COMP3422 Assignment 1

Due March 27, 2022

(Total: 100 points)

Submission: All homework and project assignments should be submitted via the BB system. The normal cut-off time would be 11:59 p.m. on the specified date using the BB clock. Late homework assignment submission will be subjected to 33% penalty each day and after 3 days the system will be closed. Grading and the late penalty will be based on your latest submission.

Format: You can write your solution in a digital document, or a scan of your handwritten work is also acceptable (should be neat and clean). Please combine all your solutions into a single *.pdf file and submit it.

Note: please show your steps in answering the questions.

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Question 1 (**15 pts**) Given the initial dictionary as follow:

Index	Entry
0	p
1	e
2	t
3	r

a) (**10pts**) Encode "peterpepper" using LZW compression and show the new dictionary.

b) (**5 pts**) Explain how LZW achieves data compression and at what circumstances it is not able to compress data very well.

Answer:

Input	Current String	Seen this before	Encoded output	New Dict / Index
p	p	yes	nothing	none
pe	pe	no	0	pe / 4
pet	et	no	0, 1	et / 5
pete	te	no	0, 1, 2	te / 6
peter	er	no	0, 1, 2, 1	er / 7
peterp	rp	no	0, 1, 2, 1, 3	rp / 8
peterpe	pe	yes	nothing	none
peterpep	pep	no	0, 1, 2, 1, 3, 4	pep / 9
peterpepp	pp	no	0, 1, 2, 1, 3, 4, 0	pp / 10
peterpeppe	pe	yes	nothing	none
peterpepper	per	no	0, 1, 2, 1, 3, 4, 0, 4	

With the last character r, the output will be 0, 1, 2, 1, 3, 4, 0, 4, 3

The dictionary is like:

Index	Entry
0	p
1	e
2	t
3	r
4	pe
5	et
6	te
7	er
8	rp
9	pep
10	pp

Since many frequently occurring substrings will be replaced by a single code, in the long haul, compression is achieved. The more repetition of the strings in the data, the better the compression. However, if there is not much repetition of the strings in the data, the LZW compression may not be that effective.

Question 2 (**35 pts**) Suppose the information source contains five symbols and the probability distribution is given as follows:

Symbol	C	O	V	I	D
Probability	0.05	0.25	0.2	0.4	0.1

a) (**5 pts**) Calculate the entropy of the given probability distribution.

b) (**10 pts**) Encode the message “VIVID” with Shannon-Fano coding and show the codeword.

c) (**10 pts**) Encode the message “VIVID” with Huffman coding and show the codeword.

d) (**10 pts**) Encode “VID” with Arithmetic Coding and show the codeword.

Q2.

(a) $\eta = -\sum_{i=1}^{n} p_i \log_2 p_i$

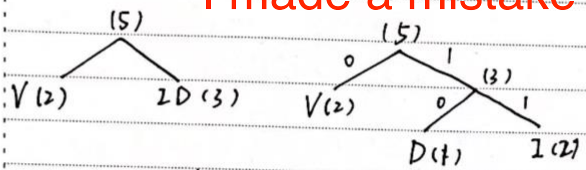
$= -\left(\frac{1}{20}\log_2\frac{1}{20} + \frac{1}{4}\log_2\frac{1}{4} + \frac{1}{5}\log_2\frac{1}{5} + \frac{2}{5}\log_2\frac{2}{5} + \frac{1}{10}\log_2\frac{1}{10}\right)$

$= 2.0414$

(b)

Symbol	V	I	D
Count	2	2	1

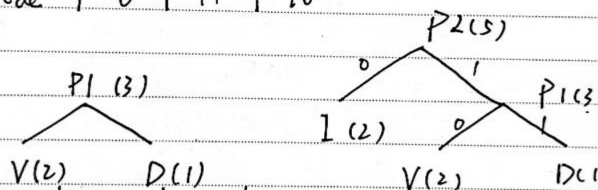
I made a mistake here!!



∴

Symbol	V	I	D
Code	0	11	10

(c)



∴

Symbol	V	I	D
Code	10	0	11

(d)

Symbol	V	I	D
Probability	1/3	1/3	1/3
Range	[0, 1/3)	[1/3, 2/3)	[2/3, 1]

For V, low range = 0, high = $1/3$, range = $1/3$

For I, low = $1/3 \cdot 1/3 = 1/9$, high = $1/3 \cdot 2/3 = \frac{2}{9}$, range = $1/9$

For D, low = $1/9 + 1/9 \cdot 2/3 = 5/27$, high = $1/9 + 1/9 \cdot 1 = 2/9$, range = $1/27$

∴ $[5/27, 2/9)$

So, in binary codeword, it is 0.0011, which is 0.1875 in decimal.

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Question 3 (**5 pts**) The decibel (dB) is a logarithmic unit used to measure the difference in sound level by describing a ration between a sound with power P_1 and another sound with power P_2, i.e., $10\log_{10}\frac{P_2}{P_1}$.

a) (**3 pts**) If the second sound produces twice as much power than the first, what is their difference in dB?

Let the standard reference sound be P. As P2 = 2P1, we have $10\log_{10}\frac{P_1}{P} = 10\log_{10}\frac{P_1}{P} + 10\log_{10} 2$. Therefore, the difference is around 3.01dB.

b) (**2 pts**) What does -10dB say about the relationship between P_1 and P_2?

$10\log_{10}\frac{P_2}{P_1}$ = -10, therefore, P2/P1 = 1/10, which means that P2 is ten times smaller than P1.

Question 4 (**10 pts**) An input signal $X = [12.8, 3.45, -5.9, -11.7, 2.35]$ is quantized by rounding to the nearest integer. For example, 1.9 would be rounded to 2.

a) (**5 pts**) What is the quantized signal Y? What is the SQNR of Y measured in dB?

$Y = [13, 3, -6, -12, 2]$

$N = [-0.2, 0.45, 0.1, 0.3, 0.35]$

$V^2_{signal} = 352.965$

$V^2_{quan\_noise} = 0.465$

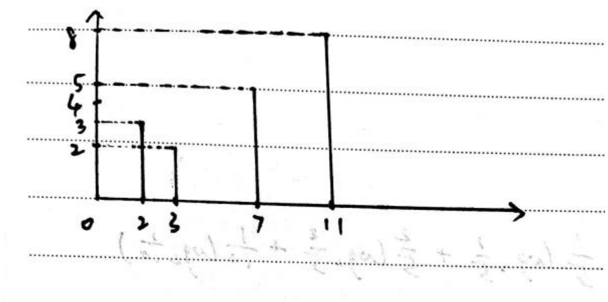
$\text{SQNR} = 10\log_{10}\frac{352.965}{0.465} = 28.80\text{dB}$

b) (**5 pts**) Suppose each quantized sample is represented by 5 bits (one bit is used to indicate the sign of the sample), what is the peak SQNR?

$\text{PSQNR} = 20\log_{10}\frac{16}{0.5} = 30.10\text{dB}$

Question 5 (**5 pts**) Consider the continuous-time signal $x(t) = 3\cos\left(2\pi t + \frac{\pi}{4}\right) + 2\cos\left(2\pi 3t + \frac{\pi}{4}\right) + 5\cos\left(2\pi 7t + \frac{\pi}{4}\right) + 8\cos\left(2\pi 11t + \frac{\pi}{4}\right)$.

a) (**3 pts**) Show the spectrum of $x(t)$ by determining the frequency of each individual sinusoid.



b) (**2 pts**) If $x(t)$ is sampled at the sampling rate fs = 10 Hz, will aliasing occur? Please explain your answer.

Yes. According to Nyquist Theorem, the sampling frequency should be at least twice the highest frequency contained in the signal. Therefore, it should be at least 22, which is larger than 10.

Question 6 (**25 pts**) Given a signal $\mathrm{x} = [98, 103, 102, 105, 97]$ $(\mathrm{x}_1 = 98,\ \mathrm{x}_2 = 103, \dots, \mathrm{x}_5 = 97)$ and a predictor:

$$\hat{x}_n = round[\tfrac{1}{3}(x_{n-1} + x_{\mathrm{n}-2} + x_{n-3})],$$

where "$round$" means rounding to the nearest integer.

a) (**10 pts**) Suppose $\mathrm{x}_\mathrm{n} = 0$ for $\mathrm{n} \le 0$. Encode x using lossless predictive coding. What are the values to be transmitted?

$\hat{x}_2 = round[1/3(98 + 0 + 0)]$ = 33	e_2=103-33=70
$\hat{x}_3 = round[1/3(98 + 103 + 0)]$ = 67	e_3=102-67=35
$\hat{x}_4 = round[1/3(98 + 103 + 102)]$ = 101	e_4=105-101=4
$\hat{x}_5 = round[1/3(105 + 103 + 102)]$ = 103	e_5=97-103=-6

Therefore, [98, 70, 35, 4, -6]

b) (**10 pts**) Suppose $\mathrm{z} = [4, 3, 5, 6, 2]$ $(\mathrm{z}_1 = 4,\ z_2 = 3, \dots, z_5 = 2)$ is the transmitted output of another signal y encoded by lossless predictive coding. With the same assumption that y_n=0 for $\mathrm{n} \le 0$, please reconstruct the original signal y.

c)

$\hat{x}_2 = round[1/3(4 + 0 + 0)]$ = 1	x_2=1+3=4
$\hat{x}_3 = round[1/3(4 + 4 + 0)]$ = 3	x_3=3+5=8
$\hat{x}_4 = round[1/3(8 + 4 + 4)]$ = 5	x_4=5+6=11
$\hat{x}_5 = round[1/3(11 + 8 + 4)]$ = 8	x_5=8+2=10

Therefore, y = [4, 4, 8, 11, 10]

d) (**5 pts**) Explain why differential coding is beneficial.

The first reason is that it uses smaller numbers to represent the codes, using fewer bits to store. The second reason is that it assigns short codewords to prevalent values and long codewords to rarely occurring ones.

Question 7 (**5 pts**) Compare the coding schemes of Shannon-Fano coding, Huffman coding, and Arithmetic coding. What are the disadvantages of Arithmetic coding compared with others? Please explain your answer.

The disadvantages are as follows:

1. Its error-tolerant ability is not good. If one bit is corrupted, the entire message will corrupt.
2. It is slower than the other two algorithms. Therefore, in practice, when we are caring about the efficiency of the encoding, it is not that practical.
3. It is fairly difficult to implement.

Witten I H, Neal R M, Cleary J G. Arithmetic coding for data compression[J]. Communications of the ACM, 1987, 30(6): 520-540.