

You need to give reasoning of how you derive your answers. No mark if no reasoning is given.

#### Question 1

Read the following payoff matrix. Select the correct statement(s) below.

		B	
		L	R
A	U	1,1	4,0
	D	4,0	3,3

1. Neither player A nor player B has a dominant strategy.
2. Player A has a dominant strategy D whereas player B has the dominant strategy R.
3. Only player B has a dominant strategy L. The best response of player A is strategy D.
4. Only player B has a dominant strategy L. The best response of player A is strategy U.

1 is correct. When B chooses L, A will choose D ( $4 > 1$ ), but when B chooses R, A will choose R ( $4 > 3$ ). Therefore, there is no dominant for A. When A chooses U, B will choose L ( $1 > 0$ ), but when A chooses D, B will choose R ( $3 > 0$ ). Therefore, there is no dominant for B.

#### Question 2

When multiple Nash Equilibria exist in a game, what should the players' action be?

1. Select the equilibrium with maximum payoff
2. Need more external information to make decisions
3. There is no rule to predict on the unique choice of the players
4. Select the equilibrium with high payoff and low risk

1. Incorrect. For some cases, there is no maximum payoff in the game. Nash equilibrium may not predict the sole outcome.  
2. Correct. When multiple Nash equilibria exist, additional information is needed to make decision.  
3. Incorrect. Nash equilibrium may help to narrow down the choices, but may not predict the sole outcome. "There is no" is too strong, for some cases, there can be rules.  
4. Incorrect. Nash equilibrium may not predict the sole outcome.

#### Question 3

Select all Nash equilibrium under the following payoff matrix.

		Player B	
		<i>L</i>	<i>R</i>
Player A	<i>U</i>	1, 1	4, 2
	<i>D</i>	3, 3	2, 2

1. (D,L)
2. (1/3, 2/3)
3. (1/2, 1/2)
4. (1/4, 3/4)
5. (U,R)
6. (D,R)

1,3 and 5 are correct.

For pure strategy,

When B chooses L, A will choose D;

When B chooses R, A will choose U.

When A chooses U, B will choose R;

When A chooses D, B will choose L.

So (D,L) and (U,R) are Nash equilibriums.

For mixed strategy,

Suppose A chooses U with probability of p, and B chooses L with probability of q.

For A:

A chooses U will get  $1 \cdot q + 4 \cdot (1-q) = 4-3q$ ;

A chooses D will get  $3 \cdot q + 2 \cdot (1-q) = 2+q$ .

$4-3q = 2+q$ , and  $q = 1/2$ .

For B:

B chooses L will get  $1 \cdot p + 3 \cdot (1-p) = 3-2p$ ;

B chooses R will get  $2 \cdot p + 2 \cdot (1-p) = 2$ .

$3-2p = 2$ , and  $p = 1/2$ .

The Nash equilibrium for mixed strategy is (1/2, 1/2).

#### Question 4

Select the Nash equilibrium for mixed strategy under the following payoff matrix.

		Player B	
		<i>L</i>	<i>R</i>
Player A	<i>U</i>	5, 6	0, 10
	<i>D</i>	4, 4	2, 2

1. (1/2, 1/2)
2. (1/3, 2/3)

3.  $(1/4, 3/4)$
4.  $(1, 0)$

2 is correct.

Suppose A chooses U with probability of  $p$ , and B chooses L with probability of  $q$ .

For A:

A chooses U will get  $5q + 0 \cdot (1 - q) = 5q$ ;

A chooses D will get  $4q + 2(1 - q) = 2 + 2q$ .

$2 + 2q = 5q$ , and  $q = 2/3$ .

For B:

B chooses L will get  $6p + 4(1 - p) = 2p + 4$ ;

B chooses R will get  $10p + 2(1 - p) = 8p + 2$ .

$8p + 2 = 2p + 4$ , and  $p = 1/3$ .

The Nash equilibrium for mixed strategy is  $(1/3, 2/3)$ .

#### Question 5

A strategy is *weakly dominant* if, regardless of what any other players do, the strategy earns a player a payoff at least as high as any other strategy, and, the strategy earns a strictly higher payoff for some profile of other players' strategies. Hence, a strategy is weakly dominant if it is always at least as good as any other strategy, for any profile of other players' actions, and is strictly better for some profile of others' strategies.

Consider the following two questions with the following game

		Player B	
		<i>L</i>	<i>R</i>
Player A	<i>U</i>	1, 1	1, 1
	<i>D</i>	0, 0	2, 1

1. \_\_\_\_\_ Nash equilibrium
2. \_\_\_\_\_ Nash equilibrium using weak dominant strategy

Which of the following is correct in filling up the two questions?

1. \_\_\_\_\_ There exists, There is no
2. \_\_\_\_\_ There is no, There is no
3. \_\_\_\_\_ There exists, There exists

3 is correct. For Nash equilibrium, when B chooses L, A will choose U; when B chooses R, A will choose D. When A chooses U, B will choose L or R; when A chooses D, B will choose R. Therefore, Nash equilibria are (U, L) and (D, R). There exists Nash equilibrium.

For Nash equilibrium using weak dominant strategy, the payoff of B choosing R is at least as high as any other strategy ( $1 = 1$  when A chooses U,  $1 > 0$  when A chooses D). Therefore, B will choose R using weak dominant strategy. When B chooses R, A will choose as best response to B choosing R ( $2 > 1$ ). Therefore, there exists Nash equilibrium (D, R) using weak dominant strategy.