

You need to give reasoning of how you derive your answers. No mark if no reasoning is given.

### Question 1

Directed edge is a link from one node to another, with the direction being important.

Assume that we request each person in the world to rank 30 friends, in descending order of how well they know them. We construct two different social networks:

(a) The “close-friend” network: from each person, we create a directed edge to their ten friends with the highest rank.

(b) The “distant-friend” network: from each person, we create a directed edge to the ten friends with the rank of 21 to 30.

Let  $C$  be the average number of people that a person can reach in six steps in the close-friend network. Let  $D$  be the average number of people that a person can reach in six steps in the distant-friend network (taking the average over all people in the world).

Researchers have found from empirical studies that either  $C$  or  $D$  is consistently greater than the other. Which of the statement is true?

- 1)  $C$  is greater than  $D$
- 2)  $D$  is greater than  $C$

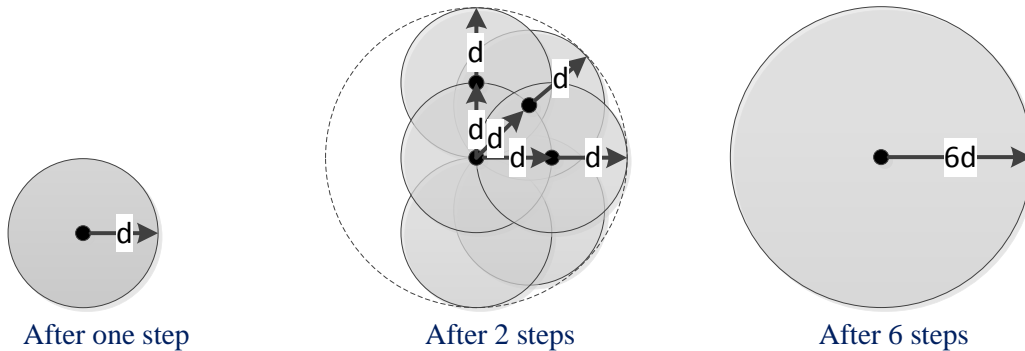
$C$  and  $D$  are the average number of people. Therefore, we should use uniform population density model to calculate the average number of people. Here, rank is in proportion to square of the distance.  $\text{Rank} \sim d^2$ .

For the “close-friend” network, after one step, the number of people it can reach is in one circle area, which is  $p = \pi d^2 \cdot \rho$ .  $p$  is the total number of people one can reach, and  $\rho$  is population density. We can get  $d = \sqrt{\frac{10}{\pi\rho}}$ , and we mark it as  $d = a\sqrt{10}$ .

After 2 steps, it will form a larger circle, which radius will be  $2d$ .

After 6 steps, it will form a big circle, which radius will be  $6d$ .

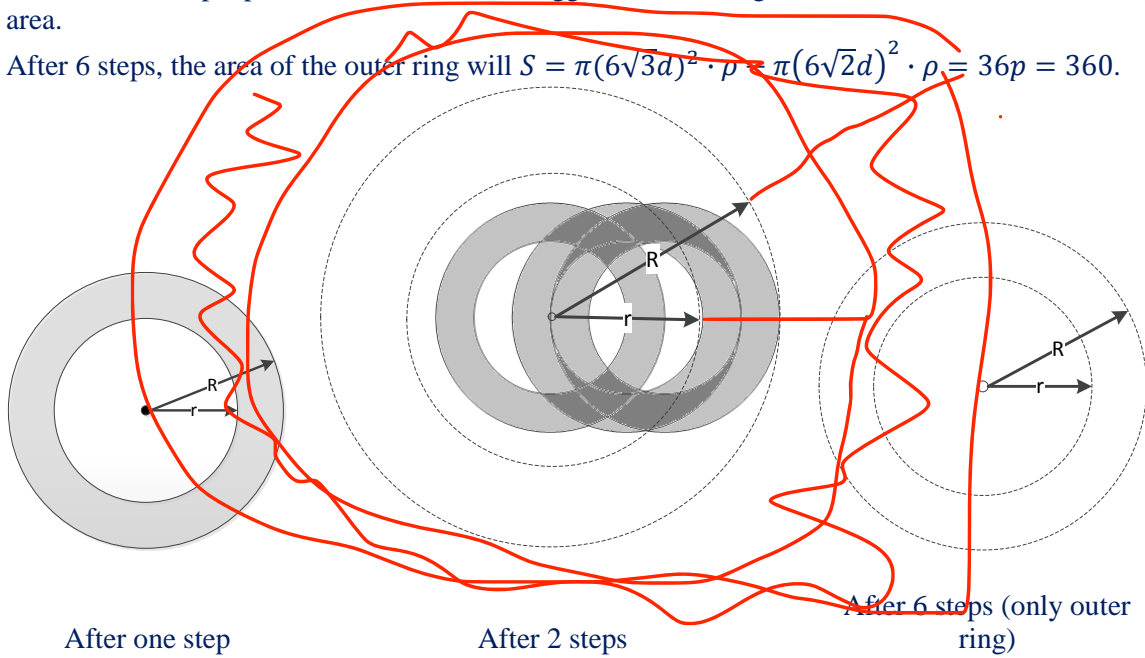
Therefore,  $C = \pi(6d)^2 \cdot \rho = 36p = 360$ .



For the “close-friend” network, after one step, the number of people it can reach is in one ring area ( $R = \sqrt{3}d$  and  $r = \sqrt{2}d$ ).

After 2 steps, the outer side will form a larger ring ( $R = 2\sqrt{3}d$  and  $r = 2\sqrt{2}d$ ). Knowing that the total number of people it can reach now is bigger than the ring area, which  $D > S$ .  $S$  is the ring area.

After 6 steps, the area of the outer ring will  $S = \pi(6\sqrt{3}d)^2 \cdot p = \pi(6\sqrt{2}d)^2 \cdot p = 36p = 360$ .



After one step  
Since  $D > S = C$ , we can get  $D > C$ .

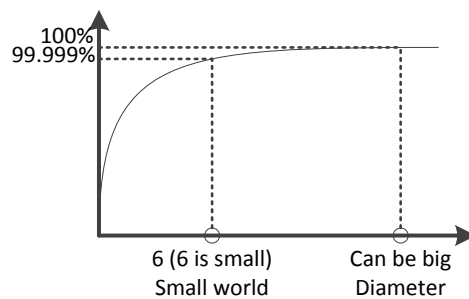
## Question 2

Which of the following statement(s) is true? The small-world phenomenon implies:

- 1) The diameter of the human social network is relatively small (diameter means the maximum distance between one node to another node in the network)
- 2) The average distance of people in human social network is relatively small
- 3) Both 1 and 2 are incorrect.

Small world phenomenon: A person can be reached in 6 steps on average.

Small world talks about the average distance, not the maximum distance. In fact, the diameter can be big; it won't infect the small world phenomenon.



2 is correct.