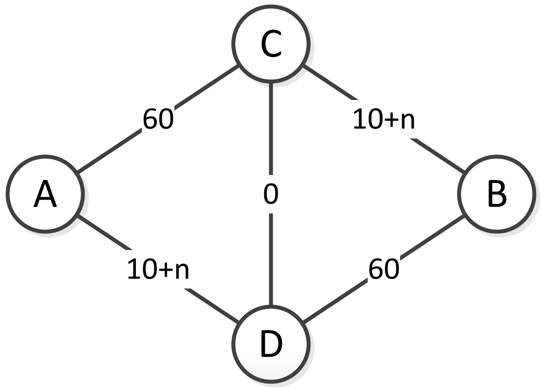
Question 1

There are two routes between A and B. There are 80 people traveling from A to B. Route I starts with a highway from A to C. It takes one hour on this highway regardless of the number of travelers on it. Then there is a local street from C to B. This local street B requires a travel time (in minutes) of 10 plus the number of travelers on this street. Route II starts with a local street from A to D, which requires a travel time (in minutes) of 10 plus the number of travelers on this street. Then there is a highway from D to B. It takes one hour on this highway regardless of the number of travelers on it. Now we build a two-way road between C and D. Find the Nash equilibrium travelling between A and B.

Based on the description, we could draw down the graph.



The Nash equilibrium is 50 people travel by A-D-C-B, and other 30 people travel by A-C-D-B
The time cost of each road is A-D-C-B: 10+50+0+10+50=120 min, A-C-D-B: 60+60=120 min. No one can get a better result when he changes his route. Changing from A-D-C-B to A-C-D-B will remain 120, while changing from A-C-D-B to A-D-C-B will be more than 120, because 10+n+0+10+n>120 when n>50. Therefore, A-D-C-B: 50 and A-C-D-B: 30 will be the Nash equilibrium.

Question 2

In this question, we study the impact of irrational behavior on other bidders in an auction. A seller sells an object using second-price, sealed-bid auction. Assume the valuations of three bidders on this object to be v1, v2 v3, which are uniformly distributed in [0,1].

Please choose the correct answers for the below situations respectively.

1. If you know that one bidder is irrational in an auction, you should bid ______ as compared to your valuation.
2. An irrational bidder will make your expected payoff _____ in a bid.

   A. higher, increase
   B. higher, decrease
   C. higher, no change
   D. normal, increase
   E. normal, decrease
   F. normal, no change
   G. lower, increase
   H. lower, decrease

I. lower, no change

For bidder i, it is a dominant strategy to choose the bid price bi=vi. That is to say, no matter what the others do, choosing the bid price bi=vi will give you the highest payoff under the current situation.
Besides, we could verify whether we could get highest payoff when we bid with bi=vi. Suppose we are bidder 1, b=max{b2, b3}.
If b1=v1 (truthful bidding):
When v1>b, payoff=v1-b>0; when v1<=b, payoff=0.
If b1>v1:
When v1<b<b1, payoff=v1-b<0 (Should lose but win. We can get 0 using truthful bidding.); else payoff= truthful bidding's payoff.
If b1<v1:
When b1<b<v1, payoff=0 (Should win but lose. We can get positive payoff using truthful bidding); else payoff= truthful bidding's payoff.
Therefore, we should bid with b1=v1.
The expected payoff for bidder 1 (us) is E(v1)=p(v1>b)*(v1-b)+p(v1<=b)*0=p(v1>b)*(v1-b).
Suppose b2 is irrational bidder, if b2>v2, our payoff will decrease or unchanged. The expected payoff will be decrease; if b2<v2, out payoff will increase or unchanged. The expected payoff will increase.
In generally, bidder 2 will have a higher change to increase his bid price, because the price should be a positive number. Therefore, our expected payoff will be more likely to decrease.
The answer is E.

Question 3

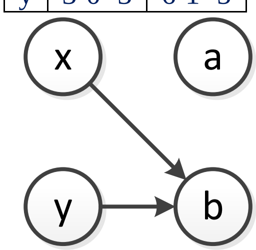
Suppose we have 2 sellers a and b, and 2 buyers x and y. Each seller has a house for sale. The valuations of the buyers are as follows.

Buyer	Value for a's house	Value for b's house
x	2	4
y	3	6

Suppose that **a** charges 0 for his house, and b charges 1 for his house. Is this set of prices market-clearing?

After get rid of the value of the houses, we can get a payoff matrix.

	a	**b**
x	2-0=2	4-1=3
y	3-0=3	6-1=5



Multiple buyers still compete for the same item. Therefore, it is not market clearing price.