1	3, 0

Suppose the payoff changes to y ($y>0$).
For pure strategy:
For A:
  When B=L, A=U
  When B=R, A=L
For B:
  When A=U, when $y>2$, B=L; when $y=2$, B=L or R; when $0<y<2$, B=R
  When A=D, B=L
Therefore, when $y>=2$, there exists Nash equilibrium (U, L) for pure strategy, when $0<y<2$, there is no Nash equilibrium for pure strategy.
Now let's consider where there exists Nash equilibrium for mixed strategy when $0<y<2$.
For B:
Choose L: $yp+(1-p)=(y-1)p+1$
Choose R: $2p$
Therefore, we can get $2p=(y-1)p+1$, $p=1/(3-y)$.
When $0<y<2$, $0<p<1$. Therefore, we can get Nash equilibrium for mixed strategy when $0<y<2$.
3 is incorrect.

		B	
		L	**R**
A	**U**	x, 3	1, 2
	D	2, 1	3, 0

Suppose the payoff changes to x ($x>0$).
For pure strategy:
For A:
  When B=L, when $x>2$, A=U; when $x=2$, A=U or D; when $0<x<2$, A=D
  When B=R, A=L
For B:
  When A=U, B=L
  When A=D, B=L
Therefore, when $x>=2$, there exists Nash equilibrium (U, L) for pure strategy, when $0<y<=2$, there exists Nash equilibrium (D, L) for pure strategy.
4 is incorrect.