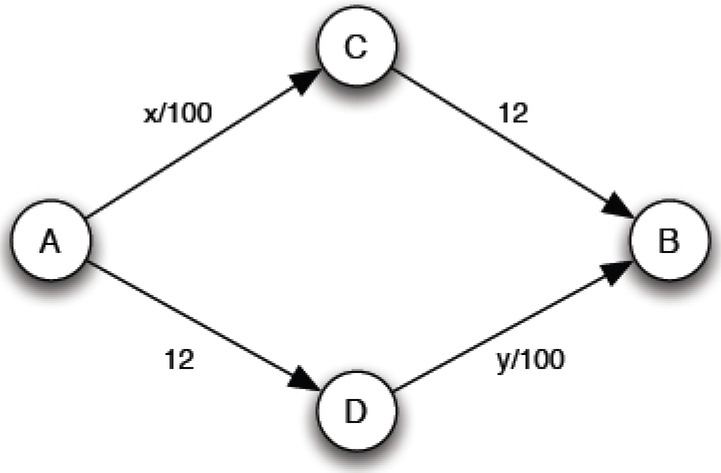
Question 1



1000 cars travel from A to B. There are two possible routes: the upper route through C or the lower route through D. Let x be the number of cars traveling on the edge (A, C) and let y be the number of cars traveling on the edge (D, B). The directed graph in the Figure indicates that the travel time of a car on edge (A, C) is x/100 if there are x cars on edge (A, C), and similarly the travel time of a car on edge (D, B) is y/100 if there are y cars on edge (D, B). The travel time of a car on edges (C, B) and (A, D) is 12 regardless of the number of cars on these edges.

Each driver wants to minimize his travel time. The drivers make simultaneous choices.

(a) Find the Nash equilibrium of x and y.

(b) Now the government builds a new one-way road from town C to town D, with a travel time of 0. Find the Nash equilibrium of x and y.

(c) Suppose now that the conditions on edges (C, B) and (A, D) improves so that the travel time on each edge is reduced to 5. The road from C to D that was constructed in part (b) is still available. Find the Nash equilibrium of x and y.

a: The Nash equilibrium is that 500 cars travel through A-C-B, and other 500 cars travel through A-D-B, x=y=500. The time cost will be 12+500/100=17 for everyone. No one can get a better payoff when they change their road (12+x/100>17 when x >500).
b: The Nash equilibrium is that all cars travel through A-C-D-B, x=y=1000. The time cost will be 1000/10+1000/10=20. No one can get a better payoff when they change their road (1000/10+12>20).
c: The Nash equilibrium is that 500 cars travel through (A-C-B), ant other 500 cars travel through (A-D-B), x=y=500. The time cost will be 5+5=500/100+500/100=10. No one can get a better payoff when they change their road (x/100+5>10 when x>500).

Question 2

Suppose we have 3 sellers a, b and c, and 3 buyers x, y and z. Each seller has a car park for sale, and the valuations of the buyers are as follows.

Buyer	Valuation on a’s car park	Valuation on b’s car park	Valuation on c’s car park
X	6	5	2
Y	7	6	3
Z	6	7	6

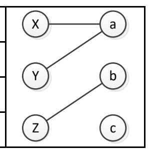
There are 4 sets of prices for a’s, b’s and c’s car parks.

1. a=0, b=0, c=0
2. a=2, b=1, c=0
3. a=0, b=1, c=0
4. a=3, b=1, c=0

How many sets are market-clearing prices?

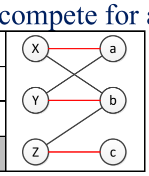
A. 0
B. 1
C. 2
D. 3
E. 4

	a	b	c
X	6	5	2
Y	7	6	3
Z	6	7	6



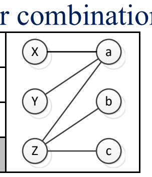
X and Y will compete for a. Not market-clearing price.

	a	b	c
X	4	4	2
Y	5	5	3
Z	4	6	6



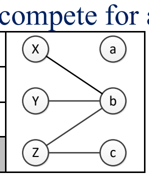
This is market-clearing price. (x,a), (y,b) and (z,c) can be one combination. ((x,b), (y,a) and (z,c) can be another combination)

	a	b	c
X	6	4	2
Y	7	5	3
Z	6	6	6



X and Y will compete for a. Not market-clearing price.

	a	b	c
X	3	4	2
Y	4	5	3
Z	3	6	6



X and Y will compete for b. Not market-clearing price.

Question 3

There are two advertising slots a and b for sale. The clickthough rate of a is 10 whereas the clickthough rate of b is 5. There are 3 advertisers x, y, and z biding for these 2 slots. The valuation per click of x is 3. The valuations per click of y and z are 2 and 1 respectively.

Which of the following statement(s) is true?

1. The VCG price is (5, 0)
2. The VCG price is (10, 5)
3. The VCG price is (15, 5)
4. The VCG price is (20, 10)

There are 3 buyers, but only 2 slots. So we suppose there is a third slot s which clickthough rate is 0. We can get:

	a	b	c
x	30	15	0
y	20	10	0
z	10	5	0

We can get the final distribution, which is (x,a), (y,b) and (z,c).
If x is not there, y can get better by choosing a, and z can get better b choosing b. The VCG price for x is 20-10+5-0=15;
If y is not there, x will remain unchanged, and z can get better by choosing b. The VCG price for y is 5-0=5;
If z is not there, x and y will remain unchanged. The VCG price for z is 0.
Therefore, The VCG price is (15,5). 3 is the correct answer.

Question 4

Let x be an item for sale. There are 3 buyers a, b, and c and their valuations for the item is 6, 3, and 1 respectively. Assume we know that:

1. If seller uses second price auction to sell x, buyer a will pay p for x
2. If seller uses VCG to sell x, buyer a will q for x.

Which of the following is/are correct?

1. p=6, q=3
2. p=6, q=6
3. p=3, q=3
4. p=3, q=6

Second price auction: winner gets the item with the second highest price. Therefore, the p=3 for a to buy x when using second price auction.
Suppose there are 2 other items y and z, the valuation for them are all 0.

	x	y	z
a	6	0	0
b	3	0	0
c	1	0	0

We can get one final distribution: (a,x), (b,y) and (c,z).
If a is not there, b can get better by choosing x, and 3. The VCG price for a is 3-0=3. Therefore, the p=3 for a to buy x when using VCG.