COMP3422 Assignment 2

Due April 24, 2022

(Total: 100 points)

Submission: All submissions including homework and project will be done online through the BB system. The normal cut-off time would be 11:59 p.m. on the specified date using the BB clock. Late homework assignment submission will be subjected to 33% penalty each day and after 3 days, the system will be closed. Grading and the late penalty will be based on your latest submission.

Format: You can write your solution in a digital document, or a scan of your handwritten work is also acceptable (should be neat and clean). Finally, combine all your solutions into a single *.pdf file and submit it.

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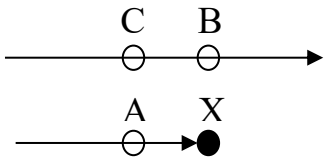
Question 1 (**20 pts**) Suppose that we have the following 4*4 image:

15	15	12	11
13	16	18	14
14	14	15	16
13	12	17	17

Three of the predictors are listed in the following table, where Max is a function that returns the maximum value of a list.



P1	A (horizontal predictor)
P2	B (vertical predictor)
P3	Max (A, B, C, A+B-C)

Encode the image as follows and calculate the prediction error image:

a) (**15 pts**) Code the first row using P1, the first column using P2, and the other pixels using P3.

	13	16	18
	14	14	15
	13	12	17

P1 = A

	15	12	11
	16	18	14
	14	15	16

P2 = B

	13	13	17
	17	16	11
	13	13	18

A+B-C

	15	15	12
	13	16	18
	14	14	15

C

	15	15	12
15	15	16	18
13	17	18	18
14	14	15	18

Prediction Image

15	0	-3	-1
-2	1	2	-4
1	-3	-3	-2
-1	-2	2	-1

Error image

b) (**5 pts**) Compute and compare the entropy of the original image and the prediction error image.

11	12	13	14	15	16	17	18
1/16	2/16	2/16	3/16	3/16	2/16	2/16	1/16

$$\eta = 2.91$$

-4	-3	-2	-1	0	1	2	15
1/16	3/16	3/16	3/16	1/16	2/16	2/16	1/16

$$\eta = 2.86$$

Question 2 (**10 pts**) An image block represented by YCbCr color model is given as follows. Now do a chroma subsampling on the image block and assume we always sample the left-up pixels.

a) (**5 pts**) What would be the sampled values for each channel with subsampling ratio J:a:b = 4:2:2? What would be the results if the subsampling ratio is 4:1:1?

J:a:b = 4:2:2

Y:

3	1	4	1
6	2	9	5

Cb:

5	5	2	2
1	1	1	1

Cr:

6	6	1	1
2	2	4	4

J:a:b = 4:1:1

Y:

3	1	4	1
6	2	9	5

Cb:

5	5	5	5
1	1	1	1

Cr:

6	6	6	6
2	2	2	2

b) (**5 pts**) Explain why color conversion is needed before chroma subsampling.

Humans are much less likely to notice the loss of very high spatial frequency components than the loss of lower frequency components. Since humans see color with much less spatial resolution than they see black and white, it makes sense to decimate the chrominance signal. Meanwhile, visual acuity is much greater for grey than for color. However, colors in RGB convention are hard to represent grey or color pixels seperately. With YCbCr convention, we can differentiate them better. By applying chroma subsampling, we want to keep as much Y information as possible and compress Cb and Cr to some extent.

Question 3 (**15 pts**) A 4×4 DCT coefficient matrix and a quantization table are given as follows.

DCT Coefficients

2	**0**	**3.2**	**0**
4.8	**1.9**	**0**	**0.3**
0	**0**	**0**	**0**
6.12	**0**	**7.24**	**6.59**

Quantization Table

1	**2**	**2**	**4**
2	**2**	**4**	**4**
2	**4**	**4**	**4**
4	**4**	**4**	**8**

(a) (**5 pts**) Fill in the following table the quantized DCT coefficients using nearest rounding.

Quantized DCT Coefficients

2	0	2	0
2	1	0	0
0	0	0	0
2	0	2	1

(b) (**5 pts**) Scan the quantized DCT coefficients in a zig-zag order (start with the top row first) and encode the AC components using run length coding as in JPEG image compression.

2	0	2	0	1	2	0	0	0	2	0	0	0	0	2	1

(R, L) => (1, 2)(1, 1)(0, 2)(3, 2)(4, 2)(0, 1)(EOB)

DC = 2

(c) (**5 pts**) Can you modify the quantization table (using element $\in \{1, 2, 4, 8\}$ and keeping the table symmetric) to achieve better compression ratio with run length coding? (you may use either the run length coding scheme in lecture 2 or the one in JPEG image compression).

Quantization Table

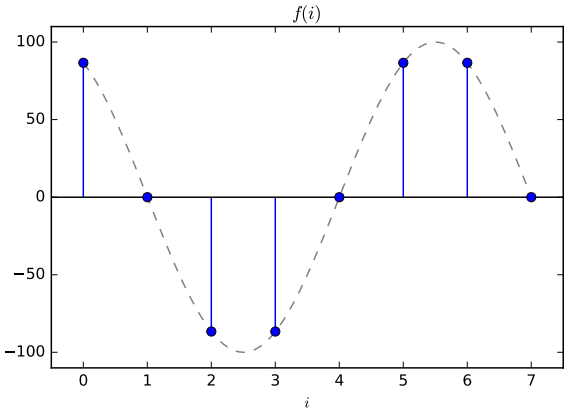
1	**2**	**4**	**4**
2	**4**	**4**	**4**
4	**4**	**4**	**4**
4	**4**	**4**	**8**

2	0	2	0	0	1	0	0	0	2	0	0	0	0	2	1

(R, L) => (1, 2)(2, 1) (3, 2)(4, 2)(0, 1)(EOB)

DC = 2

Question 4 (**20 pts**) A discrete cosine signal $f_1(i) = 100\cos\left(\frac{(2i+1)\pi}{6}\right)$ is plotted in the below figure.



(a) (**10 pts**) For the signal f_1, calculate its DCT coefficients $F(0)$ and $F(1)$.

$F(0) = \frac{\sqrt{2}}{4} \times 100 \times (\cos\frac{\pi}{6} + \cos\frac{3\pi}{6} + \cos\frac{5\pi}{6} + \cos\frac{7\pi}{6} + \cos\frac{9\pi}{6} + \cos\frac{11\pi}{6} + \cos\frac{13\pi}{6})$ *= 30.62*

$F(1) = \frac{1}{2} \times 100 \times (\cos\frac{\pi}{16}\cos\frac{\pi}{6} +$
$\cos\frac{3\pi}{16}\cos\frac{3\pi}{6} + \cos\frac{5\pi}{16}\cos\frac{5\pi}{6} + \cos\frac{7\pi}{16}\cos\frac{7\pi}{6} + \cos\frac{9\pi}{16}\cos\frac{9\pi}{6} + \cos\frac{11\pi}{16}\cos\frac{11\pi}{6} + \cos\frac{13\pi}{16}\cos\frac{13\pi}{6})$
= -50.10

(b) (**10 pts**) For another signal $f_2 = f_1 + 100$, calculate its DCT coefficients $F(0)$ and $F(1)$.

$F_3(0) = 283$
$F_3(1) = 0$
$F(0) = 283 + 30.62 = 313.62$
$F(1) = -50.1 + 0 = -50.1$

Question 5 **(20 pts)** A signal $f = [x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7]$ is decomposed into two levels by discrete Haar wavelet transform, resulting in wavelet coefficients $d = [19.5, 18.5, 1.5, -3.5, -2.5, 1.5, -1, 1]$.

(a) **(10 pts)** Can f be decomposed into more levels? If so, what are the wavelet coefficients of the fully decomposed signal? For a signal of length 1024, what is the maximum decomposition level?

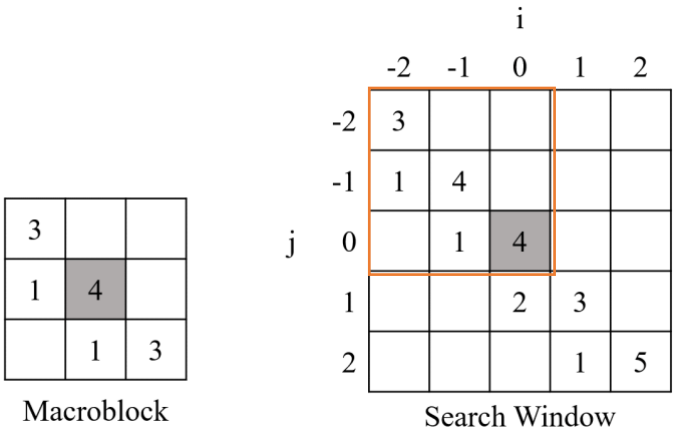
Yes, it can still be decomposed once. The result is as follows:
$d = [19, 0.5, 1.5, -3.5, -2.5, 1.5, -1, 1]$.

(b) **(10 pts)** Reconstruct the original signal f.

From
$[19.5, 18.5, 1.5, -3.5, -2.5, 1.5, -1, 1]$,
Reconstruct once:
$[21, 18, 15, 22, -2.5, 1.5, -1, 1]$.
Reconstruct once:
$[18.5,\ 23.5, 19.5,\ 16.5, 14, 16, 23, 21]$.

So, f = $[18.5,\ 23.5, 19.5,\ 16.5, 14, 16, 23, 21]$

Question 6 **(5 pts)** Given a macroblock in the target frame, please find the motion vector in the search window in the reference frame by using MAD as the similarity measurement. The blank positions are pixels with intensity value 0. The center of the macroblock is marked as grey.



The block is shown in red block which is the most similar one with the macroblock.
Therefore, we have the MV = (-1, -1)

Question 7 (**10 pts**) The display order of some MPEG-1 frames is: $I_0P_1P_2P_3I_4B_5B_6P_7B_8I_9B_{10}P_{11}$

a) (**5 pts**) Which frame(s)'s prediction depends on P_7?

$B_5B_6B_8$

b) (**5 pts**) What is the transmission order of the frames?

$I_0P_1P_2P_3I_4P_7B_5B_6I_9B_8P_{11}B_{10}$