Exercise 19.7 Suppose you are given a relation R with four attributes $ABCD$. For each of the following sets of FDs, assuming those are the only dependencies that hold for R, do the following: (a) Identify the candidate key(s) for R. (b) Identify the best normal form that R satisfies (1NF, 2NF, 3NF, or BCNF). (c) If R is not in BCNF, decompose it into a set of BCNF relations that preserve the dependencies.

1. $C \rightarrow D$, $C \rightarrow A$, $B \rightarrow C$
2. $B \rightarrow C$, $D \rightarrow A$
3. $ABC \rightarrow D$, $D \rightarrow A$
4. $A \rightarrow B$, $BC \rightarrow D$, $A \rightarrow C$
5. $AB \rightarrow C$, $AB \rightarrow D$, $C \rightarrow A$, $D \rightarrow B$

Answer 19.7

1. (a) Candidate keys: B
   (b) R is in 2NF but not 3NF.
   (c) $C \rightarrow D$ and $C \rightarrow A$ both cause violations of BCNF. One way to obtain a (lossless) join preserving decomposition is to decompose R into AC, BC, and CD.
2. (a) Candidate keys: BD
   (b) R is in 1NF but not 2NF.
   (c) Both $B \rightarrow C$ and $D \rightarrow A$ cause BCNF violations. The decomposition: AD, BC, BD (obtained by first decomposing to AD, BCD) is BCNF and lossless and join-preserving.
3. (a) Candidate keys: ABC, BCD
   (b) R is in 3NF but not BCNF.
   (c) $ABCD$ is not in BCNF since $D \rightarrow A$ and D is not a key. However if we split up R as AD, BCD we cannot preserve the dependency $ABC \rightarrow D$. So there is no BCNF decomposition.
4. (a) Candidate keys: A
   (b) R is in 2NF but not 3NF (because of the FD: $BC \rightarrow D$).
   (c) $BC \rightarrow D$ violates BCNF since BC does not contain a key. So we split up R as in: BCD, ABC.
5. (a) Candidate keys: AB, BC, CD, AD
   (b) R is in 3NF but not BCNF (because of the FD: C $\rightarrow$ A).
   (c) $C \rightarrow A$ and $D \rightarrow B$ both cause violations. So decompose into: AC, BCD but this does not preserve $AB \rightarrow C$ and $AB \rightarrow D$, and BCD is still not BCNF because $D \rightarrow B$. So we need to decompose further into: AC, BD, CD. However, when we attempt to revive the lost functioanl dependencies by adding ABC and ABD, we that these relations are not in BCNF form. Therefore, there is no BCNF decomposition.